K. McDonald
Princeton U.
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"Natural" Layout of Carbon Fiber on Cones and Bicones

1 Bicones Can Be Made From Flat Sheets

A cone can be rolled from a flat sheet without distorting the geometry on the surface of the sheet. Furthermore, if the sheet is folded along a circle about the center of the cone, a bicone results with both angles of the bicone the same as the orginal cone angle.

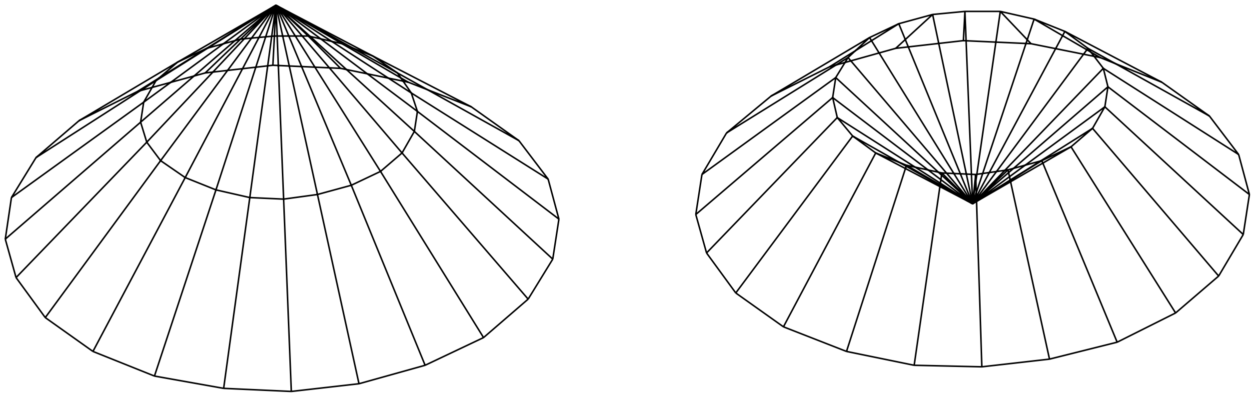

Figure 1: A cone and bicone with the same cone angle.

Thus a flexible x-y matrix of carbon fiber such at MartinPly can be laid onto a bicone without any local distortion. You can verify this by cutting out Fig. 2 and folding it along the smaller circle before rolling.

However, a lip in the form of a short cylinder concentric with the cone axis cannot be formed from the same sheet as the cone without distortion.

Thus from the point of view of carbon-fiber layout, both lipless cones and bicones are relatively straightforward.

It remains that under wire load the peak of a bicone is a high-stress and high-radial-distortion region.

2 "Natural" Layout of x-y Carbon Fiber on a Cone

A cone can be rolled from a flat circular sheet in which a sector of angle α has been removed. The resulting cone has angle θ between its base and a 'generator' (= straight line on the surface of the cone which passes through the apex) given by

$$\cos\theta = 1 - \frac{\alpha}{2\pi}.$$

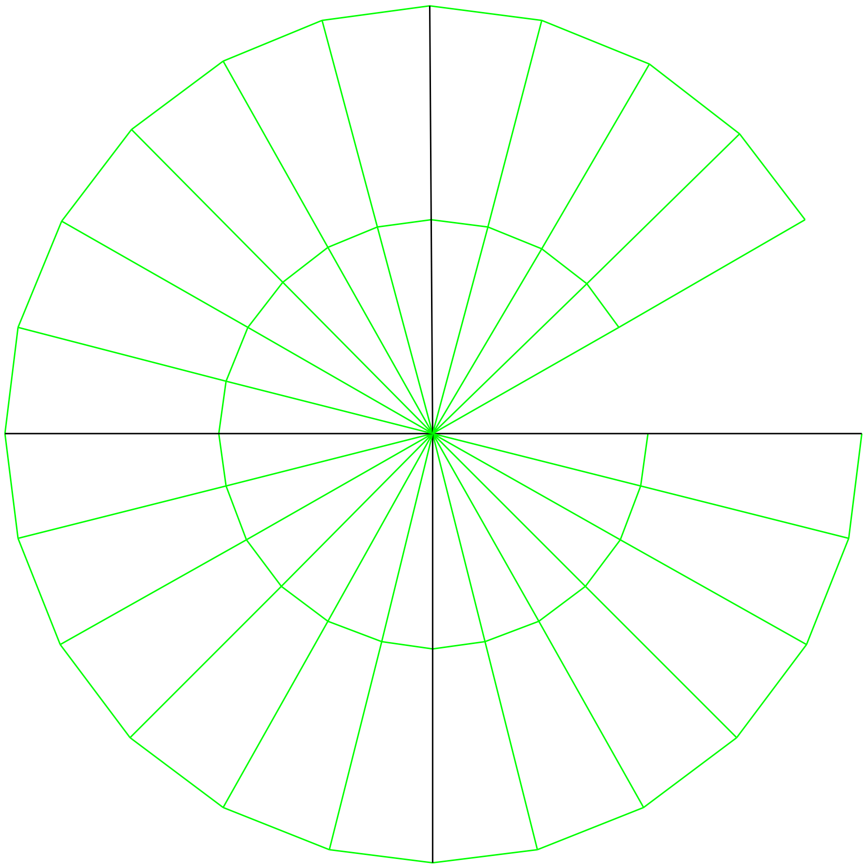

Figure 2: A flat sheet with a sector $\alpha = 30°$ removed, from which a cone or bicone of angle $\theta = 23.6°$ can be made. The dark lines indicate the directions of the x-y carbon-fiber matrix.

If the sheet has an x-y structure such as the MartinPly carbon-fiber matrix then I consider it "natural" that one axis of the matrix be aligned with one edge of the missing sector. It would be "natural" if the other edge of the missing sector were also aligned with fibers in the matrix; when the cone is rolled up the fibers on opposite sides of the gap are either parallel or perpendicular. This view leads to cones of sector angles $\pi/2$, π and $3\pi/2$, with corresponding cone angles of 41.4°, 60° and 75.5°. Unfortunately, these cones are all steeper than is appropriate for the BaBar drift chamber.

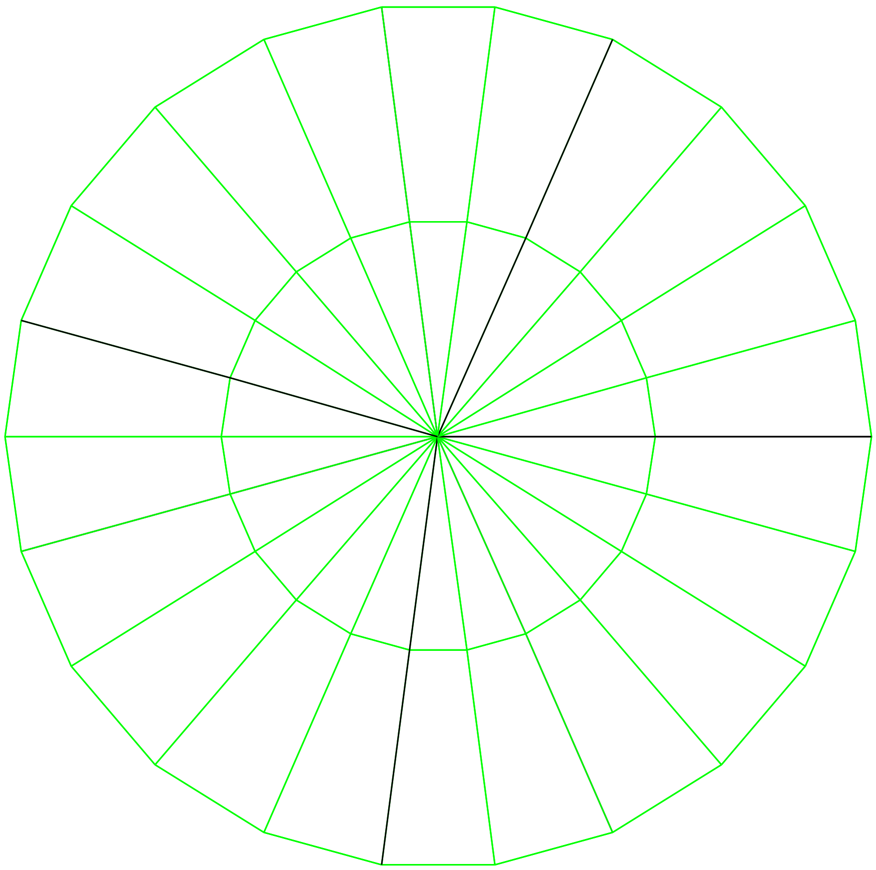

Figure 3: Top view of the cone made from the sheet shown in Fig. 2. The dark lines indicate the four generators along which carbon fibers are aligned.

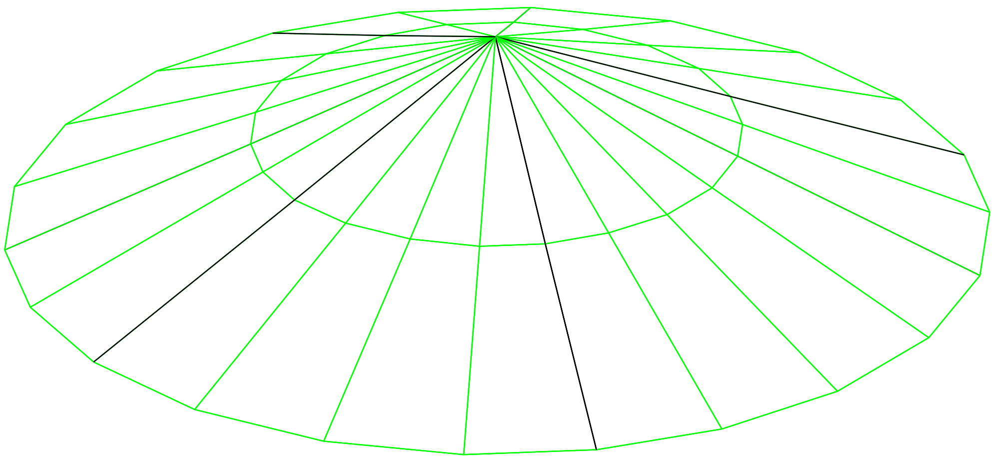

Figure 4: Another view of the cone made from Fig. 2.

To build shallower cones with reasonably "natural" layout we must expand our concept of naturalness. We propose that the sector angle be

$$\alpha = \frac{\pi}{2n} \qquad \text{for} \qquad n = 1, 2, 3, ...,$$

leading to cone angles

$$\cos\theta = 1 - \frac{1}{4n}, \qquad i.e., \qquad 41.4^\circ,\ 29.0^\circ,\ 23.6^\circ,\ 20.4^\circ,\ ...$$

The case of $n = 3$ is close to the nominal value of 22.6° in the present BABAR design.

When the cone is rolled from an x-y carbon-fiber matrix of with missing sector angle $\pi/2n$ there will be four places around the cone where the carbon fibers are aligned with generators of the cone. The azimuthal separations between these generators are 2π times $n/(4n-1)$, $n/(4n-1)$, $n/(4n-1)$ and $(n-1)/(4n-1)$. That is, the layout has a 'hidden' $(4n-1)$-fold symmetry. Correspondingly, good symmetry of the layout requires that the cone be built up of $(4n-1)$ x-y sheets, each rotated by $2\pi/(4n-1)$ with respect to the next. (Of course, layouts consisting of $m(4n-1)$ sheets also have the required symmetry for any $m = 1, 2, 3, ...$)

The completed layout would have carbon fibers aligned along (and also perpendicular to) generators of the cone every $2\pi/(4n-1)$ radians in azimuth. The seam is located at a different one of these generators in each of the $4n-1$ layers.

At any point on the cone, fibers lie along $2n$ directions. Thus it could be argued that the goal of isotropic mechanical properties of the cone is better achieved with $n > 1$.

For example, the case $n = 3$ may be best for BABAR, for which the natural symmetry of the layout is achieved with 11 x-y sheets, with fibers oriented in 6 directions at any point. A layout of 22 x-y sheets, 44 layers in all, (or 33 sheets/66 layers) consisting of two (three) sequences of the basic 11-fold pattern might be a good match to the mechanical requirements for the BABAR endplate.