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Six Ways to Measure CP-Violating Phases in B Decays

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Abstract

We review six methods for measuring the CP-violating phases ϕ_{ub} and ϕ_{td} of the CKM matrix via B decays that are free from uncertainties due to strong final-states phases:

1. B decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X \neq \overline{X}$.
   Ex: $B^+ \to DK^+$ and $B^0_d \to DK^{*0}$ measure ϕ_{ub}.
2. Neutral B-meson decays to f and $\overline{f}$ where $f \neq \overline{f}$.
   Ex: $B^0_d \to D^\pm\pi^\mp$ measures $2\phi_{td} + \phi_{ub}$ and $B^0_s \to D^\pm_s K^\mp$ measures ϕ_{ub}.
3. Neutral B-meson decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X = \overline{X}$.
   Ex: $B^0_d \to DK^0_S$ measures ϕ_{ub}, $2\phi_{ub} + \phi_{td}$ and $\phi_{ub} + \phi_{td}$, and $B^0_s \to D\phi$ measures ϕ_{ub}.
4. Neutral B-meson decays to CP eigenstates.
   Ex: $B^0_d \to J/\psi K^0_S$ measures ϕ_{td}, $B^0_d \to \pi^+\pi^-$ measures $\phi_{td} + \phi_{ub}$, and $B^0_s \to \rho^0 K^0_S$ measures ϕ_{ub}.
5. B decays to sets of final states related by isospin.
   Ex: $B^0_d \to \pi^+\pi^-$, $\pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ measure $\phi_{td} + \phi_{ub}$ free from uncertainty due to penguin contributions.
6. Angular analysis of B decays to mixtures of CP eigenstates.
   Ex: $B^0_d \to J/\psi K^0_S\pi^0$ and $D^{*+}D^{*-}$ measure ϕ_{td}, and $B^0_d \to \rho^+\rho^-$ and $\rho^0\rho^0$ measure $\phi_{td} + \phi_{ub}$.

All of these except the well-known method 4 involve non-CP eigenstates. Methods 1-3 allow extraction of ϕ_{ub} from B_u and B_d, and will require greater emphasis on Kaon identification than methods 4-6. Methods 5 and 6 require photon detections in most cases. Method 1 does not require tagging of the particle/antiparticle character of the second B, and so could be used at a symmetric e^+e^- collider without the penalty due to mixing of methods 2-6. The mode $B^0_d \to J/\psi K^0_S$ which measures ϕ_{td} via method 4 is the most accessible of all those considered here.

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1 Introduction

Discussion of prospects for measurement of CP violation in the B-meson system has often centered on analysis of decays to neutral B's to CP eigenstates because phases due to strong final-state interactions do not complicate the interpretation of the data. However, the rates to some channels of interest, particularly $B_s^0 \to \rho^0 K_S^0$ which isolates the phase of the CKM matrix element V_{ub}, may be too low for practical experimentation. Over the last two years several papers have appeared that discuss how analysis of certain groups of B decays to non-CP eigenstates can separate the strong, CP-conserving, phases from the weak, CP-violating phases of interest.

In this note we review the proposed methods of analysis, based on conversations with David London, as well as the original papers. Our discussion will emphasize decay modes that could be analyzed at a hadron collider. Recent reviews covering much of the same material have been given by Kayser [1] and by Dunietz [2]. Study of non-CP eigenstates at e^+e^- colliders has been examined in refs. [3] and [4].

1.1 The Need for Interference in CP-Violating Processes

In the Standard Model, CP violation in a process described by a single graph manifests itself only as a phase factor. If the amplitude for a single graph $B \to f$ is written

$$A(B \to f) \equiv A_f = |A_f|\, e^{i\phi_W} e^{i\delta_S}, \tag{1}$$

where ϕ_W is a phase due to the weak interaction, and δ_S is a phase due to strong final-state interactions, then the CP conjugate process has amplitude

$$A(\overline{B} \to \overline{f}) \equiv \overline{A}_{\overline{f}} = |A_f|\, e^{-i\phi_W} e^{i\delta_S}. \tag{2}$$

Hence CP violation cannot be discerned as a rate difference between a decay and its CP-conjugate decay if only a single graph contributes to the amplitude.

CP violation can only be revealed in *total-rate measurements* of $B \to f$ and $\overline{B} \to \overline{f}$ when there is interference between two or more decay amplitudes with differing weak phases *and* differing strong phases. To verify the last remark, consider the case where two graphs contribute to a decay, written as

$$A(B \to f) = |A_1|\, e^{i\phi_1} e^{i\delta_1} + |A_2|\, e^{i\phi_2} e^{i\delta_2}, \tag{3}$$

so the CP-conjugate decay has amplitude

$$A(\overline{B} \to \overline{f}) = |A_1|\, e^{-i\phi_1} e^{i\delta_1} + |A_2|\, e^{-i\phi_2} e^{i\delta_2}. \tag{4}$$

The corresponding decay rates are given by

$$\Gamma(B \to f) = |A_1|^2 + |A_2|^2 + 2\,|A_1|\,|A_2| \cos(\phi + \delta), \tag{5}$$

and

$$\Gamma(\overline{B} \to \overline{f}) = |A_1|^2 + |A_2|^2 + 2\,|A_1|\,|A_2| \cos(\phi - \delta), \tag{6}$$

where $\phi = \phi_1 - \phi_2$ and $\delta = \delta_1 - \delta_2$. Only if both ϕ and δ are nonvanishing can the interference term be determined from measurements of the two decay rates.

Even if this condition is satisfied the strong-interaction phase difference δ and the magnitudes $|A_f|$ and $\left|\overline{A}_{\overline{f}}\right|$ will not typically be known, and the CP-violating phase cannot be determined. In this note we examine six methods by which the uncertainties due to strong phases can be avoided. These are introduced in the following subsection, and then discussed in greater detail in the subsequent sections.

1.2 Six Methods for Extracting CP-Violating Phases

1. **B decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X \neq \overline{X}$**

   When a B particle can decay both to D^0X and $\overline{D}^0X$ (and so $\overline{B}$ decays to both $\overline{D}^0\overline{X}$ and $D^0\overline{X}$), then the decays

   $$B \to D^0_{1,2}X, \qquad \text{and} \qquad \overline{B} \to D^0_{1,2}\overline{X}, \qquad \text{where} \qquad D^0_{1,2} \equiv \frac{D^0 \pm \overline{D}^0}{\sqrt{2}}, \tag{7}$$

   exhibit a CP-violating asymmetry. Measurement of the six (or eight) decay modes listed will permit isolation of the CP-violating amplitude, both in magnitude and phase.

   The final state D^0X need not be self conjugate, and it is actually desirable that it not be, so that no effects of mixing are present, and no tagging of the second B is needed. Thus method 1 could be used at a symmetric e^+e^- collider without the penalty due to mixing of methods 2-6. This method works both for decays of B-mesons and b-baryons.

   The general approach of methods 1-3 was largely anticipated by Carter and Sanda [5, 6], but recent interest stems from the more specific formulation of Gronau and London [7]. Method 1 as distinct from method 3 was first examined by Gronau and Wyler [8], with further discussions given by Dunietz [9, 10]. Application of method 1 to b-baryons was first discussed by Aleksan, Dunietz and Kayser [11].

   If CP violation is found in such an analysis then it cannot be due to to superweak model, which postulates that CP violation occurs only in mixing of neutral mesons. Thus method 1 may be used to circumvent possible ambiguities [12] in the use of method 4 to prove or disprove the superweak model.

2. **Neutral B-meson decays to f and $\overline{f}$ where $f \neq \overline{f}$**

   If a neutral B-meson decays to both a final state f and its CP-conjugate state $\overline{f}$, then the interference of amplitudes needed for measurable CP violation arises due to mixing (whether or not there is CP violation in the mixing). A time-dependent analysis of the four decay modes $B(\overline{B}) \to f, \overline{f}$ can isolate the CP-violating phase.

   Tagging of the particle-antiparticle character of the second B in the event is required.

   The original paper on method 2 is by Gronau and London [7]. Discussion of method 2 as separate from method 3 was first been given by Aleksan *et al.* [13]. Method 2 is an improvement on earlier discussions by Du, Dunietz and Wu [14], and Dunietz and Rosner [15] in which only two of the four related decays were utilized.

3. **Neutral B-meson decays to $D^0 X$, $\overline{D}^0 X$, and $D^0_{1,2} X$ where $X = \overline{X}$**

   If a neutral B-mesons decays to both a final state $D^0 X$ and $\overline{D}^0 X$ where X is self conjugate ($CP(X) \equiv \overline{X} = \pm X$), then methods 1 and 2 can be combined. In a case of interest two different CP-violating phases can be determined from the time-dependent analysis of six (or eight) related decay modes.

   As previously mentioned, method 3 was first discussed by Gronau and London [7].

4. **Neutral B-meson decays to CP eigenstates**

   If a neutral B-meson decays to a final state f that is a CP eigenstate, then as in method 2, CP violation becomes observable via the interference due to mixing. But since only a single final state is involved the strong-interaction phase does not appear. Thus we recover the well-known result that a time dependent analysis of the two modes $B(\overline{B}) \to f$ can isolate the CP-violating phases.

   The advantages of measuring decays to CP eigenstates were first noted by Bigi and Sanda [16]. The important relation between decays to CP eigenstates and unitarity of the CKM matrix was first emphasized by Bjorken [17, 18], and will be reviewed in the following sec. 1.3. The measurement of the three angles of the unitarity triangle by three specific decays to CP eigenstates was first proposed by Krawczyk *et al.* [19].

5. **B decays to sets of final states related by isospin**

   In decays $B_u^+ \to f^+$ and $B_d^0 \to f^0$ where the final states each arise due to the inference of two amplitudes, and f^+ and f^0 are related by isospin, the CP-violating phase can be isolated by a detailed isospin analysis.

   The utility of the isospin analysis in removing uncertainties due to penguin diagrams in B decays was first demonstrated by Gronau and London [20]. Further discussions have been given by Nir and Quinn [21], by Lipkin *et al.* [22] and by Gronau [23].

6. **Angular analysis of B decays to mixtures of CP eigenstates**

   If a neutral B-meson decays to a self-conjugate state f, but this is not a pure CP eigenstate (as holds when f consists of two spin-1 mesons) method 4 cannot be carried out. However, a detailed analysis of the angular distribution of the secondary-decay products can separate the final state into CP(even) and CP(odd) components and the CP-violating phase extracted.

   Methods of angular analysis for B decays to mixtures of CP eigenstates have been presented for several years [24, 25, 26], with recent discussion by Kayser *et al.* [27], by Dunietz *et al.* [28], and by Kramer and Palmer [29, 30].

1.3 The Unitarity Test

In view of the variety of methods of measuring the phases of the CKM matrix elements it is useful to have an overall goal in pursuing an experimental program. This has been elegantly defined by Bjorken [17, 18] as a test of unitarity of the CKM matrix. This will provide a comprehensive test of the Standard Model view of CP violation as arising from phases in the transformation between the three generations of strong and weak quark base states.

We will discuss the CKM matrix in the Wolfenstein notation [31]:

$$
\begin{aligned}
V_{CKM} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\
&\approx \begin{pmatrix} 1-\lambda^2/2+\lambda^4/24 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2-\lambda^4(A^2/8-1/24) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2+A\lambda^4(1/2-\rho-i\eta) & 1-A^2\lambda^4/2 \end{pmatrix},
\end{aligned}
\tag{8}
$$

carrying the expansion in the parameter λ ($\approx$ the Cabibbo angle) to fourth order [19]. From measurements of the B-meson lifetime it is known that $A \approx 1$. CP violation arises in the Standard Model because $\eta \neq 0$.

The unitarity of V_{CKM} implies that

$$
\sum_k V_{ik} V^*_{jk} = \delta_{ij} = \sum_k V_{ki} V^*_{kj}. \tag{9}
$$

Of these 18 conditions the one obtained using the first and third rows (or almost equivalently, the first and third columns) is especially suitable for testing via measurements of weak phase angles:

$$
0 = V^*_{ud} V_{td} + V^*_{us} V_{ts} + V^*_{ub} V_{tb} \approx V_{td} + \lambda V_{ts} + V^*_{ub}. \tag{10}
$$

Regarding V_{td}, λV_{ts} and V_{ub} as vectors they form a closed triangle in the complex plane [32]. On dividing their lengths by $A\lambda^3$, we obtain the picture of Figure 1 in the (ρ, η) plane.

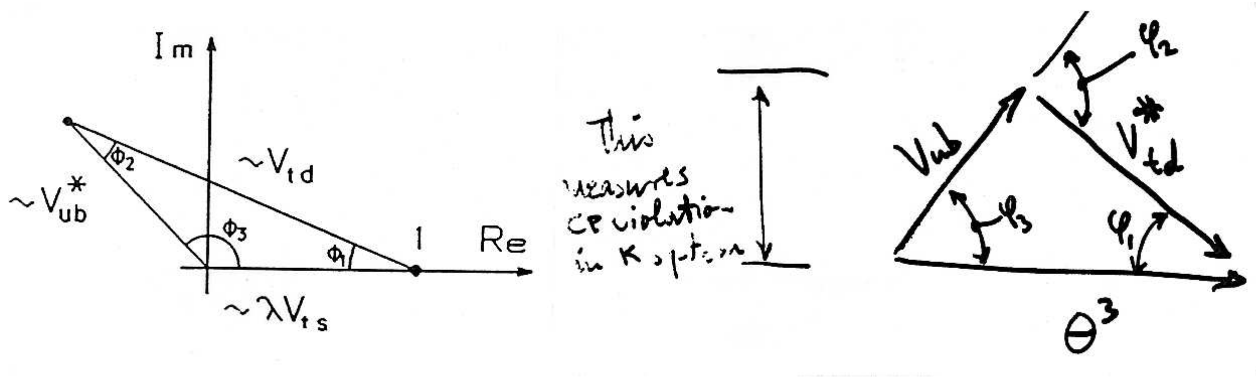

Figure 1: a) The unitarity triangle in the notation of the present work. b) The unitarity triangle as sketched by Bjorken when he first proposed the unitarity test [17].

The unitarity test then consists of measuring the magnitudes and phases of these three vectors to confirm that they form a closed triangle. It is anticipated that measurement of the magnitude of V_{td} via its role in the box diagram governing B^0_d mixing [33] will remain subject to theoretical uncertainties due to strong-interaction effects for some time to come. The insight of Bjorken was that a test of the closure of the unitarity triangle can be based on measurement of the three interior angles φ_1, φ_2, φ_3, which should sum to π. These three angles, and the area $A^2\lambda^6\eta/2$ of the unitarity triangle are invariant under the choice of representation of the CKM matrix [34].

In the Wolfenstein parametrization the three angles φ_i can be related to phases of CKM matrix elements according to

$$\begin{aligned} \varphi_1 &= 2\pi - \phi(V_{td}) && \equiv 2\pi - \phi_{td}, \\ \varphi_2 &= \pi - \varphi_1 - \varphi_3 && = -\pi + \phi_{td} + \phi_{ub}, \\ \varphi_3 &= \phi(V_{ub}^*) = -\phi(V_{ub}) && \equiv -\phi_{ub}. \end{aligned} \tag{11}$$

A favorable theoretical result is that method 4, the study of neutral B decays to CP eigenstates, can in principle determine all three angles φ_i by measurement of three different decays [19]. However, the anticipated difficulty in measuring ϕ_{ub} via decays such as $B_s^0 \to \rho^0 K_S^0$, due to the small branching ratios (and poor signal of B_s at e^+e^- colliders), has been a motivation to explore the additional methods of analysis of CP violation reviewed in this paper.

2 Method 1: B Decays to D^0X, $\overline{D}^0X$, and $D_{1,2}^0X$ where $X \neq \overline{X}$

When a B meson (*i.e.*, one that contains a $\overline{b}$-quark) decays to $\overline{D}^0X$ via a spectator graph (graphs I and II of fig. 3 in the Appendix) this involves a $\overline{b} \to \overline{c}$ transition, and hence no weak phase:

$$A(B \to \overline{D}^0X) = \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}}, \tag{12}$$

where $\delta_{\overline{f}}$ is a final-state strong-interaction phase. But when a B meson decays to D^0X this involves the transition $\overline{b} \to \overline{u}$ and hence the weak phase $-\phi_{ub}$ appears in the amplitude:

$$A(B \to D^0X) = |A_f|\, e^{-i\phi_{ub}} e^{i\delta_f}, \tag{13}$$

The two amplitudes A_f and $A_{\overline{f}}$ interfere when the D forms one of the CP eigenstates $D_{1,2}^0 = (D^0 \pm \overline{D}^0)/\sqrt{2}$:

$$A(B \to D_{1,2}^0X) = (|A_f|\, e^{-i\phi_{ub}} e^{i\delta_f} \pm \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}})/\sqrt{2}. \tag{14}$$

In eqs. (12-13) we have supposed that all graphs contributing to each decay have the same weak phases (which is not necessarily true, as discussed below).

When $X \neq \overline{X}$ the decays are self-tagging as to whether the parent was a B or a $\overline{B}$. Then even for neutral B's there is no effect due to mixing on the observed decay rates. Method 1 could be used at a symmetric e^+e^- collider without the penalty due to mixing of methods 2-6.

Assuming equal production rates for B and $\overline{B}$ the decay rates are proportional to the number of decays observed. We can therefore measure

$$\begin{aligned}
\Gamma(B \to \overline{D}^0 X) = \Gamma(\overline{B} \to D^0 \overline{X}) \ &\propto \left|A_{\overline{f}}\right|^2, \\
\Gamma(B \to D^0 X) = \Gamma(\overline{B} \to \overline{D}^0 \overline{X}) \ &\propto |A_f|^2, \\
\Gamma(B \to D^0_{1,2} X) \ &\propto (\left|A_{\overline{f}}\right|^2 + |A_f|^2)/2 \pm \left|A_{\overline{f}}\right| |A_f| \cos(\phi_{ub} + \delta), \\
\Gamma(\overline{B} \to D^0_{1,2} \overline{X}) \ &\propto (\left|A_{\overline{f}}\right|^2 + |A_f|^2)/2 \pm \left|A_{\overline{f}}\right| |A_f| \cos(\phi_{ub} - \delta),
\end{aligned} \tag{15}$$

recalling eq. (2), and defining $\delta = \delta_{\overline{f}} - \delta_f$ as the strong-interaction phase difference.

Thus there are eight possible measurements depending on the four quantities $\left|A_{\overline{f}}\right|$, $|A_f|$, $\cos(\phi_{ub} + \delta)$ and $\cos(\phi_{ub} - \delta)$. Therefore we can deduce $|\phi_{ub} \pm \delta|$ and hence determine ϕ_{ub} up to a fourfold ambiguity:

$$\phi_{ub} = \frac{\pm |\phi_{ub} + \delta| \pm |\phi_{ub} - \delta|}{2}. \tag{16}$$

The strong phase difference δ depends on the particular final state $D^0 X$ studied, so if these measurements can be carried out for different X the discrete ambiguity may be removable.

In contrast to sec. 1.1 where only two rate measurements were considered, the use of 4-8 measurements in the present method permits ϕ_{ub} to be determined whether on not the strong interaction phase difference δ is nonvanishing. Indeed, it would be preferable if δ were zero, as the discrete ambiguity is only twofold in this case.

We now consider examples of particular decay modes that might be used to implement this procedure. First, we consider the question of identifying the CP eigenstates $D^0_{1,2}$. From Table 8 of the Appendix in which the basic two-body decays of the D^0 are listed we infer that the CP(even) state D^0_1 can decay according to

$$D^0_1 \to \pi^+\pi^-,\ K^+K^-,\ K^0_S K^0_S,\ K^0_L K^0_L,\ K^0_L \pi^0,\ K^0_L \eta,\ K^0_L \rho^0,\ K^0_L \omega,\ K^0_L \phi,\ etc. \tag{17}$$

and the the CP(odd) state D^0_2 can decay to

$$D^0_2 \to K^0_S \pi^0,\ K^0_S \eta,\ K^0_S K^0_L,\ K^0_S \rho^0,\ K^0_S \omega,\ K^0_S \phi,\ etc. \tag{18}$$

Several of the decays of the D^0_2 have been observed, and the fraction of D^0's that decay as D^0_2 is at least 2%. However, all D^0_2 decays except $K^0_S \rho^0 \to \pi^+\pi^-\pi^+\pi^-$ $K^0_S \phi \to \pi^+\pi^- K^+K^-$ involve at least two final-state photons. If we suppose that only all-charged final-states will be reconstructed at a hadron collider, then only about 0.5% of all D^0's will decay to identifiable D^0_2 modes. The D^0_1 decays predominantly to all-charged daughters, but again only about 0.5% of all D^0's will decay to identifiable D^0_1 modes. In sum, about 2-5% of D^0's might be usable for the $D^0_{1,2}$ analysis at an e^+e^- collider, but only about 1% at a hadron collider.

In principle the decays $B \to D^{\star 0} X$ decays are also usable for the present analysis as $D^{\star 0}_{1,2} = (D^{\star 0} \pm \overline{D}^{\star 0})/\sqrt{2}$ are CP(even) and (odd) eigenstates, respectively. However, in the decays $D^{\star 0} \to D^0 \pi^0$ and $D^0 \gamma$ the final-state orbital angular momentum is one in both cases and so the CP eigenstates decay according to

$$D^{\star 0}_1 \to D^0_1 \pi^0, \qquad \text{but} \qquad D^{\star 0}_1 \to D^0_2 \gamma, \qquad etc. \tag{19}$$

Hence the $D^{*0}_{1,2}$ states can only be correctly identified if the single γ can be distinguished from the π^0. As both the γ and π^0 are quite soft this may be possible at an e^+e^- collider but is problematic at a hadron collider.

Finally we consider specific B-decay modes that are suitable for method 1. Referring to Tables 2-5 in the Appendix we find the following candidates:

$$\begin{aligned}
B_u^+ &\to \begin{cases} \overline{D}^0\pi^+ & [\mathrm{I}_F, \mathrm{II}_F] \\ D^0\pi^+ & [\mathrm{II}_D, \mathrm{III}_D] \end{cases}, \quad \begin{cases} \overline{D}^0K^+ & [\mathrm{I}_S, \mathrm{II}_S] \\ D^0K^+ & [\mathrm{II}_S, \mathrm{III}_S] \end{cases}, \\
B_d^0 &\to \begin{cases} \overline{D}^0K^{*0} & [\mathrm{II}_S] \\ D^0K^{*0} & [\mathrm{II}_S] \end{cases}, \\
B_s^0 &\to \begin{cases} \overline{D}^0K^{*0} & [\mathrm{II}_F] \\ D^0K^{*0} & [\mathrm{II}_D] \end{cases}, \\
B_c^+ &\to \begin{cases} \overline{D}^0D^+ & [\mathrm{II}_F, \mathrm{III}_F] \\ D^0D^+ & [\mathrm{I}_D, \mathrm{II}_D] \end{cases}, \quad \begin{cases} \overline{D}^0D_s^+ & [\mathrm{II}_S, \mathrm{III}_S] \\ D^0D_s^+ & [\mathrm{I}_S, \mathrm{II}_S] \end{cases}.
\end{aligned} \tag{20}$$

The roman numerals refer to the type of graph, as shown in fig. 3 of the Appendix, and the subscripts F, S, and D refer to CKM-favored (order λ^2), -suppressed (order λ^3), and -doubly-suppressed (order λ^4), respectively. In addition, b-baryons have suitable modes, such as $\Lambda_b^0\ (udb) \to \Lambda D^0(\overline{D}^0)$, $\Lambda_b^+\ (udb) \to \Sigma^+ D^0(\overline{D}^0)$, $\Sigma_b^0\ (usb) \to \Xi^0 D^0(\overline{D}^0)$, *etc.* [10], which we will not discuss further.

Among the candidate B-meson decays, only $B_d^0 \to D^0(\overline{D}^0)K^{*0}$ is ideally suited for method 1, as only one graph contributes to each decay and these are both singly CKM-suppressed type-II (color-suppressed) spectator graphs. These decays have not yet been observed, but should have branching ratios of order 10^{-5}. If only about 1% of the decays are useful for the $D^0_{1,2}$ analysis, the effective branching ratio is about 10^{-7}. So at least 10^9 B's must be produced to carry out method 1. Some advantage is gained by considering several channels, but since the strong phase difference varies from channel to channel, there must be enough events in each channel to carry out the analysis separately before results for ϕ_{ub} can be combined. Hence method 1 may be out of range of e^+e^- B factories with luminosity of 3×10^{33} cm^{-2}sec^{-1}, even though the method is well-suited in principle to them.

The other five candidate decay pairs listed above all suffer from the rate for D^0X being at least an order of magnitude less than that for $\overline{D}^0X$ (or *vice versa*), so the interference term in $D^0_{1,2}X$ is quite small. However, the branching fraction for $B^+ \to D^0K^+$ is likely to be very similar to that for $B_d^0 \to D^0K^{*0}$. Since the statistical accuracy of method 1 is largely set by the number of events of whichever of D^0X or $\overline{D}^0X$ has the lower branch, we conclude that $B^+ \to D^0(\overline{D}^0)K^+$ is about as useful as $B_d^0 \to D^0(\overline{D}^0)K^{*0}$.

If B_c mesons were produced as copiously as B^+ and B_d^0 then the decay pair $B_c^+ \to D^0(\overline{D}^0)D_s^+$ would be also be useful. However, B_c production is likely to be suppressed at both e^+e^- and hadron colliders.

3 Method 2: Neutral B-Meson Decays to f and $\overline{f}$ where $f \neq \overline{f}$

In the second method the needed interference arises from mixing of a B^0 and $\overline{B}^0$. The analysis is more straightforward if the final state f is not self conjugate ($f \neq \overline{f}$), but then both the B^0 and $\overline{B}^0$ must decay to both f and $\overline{f}$.

As for decay pairs suitable for method 1, one of the decay pairs (here called $\overline{f}$) proceeds via a $\overline{b} \to \overline{c}$ transition, and the other (f) via $\overline{b} \to \overline{u}$. So we may write

$$\begin{aligned}
A(B^0 \to \overline{f}) &= \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}}, \\
A(B^0 \to f) &= \left|A_f\right| e^{-i\phi_{ub}} e^{i\delta_f}, \\
A(\overline{B}^0 \to f) &= \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}}, \\
A(\overline{B}^0 \to \overline{f}) &= \left|A_f\right| e^{i\phi_{ub}} e^{i\delta_f},
\end{aligned} \tag{21}$$

using eq. (2). In writing this we must be able to assume that each amplitude is dominated by a single weak phase.

Due to mixing, a particle that was created as a B^0 (or $\overline{B}^0$) at $t = 0$ has evolved by time t to the state we label as $B^0(t)$ (or $\overline{B}^0(t)$) according to

$$\begin{aligned}
B^0(t) &= e^{-iMt} e^{-t/2} [\cos(xt/2)|B^0\rangle + i e^{2i\phi_M} \sin(xt/2)|\overline{B}^0\rangle], \\
\overline{B}^0(t) &= e^{-iMt} e^{-t/2} [i e^{-2i\phi_M} \sin(xt/2)|B^0\rangle + \cos(xt/2)|\overline{B}^0\rangle],
\end{aligned} \tag{22}$$

where throughout this paper we measure time in units of the relevant B lifetime, $x = \Delta M/\Gamma$ is the mixing parameter, and the relative amount of $|B^0\rangle$ and $|\overline{B}^0\rangle$ in the weak eigenstate B_S^0 is given by a pure phase coming from the box diagram [33], where

$$\phi_M = \begin{cases} \phi_{td}, & \text{for } B_d^0 \\ \phi_{ts} \approx 0, & \text{for } B_s^0 \end{cases} \tag{23}$$

The four time-dependent decay rates are then

$$\begin{aligned}
\Gamma(B^0(t) \to \overline{f}) &\propto e^{-t}[\left|A_{\overline{f}}\right|^2 \cos^2(xt/2) + |A_f|^2 \sin^2(xt/2) - \overline{S}\sin(xt)], \\
\Gamma(B^0(t) \to f) &\propto e^{-t}[|A_f|^2 \cos^2(xt/2) + \left|A_{\overline{f}}\right|^2 \sin^2(xt/2) - S\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to f) &\propto e^{-t}[\left|A_{\overline{f}}\right|^2 \cos^2(xt/2) + |A_f|^2 \sin^2(xt/2) + S\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to \overline{f}) &\propto e^{-t}[|A_f|^2 \cos^2(xt/2) + \left|A_{\overline{f}}\right|^2 \sin^2(xt/2) + \overline{S}\sin(xt)],
\end{aligned} \tag{24}$$

where $\delta = \delta_{\overline{f}} - \delta_f$ is the strong-interaction phase difference, and

$$\overline{S} = |A_f| \left|A_{\overline{f}}\right| \sin(2\phi_M + \phi_{ub} - \delta), \qquad \text{and} \qquad S = |A_f| \left|A_{\overline{f}}\right| \sin(2\phi_M + \phi_{ub} + \delta). \tag{25}$$

For eventual Fourier analysis it is preferable to write eqs. (24) as

$$\begin{aligned}
\Gamma(B^0(t) \to \overline{f}) &\propto e^{-t}[K + C\cos(xt) - \overline{S}\sin(xt)], \\
\Gamma(B^0(t) \to f) &\propto e^{-t}[K - C\cos(xt) - S\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to f) &\propto e^{-t}[K + C\cos(xt) + S\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to \overline{f}) &\propto e^{-t}[K - C\cos(xt) + \overline{S}\sin(xt)],
\end{aligned} \tag{26}$$

where

$$K = (\left|A_{\overline{f}}\right|^2 + |A_f|^2)/2, \qquad \text{and} \qquad C = (\left|A_{\overline{f}}\right|^2 - |A_f|^2)/2. \tag{27}$$

From measurement of these four time-dependent decay rates one deduces the four quantities $|A_f|$, $\left|A_{\overline{f}}\right|$, $\sin(2\phi_M + \phi_{ub} + \delta)$, and $\sin(2\phi_M + \phi_{ub} - \delta)$. Thus we can measure $|\pi/2 - 2\phi_M - \phi_{ub} \pm \delta|$ and thereby determine $2\phi_M + \phi_{ub}$ up to a fourfold ambiguity:

$$2\phi_M + \phi_{ub} = \frac{\pi \pm |\pi/2 - 2\phi_M - \phi_{ub} + \delta| \pm |\pi/2 - 2\phi_M - \phi_{ub} \pm \delta|}{2}. \tag{28}$$

As for method 1, the use of four rate measurements permits the weak phase $2\phi_M + \phi_{ub}$ to be extracted even when the strong phase difference δ vanishes.

To carry out the above analysis we must know for each decay whether the B was created as a B^0 or a $\overline{B}^0$. The decays are not self tagging since both B^0 and $\overline{B}^0$ can decay to both f and $\overline{f}$, so in method 2 (as well as methods 3-6) one must tag the particle/antiparticle character of the second B in the event. As that B may also be subject to mixing, a dilution of the statistical power of the method results. In particular, it is well-known that at an e^+e^- collider when the B-$\overline{B}$ pair is produced in a C(odd) state the interesting terms in $\sin(xt)$ in eqs. (28) cannot be measured unless the B's have relativistic velocity in the lab frame. This 'penalty' due to mixing can only be overcome by use of an asymmetric e^+e^- collider if the center-of-mass energy is that of the $\Upsilon(4S)$.

From Tables 3 and 4 of the Appendix we find that there are 4 candidate decays pairs for implementing method 2:

$$\begin{aligned}
B_d^0 \to &\left\{ \begin{array}{ll} D^-\pi^+ & [\mathrm{I}_F,\ \mathrm{IV}_F] \\ D^+\pi^- & [\mathrm{I}_D,\ \mathrm{IV}_D] \end{array} \right., \qquad \left\{ \begin{array}{ll} D_s^- K^+ & [\mathrm{IV}_F] \\ D_s^+ K^- & [\mathrm{IV}_D] \end{array} \right., \\
B_s^0 \to &\left\{ \begin{array}{ll} D_s^- K^+ & [\mathrm{I}_S,\ \mathrm{IV}_S] \\ D_s^+ K^- & [\mathrm{I}_S,\ \mathrm{IV}_S] \end{array} \right., \qquad \left\{ \begin{array}{ll} D^-\pi^+ & [\mathrm{IV}_S] \\ D^+\pi^- & [\mathrm{IV}_S] \end{array} \right.,
\end{aligned} \tag{29}$$

In each example the lower decay depends on the weak phase $-\phi_{ub}$. The type-IV W-exchange graphs (fig. 3) may well be highly suppressed compared to the type-I spectator graphs. Thus of the four candidates, only $B_s^0 \to D_s^\pm K^\mp$ is likely to have reasonably large ($\sim 10^{-4}$) branching ratios for both channels. This renders method 2 largely unsuitable for an e^+e^- collider, where production of B_s mesons will be low. At a hadron collider where only all-charged daughters are used in reconstructing the B_s about 5-10% of the D_s decays will be useful. Accounting for dilutions due to mixing of the second B at a hadron collider, some 10^8-10^9 B_s are needed to implement method 2.

If method 2 is used for the decays $B_s^0 \to D_s^\pm K^\mp$ the weak phase that is measured in just ϕ_{ub}, since the mixing phase ϕ_M vanishes for B_s^0.

4 Method 3: Neutral B-Meson Decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X = \overline{X}$

Aspects of methods 1 and 2 are combined when a neutral B-meson decays to final state D^0X where X is self conjugate ($X = \overline{X}$). Now interference arises both from mixing and from the use of $D^0_{1,2}$ channels.

Following eqs. (12-14) and (21) we write the eight related decay amplitudes as

$$\begin{aligned}
A(B^0 \to \overline{D}^0X) &= \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}}, \\
A(B^0 \to D^0X) &= \left|A_f\right| e^{-i\phi_{ub}} e^{i\delta_f}, \\
A(\overline{B}^0 \to D^0X) &= \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}}, \\
A(\overline{B}^0 \to \overline{D}^0X) &= \left|A_f\right| e^{i\phi_{ub}} e^{i\delta_f}, \\
A(B^0 \to D^0_{1,2}X) &= (\left|A_f\right| e^{-i\phi_{ub}} e^{i\delta_f} \pm \left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}})/\sqrt{2} \equiv A_{1,2}, \\
A(\overline{B}^0 \to D^0_{1,2}X) &= (\left|A_{\overline{f}}\right| e^{i\delta_{\overline{f}}} \pm \left|A_f\right| e^{i\phi_{ub}} e^{i\delta_f})/\sqrt{2} \equiv \overline{A}_{1,2}.
\end{aligned} \tag{30}$$

Because both D^0X and $\overline{D}^0X$ can be reached from both B^0 and $\overline{B}^0$, mixing must always be taken into account. In addition to the four time-dependent decay rates given in eq. (26), there are four more involving $D^0_{1,2}$ obtained by combining eqs. (22) and (30):

$$\begin{aligned}
\Gamma(B^0(t) \to D^0_{1,2}X) &\propto e^{-t}[\left|A_{1,2}\right|^2 \cos^2(xt/2) + \left|\overline{A}_{1,2}\right|^2 \sin^2(xt/2) - S_{1,2}\sin(xt)], \\
&= e^{-t}[K_{1,2} - C_{1,2}\cos(xt) - S_{1,2}\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to D^0_{1,2}X) &\propto e^{-t}[\left|\overline{A}_{1,2}\right|^2 \cos^2(xt/2) + \left|A_{1,2}\right|^2 \sin^2(xt/2) + S_{1,2}\sin(xt)], \\
&= e^{-t}[K_{1,2} + C_{1,2}\cos(xt) + S_{1,2}\sin(xt)],
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
\left|A_{1,2}\right|^2 &= (\left|A_{\overline{f}}\right|^2 + \left|A_f\right|^2)/2 \pm \left|A_{\overline{f}}\right| \left|A_f\right| \cos(\phi_{ub} + \delta), \\
\left|\overline{A}_{1,2}\right|^2 &= (\left|A_{\overline{f}}\right|^2 + \left|A_f\right|^2)/2 \pm \left|A_{\overline{f}}\right| \left|A_f\right| \cos(\phi_{ub} - \delta), \\
K_{1,2} &= (\left|A_{\overline{f}}\right|^2 + \left|A_f\right|^2)/2 \pm \left|A_{\overline{f}}\right| \left|A_f\right| \cos\phi_{ub}\cos\delta, \\
C_{1,2} &= \pm \left|A_{\overline{f}}\right| \left|A_f\right| \sin\phi_{ub}\sin\delta, \\
S_{1,2} &= 2\left|A_f\right| \left|A_{\overline{f}}\right| \sin(2\phi_M + \phi_{ub})\cos\delta \pm \left|A_f\right|^2 \sin 2(\phi_M + \phi_{ub}) \pm \left|A_{\overline{f}}\right|^2 \sin 2\phi_M,
\end{aligned} \tag{32}$$

and $\delta = \delta_{\overline{f}} - \delta_f$ is the strong-interaction phase difference.

From analysis of the four time-dependent rates (26) we deduce $\left|A_f\right|$, $\left|A_{\overline{f}}\right|$, $\sin(2\phi_M + \phi_{ub} + \delta)$ and $\sin(2\phi_M + \phi_{ub} - \delta)$. Then from the coefficients $K_{1,2}$ and $C_{1,2}$ of eqs. (31) we also extract $\cos\phi_{ub}\cos\delta$ and $\sin\phi_{ub}\sin\delta$. Finally, from the coefficient of $\sin(xt)$ we can extract $\sin(2\phi_M + \phi_{ub})\cos\delta$, $\sin 2(\phi_M + \phi_{ub})$ and $\sin 2\phi_M$.

Thus method 3 leads to the simultaneous measurement of ϕ_M, ϕ_{ub}, $\phi_M+\phi_{ub}$ and $2\phi_M+\phi_{ub}$. In case of B_d^0 mesons for which $\phi_M = \phi_{td}$ (see eq. (23)) ϕ_{td} and ϕ_{ub} and $\phi_{td}+\phi_{ub}$ are measured at once. It is remarkable that all three of the phase angles of the unitarity triangle can be extracted from the analysis of a single family of B_d^0 decays.

From Tables 3 and 4 of the Appendix we find that there are six candidate decays pairs for implementing method 3:

$$\begin{array}{llll} B_d^0 \to & \left\{ \begin{array}{ll} \overline{D}^0 K_{S,L}^0 & [\mathrm{II}_S] \\ D^0 K_{S,L}^0 & [\mathrm{II}_S] \end{array} \right. , & \left\{ \begin{array}{ll} \overline{D}^0 \rho^0 & [\mathrm{II}_F,\ \mathrm{IV}_F] \\ D^0 \rho^0 & [\mathrm{II}_D,\ \mathrm{IV}_D] \end{array} \right. , & \left\{ \begin{array}{ll} \overline{D}^0 J/\psi & [\mathrm{IV}_F] \\ D^0 J/\psi & [\mathrm{IV}_D] \end{array} \right. , \\ B_s^0 \to & \left\{ \begin{array}{ll} \overline{D}^0 \phi & [\mathrm{II}_S] \\ D^0 \phi & [\mathrm{II}_S] \end{array} \right. , & \left\{ \begin{array}{ll} \overline{D}^0 K_{S,L}^0 & [\mathrm{II}_F] \\ D^0 K_{S,L}^0 & [\mathrm{II}_D] \end{array} \right. , & \left\{ \begin{array}{ll} \overline{D}^0 J/\psi & [\mathrm{IV}_S] \\ D^0 J/\psi & [\mathrm{IV}_S] \end{array} \right. . \end{array} \tag{33}$$

In each example the lower decay depends on the weak phase $-\phi_{ub}$. The type-IV W-exchange graphs (fig. 3) may well be highly suppressed compared to the type-II spectator graphs, although in view of the easy trigger for $J/\psi D$ these modes should be searched for. Among the eight candidates, $B_d^0 \to DK_{S,L}^0$ and $B_s^0 \to D\phi$ are the best in terms of size of the smaller branching ratio of the pair, which should be of order 10^{-5}. Since a very intricate time-dependent analysis is required to extract the full information from method 3, the B_s decays, for which the mixing parameter x_s is expected to be 10 or more, are likely to be less useful than the B_d decays.

At a hadron collider where only all-charged daughters are used in reconstructing the B^0 about 1% of the D^0 decays will be useful. Accounting for dilutions due to mixing of the second B at a hadron collider, some 10^{10}-10^{11} B's are needed to implement method 3. At an e^+e^- collider, result of comparable statistical precision can likely be had with one order of magnitude less B's, but still a rather large number.

5 Method 4: Neutral B-Meson Decays to CP Eigenstates

The most well-known method for extracting CP-violating phases uses neutral B mesons that decay to CP eigenstates f. In this case

$$|\overline{f}\rangle \equiv CP|f\rangle = \eta|f\rangle \qquad \text{where} \qquad \eta = \left\{ \begin{array}{ll} +1 & CP(\text{even}) \\ -1 & CP(\text{odd}) \end{array} \right. . \tag{34}$$

The decay amplitude can be written

$$A(B^0 \to f) = |A|\, e^{-i\phi_D} e^{i\delta}, \tag{35}$$

where δ is a strong-interaction phase, and the weak-interaction phase ϕ_D depends on whether the decay proceeds via a $\overline{b} \to \overline{c}$ or $\overline{u}$ transition:

$$\phi_D = \left\{ \begin{array}{ll} \phi_{cb} = 0, & b \to c \\ \phi_{ub}, & b \to u \end{array} \right. . \tag{36}$$

Table 1: The 23 basic neutral-B decays to CP eigenstates. The graphs associated with each decay mode are shown in fig. 3. The subscripts F, S, and D refer to CKM-favored (amplitude $\propto \lambda^2$), -suppressed ($\propto \lambda^3$), and -doubly-suppressed ($\propto \lambda^4$), respectively. The weak-interaction phase $\phi_M + \phi_D$ is shown in parentheses after each graph type, where ϕ_M is the phase due to mixing and ϕ_D is the phase due to $\overline{b}$-quark decay. Penguin graphs (V-VII) are included in classes 1-4 if they lead to the same final state as the nominal graphs for that class, even though their topology is different. Classes 1a and 4a are pure penguin graphs. Within each class the modes are ranked roughly in order of decreasing branching ratio. A final-state π^0 could be replaced by an η, ρ^0, ω, *etc.*, and a J/ψ could be replaced by an η_c, χ, ψ', *etc.*, but final states with two spin-1 particles must be analyzed according to method 6.

Class	B^0	$\overline{b} \to \overline{q}$	Modes	Graph$(\phi_M + \phi_D)$
1	B_d^0	$\overline{b} \to \overline{c}$	$J/\psi K^0_{S,L}$	$\mathrm{II}_F(\phi_{td})$, $\mathrm{VI}_F(\phi_{td})$
			D^+D^-	$\mathrm{I}_S(\phi_{td})$, $\mathrm{IV}_S(\phi_{td})$, V_S, VII_S
			$J/\psi\pi^0$	$\mathrm{II}_S(\phi_{td})$, VI_S
			$D_s^+D_s^-$	$\mathrm{IV}_S(\phi_{td})$, V_S
			$\phi K^0_{S,L}$	$\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$
2	B_d^0	$\overline{b} \to \overline{u}$	$\pi^+\pi^-$	$\mathrm{I}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VII_S
			$\pi^0\pi^0$	$\mathrm{II}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VI_S, VII_S
			$\rho^0 K^0_{S,L}$	$\mathrm{II}_D(\phi_{td}+\phi_{ub})$, $\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$
			$D^0\overline{D}^0$	$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S
			K^+K^-	$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S
3	B_s^0	$\overline{b} \to \overline{u}$	$\rho^0 K^0_{S,L}$	$\mathrm{II}_S(\phi_{ub})$, $\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$
			K^+K^-	$\mathrm{I}_D(\phi_{ub})$, $\mathrm{IV}_D(\phi_{ub})$, V_F, VII_F
			$\phi\pi^0$	$\mathrm{II}_D(\phi_{ub})$, VI_F
			$\pi^+\pi^-$	$\mathrm{IV}_D S(\phi_{ub})$, V_F,
			$\pi^0\pi^0$	$\mathrm{IV}_D S(\phi_{ub})$, V_F,
4	B_s^0	$\overline{b} \to \overline{c}$	$D_s^+D_s^-$	I_F, IV_F, V_F, VII_F
			$J/\psi K^0_{S,L}$	II_S, $\mathrm{VI}_S(\phi_{td})$
			$D^0\overline{D}^0$	IV_F, $\mathrm{IV}_D(\phi_{ub})$, V_F, V_S
			D^+D^-	IV_F, V_F
			$K^0\overline{K}^0$	V_F, VII_F
1a	B_s^0	$\overline{b} \to \overline{s}$	$\phi K^0_{S,L}$	$\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$
4a	B_d^0	$\overline{b} \to \overline{u}$	$\phi\pi^0$	VI_S
			$K^0\overline{K}^0$	V_S, VII_S

Following eq. (2) we can write the amplitude for the CP-conjugate process as

$$A(\overline{B}^0 \to \overline{f}) = \eta A(\overline{B}^0 \to f) = |A|\, e^{i\phi_D} e^{i\delta}, \quad \text{and hence} \quad A(\overline{B}^0 \to f) = \eta\, |A|\, e^{i\phi_D} e^{i\delta}, \tag{37}$$

using eq. (34). Combining eqs. (35-37) with (22) we arrive at the time-dependent decay rates

$$\begin{aligned} \Gamma(B^0(t) \to f) &\propto |A|^2\, e^{-t}[1 - \eta \sin(xt) \sin 2(\phi_M + \phi_D)], \\ \Gamma(\overline{B}^0(t) \to f) &\propto |A|^2\, e^{-t}[1 + \eta \sin(xt) \sin 2(\phi_M + \phi_D)]. \end{aligned} \tag{38}$$

If, as we have assumed, only a single graph contributes to $B^0 \to f$, then there is only a single strong-interaction phase δ in both this and the conjugate reaction $\overline{B}^0 \to f$. This single phase does not appear at all in the interference term in eq. (38).

Both ϕ_M and ϕ_D can take on two values depending on the decay considered, according to eqs. (23) and (36), so there are four classes of phase angles explored by method 4 as listed in Table 1. Classes 1, 2 and 3 provide measurements of φ_1, φ_2 and φ_3, respectively, of the unitarity test. Class-4 decays should show very little CP violation, but not necessarily zero, as they depend on V_{ts} which has a CP-violating phase at higher order (see eq. (8)). Any difference in the size of the CP violation between class 1 and class 2, or between class 3 and class 4 would indicate that the superweak model is not the source of that effect.

The class-1 decay $B_d^0 \to J/\psi K_S^0$ is particularly easy to trigger on and identify, and may provide the first evidence for CP violation in the B system. The most prominent class-2 and -3 decays, $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to \rho^0 K_S^0$, respectively, both have smaller branching ratios and in particular it may prove elusive to measure φ_3 with $B_s^0 \to \rho^0 K_S^0$.

Another potential difficulty is that with the exception of $B_d^0 \to J/\psi K_S^0$, all other decays to CP eigenstates have admixtures of penguin diagrams with different weak phases than the dominant tree diagram [35]. Hence it is useful to have other procedures than method 4 to measure φ_2 and φ_3.

6 Method 5: *B* Decays to Sets of Final States Related by Isospin

In Table 1 we see that the decay $B_d^0 \to \pi^+\pi^-$ that can be used to determine φ_2 has contributions both from spectator diagrams and penguin diagrams. However, the penguin diagrams have no weak phase [35] in this case, and to the extent that they are significant, the measurement of φ_2 is compromised.

By measurement of the related decays $B_u^+ \to \pi^+\pi^0$, $B_d^0 \to \pi^+\pi^-$, $\pi^0\pi^0$, the weak phase φ_2 can be isolated from the strong phase of the penguin diagram (which latter phase is not determined). The separation is aided by the fact that the spin-0 $\pi\pi$ final states can only be in isospin $I = 0$ or 2 states due to Bose statistics, and by the result that the penguin graphs can only lead to the $I = 0$ states [36].

The exchange-symmetric $\pi\pi$ isospin states of interest are

$$\begin{aligned}
\sqrt{\tfrac{1}{2}}(|\pi^+\pi^0\rangle + |\pi^0\pi^+\rangle) &= |2,1\rangle, \\
\sqrt{\tfrac{1}{2}}(|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle) &= \sqrt{\tfrac{1}{3}}|2,0\rangle + \sqrt{\tfrac{2}{3}}|0,0\rangle, \\
|\pi^0\pi^0\rangle &= \sqrt{\tfrac{2}{3}}|2,0\rangle - \sqrt{\tfrac{1}{3}}|0,0\rangle, \\
\sqrt{\tfrac{1}{2}}(|\pi^-\pi^0\rangle + |\pi^0\pi^-\rangle) &= |2,-1\rangle,
\end{aligned} \tag{39}$$

via the relevant Clebsch-Gordon coefficients. The decays of a $B_d^0 = |\frac{1}{2}, -\frac{1}{2}\rangle$ or $B_u^+ = |\frac{1}{2}, \frac{1}{2}\rangle$ to these states involve $\Delta I_3 = \frac{1}{2}$ which can occur via either $\Delta I = \frac{1}{2}$ or $\frac{3}{2}$ transitions. We use the 'spurion' notation to write the weak Hamiltonian for these transitions as

$$H_{\mathrm{weak}} = \quad H_{1/2}|\tfrac{1}{2}, \tfrac{1}{2}\rangle + H_{3/2}|\tfrac{3}{2}, \tfrac{1}{2}\rangle. \tag{40}$$

Then the $\pi\pi$ isospin states obtained in the B decays are

$$\begin{aligned}
H_{1/2}|\tfrac{1}{2}, \tfrac{1}{2}\rangle|B_d^0\rangle &= \sqrt{\tfrac{1}{2}}H_{1/2}|1,0\rangle + \sqrt{\tfrac{1}{2}}H_{1/2}|0,0\rangle, \\
H_{3/2}|\tfrac{3}{2}, \tfrac{1}{2}\rangle|B_d^0\rangle &= \sqrt{\tfrac{1}{2}}H_{3/2}|2,0\rangle + \sqrt{\tfrac{1}{2}}H_{3/2}|0,0\rangle, \\
H_{1/2}|\tfrac{1}{2}, \tfrac{1}{2}\rangle|B_u^+\rangle &= H_{1/2}|1,1\rangle, \\
H_{3/2}|\tfrac{3}{2}, \tfrac{1}{2}\rangle|B_u^+\rangle &= \sqrt{\tfrac{3}{4}}H_{3/2}|2,1\rangle - \sqrt{\tfrac{1}{4}}H_{3/2}|1,1\rangle.
\end{aligned} \tag{41}$$

The transition amplitudes are then

$$\begin{aligned}
A(B_d^0 \to \pi^+\pi^-) &\equiv A^{+-} &&= \sqrt{\tfrac{1}{6}}\langle \pi\pi, I=2|H_{3/2}|B\rangle + \sqrt{\tfrac{1}{3}}\langle \pi\pi, I=0|H_{1/2}|B\rangle, \\
A(B_d^0 \to \pi^0\pi^0) &\equiv A^{00} &&= \sqrt{\tfrac{1}{3}}\langle \pi\pi, I=2|H_{3/2}|B\rangle - \sqrt{\tfrac{1}{6}}\langle \pi\pi, I=0|H_{1/2}|B\rangle, \\
A(B_u^+ \to \pi^+\pi^0) &\equiv A^{+0} &&= \sqrt{\tfrac{3}{4}}\langle \pi\pi, I=2|H_{3/2}|B\rangle.
\end{aligned} \tag{42}$$

Following ref. [20] we define

$$A_2 \equiv \quad \sqrt{\tfrac{1}{12}}\langle \pi\pi, I=2|H_{3/2}|B\rangle, \qquad \text{and} \qquad A_2 \equiv \quad -\sqrt{\tfrac{1}{6}}\langle \pi\pi, I=0|H_{1/2}|B\rangle, \tag{43}$$

so we can write the three B-decay amplitudes (and the corresponding three $\overline{B}$ amplitudes) as

$$\begin{aligned}
A^{+0} &= 3A_2, & \overline{A}^{-0} &= 3\overline{A}_2, \\
A^{+-} &= \sqrt{2}(A_2 - A_0), & \overline{A}^{+-} &= \sqrt{2}(\overline{A}_2 - \overline{A}_0), \\
A^{00} &= 2A_2 + A_0, & \overline{A}^{00} &= 2\overline{A}_2 + \overline{A}_0.
\end{aligned} \tag{44}$$

Thus the six decay amplitudes are related by the two constraints

$$\sqrt{\tfrac{1}{2}}A^{+-} + A^{00} = A^{+0}, \qquad \sqrt{\tfrac{1}{2}}\overline{A}^{+-} + \overline{A}^{00} = \overline{A}^{-0}. \tag{45}$$

As isospin amplitudes A_2 and $\overline{A}_2$ contain only spectator graphs their phase structure can be written

$$A_2 = |A_2|\, e^{i\phi_2} = |A_2|\, e^{-i\phi_{ub}} e^{i\delta_2}, \qquad \overline{A}_2 = |A_2|\, e^{i\overline{\phi}_2} = |A_2|\, e^{i\phi_{ub}} e^{i\delta_2}, \tag{46}$$

noting that the spectator graphs for A_2 involve a $\overline{b} \to \overline{u}$ transition, and defining δ_2 as the strong-interaction phase of the isospin-2 spectator graph. The amplitudes A_0 (later written $|A_0|\, e^{i\phi_0}$) and $\overline{A}_0$ $(= \left|\overline{A}_0\right| e^{i\overline{\phi}_0})$ contain both spectator and penguin graphs, but it will not be possible to separate these amplitudes in this analysis, so we do not write the equivalent of eq. (46) for them.

The decay rates are

$$\begin{aligned}
\Gamma(B^+ \to \pi^+\pi^0) &= \Gamma(B^- \to \pi^-\pi^0) \propto |A_2|^2\,, \\
\Gamma(B^0(t) \to \pi^+\pi^-) &\propto e^{-t}[K^{+-} - C^{+-}\cos(xt) - S^{+-}\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) &\propto e^{-t}[K^{+-} + C^{+-}\cos(xt) + S^{+-}\sin(xt)], \\
\Gamma(B^0(t) \to \pi^0\pi^0) &\propto e^{-t}[K^{00} - C^{00}\cos(xt) - S^{00}\sin(xt)], \\
\Gamma(\overline{B}^0(t) \to \pi^0\pi^0) &\propto e^{-t}[K^{00} + C^{00}\cos(xt) + S^{00}\sin(xt)],
\end{aligned} \tag{47}$$

using eq. (22) and defining

$$\begin{aligned}
K^{+-} &= (\left|\overline{A}^{+-}\right|^2 + |A^{+-}|^2)/2, \\
C^{+-} &= (\left|\overline{A}^{+-}\right|^2 - |A^{+-}|^2)/2, \\
S^{+-} &= \mathrm{Im}(A^{*+-} e^{2i\phi_{td}} \overline{A}^{+-}) \\
&= 2\,|A_2|^2\, \mathrm{Im}\left[e^{2i(\phi_{td}+\phi_{ub})}\left(1 - \tfrac{|A_0|}{|A_2|} e^{i(\phi_2-\phi_0)}\right)\left(1 - \tfrac{|\overline{A}_0|}{|A_2|} e^{-i(\overline{\phi}_2-\overline{\phi}_0)}\right)\right], \\
K^{00} &= (\left|\overline{A}^{00}\right|^2 + |A^{00}|^2)/2, \\
C^{00} &= (\left|\overline{A}^{00}\right|^2 - |A^{00}|^2)/2, \\
S^{00} &= \mathrm{Im}(A^{*00} e^{2i\phi_{td}} \overline{A}^{00}) \\
&= 4\,|A_2|^2\, \mathrm{Im}\left[e^{2i(\phi_{td}+\phi_{ub})}\left(1 + \tfrac{1}{2}\tfrac{|A_0|}{|A_2|} e^{i(\phi_2-\phi_0)}\right)\left(1 + \tfrac{1}{2}\tfrac{|\overline{A}_0|}{|A_2|} e^{-i(\overline{\phi}_2-\overline{\phi}_0)}\right)\right],
\end{aligned} \tag{48}$$

where we have used eqs. (44) in obtaining the second forms for the coefficients S.

Assuming the Fourier analysis of the time-dependent neutral B-decays rates (47) can be performed, the coefficients K and C determine the magnitudes $|A^{+-}|$, $\left|\overline{A}^{+-}\right|$, $|A^{00}|$ and $\left|\overline{A}^{00}\right|$. From $\Gamma(B^\pm \to \pi^\pm\pi^0)$ we know $|A^{+0}| = \left|\overline{A}^{-0}\right| = |A_2|$. Thus the magnitudes of all six quantities in the constraint equations (45) are known. Interpreting these constraints as triangles in the complex plane as shown in fig. 2, we can calculate the phase differences $|\phi_2 - \phi^{+-}|$, $|\phi_2 - \phi^{00}|$, $\left|\overline{\phi}_2 - \overline{\phi}^{+-}\right|$ and $\left|\overline{\phi}_2 - \overline{\phi}^{00}\right|$ using the cosine law. Then using the second (or third) of eqs. (44) we can calculate $|A_0|$, $|\phi_2 - \phi_0|$, $\left|\overline{A}_0\right|$ and $\left|\overline{\phi}_2 - \overline{\phi}_0\right|$.

Thus we know the magnitudes of all quantities appearing in the expressions for S^{+-} and S^{00}, but there remains a fourfold ambiguity as to the phase, since only the absolute values

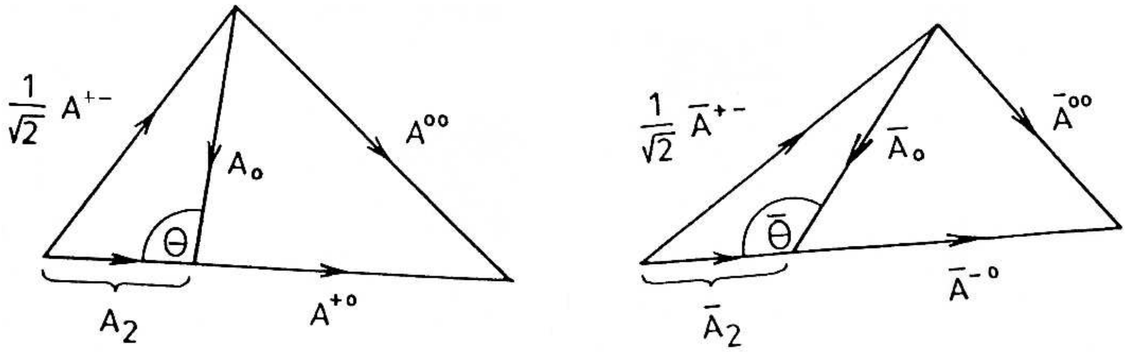

Figure 2: The isospin decomposition (44) of the $B \to \pi\pi$ decay amplitudes, and the constraint relations (45) are shown as triangles on the complex plane. An ambiguity remains as to the signs of the phase differences $\phi_2 - \phi_0$ and $\overline{\phi}_2 - \overline{\phi}_0$ as each triangle could be reflected about the A_2 or $\overline{A}_2$ axis.

of $\phi_2 - \phi_0$ and $\overline{\phi}_2 - \overline{\phi}_0$ have been determined. Therefore we can obtain two sets of four solutions for $\sin 2(\phi_{td} + \phi_{ub}) = \sin 2\varphi_2$. The true solution should be the only common value in both sets. In principle this method removes the uncertainty in the measurement of φ_2 due to penguin graphs.

In practice method 5 will be difficult to implement. The spectator graph for $B_d^0 \to \pi^0\pi^0$ is type-II, color-suppressed so the branching ratio may well be an order of magnitude smaller than that for $B_d^0 \to \pi^+\pi^-$. As method 5 depends heavily on reconstruction of B decays with final-state π^0's for which no secondary-vertex information will be available, it may be impossible to implement it at a hadron collider and it will be experimentally challenging at an e^+e^- collider. Searches for other final states than $\pi\pi$ for use with the isospin method have, however, not yielded any better candidate thus far [21, 22, 23].

7 Method 6: Angular Analysis of B Decays to Mixtures of CP Eigenstates

When applying method 4 to neutral B-mesons decays to CP eigenstates we cannot immediately use self-conjugate final states that consist of a pair of vector mesons (such as $D^\star\overline{D}^\star$), or of three or more mesons (such as $J/\psi K_S^0\pi^0$). Depending on whether the orbital angular momentum is even or odd the CP of the final state changes sign. If we know the fraction p of decays to the CP(even) final state we can write eq. (38) as

$$\begin{aligned} \Gamma(B^0(t) \to f) &\propto |A|^2\, e^{-t}[1 + (1-2p)\sin(xt)\sin 2(\phi_M + \phi_D)], \\ \Gamma(\overline{B}^0(t) \to f) &\propto |A|^2\, e^{-t}[1 - (1-2p)\sin(xt)\sin 2(\phi_M + \phi_D)], \end{aligned} \tag{49}$$

and a measurement of $\sin 2(\phi_M + \phi_D)$ can be made subject to the dilution factor $1 - 2p$. The fraction p can in general be determined by analysis of the angular distribution of the sequential decays of the final-state mesons, as discussed in detail in ref. [28] and references therein. Such an angular analysis will require sizable event samples, perhaps an order of magnitude larger than needed for method 4.

A simplified angular analysis will suffice if the final state consists of a vector meson plus two spinless mesons. When all three mesons are self conjugate (such as $J/\psi K_S^0\pi^0$), helicity-zero decays have definite CP and their abundance determined from a single angular distribution [27]. When the spin-0 mesons come from the decay of a spin-1 meson, and the two spin-1 mesons are each self conjugate (such as $D^{\star 0}\rho^0$ or $J/\psi\phi$), or the two vector mesons are antiparticles (such as $D^{\star +}D^{\star -}$) the so-called transversity analysis can be used to extract p [28].

Referring to Table 1 we see that the most interesting candidates for angular analysis are the decays $B_d^0 \to J/\psi K_S^0\pi^0$ and $D^{\star +}D^{\star -}$ from class 1, $B_d^0 \to \rho^+\rho^-$ and $\rho^0\rho^0$ from class 2, $B_s^0 \to \rho^0 K_S^0\pi^0$ from class 3, and $B_s^0 \to D_s^{\star +}D_s^{\star -}$ and $J/\psi\phi$ from class 4. It is notable that most of these decays require photon detection.

8 Appendix: Nonleptonic Decay Modes of the B Mesons

A survey of seven possible graphs describing B-meson decay indicates that the B_u will have 21 basic 2-body nonleptonic decays, the B_d will have 27, the B_s will have 29, and the B_c will have 21 (see Tables 2-5). This contrasts with the case for the K_u $(= K^+)$ and K_d $(= K^0)$ which each only have 2 such decays (not all distinct!). In the B system there are 24 basic decays to CP eigenstates compared to the 2 in the K system. All 98 of the basic two-body decays of the B-meson system have all-charged final states (at some price in secondary branching fraction), while only 1 of the basic K decays is all charged.

We have not displayed the catalog of decays of the B_c^+ $(= \overline{b}c)$, in which the charm quark decays before the b-quark, as is expected to happen in the majority of decays. Both the B_s and the B_c will be better studied at a hadron collider than at a low-energy e^+e^- collider.

The Tables refer to seven kinds of graphs, two spectator, annihilation, exchange, penguin/annihilation, and two penguin/spectator, as shown in Fig. 3. We can roughly estimate that for spectator graphs I:

CKM-favored decays have amplitudes $\propto \lambda^2$, and branching fractions of 10^{-2}-10^{-3};

CKM-suppressed decays have amplitudes $\propto \lambda^3$, and branching fractions of 3×10^{-4}-3×10^{-5};

CKM-doubly-suppressed decays have amplitudes $\propto \lambda^4$, and branching fractions of 10^{-5}-10^{-6}.

Graphs II, III and IV are 'color suppressed' in that only 1/3 of the quark pairs created by the W or gluons will have the proper color to match the other final-state quark pair, and so the rates are typically suppressed by a factor of 1/10 compared to graph I at the same order in λ.

The annihilation graph III and the exchange graph IV are controversial and both amy be heavily suppressed.

Graphs V-VII are 'penguins,' which have yet to be observed in the laboratory. This suggests that they are suppressed by a factor of order 0.01 compared to graphs I and II at the same order in λ. Graphs V and VII are color-suppressed compared to graph VI. The weak phase of the amplitude for a penguin graph is ϕ_{td} if the transition is $\overline{b} \to \overline{d}$ (CKM

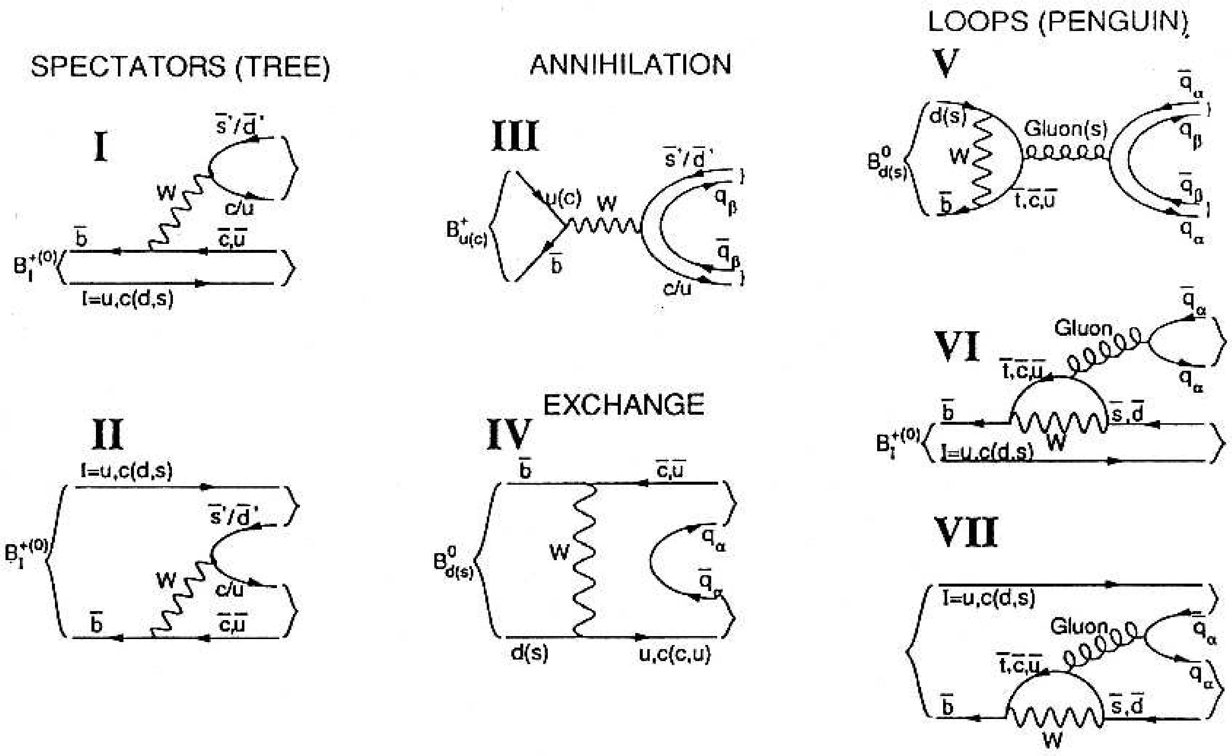

Figure 3: Seven graphs for the nonleptonic decays of B mesons.

suppressed), and 1 for $\bar{b} \to \bar{s}$, as discussed by London and Peccei [35].

The two-body final states listed in the Tables are representative of the particular $q\bar{q}/q\bar{q}$ combination for each entry. All final states could be augmented by $n(\pi^+\pi^-)$, with possibly larger branching fractions. Likewise, every spin-0 final-state particle could be replaced by its spin-1 partner, and *vice versa.* Typically the branch to the spin-1 meson will be 3 times that to the spin-0 partner.

The secondary decays used in constructing the last column of the Tables are:

Decay Mode	Branching Ratio
$K^0_S \to \pi^+\pi^-$	0.69
$\rho^0 \to \pi^+\pi^-$	1.00
$K^{*0} \to K^+\pi^-$	0.67
$\phi \to K^+K^-$	0.50
$D^+ \to K^-\pi^+\pi^+$	0.08
$D^0 \to K^-\pi^+$	0.04
$D^+_s \to \phi\pi^+$	0.03
$D^+_s \to \phi\pi^+\pi^+\pi^-$	0.04
$J/\psi \to e^+e^-$	0.07

For completeness will list the basic two-body nonleptonic decays of the D^+, D^+_s, and D^0 mesons in Tables 6-8.

Table 2: The 21 basic 2-body nonleptonic decays of the B_u^+ ($= \overline{b}u$). Figure 3 illustrates the seven types of graphs. The subscripts F, S, and D to the type of graph in this and following three tables refer to CKM-favored (ampli $\propto \lambda^2$), -suppressed (ampli $\propto \lambda^3$), and -doubly-suppressed (ampli $\propto \lambda^4$), respectively. If the decay amplitude depends on a CP-violating phase, the relevant phase of a CKM matrix element is indicated in parentheses. The decay modes are listed roughly in order of decreasing branching fraction.

Graph	Final Quarks	Final State	All-Charged Daughters
I_F, II_F	$u\overline{c}/u\overline{d}$	$\overline{D}^0\pi^+$	$K^+\pi^-\pi^+$
I_F, $\mathrm{III}_D(\phi_{ub})$, VII_F	$c\overline{s}/u\overline{c}$	$D_s^+\overline{D}^0$	$K^+K^-\pi^+K^+\pi^-$
II_F, VI_F	$c\overline{c}/u\overline{s}$	$J/\psi K^+$	$e^+e^-K^+$
I_S	$c\overline{s}/u\overline{u}$	$D_s^+\rho^0$	$K^+K^-\pi^+\pi^+\pi^-$
I_S, II_S	$u\overline{c}/u\overline{s}$	$\overline{D}^0K^+$	$K^+\pi^-K^+$
I_S, $\mathrm{III}_S(\phi_{ub})$, $\mathrm{VII}_S(\phi_{td})$	$c\overline{d}/u\overline{c}$	$D^+\overline{D}^0$	$K^-\pi^+\pi^+K^+\pi^-$
$\mathrm{I}_S(\phi_{ub})$, $\mathrm{II}_S(\phi_{ub})$, $\mathrm{III}_S(\phi_{ub})$, VI_F, VII_F	$u\overline{u}/u\overline{d}$	$\rho^0\pi^+$	$\pi^+\pi^-\pi^+$
II_S, $\mathrm{VI}_S(\phi_{td})$	$c\overline{c}/u\overline{d}$	$J/\psi\pi^+$	$e^+e^-\pi^+$
$\mathrm{II}_S(\phi_{ub})$, $\mathrm{III}_S(\phi_{ub})$	$c\overline{u}/u\overline{s}$	D^0K^+	$K^-\pi^+K^+$
$\mathrm{I}_D(\phi_{ub})$, $\mathrm{II}_D(\phi_{ub})$, $\mathrm{III}_D(\phi_{ub})$, VI_F, VII_F	$u\overline{u}/u\overline{s}$	ρ^0K^+	$\pi^+\pi^-K^+$
$\mathrm{I}_D(\phi_{ub})$, $\mathrm{III}_D(\phi_{ub})$	$c\overline{d}/u\overline{u}$	$D^+\rho^0$	$K^-\pi^+\pi^+\pi^+\pi^-$
$\mathrm{II}_D(\phi_{ub})$, $\mathrm{III}_D(\phi_{ub})$	$c\overline{u}/u\overline{d}$	$D^0\pi^+$	$K^-\pi^+\pi^+$
$\mathrm{III}_S(\phi_{ub})$, $\mathrm{VII}_S(\phi_{td})$	$u\overline{s}/s\overline{d}$	$K^+\overline{K}^{*0}$	$K^+K^-\pi^+$
$\mathrm{III}_S(\phi_{ub})$	$c\overline{d}/d\overline{s}$	D^+K^{*0}	$K^-\pi^+\pi^+K^+\pi^-$
$\mathrm{III}_S(\phi_{ub})$	$c\overline{s}/s\overline{s}$	$D_s^+\phi$	$K^+K^-\pi^+K^+K^-$
$\mathrm{III}_S(\phi_{ub})$	$c\overline{c}/c\overline{s}$	$J/\psi D_s^+$	$e^+e^-K^+K^-\pi^+$
$\mathrm{III}_D(\phi_{ub})$, VII_F	$d\overline{s}/u\overline{d}$	$K^{*0}\pi^+$	$K^+\pi^-\pi^+$
$\mathrm{III}_D(\phi_{ub})$, VI_F, VII_F	$s\overline{s}/u\overline{s}$	ϕK^+	$K^+K^-K^+$
$\mathrm{III}_D(\phi_{ub})$	$c\overline{s}/s\overline{d}$	$D_s^+\overline{K}^{*0}$	$K^+K^-\pi^+K^-\pi^+$
$\mathrm{III}_D(\phi_{ub})$	$c\overline{c}/c\overline{d}$	$J/\psi D^+$	$e^+e^-K^-\pi^+\pi^+$
$\mathrm{VI}_S(\phi_{td})$	$s\overline{s}/u\overline{d}$	$\phi\pi^+$	$K^+K^-\pi^+$

Table 3: The 27 basic 2-body nonleptonic decays of the B_d^0 $(= \overline{b}d)$. The numbers in the 'CP Eigenstate' column refer to the classification described in sec. 1.3 regarding the relevant CKM phases governing the decay asymmetries. Graphs leading to CP eigenstates includes a phase factor in ϕ_{td} due to mixing. However, in penguin graphs with $\overline{b} \to \overline{d}$ transitions to CP eigenstates, the two phase factors in ϕ_{td} cancel.

Graph	Final Quarks	Final State	CP Eigenstate	All-Charged Daughters
I_F, IV_F	$d\overline{c}/u\overline{d}$	$D^-\pi^+$		$K^+\pi^-\pi^-\pi^+$
I_F, VII_F	$c\overline{s}/d\overline{c}$	$D_s^+D^-$		$K^+K^-\pi^+K^+\pi^-\pi^-$
II_F, IV_F	$u\overline{c}/d\overline{d}$	$\overline{D}^0\rho^0$		$K^+\pi^-\pi^+\pi^-$
$\mathrm{II}_F(\phi_{td})$, $\mathrm{VI}_F(\phi_{td})$	$c\overline{c}/d\overline{s}$	$J/\psi K_S^0$	1	$e^+e^-\pi^+\pi^-$
I_S	$d\overline{c}/u\overline{s}$	D^-K^+		$K^+\pi^-\pi^-K^+$
$\mathrm{I}_S(\phi_{td})$, $\mathrm{IV}_S(\phi_{td})$, V_S, VII_S	$c\overline{d}/d\overline{c}$	D^+D^-	1, 4	$K^-\pi^+\pi^+K^+\pi^-\pi^-$
$\mathrm{I}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VII_S	$u\overline{d}/d\overline{u}$	$\pi^+\pi^-$	2, 4	$\pi^+\pi^-$
$\mathrm{I}_S(\phi_{ub})$	$c\overline{s}/d\overline{u}$	$D_s^+\pi^-$		$K^+K^-\pi^+\pi^-$
II_S	$u\overline{c}/d\overline{s}$	$\overline{D}^0K^{\star 0}$		$K^+\pi^-K^+\pi^-$
$\mathrm{II}_S(\phi_{ub})$	$c\overline{u}/d\overline{s}$	$D^0K^{\star 0}$		$K^-\pi^+K^+\pi^-$
$\mathrm{II}_S(\phi_{td})$, VI_S	$c\overline{c}/d\overline{d}$	$J/\psi\rho^0$	1, 4	$e^+e^-\pi^+\pi^-$
$\mathrm{II}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VI_S, VII_S	$u\overline{u}/d\overline{d}$	$\rho^0\rho^0$	2, 4	$\pi^+\pi^-\pi^+\pi^-$
$\mathrm{I}_D(\phi_{ub})$, VII_F	$u\overline{s}/d\overline{u}$	$K^+\pi^-$		$K^+\pi^-$
$\mathrm{I}_D(\phi_{ub})$, $\mathrm{IV}_D(\phi_{ub})$	$c\overline{d}/d\overline{u}$	$D^+\pi^-$		$K^-\pi^+\pi^+\pi^-$
$\mathrm{II}_D(\phi_{td}+\phi_{ub})$, $\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$	$u\overline{u}/d\overline{s}$	$\rho^0K_S^0$	2, 1	$\pi^+\pi^-\pi^+\pi^-$
$\mathrm{II}_D(\phi_{ub})$, $\mathrm{IV}_D(\phi_{ub})$	$c\overline{u}/d\overline{d}$	$D^0\rho^0$		$K^-\pi^+\pi^+\pi^-$
IV_F	$s\overline{c}/u\overline{s}$	$D_s^-K^+$		$K^+K^-\pi^-K^+$
IV_F	$c\overline{c}/u\overline{c}$	$J/\psi\overline{D}^0$		$e^+e^-K^+\pi^-$
$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S	$c\overline{u}/u\overline{c}$	$D^0\overline{D}^0$	2, 4	$K^-\pi^+K^+\pi^-$
$\mathrm{IV}_S(\phi_{td})$, V_S	$c\overline{s}/s\overline{c}$	$D_s^+D_s^-$	1, 4	$K^+K^-\pi^+K^+K^-\pi^-$
$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S	$u\overline{s}/s\overline{u}$	K^+K^-	2, 4	K^+K^-
$\mathrm{IV}_D(\phi_{ub})$	$c\overline{s}/s\overline{u}$	$D_s^+K^-$		$K^+K^-\pi^+K^-$
$\mathrm{IV}_D(\phi_{ub})$	$c\overline{c}/c\overline{u}$	$J/\psi D^0$		$e^+e^-K^-\pi^+$
$\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$	$s\overline{s}/d\overline{s}$	ϕK_S^0	1	$K^+K^-\pi^+\pi^-$
V_S	$s\overline{s}/s\overline{s}$	$\phi\phi$	4	$K^+K^-K^+K^-$
VI_S	$s\overline{s}/d\overline{d}$	$\phi\rho^0$	4	$K^+K^-\pi^+\pi^-$
V_S, VII_S	$s\overline{d}/d\overline{s}$	$\overline{K}^{\star 0}K^{\star 0}$	4	$K^-\pi^+K^+\pi^-$

Table 4: The 29 basic 2-body nonleptonic decays of the B_s^0 $(=\overline{b}s)$.

Graph	Final Quarks	Final State	CP Eigenstate	All-Charged Daughters
I_F	$s\overline{c}/u\overline{d}$	$D_s^-\pi^+$		$K^+K^-\pi^-\pi^+$
I_F, IV_F, V_F, VII_F	$c\overline{s}/s\overline{c}$	$D_s^+D_s^-$	4	$K^+K^-\pi^+K^+K^-\pi^-$
II_F	$u\overline{c}/s\overline{d}$	$\overline{D}^0\overline{K}^{\star 0}$		$K^+\pi^-K^-\pi^+$
II_F, VI_F	$c\overline{c}/s\overline{s}$	$J/\psi\phi$	4	$e^+e^-K^+K^-$
I_S, IV_S	$s\overline{c}/u\overline{s}$	$D_s^-K^+$		$K^+K^-\pi^-K^+$
I_S, $\mathrm{VII}_S(\phi_{td})$	$s\overline{c}/c\overline{d}$	$D_s^-D^+$		$K^+K^-\pi^-K^-\pi^+\pi^+$
I_S, $\mathrm{VII}_S(\phi_{td})$	$s\overline{u}/u\overline{d}$	$K^-\pi^+$		$K^-\pi^+$
$\mathrm{I}_S(\phi_{ub})$, $\mathrm{IV}_S(\phi_{ub})$	$c\overline{s}/s\overline{u}$	$D_s^+K^-$		$K^+K^-\pi^+K^-$
II_S	$u\overline{c}/s\overline{s}$	$\overline{D}^0\phi$		$K^+\pi^-K^+K^-$
$\mathrm{II}_S(\phi_{ub})$	$c\overline{u}/s\overline{s}$	$D^0\phi$		$K^-\pi^+K^+K^-$
II_S, $\mathrm{VI}_S(\phi_{td})$	$c\overline{c}/s\overline{d}$	$J/\psi K_S^0$	4, 1	$e^+e^-\pi^+\pi^-$
$\mathrm{II}_S(\phi_{ub})$, $\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$	$u\overline{u}/s\overline{d}$	$\rho^0K_S^0$	3, 1	$\pi^+\pi^-\pi^+\pi^-$
$\mathrm{I}_D(\phi_{ub})$, $\mathrm{IV}_D(\phi_{ub})$, V_F, VII_F	$u\overline{s}/s\overline{u}$	K^+K^-	3, 4	K^+K^-
$\mathrm{I}_D(\phi_{ub})$	$c\overline{d}/s\overline{u}$	D^+K^-		$K^-\pi^+\pi^+K^-$
$\mathrm{II}_D(\phi_{ub})$, VI_F	$s\overline{s}/u\overline{u}$	$\phi\rho^0$	3, 4	$K^+K^-\pi^+\pi^-$
$\mathrm{II}_D(\phi_{ub})$	$c\overline{u}/s\overline{d}$	$D^0\overline{K}^{\star 0}$		$K^-\pi^+K^-\pi^+$
IV_F, $\mathrm{IV}_D(\phi_{ub})$ V_F, V_S	$c\overline{u}/u\overline{c}$	$D^0\overline{D}^0$	4, 3	$K^-\pi^+K^+\pi^-$
IV_F, V_F	$c\overline{d}/d\overline{c}$	D^+D^-	4	$K^-\pi^+\pi^+K^+\pi^-\pi^-$
IV_S	$d\overline{c}/u\overline{d}$	$D^-\pi^+$		$K^+\pi^-\pi^-\pi^+$
IV_S	$u\overline{c}/u\overline{u}$	$\overline{D}^0\rho^0$		$K^+\pi^-\pi^+\pi^-$
IV_S	$c\overline{c}/u\overline{c}$	$J/\psi\overline{D}^0$		$e^+e^-K^+\pi^-$
$\mathrm{IV}_S(\phi_{ub})$	$c\overline{d}/d\overline{u}$	$D^+\pi^-$		$K^-\pi^+\pi^+\pi^-$
$\mathrm{IV}_S(\phi_{ub})$	$c\overline{u}/u\overline{u}$	$D^0\rho^0$		$K^-\pi^+\pi^+\pi^-$
$\mathrm{IV}_S(\phi_{ub})$	$c\overline{c}/c\overline{u}$	$J/\psi D^0$		$e^+e^-K^-\pi^+$
$\mathrm{IV}_D(\phi_{ub})$, V_F	$u\overline{d}/d\overline{u}$	$\pi^+\pi^-$	3, 4	$\pi^+\pi^-$
$\mathrm{IV}_D(\phi_{ub})$, V_F	$u\overline{u}/u\overline{u}$	$\rho^0\rho^0$	3, 4	$\pi^+\pi^-\pi^+\pi^-$
V_F, VII_F	$s\overline{s}/s\overline{s}$	$\phi\phi$	4	$K^+K^-K^+K^-$
V_F, VII_F	$d\overline{s}/s\overline{d}$	$K^{\star 0}\overline{K}^{\star 0}$	4	$K^+\pi^-K^-\pi^+-$
$\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$	$s\overline{s}/s\overline{d}$	ϕK_S^0	1	$K^+K^-\pi^+\pi^-$

Table 5: The 21 basic 2-body nonleptonic decays of the B_c^+ ($=\overline{b}c$) in which the $\overline{b}$-quark decays before the c quark.

Graph	Final Quarks	Final State	All-Charged Daughters
I_F	$c\overline{c}/u\overline{d}$	$J/\psi\pi^+$	$e^+e^-\pi^+$
I_F, II_F, III_F, VI_F, VII_F	$c\overline{c}/c\overline{s}$	$J/\psi D_s^+$	$e^+e^-K^+K^-\pi^+$
II_F, III_F	$c\overline{d}/u\overline{c}$	$D^+\overline{D}^0$	$K^-\pi^+\pi^+K^+\pi^-$
I_S	$c\overline{c}/u\overline{s}$	$J/\psi K^+$	$e^+e^-K^+$
I_S, II_S, III_S, $\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$	$c\overline{c}/c\overline{d}$	$J/\psi D^+$	$e^+e^-K^-\pi^+\pi^+$
$\mathrm{I}_S(\phi_{ub})$, III_S, $\mathrm{VII}_S(\phi_{td})$	$c\overline{u}/u\overline{d}$	$D^0\pi^+$	$K^-\pi^+\pi^+$
$\mathrm{I}_S(\phi_{ub})$, $\mathrm{II}_S(\phi_{ub})$	$c\overline{s}/c\overline{u}$	$D_s^+D^0$	$K^+K^-\pi^+K^-\pi^+$
II_S, III_S	$c\overline{s}/u\overline{c}$	$D_s^+\overline{D}^0$	$K^+\pi^-K^+K^-\pi^+$
$\mathrm{II}_S(\phi_{ub})$, III_S, $\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$	$c\overline{d}/u\overline{u}$	$D^+\rho^0$	$\pi^+\pi^-K^-\pi^+\pi^+$
$\mathrm{I}_D(\phi_{ub})$, III_F, VII_F	$c\overline{u}/u\overline{s}$	D^0K^+	$K^-\pi^+K^+$
$\mathrm{I}_D(\phi_{ub})$, $\mathrm{II}_D(\phi_{ub})$	$c\overline{d}/c\overline{u}$	D^+D^0	$K^-\pi^+\pi^+K^-\pi^+$
$\mathrm{II}_D(\phi_{ub})$, VI_F	$c\overline{s}/u\overline{u}$	$D_s^+\rho^0$	$K^+K^-\pi^+\pi^+\pi^-$
III_F	$u\overline{u}/u\overline{d}$	$\rho^0\pi^+$	$\pi^+\pi^-\pi^+$
III_F	$u\overline{s}/s\overline{d}$	$K^+K_S^0$	$K^+\pi^+\pi^-$
III_F, VII_F	$c\overline{d}/d\overline{s}$	$D^+K_S^0$	$K^-\pi^+\pi^+\pi^+\pi^-$
III_F, VI_F, VII_F	$c\overline{s}/s\overline{s}$	$D_s^+\phi$	$K^+K^-\pi^+K^+K^-$
III_S	$u\overline{u}/u\overline{s}$	ρ^0K^+	$\pi^+\pi^-K^+$
III_S	$d\overline{s}/u\overline{d}$	$K_S^0\pi^+$	$\pi^+\pi^-\pi^+$
III_S	$s\overline{s}/u\overline{s}$	ϕK^+	$K^+K^-K^+$
III_S, $\mathrm{VII}_S(\phi_{td})$	$c\overline{s}/s\overline{d}$	$D_s^+K_S^0$	$K^+K^-\pi^+\pi^+\pi^-$
$\mathrm{VI}_S(\phi_{td})$	$c\overline{d}/s\overline{s}$	$D^+\phi$	$K^-\pi^+\pi^+K^+K^-$

Table 6: The 7 basic 2-body nonleptonic decays of the D^+ ($= c\overline{d}$). In this and the following two tables penguin contributions are ignored.

Graph	Final Quarks	Final State	All-Charged Daughters
I_F, II_F	$s\overline{d}/u\overline{d}$	$\overline{K}^{\star 0}\pi^+$	$K^-\pi^+\pi^+$
I_S, III_S	$s\overline{d}/u\overline{s}$	$\overline{K}^{\star 0}K^+$	$K^-\pi^+K^+$
I_S, II_S, III_S	$d\overline{d}/u\overline{d}$	$\rho^0\pi^+$	$\pi^+\pi^-\pi^+$
II_S	$s\overline{s}/u\overline{d}$	$\phi\pi^+$	$K^+K^-\pi^+$
I_D, III_D	$d\overline{d}/u\overline{s}$	ρ^0K^+	$\pi^+\pi^-K^+$
II_D, III_D	$d\overline{s}/u\overline{d}$	$K^{\star 0}\pi^+$	$K^+\pi^-\pi^+$
III_D	$s\overline{s}/u\overline{s}$	ϕK^+	$K^+K^-K^+$

Table 7: The 6 basic 2-body nonleptonic decays of the D_s^+ ($= c\overline{s}$).

Graph	Final Quarks	Final State	All-Charged Daughters
I_F	$s\overline{s}/u\overline{d}$	$\phi\pi^+$	$K^+K^-\pi^+$
II_F, III_F	$s\overline{d}/u\overline{s}$	$\overline{K}^{\star 0}K^+$	$K^-\pi^+K^+$
I_S, II_S, III_S	$s\overline{s}/u\overline{s}$	ϕK^+	$K^+K^-K^+$
I_S, III_S	$d\overline{s}/u\overline{d}$	$K^{\star 0}\pi^+$	$K^+\pi^-\pi^+$
II_S, III_S	$d\overline{d}/u\overline{s}$	ρ^0K^+	$\pi^+\pi^-K^+$
III_F	$u\overline{u}/u\overline{d}$	$\rho^0\pi^+$	$\pi^+\pi^-\pi^+$

Table 8: The 11 basic 2-body nonleptonic decays of the D^0 $(= c\overline{u})$.

Graph	Final Quarks	Final State	All-Charged Daughters
I_F, IV_F	$s\overline{u}/u\overline{d}$	$K^-\pi^+$	$K^-\pi^+$
II_F	$s\overline{s}/u\overline{u}$	$\phi\rho^0$	$K^+K^-\pi^+\pi^-$
I_S, IV_S	$u\overline{d}/d\overline{u}$	$\pi^+\pi^-$	$\pi^+\pi^-$
I_S, IV_S	$u\overline{s}/s\overline{u}$	K^+K^-	K^+K^-
II_S, IV_S	$s\overline{d}/u\overline{u}$	$\overline{K}^{\star 0}\rho^0$	$K^-\pi^+\pi^+\pi^-$
II_S, IV_S	$d\overline{s}/u\overline{u}$	$K^{\star 0}\rho^0$	$K^+\pi^-\pi^+\pi^-$
II_S, IV_S	$d\overline{u}/u\overline{d}$	$\rho^0\rho^0$	$\pi^+\pi^-\pi^+\pi^-$
I_D, IV_D	$u\overline{s}/d\overline{u}$	$K^+\pi^-$	$K^+\pi^-$
IV_F	$s\overline{s}/s\overline{d}$	$\phi\overline{K}^{\star 0}$	$K^+K^-K^-\pi^+$
IV_S	$d\overline{s}/s\overline{d}$	$K^{\star 0}\overline{K}^{\star 0}$	$K^+\pi^-K^-\pi^+$
IV_D	$s\overline{s}/d\overline{s}$	$\phi K^{\star 0}$	$K^+K^-K^+\pi^-$

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