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Choosing Radiators for the BCD RICH Counters

These remarks follow the thinking of the notes of Tom Ypsilantis from Snowmass '88.

Gas Radiator

Tom emphasizes that the ultimate limit to performance of a RICH counter is the dispersion in Čerenkov angle due to the variation of the index of refraction with photon energy (chromatic aberration). He claims that for many interesting gases

$$\frac{\Delta\theta_C}{\theta_C} \sim 5 \times 10^{-3}.$$

The Čerenkov angle is given by

$$\sin^2\theta_C = \frac{1}{\beta^2}\left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right), \qquad \text{so} \qquad \theta_C \approx \frac{1}{\gamma_t}$$

for large γ. The minimum value of γ_t for any practical gas is 17, achieved by C_5F_{12}. If we consider a RICH counter of, say, 100 cm in length, the chromatic aberration implies the Čerenkov ring has a spatial extent of

$$\sigma_r \approx 100\Delta\theta_C < 100\frac{5\times 10^{-3}}{17} = .03 \text{ cm}.$$

This is likely smaller than the spatial resolution of an affordable readout of the RICH counters. Hence it is useful to display the limits of RICH-counter performance set by detector resolution (rather than by chromatic aberration as Ypsilantis does).

For a counter whose light is focused by a mirror of focal L (usually just the length of the counter itself), the radius of the Čerenkov ring of a particle of mass M is

$$r = L\tan\theta_C \approx \frac{L}{\gamma_t}\sqrt{1 - \frac{\gamma_t^2}{\gamma^2}} \approx \frac{L}{\gamma_t} - \frac{L\gamma_t^2}{2\gamma^2}.$$

Noting that $1/\gamma = M/P$, we have

$$r_\pi - r_K = L\gamma_t\frac{M_K^2 - M_\pi^2}{2P^2}.$$

If the position of each photoelectron is measured to accuracy σ_r, and N photoelectrons are observed, then the number of standard deviations S between the Čerenkov rings for pions and Kaons of momentum P is

$$S = \frac{r_\pi - r_K}{\sqrt{N}\sigma_r}.$$

The number of photoelectrons N is given by the expression

$$N = \epsilon \frac{\alpha}{\hbar c} \sin^2 \theta_C L dE \approx 370[\mathrm{cm}^{-1}\mathrm{eV}^{-1}] \frac{\epsilon L dE}{\gamma_t^2},$$

where ϵ is the efficiency for collecting the light and converting it to electrons, and dE is the range of photon energies collected. With $\epsilon \sim 0.3$ and $dE \sim 1$ eV we have

$$N \approx \frac{N_0 L}{\gamma_t^2},$$

where $N_0 \sim 100$/cm. Using this last relation for N, the momentum at which pions and Kaons can be separated to S standard deviations is

$$P[\mathrm{GeV}/c] \approx M_K \frac{N_0^{1/4} L^{3/4}}{\sqrt{2S\sigma_r}} \approx 2(L[\mathrm{cm}])^{3/4},$$

for $N_0 = 100$/cm, $S = 3$ standard deviations, and $\sigma_r = 1$ mm. Thus if we wish to build a RICH counter of a fixed length L, the same π-K separation can, in principle, be obtained with any gas! It seems wise to have a large number N of photoelectrons, so we should pick the gas with the minimum γ_t such that pions produce detectable signals over the entire relevant momentum range.

However, if we wish to design the counter to provide a given number of photoelectrons, we can replace L by its dependence of N in the above to find

$$P \approx M_K \frac{\gamma_t^{3/2} N^{3/4}}{\sqrt{2SN_0\sigma_r}}.$$

In this case it is advantageous to use a gas with large γ_t, but the needed length L to generate the N photoelectrons may become quite large.

From the above, we see that a counter of $L = 100$ cm could perform π-K separation (at the 3-σ level) up to $P \approx 63$ GeV/c, which is well matched to our requirements in the Intermediate region. With $L = 400$ cm we have separation up to $P = 180$ GeV/c, which is perhaps a bit low for the Forward region. With

$L = 600$ cm, we would have separation up to $P = 240$ Gev/c, a better match. Hence I concur with Jorge that **we should lengthen the Forward RICH to 6 m.** If we desire $N = 25$ photoelectrons then we need a gas with $\gamma_t = \sqrt{N_0 L/N} = 2\sqrt{L}$. Hence in the Intermediate region we need $\gamma_t = 20$, so we could use C_5F_{12} or C_4F_{10}. In the Forward region we need $\gamma_t = 49$, and we could use argon, nitrogen, or CF_4. The latter has 1/3 the chromatic aberration of the first two, as well as a lower γ_t, so is probably the best choice.

Solid Radiator

In the Central region of the BCD the minimum pion momentum of interest is too small to be detected by a gas RICH counter. Suppose instead we use a solid (or liquid, or aerogel) radiator of thickness T and index of refraction n, followed by a drift of distance L.

We consider the use of 'proximity focusing,' *i.e.*, no focusing at all. Then the Čerenkov ring has radius

$$r = L \tan\theta,$$

where

$$\sin\theta = n \sin\theta_C$$

for normal incidence, according to Snell's law. If $n \sin\theta_C > 1$ then the light will not come out of the radiator!

We consider media with index n significantly larger than 1, and particles with $\gamma \gg 1$. In this regime

$$\cos\theta_C = \frac{1}{n\beta} \to \frac{1}{n},$$

$$\sin\theta_C = \sqrt{1 - \left(\frac{1}{n\beta}\right)^2} \to \frac{\sqrt{n^2 - 1}}{n},$$

$$\tan\theta_C = \sqrt{n^2\beta^2 - 1} \to \sqrt{n^2 - 1}.$$

From this we deduce that total internal reflection will occur inside the radiator if $\sqrt{n^2 - 1} > 1$, or $n > \sqrt{2}$.

A UV-transparent solid with a low index is NaF ($n = 1.38$ at 6.5 eV, R. Arnold *et al.*, N.I.M. **A273**, 466 (1988). The chromatic aberration of NaF is fairly large, and Ypsilantis concludes that it can't do π-K separation much above 3 GeV/c. A UV-transparent liquid is C_6F_{14} ($n = 1.278$). Its chromatic aberration is somewhat better, and it is used by SLD and Delphi.

Again we estimate the significance of π-K separation,

$$S = \frac{r_\pi - r_K}{\sigma_r \sqrt{N}},$$

supposing that spatial resolution is more important than chromatic aberration. For a proximity 'focussed' device, the effective spatial resolution is set by the finite width of the unfocussed Čerenkov ring.

The full radial extent of the Čerenkov ring is $T \tan\theta_C$ (not $T \tan\theta$), so the r.m.s. spread in the radii of the photoelectrons is

$$\sigma_r = \frac{T \tan\theta_C}{\sqrt{12}}.$$

The number of photoelectrons observed is

$$N = N_0 T \sin^2\theta_C.$$

We need to expand $\tan\theta$ to first order using $\beta^2 = P^2/E^2 \approx 1 - M^2/P^2$. Some steps are

$$\tan\theta = \frac{n \sin\theta_C}{\sqrt{1 - n^2 \sin^2\theta_C}} = \sqrt{\frac{n^2\beta^2 - 1}{1 - \beta^2(n^2 - 1)}}$$
$$\approx \sqrt{\frac{n^2 - 1}{2 - n^2}} \left(1 - \frac{M^2}{2P^2} \frac{1}{(n^2 - 1)(2 - n^2)}\right).$$

The difference in radii of the Čerenkov rings from pions and Kaons of the same momentum is

$$r_\pi - r_K \approx L \frac{M_K^2 - M_\pi^2}{2P^2} \sqrt{\frac{n^2 - 1}{2 - n^2}} \frac{1}{(n^2 - 1)(2 - n^2)}.$$

The number of standard deviations S to which this separation is determined by the observation of N photoelectron is

$$S = \frac{r_\pi - r_K}{\sigma_r/\sqrt{N}} \approx \frac{\sqrt{12} N_0 L}{2n^2(2 - n^2)^{3/2}\sqrt{N}} \frac{M_K^2 - M_\pi^2}{P^2},$$

where we have noted that the thickness T of the radiator can be written

$$T = \frac{N}{N_0 \sin^2\theta_C} \approx \frac{N}{N_0} \frac{n^2}{n^2 - 1}.$$

The momentum at which an S-σ separation of pions and Kaons can be made is then

$$P[\mathrm{GeV}/c] \approx M_K \sqrt{\frac{\sqrt{12} N_0 L}{2Sn^2(2 - n^2)^{3/4}\sqrt{N}}} \approx \sqrt{\frac{1.5L[\mathrm{cm}]}{n^2(2 - n^2)^{3/2}}},$$

for a 3-σ separation, $N = 25$ photoelectrons, and $N_0 \sim 50$/cm as reported by Delphi in their liquid RICH.

For C_6F_{14} with $n = 1.27$, for which the effective Čerenkov angle is 52°, and we allocate $L = 20$ cm, then we have good π-K separation up to $P = 9$ GeV/c. This would be quite adequate for the BCD. The radiator thickness should be 1 cm to provide the 25 photoelectrons. The Čerenkov intensity has reached 80% of maximum for 250 MeV/c pions, and the r.m.s. width of the Čerenkov rings would be 3 mm.

For comparison, we make a similar derivation for the effect of chromatic aberration. From $\cos\theta_C = 1/n$ we find

$$d\theta_C = \frac{dn}{n^2 \sin\theta_C} = \frac{dn}{n\sqrt{n^2 - 1}}.$$

The index for C_6F_{14} varies as

$$n = 1.2733 + 9.3 \times 10^{-3}(E[\mathrm{eV}] - 6),$$

and the useful photon-energy interval is 1 eV, so $dn \approx 10^{-2}$ and $d\theta_C \approx 0.01$. Then $\sigma_{\theta_C} = d\theta/\sqrt{12} \approx 0.003$. This leads to an uncertainty in the radius $r = L\tan\theta$ of the Čerenkov ring given by

$$\sigma_r = \frac{L\sigma_\theta}{\cos^2\theta} = \frac{nL\cos\theta_C\sigma_{\theta_C}}{\cos^3\theta} = \frac{L\sigma_{\theta_C}}{(2 - n^2)^{3/2}} \approx 0.012L$$

for C_6F_{14}. We can use the expression for σ_r from chromatic aberration to deduce the maximum momentum for π-K separation:

$$P = M_K\sqrt{\frac{\sqrt{N}}{2S\sqrt{n^2 - 1}\sigma_{\theta_C}}} \approx 9.5 \text{ GeV}/c,$$

for $S = 3$ standard deviations. This is very comparable to the limit due to the use of proximity focussing for $L = 25$ cm. Probably we should use $L = 50$ cm, in which case we could hope for π-K separation up to 9 GeV/c.

Comments added 8/15/90:

The above argument is for normally incident particles. When the angle of incidence i is big enough, not all the Čerenkov light will come out of the radiator, some being trapped by internal reflection. This starts happening when

$$\sin(i + C) = \frac{1}{n},$$

writing C for the Čerenkov angle. This can be rewritten as

$$i = \sin^{-1} \frac{1}{n} - \cos^{-1} \frac{1}{n}.$$

For NaF with $n = 1.38$ we have $i = 2.8°$, and for C_6F_{14} with $n = 1.27$ we have $i = 14°$.

Perhaps of greater interest is the angle of incidence i at which only one half of the Čerenkov cone emerges. After some geometry we find this condition is

$$i = \sin^{-1} \sqrt{2 - n^2}.$$

For NaF this implies $i = 18°$ and for C_6F_{14} we have $i = 38°$. Note that when $i > 0$ the path length in the radiator is longer, so additional Čerenkov light is emitted, according to $1/\cos i$. This doesn't help NaF much, but is significant for C_6F_{14}. The geometry is complicated, and I don't have an simple analytic expression – but it can readily be calculated on an computer.

Similarly, the expression for the significance of π/K separation is actually a function of the angle of incidence, and should be calculated in the future.