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# THE RELATION BETWEEN ELECTRODISINTEGRATION AND PHOTODISINTEGRATION OF HELIUM-3*

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In this paper we investigate the possibility of extracting information about photodisintegration of Helium-3, $\gamma + He^3 \to p + d$, from the reaction $e + He^3 \to e' + p + d$. If the latter reaction involves the exchange of a single photon, we may consider the reaction to take place in two steps: $e \to e' + \gamma$ followed by $\gamma + He^3 \to p + d$. The second step is called virtual photodisintegration since the photon is off the mass-shell. Hence an extrapolation to real photodisintegration is necessary.

If the final state electron and proton are detected in the reaction $e + He^3 \to e' + p + d$, we may write (in the one-photon approximation):

$$\frac{d^3\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_p} = \Gamma \frac{d\sigma}{d\Omega_p} .$$

$\Gamma$ is the virtual photon spectrum factor given by

$$\Gamma = \frac{\alpha}{2\pi^2}\frac{E_{e'}}{E_e}\,\frac{K}{q^2}\,\frac{1}{1-\varepsilon} ,$$

where

$$q^2 = 4E_e\, E_{e'} \sin^2\frac{\theta_{e'}}{2} = -\,(\text{4-momentum transfer})^2 ,$$

$$K = \frac{W^2 - M^2_{He^3}}{2M_{He^3}} = \text{Real photon energy needed to produce the p+d final state,}$$

$$W = (M^2_{He^3} + 2\nu\, M_{He^3} - q^2)^{1/2} = \text{Mass of p+d system,}$$

$$\nu = E_e - E_{e'} ,$$

$$\varepsilon = 1/(1 + 2(1 + \nu^2/q^2)\tan^2\frac{\theta_{e'}}{2}) = \text{Polarization parameter.}$$

The virtual photodisintegration cross-section can be further expressed

$$\frac{d\sigma}{d\Omega_p} = A + \varepsilon B \sin^2\theta_p \cos 2\phi + \varepsilon C + \sqrt{\varepsilon(1+\varepsilon)}\, D \sin\theta_p \cos\phi \quad ,$$

where

$\theta_p$ = Proton angle in the $\gamma$-$He^3$ c.m. frame.

$\phi$ = Angle between the plane $e \to e' + \gamma$ and the plane $\gamma + He^3 \to p + d$ .

A, B, C, and D are functions of $q^2$, W and $\theta_p$. See the Appendix for comments on the derivation of the above expressions. In the limit of $q^2 \to 0$

$$A \to \frac{d\sigma_\parallel}{d\Omega_p} + \frac{d\sigma_\perp}{d\Omega_p}$$

$$B \to \frac{d\Omega_\parallel}{d\Omega_p} - \frac{d\Omega_\perp}{d\Omega_p}$$

$$C \to 0$$

$$D \to 0 \quad ,$$

where $\frac{d\sigma_\perp}{d\Omega_p}$ and $\frac{d\sigma_\parallel}{d\Omega_p}$ are the cross-sections for photodisintegration by real polarized photons.

The only data for $e + He^3 \to e' + p + d$ is that of Johansson[1)].

$E_e$ = 550 MeV $\quad E_{e'}$ = 443 MeV $\quad \theta_{e'} = 51.7^o$

| | | | | |
|---|---|---|---|---|
| $\theta_p$(LAB) = | $44.2^o$ | $51.7^o$ | $56.7^o$ | $62.0^o$ |
| $\frac{d^3\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_p} \left(\frac{10^{32}cm^2}{Sr\ MeV}\right)$ | 29.6 | 7.32 | 5.17 | 0.88 |
| Statistical Error (%) = | 15 | 6.3 | 11 | 41 |

$\nu = 105$ MeV $\qquad q^2 = 1.85 \times 10^5$ MeV

$K = 74$ MeV $\qquad \varepsilon = .667$

$\Gamma = 3.57 \times 10^{-7}$ MeV$^{-1}$

Note that the data were taken at only a single $q^2$, making the extrapolation to $q^2 = 0$ rather uncertain. The effective real photon energy, 74 MeV, is well below the region of $N^*$ production in $\gamma + He^3 \to p + d$ .

To compare these data to those for $\gamma + He^3 \to p + d$ we need to cast the p+d system into the $\gamma$-$He^3$ c.m. frame. The lab angle of the photon is

$$\theta_\gamma = \tan^{-1}\left(\frac{E_{e'} \sin\theta_{e'}}{E_e - E_{e'} \cos\theta_{e'}}\right) = 51.6^\circ .$$

Thus the data for $\gamma + He^3 \to p + d$ will be all in the forward direction.

The results of the LAB to c.m. transformation are

| $\theta_p$(LAB) | $\theta_p$(c.m.) | $\phi$ | $\dfrac{d\Omega_p(\text{LAB})}{d\Omega_p(\text{c.m.})}$ | $\dfrac{1}{\Gamma} \cdot \dfrac{d^3\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_p(\text{c.m.})}$ |
|---|---|---|---|---|
| 51.7 | 0.15 | $0^\circ$ | .430 | $1.7 \times 10^{-27}$ |
| 56.7 | 7.8 | $0^\circ$ | .431 | 1.2 |
| 44.2 | 11.3 | $180^\circ$ | .433 | $6.7 \times 10^{-26}$ |
| 62.0 | 15.8 | $0^\circ$ | .436 | 2.0 |

We can now make estimates of the functions A, B, C, and D mentioned above. Because of the small $\theta_p$ involved I neglect B and D. Also, the $\theta_p(\text{LAB}) = 44^\circ$ data show that a term in $\cos\phi$ cannot be very important. For term A, which is the cross-section for photodisintegration due to transversely polarized photons (see Appendix), we approximate $A(q^2) = A(0)$ . This is indicated by experience with e-p inelastic scattering[2]. Term C is related to the cross-section for longitudinally polarized photons by

$$C = \frac{q^2}{q_o^2} \frac{d\sigma_L}{d\Omega}$$

where $q_o$ is the energy of the photon in the $\gamma$-$He^3$ c.m. frame. For the present case $q^2/q_o^2 = 116$. Thus even if $\frac{d\sigma_T}{d\Omega} \sim \frac{d\sigma_L}{d\Omega}$, term C could be dominant.

The experimental data nearest to $E_\gamma = 74$ MeV is that of O'Fallon et al. reported by Carron[3]. The data are characterized by an order of magnitude of $10^{-30}$ $cm^2/Sr$ for the cross-section, which shows a marked forward dip. This is rather spectacularly different from the virtual photon cross-sections found above.

To get a more quantitative comparison between real and virtual photo-disintegration, I have done a simple partial wave analysis. I assume the $He^3$ and d ground state wave functions to be pure S-waves. The only important electromagnetic transitions at the fairly low energy under consideration are

$$E1 \rightarrow {}^2P_{3/2}\,,\ {}^2P_{1/2}$$

$$M1 \rightarrow {}^4S_{3/2}$$

$$E2 \rightarrow {}^2D_{5/2}\,,\ {}^2D_{3/2}\qquad ,$$

where S, P, D label the relative p-d angular momentum, etc. Further, the 2 electric dipole amplitudes should be equal, and the 2 electric quadrupole amplitudes should also be equal. Thus there are only 3 amplitudes to consider, which I call S, P and D. Then

$$\frac{d\sigma_T}{d\Omega} \sim S^2 + \sin^2\theta(3P^2 + 2\sqrt{6}\ \mathrm{Re}\ P^* D \cos\theta + 2 D^2 \cos^2\theta)$$

$$\frac{d\sigma_L}{d\Omega} \sim \frac{1}{3} S^2 + 3P^2 + \frac{2}{3} D^2 + \sqrt{2}\ \mathrm{Re}\ P^* D \cos\theta +$$

$$+ \sin^2\theta(-3P^2 - \frac{1}{2} D^2 + \frac{3}{2}\sqrt{2}\ \mathrm{Re}\ P^* D \cos\theta - \frac{3}{2} D^2 \cos^2\theta) \quad .$$

A fit to the O'Fallon data, interpolated to 74 MeV is

$$\frac{d\sigma_T}{d\Omega} = (0.4 + 0.6 \sin^2\theta + 1.33 \sin^2\theta \cos\theta + 1.0 \sin^2\theta \cos^2\theta) \times 3 \times 10^{-30}\ \mathrm{cm}^2/\mathrm{Sr} \quad .$$

Then

$$\frac{d\sigma_L}{d\Omega} = (1.0 + 0.4 \cos\theta - 0.85 \sin^2\theta + 0.6 \sin^2\theta \cos\theta - 0.75 \sin^2\theta \cos^2\theta) \times 3 \times 10^{-30}$$

Note that $d\sigma_L/d\Omega$ is peaked in the forward direction.

If we estimate $\frac{1}{\Gamma} d^3\sigma/dE_{e'}\, d\Omega_e\, d\Omega_p$ by

$$\frac{q^2}{q_o^2} \frac{d\sigma_L}{d\Omega} \quad ,$$

we get $3.3 \times 10^{-28}$ cm$^2$/Sr at $0^\circ$, a factor of 5 below the electrodisintegration data. The angular dependence of the fit for $\sigma_L$ is not rapid enough to match the forward peak seen in the electrodisintegration: at $15.8^\circ$ we have $3 \times 10^{-28}$ cm$^2$/Sr .

Thus the attempt to relate real to virtual photodisintegration of $He^3$ has been only qualitatively successful. The dominance of $\sigma_L$ over $\sigma_T$ combined with the uncertainty of the extrapolation to $q^2 = 0$ do not allow any precise constraint to be placed on real photodisintegration amplitudes by means of virtual photodisintegration data.

## APPENDIX

In this Appendix we outline the decomposition of electrodisintegration into virtual photodisintegration. This is taken mainly from Reference 4; see also Reference 5.

The basic cross-section for electrodisintegration is

$$d\sigma = \frac{1}{V_e}\,\frac{1}{2E_e}\,\frac{1}{2M_H}\,\frac{1}{(2\pi)^5}\,\frac{d^3\vec{P}_{e'}}{dE_{e'}}\,\frac{d^3\vec{P}_p}{2E_p}\,\frac{d^3\vec{P}_d}{2E_d}\sum_{\text{spins}}|M|^2 \quad .$$

$e$ = Incident electron

$e'$ = Recoil electron

$H$ = $He^3$

$p$ = Proton

$d$ = Deuteron .

We neglect the mass of the electron.

$M = \frac{1}{k^2}\, j_\mu\, J_\mu$ is the matrix element in the one-photon exchange approximation where $k_\mu$ is the virtual photon 4-vector.

$$\sum_{\text{spins}} |M|^2 = \frac{1}{k^4}\,\frac{1}{2}\sum_{\substack{\text{electron}\\ \text{spins}}} j_\mu\, j_\nu^+\;\frac{1}{2}\sum_{\substack{\text{hadron}\\ \text{spins}}} J_\mu\, J_\nu^+ \equiv \frac{1}{k^4}\, L_{\mu\nu}\, T_{\mu\nu} \quad .$$

$L_{\mu\nu}$ is well-known:

$$L_{\mu\nu} = 8\pi\alpha\left(e_\mu\, e'_\nu + e_\nu\, e'_\mu + \frac{k^2}{2}\,\delta_{\mu\nu}\right) \equiv 8\pi\alpha\; \ell_{\mu\nu} \quad .$$

We can now separate the cross-section into electron and hadron parts:

$$d\sigma = \left[\frac{\alpha}{4\pi^2}\,\frac{E_{e'}}{E_e}\, dE_{e'}\; d\Omega_{e'}\;\frac{\ell_{\mu\nu}}{k^4}\right]\cdot\left[\frac{1}{2M_H}\,\frac{1}{(2\pi)^2}\,\frac{d^3\vec{P}_p}{2E_p}\,\frac{d^3\vec{P}_d}{2E_d}\, T_{\mu\nu}\right] \quad .$$

The second bracket would be the cross-section for photodisintegration if it also contained the factor

$$\frac{1}{V(\gamma\text{-He}^3)_{REL}} \cdot \frac{1}{2E_\gamma} = \frac{k_o}{|\vec{k}|}\frac{1}{2k_o} = \frac{1}{2|\vec{k}|} \quad .$$

This introduces a factor $2|\vec{k}|$ into the left bracket. $|\vec{k}|$ is a function of $k^2(= -q^2)$ and its physical significance is unclear. We replace it by

$$K = \frac{W^2 - M_H^2}{2M_H} \quad ,$$

following Hand[6]. K and $|\vec{k}|$ approach each other as $k^2$ goes to zero, but K is independent of $k^2$.

$\ell_{\mu\nu}$ carries the information about the polarization of the virtual photon. The polarization deisity matrix, $\varepsilon_{\mu\nu}$ , is related by

$$\frac{\varepsilon_{\mu\nu}}{1-\varepsilon} = \frac{1}{q^2} \ \ell_{\mu\nu} \quad ,$$

where $\varepsilon$ is the polarization parameter mentioned previously. The separation of $1/(1-\varepsilon)$ out of $\varepsilon_{\mu\nu}$ is a convention suggested by the explicit evaluation of $\ell_{\mu\nu}$ . The factor $q^2$ is needed to normalize $\varepsilon_{\mu\nu}$.

Now we have

$$d\sigma = \frac{\alpha}{2\pi^2} \frac{E_{e'}}{E_e} \frac{K}{q^2} \frac{1}{1-\varepsilon} \, dE_{e'} \, d\Omega_{e'} \left[\frac{1}{2K} \frac{1}{2M_H} \frac{1}{(2\pi)^2} \frac{d^3\vec{P}_p}{2E_p} \frac{d^3\vec{P}_d}{2E_d} \varepsilon_{\mu\nu} T_{\mu\nu}\right] \quad .$$

We note that we need only be concerned with the spatial components of $\varepsilon_{\mu\nu}$ and $T_{\mu\nu}$ by invoking current conservation

$$k_\mu j_\mu = 0 \qquad\qquad k_\mu J_\mu = 0 \quad ,$$

or

$$j_o = \frac{\vec{k} \cdot \vec{j}}{k_o} \qquad J_o = \frac{\vec{k} \cdot \vec{J}}{k_o} \quad .$$

In the $\gamma$-$He^3$ c.m. frame, $k_z$ is the only non-zero component of $\vec{k}$, choosing the z axis in the photon's direction. Thus

$$- j_\mu J_\mu = j_x J_x + j_y J_y + \frac{k^2}{k_o^2} j_z J_z \quad .$$

We can write $\varepsilon_{\mu\nu} T_{\mu\nu} = \varepsilon_{ij} T_{ij}$ provided the z-components of $\varepsilon_{ij}$ are $k^2/k_o^2$ times those of $\varepsilon_{\mu\nu}$ (so $\varepsilon_{zz}$ is $k^4/k_o^4$ times that of $\varepsilon_{\mu\nu}$). An evaluation of $\varepsilon_{ij}$ in the $\gamma$-$He^3$ c.m. frame gives

$$\varepsilon_{ij} = \begin{pmatrix} (1+\varepsilon)/2 & 0 & \sqrt{\varepsilon_L (1+\varepsilon)/2} \\ 0 & (1-\varepsilon)/2 & 0 \\ \sqrt{\varepsilon_L (1+\varepsilon)/2} & 0 & \varepsilon_L \end{pmatrix}$$

where $\varepsilon_L = \frac{q^2}{k_o^2} \varepsilon$ and $\varepsilon$ is given as before in terms of laboratory quantities. The axes are chosen with the photon in the +z direction, and the plane of interaction as the x-z plane. If $\varepsilon_L$ were zero, the remaining $\varepsilon_{ij}$ would be the density matrix for a photon beam with $(1+\varepsilon)/2$ of the photons polarized in the x-direction, and $(1-\varepsilon)/2$ in the y-direction. Hence the description of $\varepsilon$ as the amount of transverse linear polarization.

We can now write the electrodisintegration cross-section as

$$d^2\sigma/dE_{e'} \, d\Omega_{e'} = \Gamma d\sigma_\gamma$$

with $\Gamma$ as given above and

$$d\sigma_\gamma = (d\sigma_{xx} + d\sigma_{yy})/2 + \varepsilon(d\sigma_{xx} - d\sigma_{yy})/2 + \varepsilon_L\, d\sigma_{zz} + \sqrt{2\varepsilon_L(1+\varepsilon)}\, d\sigma_{xz} \quad ,$$

using the fact that $T_{ij}$ is symmetric. In another common notation

$$d\sigma_{xx} = d\sigma_{\parallel}$$
$$d\sigma_{yy} = d\sigma_{\perp}$$
$$d\sigma_{zz} = d\sigma_L \quad .$$

To exhibit the angular factors in $d\sigma_\gamma$ mentioned in the main text, we need to convert from spatial polarizations to helicities:

$$|x\rangle = -(|+\rangle - |-\rangle)/\sqrt{2} \qquad |y\rangle = i(|+\rangle + |-\rangle)/\sqrt{2}$$

$$|z\rangle = |0\rangle$$

$$d\sigma_\gamma = (d\sigma_{++} + d\sigma_{--})/2 - \varepsilon d\sigma_{+-} + \varepsilon_L d\sigma_{oo} + \sqrt{\varepsilon_L(1+\varepsilon)}\,(d\sigma_{+o} - d\sigma_{-o}) \quad .$$

Note that $d\sigma_{\lambda\mu}$ is the sum of terms like

$$\langle p,d|S^{+}|\gamma_\lambda, He^3\rangle\langle\gamma_\mu, He^3|S|p,d\rangle \quad ,$$

where the p,d and $He^3$ have the same helicities in both amplitudes, but the photons have helicities $\lambda$ and $\mu$. The angular dependence of $d\sigma_{\lambda\mu}$ is then a sum of terms like

$$Y^{*}_{\ell,m}(\theta,\phi) \cdot Y_{\ell',m+\lambda-\mu}(\theta,\phi) \quad .$$

Hence $d\sigma_{++}$, $d\sigma_{oo}$, and $d\sigma_{--}$ have no particular $\theta$ or $\phi$ dependence. However, $d\sigma_{+-}$ will have a factor $\sin^2\theta \cos^2\phi$ [7] and $d\sigma_{+o}$, $d\sigma_{-o}$ will have $\sin\theta \cos\phi$. Thus we can also write

$$d\sigma_\gamma = A + \varepsilon B \sin^2\theta \cos^2\phi + \varepsilon C + \sqrt{\varepsilon(1+\varepsilon)}\; D \sin\theta \cos\phi \quad ,$$

where

$$A = (d\sigma_{\parallel} + d\sigma_{\perp})/2$$

$$B \sin^2\theta \cos^2\phi = (d\sigma_{\parallel} - d\sigma_{\perp})/2$$

$$C = q^2/k_o^2 \, d\sigma_L$$

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