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EFFECT OF MULTIPLE COULOMB SCATTERING ON SPARK CHAMBER TRACK RECONSTRUCTION*

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In this report I calculate the effect of multiple Coulomb scattering on the slope and intercept of a track of a charged particle in a 2-plane spark chamber system. The result is not greatly different from an estimate based on the cumulative effect evaluated midway between the planes. This is important because a precise calculation for a system involving 3 or more planes is lengthy.

The following calculations are made for the multiple scattering projected onto a plane. The z axis is in the direction of the incident particle.

The cumulative angle and displacement of the particle is the result of many successive scatterings. For a particle incident at z=0, the cumulative angle, θ, and transverse displacement, x, at position z are given by

$$\theta = \sum_i d\theta_i$$

$$x = \sum_i (z - z_i)\, d\theta_i \qquad , \qquad (1)$$

where a scattering of angle $d\theta_i$ occurs at position z_i. The $d\theta_i$ are randomly distributed so that the expectation values of θ and x obey

$$\langle\theta\rangle = 0 \qquad \langle x\rangle = 0 \qquad .$$

However, θ and x have non-zero mean square deviations of

$$\langle\theta^2\rangle = \langle\sum_i d\theta_i^2\rangle$$

$$\langle x^2\rangle = \langle\sum_i d\theta_i^2 \,(z - z_i)^2\rangle \qquad . \qquad (2)$$

For a single material, the number of scatters per unit length is essentially constant for laboratory size lengths. Thus,

$$\langle\theta^2\rangle = Kz \qquad \text{and} \qquad \langle x^2\rangle = \frac{z^2}{3}\langle\theta^2\rangle \quad .$$

The constant K is usually estimated by the Rossi formula:

$$K = \left(\frac{15}{P\beta}\right)^2 \frac{1}{L_{RAD}} \quad .$$

P = momentum in MeV/c.

β = velocity.

L_{RAD} = radiation length.

For the case of several materials, or one material so long the momentum variation must be taken into account, it is easy to numerically integrate Equations (2) using a step size small enough that the Rossi formula holds for that step.

The quantities x and θ are correlated. From Equations (1),

$$\langle\theta x\rangle = z\langle\theta^2\rangle - \langle\sum_i z_i d\theta_i^2\rangle \quad . \qquad (3)$$

For a single material this reduces to

$$\langle\theta x\rangle = \tfrac{1}{2}\, z\langle\theta^2\rangle \quad .$$

In a spark chamber system, x is observed for several z. The multiple scattering introduces a correlation between the various observations. Suppose x_1 and x_2 are observed at z_1 and z_2. Then

$$x_2 = x_1 + (z_2 - z_1)\,\theta_1 + \sum_j (z_2 - z_j)\, d\theta_j \quad ,$$

where the range of j obeys $z_1 < z_j < z_2$.

Hence

$$\langle x_1 x_2 \rangle = \langle x_1^2 \rangle + (z_2 - z_1) \langle \theta_1 x_1 \rangle \quad . \tag{4}$$

Also

$$\langle x_2^2 \rangle = \langle x_1^2 \rangle + (z_2 - z_1)^2 \langle \theta_1^2 \rangle + 2(z_2 - z_1) \langle \theta_1 x_1 \rangle + \langle \sum_j d\theta_j^2 (z_2 - z_j)^2 \rangle \ . \tag{5}$$

The best estimates as to the particle trajectory's slope and intercept are made using a least squares fit of the observed displacements. The usual error estimates from a least squares fit presume the various observations going into the fit have uncorrelated errors. This is not true for multiple scattering as demonstrated above. The inclusion of the correlations makes the errors estimates rather lengthy if more than 2 spark chamber planes are involved.

For the case of only 2 planes, the least squares fit is exact giving the slope, M, and intercept, b:

$$M = \frac{x_2 - x_1}{z_2 - z_1} \qquad\qquad b = \frac{x_1 z_2 - x_2 z_1}{z_2 - z_1}$$

The mean square deviations of M and b can now be calculated with the help of Equations (4) and (5).

$$\langle M^2 \rangle = \langle \theta_1^2 \rangle + \frac{1}{(z_2 - z_1)^2} \langle \sum_j d\theta_i^2 (z_2 - z_j)^2 \rangle \quad . \tag{6}$$

$$z_1 < z_j < z_2 \quad .$$

$$\langle b^2 \rangle = \langle x_1^2 \rangle + z_1^2 \langle M^2 \rangle - 2 z_1 \langle \theta_1 x_1 \rangle \quad . \tag{7}$$

These may be further evaluated using Equations (2) and (3).

To understand the second term in the expression for $\langle M^2 \rangle$ consider the following two cases:

1. The material between the spark chambers is uniformly distributed.

Let θ_{12} be the multiple scattering caused by the particle moving from z_1 to z_2. Then from Equation (2), the second term in (6) is $\frac{1}{3} \langle \theta_{12}^2 \rangle$.

2. The material between the chambers is concentrated in the chamber walls.

Now all of the scattering between 1 and 2 occurs at 1, so the second term in (6) is simply $\langle \theta_{12}^2 \rangle$.

Thus $\langle M^2 \rangle$ can vary from $\langle \theta_1^2 \rangle + \frac{1}{3} \langle \theta_{12}^2 \rangle$ to $\langle \theta_1^2 \rangle + \langle \theta_{12}^2 \rangle$. A good estimate is then

$$\langle M^2 \rangle = \langle \theta_1^2 \rangle + \frac{1}{2} \langle \theta_{12}^2 \rangle \quad , \qquad (8)$$

which is the cumulative multiple scattering midway between planes 1 and 2. Equations (7) and (8) can thus be applied with good accuracy to a system of any number of planes (assuming 2 labels the last plane).

These results have been derived assuming the unscattered trajectory would have zero slope. The results hold with minor modifications for the case of arbitrary angle, α, with respect to the z-axis. For $\langle \theta^2 \rangle$ of Equations (2), the path length is $z/\cos\alpha$ instead of z. Also

$$\langle x^2 \rangle = \frac{1}{\cos^2\alpha} \langle \sum d\theta_i^2 \left(\frac{z - z_i}{\cos\alpha} \right)^2 \rangle \quad ,$$

where again the extra path length must be considered.