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Prospects for B Physics at RHIC

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1 Introduction

RHIC is a prolific bottom factory, with B-pair production rates order 10^{10} per years at a luminosity of 10^{32} cm^{-2}sec^{-1}. As such, it will provide an excellent opportunity for a study of CP violation in the B-$\bar{B}$ system.[1, 2, 3] Such a study will either confirm and refine the Standard Model, or be a sensitive indicator of new physics.

In proposing detailed studies of B physics at RHIC we must extract the B-meson signal in an environment often considered less favorable than at an e^+e^- collider. Experimental evidence that this is possible is now available in a recent, preliminary result of the CDF collaboration in which 16 ± 6 events have been reconstructed from the decay chain $B^\pm \to J/\psi K^\pm \to \mu^+\mu^- K^\pm$, as shown in Fig. 1. This is the largest sample of $B \to J/\psi K$ decays reconstructed in any experiment to date.

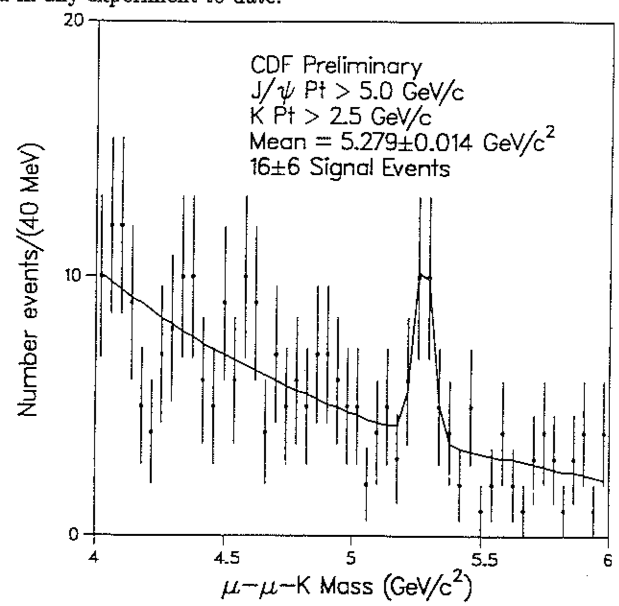

Figure 1: Preliminary analysis of the CDF collaboration showing reconstruction of the decay chain $B^\pm \to J/\psi K^\pm \to \mu^+\mu^- K^\pm$.

2 B Physics at the RHIC

Several developments during the 1980's indicate that B-meson physics will be a rich topic for study at the RHIC:

- The B-meson lifetime was first measured by the MAC[4] and Mark II[5] groups to be 1.2 picoseconds, which is longer than that for D mesons. Thus with the use of a silicon vertex detector, perfected during the early 1980's, charged secondaries from B-meson decay can be isolated from the primary interaction vertex. This will permit B physics to be done at a hadron collider, but with the important restriction that complete reconstruction will typically be possible only for all-charged decay modes.

- The cross section for $b\bar{b}$ pairs in hadron-hadron interactions is enhanced by order-α_S^3 QCD corrections[6] and is anticipated to be approximately 12.5 μbarn at RHIC. This is 10^4 times larger than the cross section for $b\bar{b}$ production for an e^+e^- collider operating at the $\Upsilon(4s)$ resonance.

- The B_d^0-$\bar{B}_d^0$ mixing parameter, $x_d = \Delta M/\Gamma$, has been measured at Argus[7] (and subsequently at CLEO[8]) to be about 0.7. This demonstrates that the neutral B-meson system, like the neutral kaon system, is rich in quantum-mechanical phenomena.

- It is expected that CP violation in the decays of neutral B mesons to CP eigenstates will have an especially clean interpretation. Such decays, when dominated by a single weak amplitude, exhibit CP violation via interference due to mixing, which results in an effect directly dependent on a phase of a CKM-matrix element without strong-interaction ambiguities. However, it is probable that discovery of this type of CP violation will require a sample of 10^8 reconstructible B decays. Detailed exploration of consistency of the Standard Model explanation of CP violation will require at least 100 times more events.

We are embarking on a long-range program, initiated at Fermilab[9] or RHIC[10] and continuing at the SSC,[11] with the ultimate goal being the detailed investigation of CP violation in the B-$\bar{B}$ system. Of all known phenomena, we believe that the existence of CP violation provides the clearest indication that new physics is to be found at energy scales above 1 TeV.

Speculations about the role of CP violation encourage study of the phenomena outside its present preserve in the K^0-$\bar{K}^0$ system:

- Cosmological models that try to explain the matter-antimatter asymmetry in the universe usually invoke CP violation and some Grand Unified Theory. Thus the existence of the universe is thought to be related to CP violation in some way. Further study is justified on these grounds alone.

- Multiple Higgs bosons can lead to relative complex phases that in turn have CP-violating effects. Thus the understanding of mass generation and CP violation is related.

- CP violation is relevant to the generation puzzle. Of the 21 free parameters in the standard model, 18 are related to the fact that we have 3 generations of families. If there were two families, no complex phase would exist in the standard model and thus no simple explanation of CP violation. With 3 families, there is one complex phase, and this is consistent with present data. If there are 4 families, then there are 3 complex phases and new CP phenomena might be expected. Phenomena involving the b-quark of the third generation should reveal the origin of CP violation more clearly than in the K-$\bar{K}$ system.

- Measurements of CP violation in the B-$\bar{B}$ system (unlike the K-$\bar{K}$ system) can determine CKM angles with little strong-interaction uncertainty. By the study of several B-decay modes, the CKM system can be overconstrained.

- When the CKM elements are well determined it may be possible to deduce regularities in the mass matrices of the quarks, and hence among their Yukawa couplings. This is an approach that might lead to discovery of a higher symmetry beyond the present standard model.

- Left-right symmetric models predict smaller CP-violating effects in the B-$\bar{B}$ system than does the standard model.

The greatest opportunity to explore CP violation in a new way, via the B-$\bar{B}$ system, is at a hadron collider. As summarized in Table 1, the cross-section for B-meson production at RHIC is about 1/3000 of the total inelastic p-p cross section, and 10^{10} B-$\bar{B}$ pairs could be produced in a year of running at a luminosity of 10^{32} cm^{-2}sec^{-1}, during which the total interaction rate would be about 4 MHz.

Table 1: B-$\bar{B}$ production at various colliders.

Collider	$\sqrt{s}$ (TeV)	$\sigma_{b\bar{b}}$ (μb)	$\sigma_{b\bar{b}}/\sigma_{tot}$	$\mathcal{L}_{ave}$ (cm^{-2}sec^{-1})	$N_{b\bar{b}}/10^7$ sec
RHIC	0.5	12.5	1/3000	10^{32}	1.25×10^{10}
TEV I	1.8	45	1/1000	5×10^{31}	2×10^{10}
SSC	40	500	1/200	10^{32}	5×10^{11}
CESR	0.01	0.001	1/5	10^{33}	10^7
LEP	0.09	0.005	1/7	10^{32}	5×10^6

The study of CP violation in the B-$\bar{B}$ system can be accomplished by measurement of an asymmetry in the decay of B mesons to all-charged final states:

$$A = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}.$$

While the asymmetry A may be as large as 30% (even after dilution due to B-$\bar{B}$ mixing), this likely occurs only in modes with branching fractions $\Gamma \sim 10^{-5}$. This requires at least 10^8

reconstructible decays for a significant signal to be discerned. Further, the cleanest signals are for modes with $f = \bar{f}$, so the particle-antiparticle character of the parent B must be 'tagged' by observation of the second B in the interaction. Of course, a detailed study should include measurement of asymmetries in several different decay modes.

Some of the elegance of measurements in the B-$\bar{B}$ system may be inferred from consideration of the CKM matrix (in the Wolfenstein notation):

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

Parameter λ is essentially the Cabibbo angle, A is known via the B lifetime, while $\eta \neq 0 \leftrightarrow CP$ violation. But, ρ and η are not well determined from the K-$\bar{K}$ system. Rather, the B-$\bar{B}$ system will be the place for detailed measurements of these parameters.

Further, unitarity of V_{CKM} implies

$$V_{td} + \lambda V_{ts} + V_{ub}^* \approx 0.$$

Hence if these three complex matrix elements are regarded as vectors they form a closed triangle in the complex plane. On dividing their lengths by $A\lambda^3$, we obtain the picture of Figure 2 in the (ρ, η) plane. Since the base is known, the experimental challenge of measuring the amplitudes V_{ub} and V_{td} is equivalent to measuring the three interior angles φ_1, φ_2, φ_3.

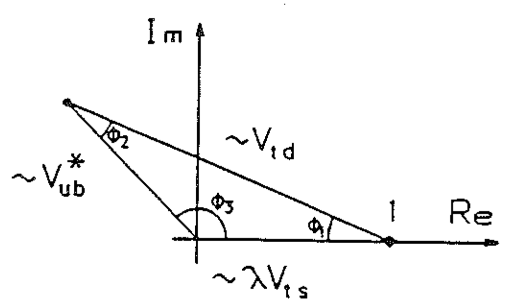

Figure 2: The unitarity triangle.

A favorable theoretical result is that for decays $B \to f$ with f a CP eigenstate, the asymmetry A depends only on one of the angles φ_i:

$A \sim \sin 2\varphi_1$ for $B_d^0 \to J/\psi K_S$, $D\bar{D}$, and $D_s^+ D_s^-$.

$A \sim \sin 2\varphi_2$ for $B_d^0 \to \pi^+\pi^-$, K^+K^-, and $\rho^0 K_S$.

$A \sim \sin 2\varphi_3$ for $B_s^0 \to \rho^0 K_S$, $\pi^+\pi^-$, and K^+K^-.

Hence the experimentally accessible asymmetries rather directly measure the V_{CKM} amplitudes. It will require a general purpose, low-P_t detector to take full advantage of this splendid opportunity.

The program to measure CP violation in the B-$\bar{B}$ system is ambitious. There will be four phases to this program.

1. Study of the branching ratios of nonleptonic decay modes of B mesons and baryons. From the associated measurement of $\sigma_{b\bar{b}}$ at a hadron collider will come determinations of the gluon structure function at very low x. The BCD will produce even larger samples of decays of charmed-particles.

2. Study of B_s-$\bar{B}_s$ mixing, which is expected to be quite large, and hence difficult to observe due to the rapid oscillations. At a hadron collider the B's are produced with sufficient laboratory energy that their time evolution may be observed in detail. (Only e^+e^- colliders more ambitious than those in Table 1 could resolve this mixing.)

3. Study of CP violation in decays $B \to f$ where $f \neq \bar{f}$, which does not require tagging of the other B in the event. This might provide the first evidence of CP violation in the B-meson system, but the interpretation of the results will be subject to theoretical uncertainty from strong-interaction effects.

4. Study of CP violation in decays of neutral B mesons to CP eigenstates, which will provide the critical test of our understanding of CP violation in the Standard Model. This difficult but rewarding task sets the standards for detector performance of the B-physics program.

In the following four subsections we elaborate on each of the four aspects of the B-physics program.

2.1 Nonleptonic Decay Modes of the B Mesons

A survey of seven possible graphs describing B-meson decay indicates that the B_u will have 21 basic 2-body nonleptonic decays, the B_d will have 28, and B_s will have 29 (see Tables 2-4). This contrasts with the case for the K_u $(= K^+)$ and K_d $(= K^0)$ which each only have 2 such decays (not all distinct!). In the B system there are 26 basic decays to CP eigenstates compared to the 2 in the K system. All 78 of the basic two-body decays of the B system have all-charged final states (at some price in secondary branching fraction), while only 1 of the basic K decays is all charged.

We have not displayed the catalog of decays of the B_c^+ $(= \bar{b}c)$, which decays to such states as $D^+\bar{D}^0$ and $J/\psi\pi^+$. Both the B_s and the B_c will be better studied at a hadron collider than at a low-energy e^+e^- collider.

The Tables refer to seven kinds of graphs, two spectator, annihilation, exchange, penguin/annihilation, and two penguin/spectator, as shown in Fig. 3. We can roughly estimate that for graphs I and II

CKM-favored decays have branching fractions of 10^{-2}-10^{-3};
CKM-suppressed decays have branching fractions of 3×10^{-4}-3×10^{-5};
CKM-doubly-suppressed decays have branching fractions of 10^{-5}-10^{-6}.

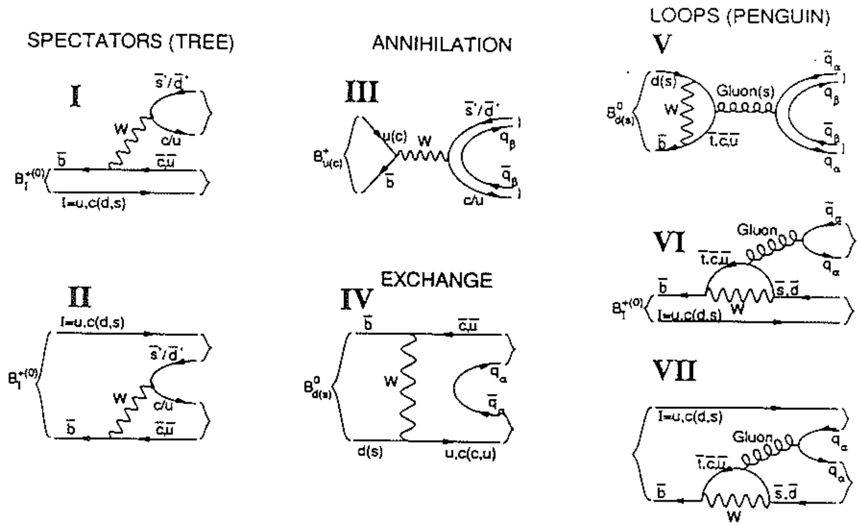

Figure 3: Seven graphs for the nonleptonic decays of B mesons. The dashed lines are W bosons; gluons are not shown.

Graphs III-VII are typically suppressed by an additional factor of 0.1-0.01 compared to graphs I and II of the same CKM coupling. However, D-meson decays show some cases where graphs III-VII give large amplitudes.

The two-body final states listed in the Tables are representative of the particular $q\bar{q}/q\bar{q}$ combination for each entry. All final states could be augmented by $n(\pi^+\pi^-)$, with possibly larger branching fractions. Likewise, every spin-0 final-state particle could be replaced by its spin-1 partner, and *vice versa*. Typically the branch to the spin-1 meson will be 3 times that to the spin-0 partner.

The secondary decays used in constructing the last column of the Tables are:

Decay Mode	Branching Ratio
$K_S^0 \to \pi^+\pi^-$	0.69
$\rho^0 \to \pi^+\pi^-$	1.00
$K^{*0} \to K^+\pi^-$	0.67
$\phi \to K^+K^-$	0.50
$D^+ \to K^-\pi^+\pi^+$	0.08
$D^0 \to K^-\pi^+$	0.04
$D_s^+ \to \phi\pi^+$	0.03
$D_s^+ \to \phi\pi^+\pi^+\pi^-$	0.04
$J/\psi \to e^+e^-$	0.07

Tables 2-4 suggest that a study of only the all-charged nonleptonic decay modes of B-mesons will be very rich, as all basic 4-quark decays of the B's can be accessed in this way.

Table 2: The 21 basic 2-body nonleptonic decays of the B_u^+ ($= \bar{b}u$). Figure 3 illustrates the seven types of graphs. The subscripts F, S, and D to the type of graph in this and following two tables refer to CKM-favored, -suppressed, and -doubly-suppressed, respectively.

Graph	Final Quarks	Final State	All-Charged Daughters
I_F, II_F	$u\bar{c}/u\bar{d}$	$\bar{D}^0\pi^+$	$K^+\pi^-\pi^+$
I_F, III_D, VII_F	$c\bar{s}/u\bar{c}$	$D_s^+\bar{D}^0$	$K^+K^-\pi^+K^+\pi^-$
II_F, VI_F	$c\bar{c}/u\bar{s}$	$J/\psi K^+$	$e^+e^-K^+$
III_D, VII_F	$d\bar{s}/u\bar{d}$	$K^{*0}\pi^+$	$K^+\pi^-\pi^+$
III_D, VI_F, VII_F	$s\bar{s}/u\bar{s}$	ϕK^+	$K^+K^-K^+$
I_S, II_S, III_S, VI_F, VII_F	$u\bar{u}/u\bar{d}$	$\rho^0\pi^+$	$\pi^+\pi^-\pi^+$
I_D, II_D, III_D, VI_F, VII_F	$u\bar{u}/u\bar{s}$	ρ^0K^+	$\pi^+\pi^-K^+$
I_S, II_S	$u\bar{c}/u\bar{s}$	$\bar{D}^0K^+$	$K^+\pi^-K^+$
I_S, III_S, VII_S	$c\bar{d}/u\bar{c}$	$D^+\bar{D}^0$	$K^-\pi^+\pi^+K^+\pi^-$
I_S	$c\bar{s}/u\bar{u}$	$D_s^+\rho^0$	$K^+K^-\pi^+\pi^+\pi^-$
II_S, VI_S	$c\bar{c}/u\bar{d}$	$J/\psi\pi^+$	$e^+e^-\pi^+$
II_S, III_S	$c\bar{u}/u\bar{s}$	D^0K^+	$K^-\pi^+K^+$
III_S, VII_S	$u\bar{s}/s\bar{d}$	K^+K^{*0}	$K^+K^+\pi^-$
III_S	$c\bar{d}/d\bar{s}$	D^+K^{*0}	$K^-\pi^+\pi^+K^+\pi^-$
III_S	$c\bar{s}/s\bar{s}$	$D_s^+\phi$	$K^+K^-\pi^+K^+K^-$
III_S	$c\bar{c}/c\bar{s}$	$J/\psi D_s^+$	$e^+e^-K^+K^-\pi^+$
VI_S	$s\bar{s}/\bar{d}u$	$\phi\pi^+$	$K^+K^-\pi^+$
I_D, III_D	$c\bar{d}/u\bar{u}$	$D^+\rho^0$	$K^-\pi^+\pi^+\pi^+\pi^-$
II_D, III_D	$c\bar{u}/u\bar{d}$	$D^0\pi^+$	$K^-\pi^+\pi^+$
III_D	$c\bar{s}/s\bar{d}$	$D_s^+K^{*0}$	$K^+K^-\pi^+K^+\pi^-$
III_D	$c\bar{c}/c\bar{d}$	$J/\psi D^+$	$e^+e^-K^-\pi^+\pi^+$

Clearly a thorough study will require charged particle identification: $\pi^\pm$, $K^\pm$, $e^\pm$, and also $\mu^\pm$ should all be recognized in the apparatus.

We will use 11 of the more interesting basic nonleptonic decay modes to guide our thinking about the design of the apparatus. The ISAJET Monte Carlo program was used to generate samples of the production and decay of B mesons at the RHIC, by numerical calculation of the gluon-gluon production process.

Prominent results of such simulation is that the B-decay products are spread over $\sim \pm 3.5$ units of pseudorapidity η at RHIC, and that the typical transverse momentum of the decay products is less than the B mass, 5 GeV (see Fig. 4) Now $\eta = -\ln\tan\theta/2$ where θ is the polar angle with respect to the beam direction, so that $d\eta = d\theta/\theta$. Hence the population of decay products is roughly $dN/d\theta \sim k/\theta$, where a value of $k \sim 3$ is expected at the RHIC. The relevant $\theta_{\min}$ corresponding to $\eta = 3.5$ is 60 mrad. Thus a detector for B physics must emphasize low transverse momentum and forward angles, in contrast to a detector for Higgs

Table 3: The 28 basic 2-body nonleptonic decays of the B_d^0 $(= \bar{b}d)$. The numbers in the 'CP Eigenstate' column refer to the classification described in section 2.4 regarding the relevant CKM phases governing the decay asymmetries.

Graph	Final Quarks	Final State	CP Eigenstate	All-Charged Daughters
I_F, IV_F	$d\bar{c}/u\bar{d}$	$D^-\pi^+$		$K^+\pi^-\pi^-\pi^+$
I_F, VII_F	$c\bar{s}/d\bar{c}$	$D_s^+D^-$		$K^+K^-\pi^+K^+\pi^-\pi^-$
II_F	$u\bar{c}/d\bar{d}$	$\bar{D}^0\rho^0$		$K^+\pi^-\pi^+\pi^-$
II_F, VI_F	$c\bar{c}/d\bar{s}$	$J/\psi K_S^0$	1	$e^+e^-\pi^+\pi^-$
IV_F	$u\bar{c}/u\bar{u}$	$\bar{D}^0\rho^0$		$K^+\pi^-\pi^+\pi^-$
IV_F	$s\bar{c}/u\bar{s}$	$D_s^-K^+$		$K^+K^-\pi^-K^+$
IV_F	$c\bar{c}/u\bar{c}$	$J/\psi\bar{D}^0$		$e^+e^-K^+\pi^-$
II_D, VI_F, VII_F	$u\bar{u}/d\bar{s}$	$\rho^0K_S^0$	2, 1	$\pi^+\pi^-\pi^+\pi^-$
VI_F, VII_F	$s\bar{s}/d\bar{s}$	ϕK_S^0	1	$K^+K^-\pi^+\pi^-$
I_D, VII_F	$u\bar{s}/d\bar{u}$	$K^+\pi^-$		$K^+\pi^-$
I_S	$d\bar{c}/u\bar{s}$	D^-K^+		$K^+\pi^-\pi^-K^+$
I_S, IV_S, V_S,VII_S	$c\bar{d}/d\bar{c}$	D^+D^-	1, 4	$K^-\pi^+\pi^+K^+\pi^-\pi^-$
I_S, IV_S, V_S,VII_S	$u\bar{d}/d\bar{u}$	$\pi^+\pi^-$	2, 4	$\pi^+\pi^-$
I_S	$c\bar{s}/d\bar{u}$	$D_s^+\pi^-$		$K^+K^-\pi^+\pi^-$
II_S	$u\bar{c}/d\bar{s}$	$\bar{D}^0K^{*0}$		$K^+\pi^-K^+\pi^-$
II_S, VI_S	$c\bar{c}/d\bar{d}$	$J/\psi\rho^0$	1, 4	$e^+e^-\pi^+\pi^-$
II_S, IV_S, V_S, VI_S, VII_S	$u\bar{u}/d\bar{d}$	$\rho^0\rho^0$	2, 4	$\pi^+\pi^-\pi^+\pi^-$
II_S	$c\bar{u}/d\bar{s}$	D^0K^{*0}		$K^-\pi^+K^+\pi^-$
IV_S, V_S	$c\bar{u}/u\bar{c}$	$D^0\bar{D}^0$	1, 4	$K^-\pi^+K^+\pi^-$
IV_S, V_S	$c\bar{s}/s\bar{c}$	$D_s^+D_s^-$	1, 4	$K^+K^-\pi^+K^+K^-\pi^-$
IV_S, V_S	$u\bar{s}/s\bar{u}$	K^+K^-	2, 4	K^+K^-
V_S	$s\bar{s}/s\bar{s}$	$\phi\phi$	4	$K^+K^-K^+K^-$
VI_S	$s\bar{s}/d\bar{d}$	$\phi\rho^0$	4	$K^+K^-\pi^+\pi^-$
V_S, VII_S	$s\bar{d}/d\bar{s}$	$K^{*0}\bar{K}^{*0}$	4	$K^+\pi^-K^-\pi^+$
I_D, IV_D	$c\bar{d}/d\bar{u}$	$D^+\pi^-$		$K^-\pi^+\pi^+\pi^-$
II_D, IV_D	$c\bar{u}/d\bar{d}$	$D^0\rho^0$		$K^-\pi^+\pi^+\pi^-$
IV_D	$c\bar{s}/s\bar{u}$	$D_s^+K^-$		$K^+K^-\pi^+K^-$
IV_D	$c\bar{c}/c\bar{u}$	$J/\psi D^0$		$e^+e^-K^-\pi^+$

physics.

A large range of angle θ cannot be covered in a single detector module, and we are led to conceive of an experiment with two angular regions whose approximate boundaries are given by:

1. the Central Region, with $|\eta| < 1.2$ ($\theta > 600$ mrad),

Table 4: The 29 basic 2-body nonleptonic decays of the B_s^0 $(= \bar{b}s)$.

Graph	Final Quarks	Final State	CP Eigenstate	All-Charged Daughters
I_F	$s\bar{c}/u\bar{d}$	$D_s^-\pi^+$		$K^+K^-\pi^-\pi^+$
I_F, IV_F, V_F, V_S,VII_F	$c\bar{s}/s\bar{c}$	$D_s^+D_s^-$	4	$K^+K^-\pi^+K^+K^-\pi^-$
II_F	$u\bar{c}/s\bar{d}$	$\bar{D}^0K^{*0}$		$K^+\pi^-K^+\pi^-$
II_F, VI_F	$c\bar{c}/s\bar{s}$	$J/\psi\phi$	4	$e^+e^-K^+K^-$
IV_F, IV_D V_F, V_S	$c\bar{u}/u\bar{c}$	$D^0\bar{D}^0$	4, 3	$K^-\pi^+K^+\pi^-$
IV_F V_F, V_S	$c\bar{d}/d\bar{c}$	D^+D^-	4	$K^-\pi^+\pi^+K^+\pi^-\pi^-$
IV_D, V_F, V_S	$u\bar{u}/u\bar{u}$	$\rho^0\rho^0$	3, 4	$\pi^+\pi^-\pi^+\pi^-$
II_D, VI_F	$s\bar{s}/u\bar{u}$	$\phi\rho^0$	3, 4	$K^+K^-\pi^+\pi^-$
V_F, V_S, VII_F	$s\bar{s}/s\bar{s}$	$\phi\phi$	4	$K^+K^-K^+K^-$
I_D, IV_D, V_F, V_S, VII_F	$u\bar{s}/s\bar{u}$	K^+K^-	3, 4	K^+K^-
V_F,V_S, VII_F	$d\bar{s}/s\bar{d}$	$K^{*0}\bar{K}^{*0}$	4	$K^+\pi^-K^-\pi^+$
IV_D, V_F, V_S	$u\bar{d}/d\bar{u}$	$\pi^+\pi^-$	3, 4	$\pi^+\pi^-$
I_S	$s\bar{c}/u\bar{s}$	$D_s^-K^+$		$K^+K^-\pi^-K^+$
I_S, VII_S	$s\bar{c}/c\bar{d}$	$D_s^-D^+$		$K^+K^-\pi^-K^-\pi^+\pi^+$
I_S, VII_S	$s\bar{u}/u\bar{d}$	$K^-\pi^+$		$K^-\pi^+$
I_S, IV_S	$c\bar{s}/s\bar{u}$	$D_s^+K^-$		$K^+K^-\pi^+K^-$
II_S	$u\bar{c}/s\bar{s}$	$\bar{D}^0\phi$		$K^+\pi^-K^+K^-$
II_S, VI_S	$c\bar{c}/s\bar{d}$	$J/\psi K_S^0$	4, 1	$e^+e^-\pi^+\pi^-$
II_S, VI_S, VII_S	$u\bar{u}/s\bar{d}$	$\rho^0K_S^0$	3, 1	$\pi^+\pi^-\pi^+\pi^-$
II_S	$c\bar{u}/s\bar{s}$	$D^0\phi$		$K^-\pi^+K^+K^-$
IV_S	$u\bar{c}/u\bar{u}$	$\bar{D}^0\rho^0$		$K^+\pi^-\pi^+\pi^-$
IV_S	$d\bar{c}/u\bar{d}$	$D^-\pi^+$		$K^+\pi^-\pi^-\pi^+$
IV_S	$c\bar{c}/u\bar{c}$	$J/\psi\bar{D}^0$		$e^+e^-K^+\pi^-$
IV_S	$c\bar{u}/u\bar{u}$	$D^0\rho^0$		$K^-\pi^+\pi^+\pi^-$
IV_S	$c\bar{d}/d\bar{u}$	$D^+\pi^-$		$K^-\pi^+\pi^+\pi^-$
IV_S	$c\bar{s}/s\bar{u}$	$D_s^+K^-$		$K^+K^-\pi^+K^-$
VI_S, VII_S	$s\bar{s}/s\bar{d}$	ϕK_S^0	1	$K^+K^-\pi^+\pi^-$
I_D	$c\bar{d}/s\bar{u}$	D^+K^-		$K^-\pi^+\pi^+K^-$
II_D	$c\bar{u}/s\bar{d}$	D^0K^{*0}		$K^-\pi^+K^+\pi^-$

2. the Forward Region, with $1.2 < |\eta| < 3.5$ (60 mrad $< \theta <$ 600 mrad),

Table 5 lists the geometrical acceptance for various basic nonleptonic decay modes, requiring that all decay products be accepted. Results are presented for of the detector regions separately, and in combination. We have applied a few additional cuts, anticipating some details of a realistic detector:

1. In the Central Region, C_D, the azimuthal regions within $\pm 15°$ from the vertical are

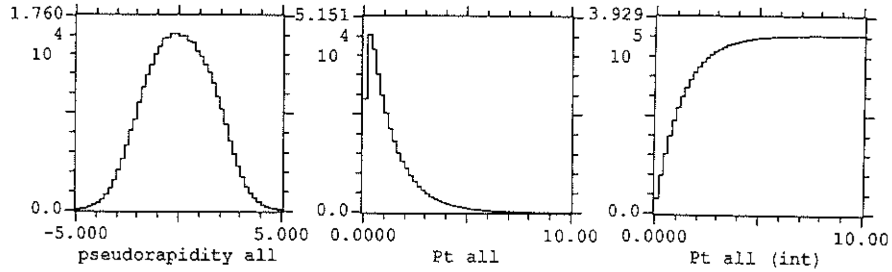

Figure 4: **a)** Distribution in pseudorapidity η, **b)** the P_t distribution , and **c)**, the integral of the P_t distribution for B-decay products at RHIC, according to an ISAJET simulation that averages over the 12 decay modes in Table 5. 87% of all B-decay products have $P_t < 2.5$ GeV/c.

excluded, anticipating an effective dead region due to use of a transverse dipole analysis magnet.

2. The P_t of all detected $\pi^\pm$ and $K^\pm$ must be greater than 0.3 GeV/c,
3. the P_t of all detected electrons (or muons) must be greater than 1.0 GeV/c,
4. Any K_S^0 must decay before leaving the silicon vertex detector. If this cut is not applied it is likely that the K_S cannot be associated reliably with the B-decay vertex.
5. An alternative Central detector configuration, C$_S$, usesa solenoid magnet to maintain full azimuthal coverage. There would be no Forward detector associated with this configuration.

The eventual goal of the BCD is to study CP violation in neutral B decays, which requires tagging of the second B in the event. Tagging will imply an additional loss of acceptance, especially for detector configurations with only partial coverage. While the precise tagging strategy will evolve over time, we estimate the effect of tagging on acceptance by supposing that the second B in the event decays according to $B^0 \to D^{*-}e^+\nu$, with $D^{*-} \to \bar{D}^0\pi^-$ and $\bar{D}^0 \to K^+\pi^-$, and that we must accept the electron and two of three daughter hadrons for a valid tag. Table 6 presents calculations for the acceptance for this tag. and then Table 7 extends the results of Table 5 to include the requirement of a tag.

We draw our main conclusions as to how the acceptance influences the detector configuration from Table 7:

1. Of the various regions, a Central detector with a solenoid magnet would be the most efficient by itself. However, it is only about 1.5 times as efficient as a Forward-only detector.

Table 5: Geometric acceptance for single B decays, estimated with an ISAJET simulation. The geometric (and P_t) cuts are described in the text. Region C$_D$ = Central Dipole, F = Forward, and C$_S$ = Central Solenoid.

Decay Mode	All-Charged Daughters	Detector Region			
		C$_D$	F	C$_D$+F	C$_S$
$B^+ \to \bar{D}^0\pi^+$	$K^+\pi^-\pi^+$	0.129	0.151	0.605	0.229
$B^+ \to D_s^+\bar{D}^0$	$K^+K^-\pi^+K^+\pi^-$	0.062	0.091	0.359	0.135
$B^+ \to J/\psi K^+$	$e^+e^-K^+$	0.092	0.106	0.384	0.163
$B_d^0 \to D^-\pi^+$	$K^+\pi^-\pi^-\pi^+$	0.074	0.103	0.374	0.150
$B_d^0 \to J/\psi K_S^0$	$e^+e^-\pi^+\pi^-$	0.044	0.051	0.192	0.087
$B_d^0 \to \pi^+\pi^-$	$\pi^+\pi^-$	0.227	0.225	0.783	0.327
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+$	0.102	0.142	0.486	0.190
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+\pi^-\pi^+$	0.034	0.044	0.143	0.063
$B_s^0 \to D_s^-\pi^+\pi^+\pi^-$	$K^+K^-\pi^-\pi^+\pi^-\pi^+\pi^+\pi^-$	0.008	0.010	0.043	0.019
$B_s^0 \to \bar{D}^0 K^{*0}$	$K^+\pi^-K^+\pi^-$	0.084	0.116	0.436	0.166
$B_s^0 \to \rho^0 K_S^0$	$\pi^+\pi^-\pi^+\pi^-$	0.069	0.080	0.288	0.126
$B_s^0 \to K^+K^-$	K^+K^-	0.227	0.225	0.783	0.327
Average		0.096	0.112	0.406	0.165

Table 6: Geometric acceptance for the tagging decay $B_d^0 \to D^{*-}e^+\nu$, estimated with an ISAJET simulation. We require the e^+ and only two of three hadrons.

Decay Mode	All-Charged Daughters	Detector Region			
		C$_D$	F	C$_D$+F	C$_S$
$B_d^0 \to D^{*-}e^+\nu$	$K^+\pi^-\pi^-e^+$	0.105	0.111	0.389	0.168

2. While a Central detector with a dipole magnet is less efficient by itself compared to a solenoid magnet, the dipole magnet permits the addition of a Forward detector. The combination of a Central dipole and Forward detector is about 5 times more efficient than a Central solenoid.
3. Because of the P_t cuts the acceptance is lower for high-multiplicity decays, and for decays with $J/\psi \to e^+e^-$ or $\mu^+\mu^-$. The soft pion from D^* decays typically fails the P_t cut.

We now estimate how many B decays into various modes might be reconstructed in a year of running at RHIC. We will consider only the combination of a Central dipole detector along with a Forward detector from now on.

Table 7: Geometric acceptance for tagged B decays, estimated with an ISAJET simulation. We assume that all decays of the second B are useful for tagging, but use the decay $B_d^0 \to D^{*-}e^+\nu$ to estimate the correlation in acceptance of the tagging and tagged B's.

Decay Mode	All-Charged Daughters	Detector Region C_D	F	C_D+F	C_S
$B^+ \to \bar{D}^0\pi^+$	$K^+\pi^-\pi^+$	0.015	0.020	0.238	0.046
$B^+ \to D_s^+\bar{D}^0$	$K^+K^-\pi^+K^+\pi^-$	0.007	0.013	0.139	0.027
$B^+ \to J/\psi K^+$	$e^+e^-K^+$	0.011	0.017	0.146	0.032
$B_d^0 \to D^-\pi^+$	$K^+\pi^-\pi^-\pi^+$	0.010	0.014	0.139	0.032
$B_d^0 \to J/\psi K_S^0$	$e^+e^-\pi^+\pi^-$	0.005	0.008	0.074	0.018
$B_d^0 \to \pi^+\pi^-$	$\pi^+\pi^-$	0.025	0.035	0.301	0.063
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+$	0.012	0.021	0.190	0.039
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+\pi^-\pi^+$	0.005	0.006	0.059	0.013
$B_s^0 \to D_s^-\pi^+\pi^+\pi^-$	$K^+K^-\pi^-\pi^+\pi^-\pi^+\pi^+\pi^-$	0.001	0.002	0.018	0.003
$B_s^0 \to \bar{D}^0K^{*0}$	$K^+\pi^-K^+\pi^-$	0.011	0.019	0.166	0.031
$B_s^0 \to \rho^0K_S^0$	$\pi^+\pi^-\pi^+\pi^-$	0.011	0.012	0.105	0.028
$B_s^0 \to K^+K^-$	K^+K^-	0.025	0.035	0.301	0.063
Average		0.012	0.017	0.156	0.033

For this we need three further ingredients. First, we suppose that a $b\bar{b}$ pair materializes only into mesons, and in the ratios

$$B_u : B_d : B_s = \frac{1-\epsilon}{2} : \frac{1-\epsilon}{2} : \epsilon.$$

We estimate that $\epsilon = 0.25$, and hence

$$B_u : B_d : B_s = 0.375 : 0.375 : 0.25.$$

We also need an estimate of the efficiency of the tracking and vertexing of the B decay, which we take to be 0.33. This is a representative value emerging from simulations of the performance of the silicon vertex detector.

Finally, we need estimates of the branching ratios of the B's to the modes of interest, as well as for the secondary decays. The latter branching ratios have been already listed.

A sample rate calculation follows:

- Luminosity .... 10^{32} cm^{-2}sec^{-1}
- Standard running year of 10^7 sec $\Rightarrow$ .... 1 fb^{-1}
- $\sigma_{b\bar{b}} = 12.5$ μb $\Rightarrow$ .... 1.25×10^{10} B-$\bar{B}$ pairs

- 3/4 $B^\pm$ per B-$\bar{B}$ pair $\Rightarrow$ .... 9.38×10^9 $B^\pm$
  3/4 B_d^0 or $\bar{B}_d^0$ per B-$\bar{B}$ pair $\Rightarrow$ .... 9.38×10^9 B_d^0
  1/2 B_s^0 or $\bar{B}_s^0$ per B-$\bar{B}$ pair $\Rightarrow$ .... 6.25×10^9 B_s^0
- B.R. for $B^+ \to \bar{D}^0\pi^+$; $\bar{D}^0 \to K^+\pi^-$: $(0.004)(0.04) = 1.6 \times 10^{-4}$
  $\Rightarrow$ .... 1.5×10^6 $B^+ \to K^+\pi^-\pi^+$
- Geometric acceptance (column C_D+F of Table 5) = 0.605;
  Vertexing and tracking efficiency = 0.33;
  $\Rightarrow$ Overall efficiency = 0.2 $\Rightarrow$ .... 3×10^5 reconstructed $B^+ \to K^+\pi^-\pi^+$

In this manner we obtain the rate estimates of Table 8, taking branching ratios for B decay from the model calculations of Bauer, Stech, and Wirbel.[12] That paper does not directly predict branching ratios for the B_s. In the spectator model we expect that B.R.$(B_s \to D_s^-\pi^+)$ = B.R.$(B_d \to D^-\pi^+)$, B.R.$(B_s \to \rho^0K_S^0)$ = B.R.$(B_d \to \rho^0\pi^0)$, and B.R.$(B_s \to K^+K^-)$ = B.R.$(B_d \to K^+\pi^-) \approx \lambda^2$B.R.$(B_d \to \pi^+\pi^-)$, which leads to the values in Table 8. According to Ref. [12], B.R.$(B_s \to \rho^0K_S^0)$ will be suppressed by strong-interaction effects and will be comparable to that for $B_s \to K^+K^-$, although the latter is doubly-CKM-suppressed. For $B_s \to K^+K^-$ we are ignoring the possibility of an important contribution from penguin graphs. This contrasts with sec. 2.3 below where effects of large penguin contributions are considered.

An important first phase of the BCD will be the measurement of the branching ratios of the rarer decays modes that are of interest for studies of CP violation. Many of these modes, such as those for B_s decay and for modes with significant penguin contributions, will likely only be studied at a hadron collider.

2.2 B_s-$\bar{B}_s$ Mixing

The neutral B-meson systems B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ each exhibit mixing, characterized by the parameters $x = \Delta M/\Gamma$. Theoretically, the mixing arises from a box diagram that is dominated by the top quark, and that is the same for B_d and B_s except for $d \leftrightarrow s$ interchange.[13] Then we can write

$$x_s = x_d \left|\frac{V_{ts}}{V_{td}}\right|^2 \left[\frac{\tau_s f_s^2 B_s}{\tau_s f_d^2 B_d}\right],$$

where τ, f, and B are the lifetime, decay constant and bag constant. The quantity in brackets is near unity, and may eventually be calculable in lattice gauge models. If so, mixing measurement will provide extra constraints on the CKM matrix elements. For now we suppose this quantity is unity.

We now have

$$x_s \simeq \frac{x_d}{\lambda^2((1-\rho)^2+\eta^2)} \simeq \frac{14.5}{(1-\rho)^2+\eta^2},$$

referring to our previous parametrization of the CKM matrix, and noting the experimental results that $\lambda = 0.22$ and $x_d \simeq 0.7$. Present knowledge of parameters ρ and η then suggests that $8 < x_s < 20$ for $100 < M_t < 150$ GeV$/c^2$.

At a hadron collider the measurement of B_s mixing previews many of the challenges of CP violation in neutral B's. This is because we must know the particle/antiparticle

Table 8: Rate estimates for reconstructed B decays. B.R.(B) is the branching ratio for the two-body B decay, estimated according to Bauer *et al.*[12] B.R.(Tot) is the product of B.R.(B) and the secondary branching ratios. Eff. is the product of the geometric acceptance from column C_D+F of Table 5 and a factor 0.33 for the efficiency of tracking and vertexing. The reconstructed-event samples are for an integrated luminosity of 1 fb^{-1}, collectable in 1 year of running at a luminosity of 10^{32} $\text{cm}^{-2}\text{sec}^{-1}$.

Decay Mode	All-Charged Daughters	B.R.(B)	B.R.(Tot)	Eff.	Recon. Decays
$B^+ \to \bar{D}^0\pi^+$	$K^+\pi^-\pi^+$	0.004	1.6×10^{-4}	0.20	3×10^5
$B^+ \to D_s^+\bar{D}^0$	$K^+K^-\pi^+K^+\pi^-$	0.008	4.8×10^{-6}	0.12	5.4×10^3
$B^+ \to J/\psi K^+$	$e^+e^-K^+$	6×10^{-4}	4.2×10^{-5}	0.13	5.1×10^4
$B_d^0 \to D^-\pi^+$	$K^+\pi^-\pi^-\pi^+$	0.006	4.8×10^{-4}	0.12	5.4×10^5
$B_d^0 \to J/\psi K_S^0$	$e^+e^-\pi^+\pi^-$	3×10^{-4}	1.4×10^{-5}	0.064	8.4×10^3
$B_d^0 \to \pi^+\pi^-$	$\pi^+\pi^-$	2×10^{-5}	2×10^{-5}	0.26	4.9×10^4
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+$	0.005	1.5×10^{-4}	0.16	1.5×10^5
$B_s^0 \to D_s^-\pi^+$	$K^+K^-\pi^-\pi^+\pi^-\pi^+$	0.005	2×10^{-4}	0.048	6.0×10^4
$B_s^0 \to D_s^-\pi^+\pi^+\pi^-$	$K^+K^-\pi^-\pi^+\pi^-\pi^+\pi^+\pi^-$	0.01	4×10^{-4}	0.014	3.5×10^4
$B_s^0 \to \bar{D}^0K^{*0}$	$K^+\pi^-K^+\pi^-$	0.005	1.3×10^{-4}	0.15	1.2×10^5
$B_s^0 \to \rho^0K_S^0$	$\pi^+\pi^-\pi^+\pi^-$	10^{-6}	7×10^{-7}	0.096	420
$B_s^0 \to K^+K^-$	K^+K^-	8×10^{-7}	8×10^{-7}	0.26	1.3×10^3

character of the B when it is produced to complete the analysis. To study mixing, we must also know the particle/antiparticle character of the B when it decays. Hence the observed decay mode must not be self-conjugate. We will use the modes $B_s \to D_s^-\pi^+$ and $\bar{D}^0K^{*0}$ as examples.

If we start with N B_s mesons at $t = 0$, at time t we have

$$N_{B(\bar{B})} = (N/2)e^{-t}(1 \pm \cos xt),$$

where in this section we measure the proper time t in units of the B lifetime (assumed equal for the two mass eigenstates). We actually start with an equal number of B and $\bar{B}$ mesons at $t = 0$, and so collect four decay distributions, labelled by the particle/antiparticle character at creation and decay. By taking the appropriate sum and difference of these fours distributions we can isolate the distribution

$$D(t) = Ne^{-t}\cos xt,$$

while the number of B's left at time t is, of course, Ne^{-t}.

The value of x can be determined if the oscillations of $D(t)$ are well resolved. First we consider the statistical power required to determine x, supposing that the time resolution is

perfect. A simple criterion is that the size of the oscillations must be clearly non-zero; we require that the first few quarter cycles each to have 25 events to maintain a 5-σ distinction. In, say, the 8th quarter cycle there are $\sim (N\pi/2x)e^{-4\pi/x}$ events, so we need $N \gtrsim 50xe^{4\pi/x}/\pi$. Then about 600 events would be needed to measure the oscillations for any x from 8 to 25.

However, if the time resolution deteriorates to a quarter cycle, the oscillation will become unrecognizable. Thus we desire an resolution $\sigma_s < \pi/2x$, and hence $\sigma_s < 1/16$ to resolve x up to 25. The achieved value of σ_s depends on the quality of the vertex detector. To a good approximation $\sigma_s = \sigma_z/c\tau$ where σ_z is the spatial resolution of the vertex detector, and τ is the B lifetime. Since $c\tau = 360$ μm for B's, and $\sigma_z \sim 20$ μm is typically obtained with silicon vertex detectors, we expect that mixing parameters of $x \lesssim 25$ should indeed be resolvable. Extreme care in the construction of the vertex detector will be required to resolve larger values of x_s.

Thus 600 B_s decays will be sufficient to measure x_s over its presently expected range, provided each decay is tagged as to whether the B started as a particle or antiparticle. For the tag we must look at the other B in the event. If we are to maintain a reasonable tagging efficiency, we must identify decays of the other B that contain neutrals. To have confidence that we are associating tracks with a B decay and not the primary hadronic interaction, the tracks of the second B must come from a secondary vertex. Neutral tracks are not useful for this. Hence we will have only a partial reconstruction of the second B in general. Without a mass constraint we cannot be sure that all charged tracks from the second B have been found, and so we will not be able, in general, to tell whether the second B was a B_u, B_d, or B_s.

If the second B is neutral, it may have oscillated before we detect it. Thus the tag cannot be perfect. Indeed, a neutral B with mixing parameter x decays in the opposite particle/antiparticle state to that in which it was created with probability $x^2/2(1+x^2)$. For a B_d with $x_d = 0.7$ the probability of a mistag is 1/6, but for a B_s with $x_s > 8$ it is essentially 1/2. The B_s oscillates so rapidly that is is useless as a tag.

If a fraction p of our events are mistagged, the distribution $D(t)$ describing the oscillation becomes $(1-2p)Ne^{-t}\cos xt$, and the error on x extracted from analysis of D grows by a factor $1/(1-2p)$. It will now require $1/(1-2p)^2$ as many events to achieve a given statistical significance. This dilution of the tag is largely unavoidable at a hadron collider.

We estimate the dilution factor by recalling our hypothesis as to the relative product of the various B's, assumed uncorrelated with the type of the other B,

$$B_u : B_d : B_s = \frac{1-\epsilon}{2} : \frac{1-\epsilon}{2} : \epsilon.$$

We can then write

$$1 - 2p = \frac{1-\epsilon}{2} + \frac{1-\epsilon}{2}\frac{1}{1+x_d^2} + \epsilon\frac{1}{1+x^2} \approx 0.6,$$

using $\epsilon = 0.25$, $x_d = 0.7$, and $x_s > 8$. Our earlier requirement of 600 perfectly tagged B_s decays now becomes 1500 decays with a diluted tag.

The question remains as to the mechanism of the tag. The most straightforward tag is on the sign of the leading lepton in a semileptonic B decay. Even if both electrons and muons are identified, only about 20% of all B's could be tagged. In Table 6 (based on calculations similar to those for Table 5) we estimated that the geometric acceptance for a semileptonic decay would be about 40%, taking into account P_t cuts. Also, only about 30% of the semileptonic decays would be clearly identified as belonging to a secondary vertex. Hence the overall efficiency of a semileptonic tag would be about 3%. Then instead of needing 1500 reconstructed B_s decays for the oscillation analysis, we will need about 56,000.

In Table 8 we estimated the yield of reconstructed B_s decays to be 2.6×10^5 in 4 modes useful for an oscillation study. Other modes can be added as well, raising the yield to at least 5×10^5, or 10 times that needed to demonstrate the oscillation signal. Equivalently, even if the RHIC ran at a luminosity of only 10^{31} cm^{-2}sec^{-1}, the BCD could resolve B_s mixing.

2.3 *CP* Violation in Self-Tagging *B* Decays

The study of CP violation in the decays of neutral B's to CP eigenstates will also require us to confront the complexities of tagging. First we consider the prospects for demonstration of CP violation in the B-meson system without the need for tagging.

Recall that the proposed method to study CP violation is measurement of the asymmetry

$$A = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}.$$

In this measurement we need only know the particle/antiparticle character of the B at the time of its decay. So if final state f is distinct from $\bar{f}$ then no tagging is needed. Decays such as $B^+ \to \bar{D}^0\pi^+$, $B_d^0 \to D^-\pi^+$, and $B_s^0 \to D_s^-\pi^+$ are all of this type, which is often called self-tagging.

For a decay rate Γ to be different from its CP-conjugate rate $\bar{\Gamma}$, the decay process must involve the interference of two amplitudes. Labelling these a_1 and a_2 and supposing a_1 to be real, we can write

$$\Gamma = \left|a_1 + a_2 e^{i\delta_{WK}} e^{i\delta_{ST}}\right|^2.$$

For the CP-conjugate decay, the weak-interaction phase changes sign while the strong-interaction phase does not (due to CP invariance of the strong interaction). Hence

$$\bar{\Gamma} = \left|a_1 + a_2 e^{-i\delta_{WK}} e^{i\delta_{ST}}\right|^2.$$

To have $\Gamma \neq \bar{\Gamma}$, not only must two amplitudes interfere, but also both the weak and strong phases must differ between the two amplitudes. For a large effect, the two amplitudes should be of comparable magnitude.

As such, the more prominent two-body nonleptonic B decays to self-tagging final states are likely to have very small CP asymmetries. Referring to Tables 2-4, we see that the decays with a high rate are due to the (tree-level) spectator graphs. Even when both spectator graphs contribute, their CKM structure is the same. The speculation is that the largest asymmetries in self-tagging modes will occur when penguin graphs interfere with suppressed spectator graphs.[14, 15, 16] Table 9 summarizes the events rates expected in BCD for three self-tagging modes whose CP asymmetries may be in the range 10-40% in the optimistic view of Ref. [15], although a more typical prediction[16] is for asymmetries of 1-3%. Even the size of the penguin contribution to the branching ratios is somewhat controversial, and

Table 9: Rate estimates for self-tagging B decays. B.R.(B) is the branching ratio for the two-body B decay, estimated according to Chau.[15] B.R.(Tot) is the product of B.R.(B) and the secondary branching ratios. Eff. is the product of the geometric acceptance in a detector configuration C$_D$F as in Table 5 and a factor 0.33 for the efficiency of tracking and vertexing. The reconstructed event samples are for an integrated luminosity of 1 fb^{-1}, collectable in 1 year of running at a luminosity of 10^{32} cm^{-2}sec^{-1}.

Decay Mode	All-Charged Daughters	B.R.(B)	B.R.(Tot)	Eff.	Recon. Decays
$B^+ \to \rho^0 K^+$	$\pi^+\pi^-K^+$	10^{-5}	10^{-5}	0.20	1.9×10^4
$B_d^0 \to K^{*+}\pi^-$	$\pi^+\pi^-\pi^+\pi^-$	10^{-4}	2.2×10^{-5}	0.09	1.9×10^4
$B_s^0 \to K^{*+}K^-$	$\pi^+\pi^-\pi^+K^-$	2×10^{-4}	6.7×10^{-5}	0.10	4.2×10^4

the values given below are considerably larger than those calculated in the model of Bauer, Stech, and Wirbel.[12]

If we collect N events in a self-tagging mode with CP asymmetry A, then the statistical significance of our measurement of A is

$$S = \frac{A}{\sigma_A} = \sqrt{N}\frac{A}{\sqrt{1-A^2}}.$$

The minimum value of A that could be resolved to S standard deviations is

$$A_{\min,S\sigma} = \frac{S}{\sqrt{N+S^2}}.$$

Thus with 2×10^4 events we could resolve an asymmetry as small as $A = 0.035$ to 5 σ.

Ref. [16] argues that decays with penguin contributions involving $b \to d$ such as $B^+ \to K^+K^0$ should have an asymmetry of 10-15%, but the branching ratios will be more like 10^{-6}. The BCD should be sensitive to asymmetries of about 2% or greater in such modes.

Should the branching ratios and CP asymmetries in self-tagging modes be as large as the most optimistic predictions they will likely provide the first evidence for CP violation in the B system. For the typical predicted asymmetry, the signal in self-tagging modes is only comparable to that in decays to CP eigenstates, discussed in the next section. The theoretical interpretation in terms of CKM-matrix parameters of an observed asymmetry in a self-tagging decay may be unclear, due to uncertainties in the calculation of the penguin graphs.

2.4 *CP* Violation in Decays of Neutral *B*'s to *CP* Eigenstates

The most detailed knowledge to be gained from the BCD is in the study of CP violation in the decays of neutral B's to CP eigenstates. This topic is interesting in part because of its intricacy. We first present a review of the context of CP violation in the B-meson system in some detail.[18]

2.4.1 A CP Primer

Perhaps the best way to understand CP violation in the B system is by comparison with the K system. In the K system, the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are eigenstates of the strong and electromagnetic interactions, which conserve strangeness. However, these states are not eigenstates of the weak interactions, which violate strangeness, and which are responsible for Kaon decay. Taking into account the weak interactions, one writes the 2×2 Hamiltonian (in the K^0-$\bar{K}^0$ basis)

$$H = M - \frac{i}{2}\Gamma \,, \tag{1}$$

where the mass matrix M and the decay matrix Γ are Hermitian. (Since neutral Kaons do decay, H itself is not Hermitian.) CPT invariance implies that the diagonal components of H are equal, and if CP is conserved M and Γ are real. Allowing for the possibility of CP violation, diagonalizing the Hamiltonian

$$H = \begin{bmatrix} m & M_{12} \\ M_{12}^* & m \end{bmatrix} - \frac{i}{2}\begin{bmatrix} \gamma & \Gamma_{12} \\ \Gamma_{12}^* & \gamma \end{bmatrix} \tag{2}$$

yields the eigenstates

$$\begin{aligned} |K_S^0\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right], \\ |K_L^0\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right], \end{aligned} \tag{3}$$

where

$$\frac{1-\epsilon}{1+\epsilon} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \,. \tag{4}$$

If M and Γ were real, then ϵ would be zero, so that a nonzero ϵ is evidence for $\Delta S = 2$ CP violation. (In the limit of vanishing ϵ, the weak states would be CP eigenstates: K_S would have CP +; K_L would have CP −.) This will be referred to as CP violation in the mixing. The mass and width differences between the states $|K_L^0\rangle$ and $|K_S^0\rangle$ are given by

$$\begin{aligned} \Delta M &= 2\,\mathrm{Re}\left[\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}, \\ \Delta\Gamma &= -4\,\mathrm{Im}\left[\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}. \end{aligned} \tag{5}$$

It is also possible to have CP violation in the decays of Kaons, parametrized by the $\Delta S = 1$ CP-violating parameter ϵ', which arises from different isospin phases in the amplitudes for the decays $K \to 2\pi$:

$$\begin{aligned} a_0 &= \langle \pi\pi,\ I = 0|H_W|K^0\rangle \,, \\ a_2 &= \langle \pi\pi,\ I = 2|H_W|K^0\rangle \,, \end{aligned} \tag{6}$$

and

$$\epsilon' \propto \mathrm{Im}\left(\frac{a_2}{a_0}\right) . \tag{7}$$

In the standard model, it is expected that $\epsilon' \ll \epsilon$, and experimentally one finds that

$$\frac{\epsilon'}{\epsilon} \lesssim \mathrm{O}(10^{-3}). \tag{8}$$

Therefore, in the Kaon system, CP violation with $\Delta S = 2$ is much larger than that with $\Delta S = 1$.

For B mesons, the mixing formalism is identical to that given in Eqs. 1-5. However, there are some significant differences between the B system and the K system. First of all, since B mesons are so heavy, the phase space for their decays is quite large. Therefore both B and $\bar{B}$ have essentially the same lifetime, so that $\Delta\Gamma/\Gamma \ll 1$. Furthermore, calculations based on the box diagram in the standard model have shown that, for the B system, $\Gamma_{12} \ll M_{12}$, which leads to the result that $\Delta\Gamma \ll \Delta M$. (We note that this is quite different than the K system, where there is a substantial lifetime difference, due to the difference between the 2π and 3π channels.) We will therefore neglect $\Delta\Gamma$ in what follows.

For the same reason, the $\Delta B = 2$ CP-violating parameter in the B system, ϵ_B, is also small:

$$\left|\frac{1+\epsilon_B}{1-\epsilon_B}\right| \simeq 1 + \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}} \,. \tag{9}$$

Using the box diagram, ϵ_B has been calculated in the Standard Model:

$$\epsilon_B = \begin{cases} \mathrm{O}(10^{-4}), & B_d, \\ \mathrm{O}(10^{-5}), & B_s. \end{cases} \tag{10}$$

It therefore seems that the prospects for observation of $\Delta B = 2$ CP-violating phenomena are essentially hopeless.

However, in the B system, the situation is reversed with respect to the Kaon system, namely, CP violation in B decays ($\Delta B = 1$) can be large. In order to observe CP violation, one needs interference between two amplitudes with different phases. The most interesting processes are those involving a final state f to which both the B^0 and $\bar{B}^0$ can decay. Then the needed interference will arise because of B^0-$\bar{B}^0$ mixing.[1] Even more interesting is the case when f is a CP eigenstate, and also when only a single weak amplitude (and its CP-conjugate) are involved, so that certain strong-interaction complications cancel, as noted later.

First, for B-$\bar{B}$ mixing the relevant parameter is x_q, the ratio of the energy of the oscillation (*i.e.*, the mass difference) and the total width for the B_q mesons ($q = d, s$):

$$x_q = \frac{\Delta M}{\Gamma} \quad \frac{(\text{transition energy})}{(\text{mean total width})} \,. \tag{11}$$

After all, mixing hardly matters if the particle decays before it has a chance to oscillate into its antiparticle.

[1] There are also decays in which the CP violation is due to final state interactions as discussed in the previous subsection, but these are less useful theoretically, since knowledge of hadronic matrix elements is needed to extract information about the CKM matrix.

This can be seen explicitly by considering the time evolution of B mesons. Because of B^0-$\bar{B}^0$ mixing, a state which starts out as a pure B^0 or $\bar{B}^0$ will evolve in time to a mixture of B^0 and $\bar{B}^0$:

$$\begin{aligned} |B^0(t)\rangle &= f_+(t)|B^0\rangle + \tfrac{q}{p} f_-(t)|\bar{B}^0\rangle \, , \\ |\bar{B}^0(t)\rangle &= \tfrac{p}{q} f_-(t)|B^0\rangle + f_+(t)|\bar{B}^0\rangle \, . \end{aligned} \tag{12}$$

Here, $|B^0\rangle$ represents a pure B^0 state at $t = 0$, $|\bar{B}^0\rangle$ represents a pure $\bar{B}^0$ state at $t = 0$,

$$\frac{q}{p} = \frac{1 - \epsilon_B}{1 + \epsilon_B} \, , \tag{13}$$

and

$$\begin{aligned} f_+(t) &= e^{-iMt} e^{-\Gamma t/2} \cos(\Delta M t/2) \, , \\ f_-(t) &= i e^{-iMt} e^{-\Gamma t/2} \sin(\Delta M t/2) \, . \end{aligned} \tag{14}$$

From Eq. 14, it is clearly seen that the competition between ΔM and Γ is the important consideration for seeing CP violation in B decays.

CP violation is manifested in a nonzero value of the asymmetry

$$A_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} \, . \tag{15}$$

If we consider a nonleptonic final state f such that both B^0 and $\bar{B}^0$ can decay both to it (and to its CP-conjugate state $\bar{f}$), this asymmetry can be calculated from Eqs. 12 and 14. We have

$$\begin{aligned} \Gamma(\overset{(-)}{B}{}^0(t) \to f) &= \left| \langle \overset{(-)}{f} | \overset{(-)}{B}{}^0(t) \rangle \right|^2 \\ &= \left| \langle \overset{(-)}{f} | \overset{(-)}{B}{}^0 \rangle \right|^2 e^{-t/\tau} \left[\cos^2 \tfrac{\Delta M t}{2} + \left| \overset{(-)}{\alpha}_f \right|^2 \sin^2 \tfrac{\Delta M t}{2} - \mathrm{Im}\, \overset{(-)}{\alpha}_f \sin \Delta M t \right] \end{aligned} \tag{16}$$

where we have introduced

$$\alpha_f = \frac{q}{p}\, \rho_f \, , \qquad \bar{\alpha}_f = \frac{p}{q}\, \bar{\rho}_f \, , \tag{17}$$

and

$$\rho_f = \frac{\langle f|\bar{B}^0\rangle}{\langle f|B^0\rangle} \, , \qquad \bar{\rho}_f = \frac{\langle \bar{f}|B^0\rangle}{\langle \bar{f}|\bar{B}^0\rangle} \, . \tag{18}$$

There are several points worth noting here. First of all, q/p **is a pure phase**. This can be seen from Eqs. 4 and 13, for $\Gamma_{12} \ll M_{12}$:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}} \, . \tag{19}$$

Secondly, **when only one amplitude contributes to** $B^0 \to f$ (and its CP-conjugate to $\bar{B}^0 \to \bar{f}$) then (recall the argument of the previous subsection about the converse)

$$\left|\langle f|B^0\rangle\right| = \left|\langle \bar{f}|\bar{B}^0\rangle\right| , \qquad \left|\langle \bar{f}|B^0\rangle\right| = \left|\langle f|\bar{B}^0\rangle\right| , \tag{20}$$

i.e., $|\rho_f| = |\bar{\rho}_f|$. If there were no mixing there would now be no CP violation. However, in the presence of mixing case Eq. 15 becomes

$$A_f(t) = \frac{-(\mathrm{Im}\,\alpha - \mathrm{Im}\,\bar{\alpha}) \sin \Delta M t}{2\cos^2(\Delta M t/2) + 2\left|\rho_f\right|^2 \sin^2(\Delta M t/2) - (\mathrm{Im}\,\alpha + \mathrm{Im}\,\bar{\alpha}) \sin \Delta M t} \tag{21}$$

There may still be complications, however, due to ρ_f (and $\bar{\rho}_f$). To see this, it is helpful to introduce the Cabibbo-Kobayashi-Maskawa (CKM) matrix here. A convenient parametrization of the CKM matrix (slightly different from that used earlier) is[19]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \rho e^{-i\delta} \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} . \tag{22}$$

In this parametrization the CKM matrix elements are expanded in powers of λ, the Cabibbo angle ($\lambda = 0.22$).[2] This is particularly convenient because one can estimate the size of certain B-decay diagrams just by counting powers of λ. For example, consider the final state $D^+\pi^-$. Here,

$$\begin{aligned} \langle D^+\pi^-|B^0\rangle &\sim V_{ub}^* V_{cd} \sim O(\lambda^4) \, , \\ \langle D^+\pi^-|\bar{B}^0\rangle &\sim V_{cb} V_{ud}^* \sim O(\lambda^2) \, . \end{aligned} \tag{23}$$

Therefore $\rho_f \sim \lambda^{-2} \sim 20$. According to Eq. 15 the asymmetry goes like $1/\rho_f$ for large ρ_f, so it will be quite small for most values of t. If we had considered the final state $D^-\pi^+$, then we would have $\rho_f \sim \lambda^2 \sim 0.05$, and the asymmetry would again be small since $\mathrm{Im}\,\alpha_f$ is proportional to ρ_f. Furthermore, for both of these final states, hadronization effects are important. For example, for the final state $D^+\pi^-$ from B^0 decay the W hadronizes into the D, while from $\bar{B}^0$ decay it hadronizes into the π. There is no reliable way to calculate these effects.

These problems can be avoided by considering final states which are CP eigenstates ($f = \pm\bar{f}$). Because of mixing, interference will occur between $\langle f|B^0\rangle$ and $\langle f|\bar{B}^0\rangle$, but now the latter is equal to $\pm\langle \bar{f}|\bar{B}^0\rangle$, which is equal in magnitude to $\langle f|B^0\rangle$ according to Eq. 20. **Hence the amplitude ratio** ρ_f **will also be a pure phase** (*i.e.*, $|\rho_f| = 1$).

For example, in the decay $B^0 \to \pi^+\pi^-$ we have

$$\begin{aligned} \langle \pi^+\pi^-|B^0\rangle &\sim V_{ub}^* V_{ud} = A\rho\lambda^3 e^{i\delta} , \\ \langle \pi^+\pi^-|\bar{B}^0\rangle &\sim V_{ub} V_{ud}^* = A\rho\lambda^3 e^{-i\delta} . \end{aligned} \tag{24}$$

In addition, any hadronization phases must cancel in ρ_f, since the two diagrams are CP conjugates of one another, and the strong interactions are CP invariant. We therefore obtain $\rho_f = \exp(-2i\delta)$. And, from $|\rho_f| = 1$ we get

$$\mathrm{Im}\,\bar{\alpha}_f = -\,\mathrm{Im}\,\alpha_f . \tag{25}$$

Eq. 25 holds for any decay to a CP eigenstate that is described by a single weak amplitude. Thus, for this class of final states we have (using Eqs. 11 and 16)

$$\Gamma(\overset{(-)}{B}{}^0_q(t) \to \overset{(-)}{f}) = \left|\langle f|B_q^0\rangle\right|^2 e^{-t/\tau} \left[1 \mp \mathrm{Im}\,\alpha_f \sin x_q \tfrac{t}{\tau}\right] \tag{26}$$

[2]Note that, in V_{CKM}, other terms have phases as well, but they are of higher order in λ. Large phases appear only in V_{ub} and V_{td}.

Now Eq. 15 assumes the elegantly simple form

$$A_f(t) = -\operatorname{Im}\alpha_f \sin x_q \frac{t}{\tau} \,. \tag{27}$$

For decays to CP eigenstates, **the CP-violating parameter α_f is therefore a pure phase**, and keeps track of all phase information in the process. This is related to the parameters of the CKM matrix as follows. The expression for α_f is (Eq. 17, repeated here for clarity)

$$\alpha_f = \frac{q}{p}\rho_f \,.$$

There are therefore two sources of phase information – in the mixing (q/p), and in the decay (ρ_f). As discussed earlier, to a good approximation, q/p is a pure phase (Eq. 19). Now, M_{12} is calculable from the Standard Model box diagram for B_q^0-$\bar{B}_q^0$ mixing $(q = d, s)$, which is dominated by t-quark exchange:

$$M_{12} \sim f(M_t)\left(V_{tb}V_{tq}^*\right)^2 \,, \tag{28}$$

and therefore the phase information from the mixing is

$$\frac{q}{p} = \begin{cases} (V_{tb}^*/V_{tb})(V_{td}/V_{td}^*) = (V_{td}/V_{td}^*) \equiv e^{-2i\phi} \,, & (B_d), \\ (V_{tb}^*/V_{tb})(V_{ts}/V_{ts}^*) \simeq 1 \,, & (B_s). \end{cases} \tag{29}$$

In Eq. 29, ϕ is the phase of V_{td}. The phase information in the decay depends on whether the b-quark decays into a c- or a u-quark:

$$\rho_f = \begin{cases} (V_{ub}/V_{ub}^*) \equiv e^{-2i\delta} \,, & b \to u, \\ (V_{cb}/V_{cb}^*) = 1 \,, & b \to c. \end{cases} \tag{30}$$

As before, δ is the phase of V_{ub}. Thus it can be immediately seen that there are four classes of decays measuring different combinations of phases in the CKM matrix:

$$\begin{array}{llll} \text{Class 1.} & B_d \text{ decays with } b \to c & (e.g.\ B_d \to J/\psi K_S): & \operatorname{Im}\alpha_1 = -\sin 2\phi \,, \\ \text{Class 2.} & B_d \text{ decays with } b \to u & (e.g.\ B_d \to \pi^+\pi^-): & \operatorname{Im}\alpha_2 = -\sin 2(\delta+\phi) \,, \\ \text{Class 3.} & B_s \text{ decays with } b \to u & (e.g.\ B_s \to \rho K_S): & \operatorname{Im}\alpha_3 = -\sin 2\delta \,, \\ \text{Class 4.} & B_s \text{ decays with } b \to c & (e.g.\ B_s \to J/\psi\phi): & \operatorname{Im}\alpha_4 \simeq 0 \,. \end{array} \tag{31}$$

The CP violation in class-1 decays originates in the mass matrix, that of class-3 decays is from the decay amplitude, while that of class-2 decays is due to both effects. In addition to measuring the decay asymmetries in classes 1-3, it will be an important test of the Standard Model to search for CP violation in class-4 decays. A nonzero measurement of a CP asymmetry in this class would be a clear signal of physics beyond the Standard Model.

We summarize the main arguments of this section, with an attempt to trace their historical development:

1. CP violation in the B system is not primarily due to the magnitude of CP violation in the mass eigenstates, $\epsilon_B \ll \epsilon_K$. However, the relative amplitudes of $|B^0\rangle$ and $|\bar{B}^0\rangle$ in the mass eigenstates is given by a pure phase (since $\Gamma_{12} \ll M_{12}$ in Eqs. 4 and 13). The phase is that of V_{td}, due to the dominance of the top-quark in the box diagram that governs B^0-$\bar{B}^0$ mixing.[20]

2. CP violation can be due to interference of two decay amplitudes with different weak phases. When comparing rates for $B^0 \to f$ and $\bar{B}^0 \to f$, the phase in the mass eigenstates can enter due to B^0-$\bar{B}^0$ mixing.[21]

3. If the final state f is a CP eigenstate then strong-interaction effects cancel, and the CP violation can be entirely due to the phase in the mass eigenstates.[22] Hence a direct measurement of the phase of a CKM-matrix element is possible.

4. There can be a large weak phase in the decay amplitudes also, that of V_{ub}. (This phase can only enter in higher order in K decays.) As a consequence there are four classes of neutral-B decays to CP eigenstates, yielding three different measures of phases in the CKM matrix.[18]

The interpretation of experimental asymmetries in terms of phases of CKM-matrix elements is dependent on the phase convention adopted. The Miani-Wolfenstein representation of the CKM matrix is convenient in that here V_{tb} has, in general, a nonzero phase, which serves to emphasize the importance of mixing in CP-violation studies.

The CKM phases that govern the three classes of nonzero asymmetries are often demonstrated using the unitarity triangle,[23] shown earlier in Fig. 2. The unitarity of V_{CKM} implies

$$V_{td} + \lambda V_{ts} + V_{ub}^* \approx 0. \tag{32}$$

Hence, if these three complex matrix elements are regarded as vectors, they form a closed triangle. The goal is then to measure the three interior angles φ_1, φ_2, φ_3, and to see if the triangle closes, *i.e.*, *to see if the angles add up to* 180°. These three angles are exactly the three CKM phases that appear in the above classification of CP asymmetries: $\varphi_1 = \phi$, $\varphi_2 = \delta + \phi$, and $\varphi_3 = \delta$.

The quantities that will be directly determined by measurement of the CP-violating asymmetries (15) in decays to CP eigenstates are

$$\sin 2\varphi_1 = \sin 2\phi = \operatorname{Im}\left(\frac{V_{td}^*}{V_{td}}\right) = \frac{2\rho\sin\delta(1-\rho\cos\delta)}{1+\rho^2-2\rho\cos\delta} \,, \tag{33}$$

$$\sin 2\varphi_2 = \sin 2(\delta+\phi) = \operatorname{Im}\left(\frac{V_{ub}^*}{V_{ub}}\right)\left(\frac{V_{td}^*}{V_{td}}\right) = \frac{2\sin\delta(\cos\delta-\rho)}{1+\rho^2-2\rho\cos\delta} \,, \tag{34}$$

$$\sin 2\varphi_3 = \sin 2\delta = \operatorname{Im}\left(\frac{V_{ub}^*}{V_{ub}}\right) . \tag{35}$$

where ρ is the CKM-matrix parameter defined in Eq. 22. It is interesting to estimate the allowed regions of the $\sin 2\varphi_i$ based on present knowledge of the CKM-matrix parameters, and on the top-quark mass. The relevant experimental results are

$\lambda = 0.22$ (the Cabibbo angle),

The CKM parameter $A = 1$ to good accuracy from the B-meson lifetime,

$|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ from K decay,

$x_d = 0.72 \pm 0.10$ from ARGUS and CLEO,

$V_{cb} = 0.046 \pm 0.010,$

$\rho = 0.52 \pm 0.08$ from $|V_{ub}/V_{cb}| = 0.115 \pm 0.018$ at ARGUS and CLEO,

$\tau_B = 1.18 \pm 0.12$ ps.

From these constraints we deduce the 90% confidence limits shown in Fig. 5 for $M_t = 150$ GeV/c^2. We infer that $\sin 2\varphi_1$ and $\sin 2\varphi_2$ are unlikely both to be small, but that φ_2 could be near 90° for $M_t \sim 150$ GeV/c^2.

The example final states given in (31) for the asymmetry classes are the most useful for BCD. From Tables 3 and 4 we find for the tree-level graphs

Class 1: $B_d^0 \to J/\psi K_S^0$, $J/\psi\rho^0$, D^+D^-, $D^0\bar{D}^0$, and $D_s^+D_s^-$. All but the first of these are CKM-suppressed. Not shown in the Tables are such related decays as χK_S^0, and $\eta_c K_S^0$. The pure penguin decays $B_d^0 \to \phi K_S^0$ and $B_s^0 \to \phi K_S^0$ also belong to this class.

Class 2: $B_d^0 \to \pi^+\pi^-$, $\rho^0\rho^0$, K^+K^-, $D^0\bar{D}^0$, and $\rho^0 K_S^0$. The first 4 are CKM-suppressed, and the fourth is doubly CKM-suppressed. Because $D^0\bar{D}^0$ belongs to both classes 1 and 2 with similar strength its utility in determining the CKM phases is limited.

Class 3: $B_s^0 \to \rho^0 K_S^0$, $\rho^0\phi$, K^+K^-, $\pi^+\pi^-$, and $\rho^0\rho^0$. The first is CKM-suppressed, and the last four are doubly CKM-suppressed (although penguin contributions to all but $\rho^0 K_S^0$ are CKM-favored).

Class 4: $B_s^0 \to D_s^+D_s^-$, $J/\psi\phi$, $D^0\bar{D}^0$, D^+D^-, and $J/\psi K_S^0$. The last is CKM-suppressed.

Note that the interesting classes 2 and 3 are always CKM suppressed.

There are some theoretical uncertainties in the modes such as $B_d \to \pi^+\pi^-$ and $B_s \to \rho K_S$ in which there is a penguin-graph contribution in addition to the favored tree-level spectator graph. The argument that led to Eqs. 26 and 27 was based on the assumption that only one weak amplitude contributes to the decay of a B meson to a CP eigenstate. Therefore the result $|\rho_f| = 1$ must be reconsidered. This has been done[17] – the effects of the penguin diagrams are difficult to calculate, but have been estimated to be not more than 20%. On the other hand, the prediction for the mode $B_d \to J/\psi K_S$ is theoretically quite safe.

The penguin graphs can also be classified on the basis of the CKM-phase information they contain

Class 1. B_d decays with $b \to s$ ($B_d \to J/\psi K_S^0$, $\rho^0 K_S^0$, ϕK_S^0, and $\pi^+\pi^-$).

Class 4. B_d decays with $b \to d$ ($B_d \to D^+D^-$, $J/\psi\rho^0$, $\rho^0\rho^0$, $D^0\bar{D}^0$, $D_s^+D_s^-$, K^+K^-, $\phi\phi$, $\phi\rho^0$, and $K^{*0}\bar{K}^{*0}$). Here, both mixing and the decay amplitudes depend on V_{td}, but the phases cancel.

Class 4. B_s decays with $b \to s$ ($B_s \to D_s^+D_s^-$, $J/\psi\phi$, $D^0\bar{D}^0$, D^+D^-, $\rho^0\rho^0$, $\phi\rho^0$, $\phi\phi$, K^+K^-, $K^{*0}\bar{K}^{*0}$, and $\pi^+\pi^-$).

Class 1. B_s decays with $b \to d$ ($B_s \to J/\psi K_S^0$, $\rho^0 K_S^0$, and ϕK_S^0).

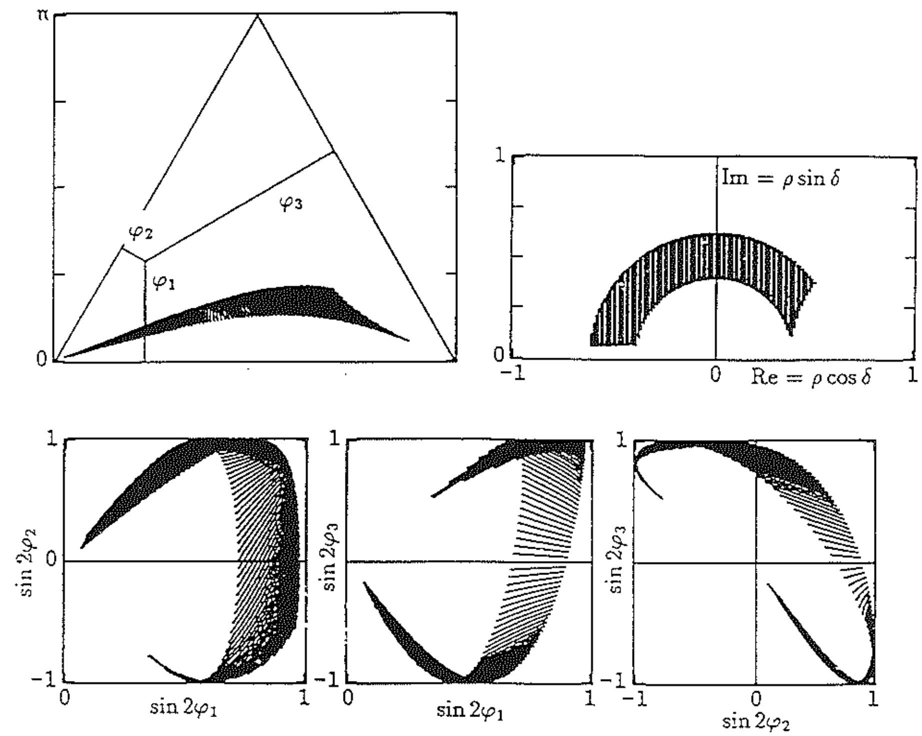

Figure 5: Allowed regions of the φ_i at 90% confidence, based on present knowledge of the CKM matrix. In the triangle plot, $\varphi_1 + \varphi_2 + \varphi_3 = \pi$. The top-quark mass was taken to be 150 GeV/c^2.

Two-body final states in which both particles have nonzero spin are not, in general, pure CP eigenstates, but mixtures of CP + and CP −, due to the different possible orbital angular momenta. For these states, such as $J/\psi\phi$, $J/\psi\rho^0$, $\phi\phi$, $\phi\rho^0$, $\rho^0\rho^0$, and also $p\bar{p}$, the simple relation between the CP asymmetry and phase of the CKM matrix no longer holds. However, for particular polarizations of the final-state particles the elegant relations can be recovered. Additional measurements of CP violation will be possible with event samples sufficiently large to permit spin-density-matrix analyses.

Although we have emphasized the time dependence of the CP asymmetries in Eq. 27, these can also be measured using time-independent techniques. Integrating over time, we find

$$A_f = -\frac{x_q}{1+x_q^2}\,\mathrm{Im}\,\alpha_f\,. \qquad (36)$$

For a time-integrated measurement to be interpreted it is necessary to have an accurate measurement of the mixing parameter x_q. At a hadron collider where a vertex detector must be used to isolate the B-decay products, the time-resolved measurements will consequently be possible. However, the large mixing parameter x_s renders time-integrated measurements useless for B_s decays.

Finally, it is interesting to examine why CP-violation measurements in the Kaon system do not directly access the phases of the CKM matrix. Many of the needed ingredients are present: $\pi\pi$ is a CP eigenstate, $|p/q|_K \simeq 1$, and a single weak amplitude dominates the decay. However, the CKM structure of the amplitude (and of K^0-$\bar{K}^0$ mixing) involves only the uc-ds submatrix to a first approximation, and the elements V_{ub} and V_{tb} that have nontrivial phases enter only as small correction. The Kaon system does not 'know' enough about the third quark generation to be an ideal laboratory for CP violation.

2.4.2 Sensitivity of the BCD to CKM Phase

The principal result of the preceding subsection is that the time-resolved CP-violating asymmetry in the decay of neutral B mesons to CP eigenstates can be written

$$A(t) = \frac{\Gamma(B_q^0(t)\to f) - \Gamma(\bar{B}_q^0(t)\to \bar{f})}{\Gamma(B_q^0(t)\to f) + \Gamma(\bar{B}_q^0(t)\to \bar{f})} = \sin 2\varphi \sin x_q t/\tau,$$

where φ is a phase directly related to CKM-matrix elements, and x_q is the mixing parameter of the B_q mesons. Strictly, this relation holds only when the decay is dominated by a single weak amplitude, and the interference required to produce an asymmetry arises from the mixing.

To extract the amplitude, $\sin 2\varphi$, of the time-resolved asymmetry $A(t)$ we must note a 'dilution factor' not present in a time-independent asymmetry (such as that discussed in sec. 2.3). For a small mixing parameter x_q, only the first wiggle in Eq. 26 will be significant (due to the relatively rapid decay), and the time-resolved asymmetry measurement reduces to integrating over this wiggle, which becomes equivalent to the time-integrated measurement (36). That is, the effective asymmetry is

$$A \to \frac{x_q}{1+x_q^2}\sin 2\varphi, \qquad \text{small } x_q.$$

For large x_q, however, $\sin x_q t/\tau$ oscillates before the B's decay, and the asymmetry measurement can be thought of as comparing alternate half cycles. In this case the effective size of the asymmetry is only reduced by the average amplitude of a half cycle of a sine wave:

$$A \to \frac{2}{\pi}\sin 2\varphi, \qquad \text{large } x_q.$$

A complete analysis shows that we can write

$$A = D(x_q)\sin 2\varphi \qquad \text{with} \qquad D = \frac{x_q}{1+x_q^2}\coth(\pi/2x_q).$$

(The dilution factor D is the Laplace transform of $|\sin x_q t/\tau|$, which arises because the effective signal is $e^{-t/\tau}\,|\sin x_q t/\tau|$ compared to exponential-decay 'background.')

Although the dilution D due to mixing is annoying, without mixing the richness of CP violation in the B system would be greatly reduced.

Assuming our vertex detector has sufficient accuracy to resolve the mixing oscillations (*cf.* sec. 2.2), the statistical significance in standard deviations of a measurement of $\sin 2\varphi$ via the effective asymmetry $A = D\sin 2\varphi$ follows from the discussion in sec. 2.3, namely

$$S = \frac{\sin 2\varphi}{\sigma_{\sin 2\varphi}} = \sqrt{N}\frac{D\sin 2\varphi}{\sqrt{1 - D\sin^2 2\varphi}},$$

where N is the total number of (true) reconstructed, tagged decays. The minimum value of $\sin 2\varphi$ that could be resolved to three standard deviations with N events is then

$$\sin 2\varphi_{\min,3\sigma} = \frac{3}{D\sqrt{N+9}}.$$

To perform the asymmetry measurement we must tag the other B in the event to determine the initial particle/antiparticle character of the B that decays to the CP eigenstate. As discussed in section 2.2 above, at a hadron collider we will likely have only a partial reconstruction of the second B, and will not distinguish B_u, B_d, and B_s. Because of the mixing of the neutral B's there will be a probability $p \approx 0.2$ of a mistag, which dilutes the significance of the asymmetry measurement.

We also consider the effect of backgrounds in our sample of CP-eigenstate decays, supposing that for each true reconstruction of $B \to f$ there are b false reconstructions that exhibit zero asymmetry. Then $1/b$ is the signal-to-noise for reconstruction of this decay mode.

Taking into the account these two effects, the apparent rate for B ($\bar{B}$) decay will be

$$\Gamma(\overset{(-)}{B}{}_q^0(t) \to \overset{(-)}{f}) = \left|\langle f|B_q^0\rangle\right|^2 e^{-t/\tau}\left[1 + b \mp (1-2p)\,\mathrm{Im}\,\alpha_f \sin x_q\frac{t}{\tau}\right],$$

and the true CP-violating asymmetry A_{CP} will be related to the asymmetry $A_{\rm obs}$ observed in the laboratory by

$$A_{CP} = \frac{1+b}{1-2p}A_{\rm obs}.$$

As a consequence, the statistical power of a sample of N true events is reduced, and we now have (assuming that b and p are well known)

$$\sin 2\varphi_{\min,3\sigma} = \frac{3(1+b)}{D(1-2p)\sqrt{N(1+b)+9}}.$$

With $p = 0.2$ we expect $1 - 2p = 0.6$, as discussed in section 2.2.

Consideration of the tagging efficiency has also been given in section 2.2, where we estimated that a tag based on the sign of the leading electron or muon in semileptonic decays would have about 3% efficiency.

The expectations for rates of three decays to CP eigenstates are

φ_1 can be determined from $B_d^0 \to J/\psi K_S^0 \to e^+e^-\pi^+\pi^-$ or $\mu^+\mu^-\pi^+\pi^-$. Table 8 indicates that the BCD could reconstruct 8,400 $e^+e^-\pi^+\pi^-$ decays in 10^7 sec of running at a luminosity of 10^{32} cm^{-2}sec^{-1}. As muons are identified only in the Forward region, the $\mu^+\mu^-\pi^+\pi^-$ is only about 0.25 times the $e^+e^-\pi^+\pi^-$ sample, for a total of 2,100 $J/\psi K_S^0$ events. The number of tagged and reconstructed events would then be 300. This mode will be relatively background free so long as the mass resolution is smaller than M_π. We estimate the background-to-signal to be $b \sim 0.1$

φ_2 can be determined from $B_d^0 \to \pi^+\pi^-$. From the 50,000 events expected in Table 8, there would be 1,500 tagged, reconstructed events. The $\pi^+\pi^-$ is one of the most background prone as a fake secondary vertex is more likely for two tracks than for three. From Monte Carlo simulations of the vertexing algorithm we estimate that $b \sim 1$.

φ_3 can be determined from $B_s^0 \to \rho^0 K_S^0 \to \pi^+\pi^-\pi^+\pi^-$. From the 420 events expected in Table 8, only about 12 remain after the tagging requirement. It seems likely that a determination of $\sin\varphi_3$ must await the SSC.

Table 10: The minimum values of $\sin 2\varphi$ resolvable to three standard deviations in 10^7 sec of running at luminosity of 10^{32} cm^{-2}sec^{-1}. The dilution factor D due to mixing is given by $x_q \coth(\pi/2x_q)/(1+x_q^2)$.

Angle	Mode	Tag	Tagged Events	$1-2p$	b	x_q	D	$\sin 2\varphi_{\min,3\sigma}$
φ_1	$B_d^0 \to J/\psi K_S^0$	$e^\pm$	300	0.60	0.1	0.7	0.47	0.64
φ_1	$B_d^0 \to J/\psi K_S^0$	$K^\pm$	2,400	0.40	0.1	0.7	0.47	0.34
φ_2	$B_d^0 \to \pi^+\pi^-$	$e^\pm$	1,500	0.60	1.0	0.7	0.47	0.39

Table 10 indicates the smallest values of the φ_i that could be resolved in one year of running at the BCD. In view of the limited accuracy, it would be desirable to do even better. A significant loss of statistical power occurs because of the tagging requirement, whose overall efficiency is about 3% if the only tag is on the charge of the leading electron or muon in semileptonic B decays. Clearly it will be very advantageous to devise a tag based on some substantial fraction of the nonleptonic decays that comprise 70% of all B decays.

An alternative tag might well be applied for the decays $B_d^0 \to J/\psi K_S^0$, for which a trigger could be made on the dilepton decay of the J/ψ/ This tag would be based on the sign of the Kaon arising in the decay chain

$$b \to c \to s \to K^- \quad \text{or} \quad \bar{K}^0.$$

This chain occurs $\sim 100\%$ of the time. However, the $b \to c$ transition includes the emission of a W^-, which can decay to a $\bar{c}s$ combination about 33% of the time; then the $\bar{c}$ decays to a $\bar{s}$ essentially 100% of the time. The $s(\bar{s})$ quark emerges as a $K^-(K^+)$ 50% of the time. The presence of a K^+ could lead to a mistag. In the case of multiple Kaons, we might just choose one at random to be the tagging Kaon, and suffer the consequences. Table 11 summarizes the probabilities of various qualities of tags occurring.

Table 11 ignores the small probability that an $s\bar{s}$ pair is created from glue. This, and some of the 'Bad Tags' listed in the Table 11, could likely be suppressed by a momentum cut, not explored here. There is typically an extra Kaon in B_s decays, which would lead to bad tags. However, B_s decays are useless as tags because of their rapid oscillations; this dilution is already accounted for in the 20% mistagging probability due to mixing.

From Table 11 we see that $45/72 = 62\%$ of all B decays could yield a Kaon tag, but that $7/45 = 16\%$ of these would be a mistag. Actually, we must combine the mistags due to the wrong-sign Kaon with the mistags due to mixing. The total mistagging probability is $(16\%)(80\%) + (84\%)(20\%) = 30\%$. The tagging efficiency is then $1 - 2p = 40\%$. We estimate that the geometrical acceptance for the Kaon tag would be more like 70%, as we wouldn't need as strong a P_t cut as for the leptons. The vertexing efficiency is again about 33%. Hence the fraction of B events that could have a Kaon tag is $(70\%)(33\%) = 23\%$.

Table 11: Estimates of the efficiency of a tag on the particle/antiparticle character of a B meson based on the sign of Kaons in the B decay.

$b \to c \to s \to$	$b \to W^- \to$	Good Tag Prob.	Bad Tag Prob.	No Tag Prob.
K^-	other	1/3		
K^-	$K^+\bar{K}^0$	1/48	1/48	
K^-	K^+K^-	2/72	1/72	
K^-	$K^0\bar{K}^0$	1/24		
K^-	K^0K^-	1/24		
$\bar{K}^0$	other			1/3
$\bar{K}^0$	$K^+\bar{K}^0$		1/24	
$\bar{K}^0$	K^+K^-	1/48	1/48	
$\bar{K}^0$	$K^0\bar{K}^0$			1/24
$\bar{K}^0$	K^0K^-	1/24		
Total		38/72	7/72	27/72

If a secondary-vertex trigger could be implemented, we might get this improvement in all tagged, reconstructed event samples. We anticipate an evolution of thinking about these critical issues in the coming years. A B-physics experiment at RHIC would provide the much-needed opportunity to explore triggering and tagging prior to a larger experiment at the SSC.

However, even with the advantages of a Kaon tag, only the strongest signals of CP violation in the B system could be resolved at RHIC.

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The role of scalar resonances in $\Delta I = 1/2$ rule*

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Abstract

The purpose of my talk is to review the results of the evaluation of hadronic matrix elements in $K \to \pi\pi$ decays with linear σ model. We have obtained the large enhancement factor for $\Delta I = \frac{1}{2}$ amplitude.

1 Introduction

Experimentally $\Delta I = \frac{1}{2}$ rule appears typically in $K \to \pi\pi$ decays:

$$\begin{aligned} \Gamma[K^+ \to \pi^+\pi^0] &= 1.1 \times 10^{-14}(MeV) \\ \Gamma[K_s \to \pi^0\pi^0] &= 2.3 \times 10^{-12}(MeV) \\ \Gamma[K_s \to \pi^+\pi^-] &= 5.1 \times 10^{-12}(MeV) \end{aligned} \quad (1)$$

From these data, we can extract a ratio for amplitudes,

$$\left| \frac{A_0(K_0 \to (\pi^0\pi^0)_{I=0})}{A_2(K_0 \to (\pi^0\pi^0)_{I=2})} \right| = 22 \quad (2)$$

In most of the theoretical frameworks to evaluate the non leptonic decays consist of the following two steps.

1) The perturbative QCD corrections

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