Target-Magnet Field Profile that Ramps from 20 T to 1.5 T at 7 m

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**Table I: Parameters of Target Magnet “IDS120'20to1p5T7m.xlsx” of 4/8/2013**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Current density jcoil | kA/cm2 | 2.078 | 1.665 | 1.513 | 1.393 | 1.294 | **1.911** | **3.451** | **4.131** | **4.343** |
| Coil length | cm | 129.1 | 165.4 | 165.4 | 165.4 | 165.4 | 397.53 | 49.33 | 46.80 | 32.99 |
| Gap between coils | cm |  |  |  |  |  |  | 122.29 | 88.80 | 90.98 |
| Upstream end | cm | **-86.7** | **-123.0** | -123.0 | -123.0 | -123.0 | **-238.2** | **281.6** | **418.8** | **556.5** |
| Downstream end | cm | **42.4** | 42.4 | 42.4 | 42.4 | 42.4 | **159.3** | **331.0** | **465.6** | **589.5** |
| Inner radius [cm] | 0.50 | 18.28 | 23.75 | 29.78 | 36.09 | 42.66 | 120.0 | 120.0 | 120.0 | 120.0 |
| Radial depth of conductor | cm | 4.760 | 5.318 | 5.579 | 5.815 | 6.032 | **79.63** | **17.19** | **8.386** | **10.16** |
| Outer radius [cm] | 50.0 | 23.04 | 29.07 | 35.35 | 41.90 | 48.69 | 199.63 | 137.19 | 128.39 | 130.16 |
| Maximum on-axis field | T | 20.00 | 18.83 | 17.77 | 16.78 | 15.86 | 15.00 | 4.86 | 2.76 | 1.84 |
| Current density jcoil | kA/cm2 | **4.570** | **4.588** | **4.548** | **4.673** | **4.715** | **4.715** | 4.715 | 4.715 | 4.715 |
| Coil length | cm | 122.90 | 323.49 | 30.00 | 20.00 | 479.49 | 20.00 | 20.00 | 479.49 | 20.00 |
| Gap between coils | cm | 70.00 | **10.00** | **24.08** | 50.00 | **20.25** | 20.25 | 40.00 | 20.25 | 20.25 |
| Upstream end | cm | 659.5 | 792.4 | 1140.0 | 1220.0 | 1260.3 | 1760.0 | 1820.0 | 1860.3 | 2360.0 |
| Downstream end | cm | **782.4** | 1115.9 | 1170.0 | 1240.0 | 1739.7 | 1780.0 | 1840.0 | 2339.7 | 2380.0 |
| Inner radius | cm | 90.0 | 90.0 | 90.0 | 60.0 | 60.0 | 60.0 | 60.0 | 60.0 | 60.0 |
| Radial depth of conductor | cm | **2.252** | **2.434** | **8.713** | **5.775** | **2.434** | **7.623** | 7.623 | 2.434 | 7.623 |
| Outer radius | cm | 92.25 | 92.43 | 98.71 | 65.77 | 62.43 | 67.62 | 67.62 | 62.43 | 67.62 |
| Maximum on-axis field | T | 1.51 | 1.48 | 1.52 | 1.51 | 1.51 | 1.52 | 1.51 | 1.51 | 1.52 |

The “desired field” is the inverse-polynomial B(u) = 180/[9+37u2(4-u6)], where u ≡ x/L, x ≡ z+37.5 cm, and L ≡ 737.5 cm. B(u) involves only even powers of u, and therefore is symmetric about u=0—i.e., z = −37.5 cm. The derivative, dB/du is 53280u(u2−1)[(u2+1)2−u2]/ (37u8−148u2−9)2, which is zero at u = 0 and u = 1—i.e., x = 737.5 cm, or z = 700 cm.

A more general expression, likewise with zero slope at x = 0 and x = L, is B(u) = nB0/[n+bu2(n+2−2un)], where B0 ≡ B(u=0) and b ≡ [B0/B(L)]−1. Its first derivative is n[2bu(2un−n−2)+2bnun+1]/[bu2(2u2−n+2)−n]2. The equation for the 2nd derivative is two lines long; that for the 3rd derivative takes five lines—rather inconvenient for analytic prediction of the paraxial field by a power-series expansion in Legendre polynomials.

A form more amenable to analytic differentiation is B(u) = B0−∆B(au2+bup+cuq), where the parameters q and p need not be integers but neither should be less than 2.0, if the field near u=0 is to be dominated by the quadratic term. The three parameters a, b & c enable the expression to achieve at the end of the ramp not only the desired field and slope (zero), but also **zero curvature**. The figure plots two illustrative field profiles.



Fig. 1. Illustrative field profiles with zero 1st and 2nd derivatives at end of ramp.