

# AN INTENSE LOW ENERGY MUON SOURCE FOR THE MUON COLLIDER

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## Abstract

A scheme for obtaining an intense source of low energy muons is described. It is based on the production of pions in a high field magnetic bottle trap. By ensuring efficient slowing down and extraction of the decay muons an intense intermediate energy muon beam is obtained. For the specific case of negative muons a novel technique called frictional accumulation provides efficient conversion into a 10 keV  $\mu^-$  beam whose emittance is then reduced in a configuration providing extended frictional cooling. The result is a beam of very small transverse and longitudinal emittance that can be used together with an equivalent  $\mu^+$  beam as a compact intense muon source for the  $\mu^+\mu^-$  collider. A final luminosity around  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  is expected to be obtained at 2 TeV.

## 1. Introduction

The muon source for the  $\mu^+\mu^-$  collider is still in its conceptual stage and the door has not yet been fully closed to competing ideas. It is therefore timely to throw into the race what may become a real alternative to the standard high energy muon production concept, namely the pathway of low energy muon production and cooling. Although this pathway has been considered and discussed in the initial meetings on the muon collider, it suffered from the absence of a scheme providing simultaneous high intensity and low emittance beams. The present contribution aims to improve the situation by presenting basic schemes for efficient muon production, slowing down and cooling. It has quite a preliminary aspect because no full Monte-Carlo simulation of the various stages considered have yet been undertaken so that a reliable final value for the muon intensity is not available. Nevertheless it should provide sufficient motivation for a more detailed investigation and lead to optimized most efficient designs.

Because of the radically different interaction of low energy positive and negative muons with matter the required final low emittance beams will be obtained via quite different pathways. For the  $\mu^+$  the existing scheme of muonium formation and ionization [1] can in principle provide a phase space compression (PSC) factor

of  $10^9$  in one step. No such effective cooling operation is available for the  $\mu^-$  branch. We therefore concentrate here on the apparently handicapped  $\mu^-$  branch leaving the  $\mu^+$  sector to a short discussion near the end of the paper.

## 2. Basic aspects of the $\mu^-$ source

One most useful technique to obtain an efficient and compact muon source is to force pion decay to take place within a trapping configuration. With a magnetic bottle trapping both pions and muons the intrinsic high longitudinal phase space density of a trap [2] can be made used of to provide efficient slowing down of the high energy muons [3]. This is done by letting the muons lose their energy in an suitable moderator material. The associated loss in transverse energy provides an efficient transverse PSC. With  $B_0$  being the magnetic field in the central trap region and  $B_m$  the field maximum at both ends, the mirror action of the field increase results in trapping all muons whose angle  $\theta$  to the axis at the center of the trap is greater than a cut-off angle

$$\theta_c = \arcsin \sqrt{B_0 / B_m} . \quad (1)$$

In order to induce muon escape during slowing down an electric potential difference  $\Delta V$  is applied at one side of the trap, modifying thereby the escape condition to [4].

$$\theta > \theta'_c = \arcsin \sqrt{B_0 / B_m (1 + e\Delta V / E)} \quad (2)$$

where  $E$  is the energy of the muon as it comes out from a moderator assumed to be placed at  $B=B_0$ . [4]. This leads to a selective extraction of the lower energy muons enabling their escape before stopping in the moderator.

For efficient  $\mu^-$  production a most intense deuterium beam at energy around 2 GeV will be shot on a thin carbon rod at the center of the trap, and by operation at high magnetic field and a relatively high  $B_m/B_0$ . (of the order of 2), it will be possible to trap a large fraction of the produced  $\pi^-$ . By using a relatively short trap, a large number of round trips will be made possible for the decay muon so that a quite high initial muon energy can be slowed down to the escape before muon decay. A schematic configuration for this muon production stage will be presented in § 3.

The function of the next stage is best understood if we jump first to the last stage and explain how a minimal final beam emittance is obtained. In a recent experiment at PSI [5] it has been demonstrated that the quality of a low energy  $\mu^-$  beam can be improved by letting it pass a stack of thin foils with an accelerating electric field between them. Both longitudinal and transverse cooling have been observed at energies below 10 keV where the stopping power falls with energy. This cooling effect (called ionization cooling in this book or frictional cooling by its promoter [6]), which required the use of a large magnetic field to prevent beam size blow-up, has here to be implemented in a much more efficient configuration in order to provide the very large PSC factors needed to convert the wide  $\mu^-$  beam exiting the magnetic trap into a beam fulfilling the collider requirements. The way to do this is to extract the muons from their magnetic field entanglement (a non-

trivial task by itself) and form muon beams that can be focused by lenses from one foil to the next. By making the operation at optimal beam energy and divergence and allowing for adequate demagnification between one foil and the next, the beam size can be reduced step by step and after a large number of cooling stages the beam emittance can be brought to its final small value.

Furthermore, by converting the few  $\mu\text{s}$  long muon pulse into a large number of microbunches the time spread of each microbunch can be reduced between one foil and the next. The associated increase in energy spread is then cooled down in the next foil crossing. Repeated bunching between two successive foils leads to the full conversion of the energy cooling capacity of the foils into longitudinal emittance decrease.

This extended cooling concept will be shown in §6 to result in a final 6-dimensional emittance that is in principle by far better than what is required for the muon collider. On the other hand the beam intensity budget (§8) comes out somewhat poorer than what the standard high energy muon beam concept obtains. Because of the well known tune shift problematic in colliders, the low intensity results in strict luminosity limitations that cannot be compensated by a small emittance are used. It is therefore essential to minimize the muon (and pion) losses.

In this context the intermediate stage between the initial muon production bottle and the final cooling operation plays an essential role. It is made necessary because efficient final cooling requires muons of energy around 10 keV and below while efficient muon production in the considered trap requires the extraction to take place at energies greater than the MeV. How can the MeV to keV energy conversion be efficiently achieved? Here steps in a recently proposed technique called frictional accumulation [7]. It makes use of a stack consisting of a large number (50 - 100) of thin foils to which a monotonically increasing voltage is applied. High energy muons that are made to cross this stack (one or many times) along a magnetic field perpendicular to it, slowly lose their energy until they reach such a low energy that they are on the verge of stopping in one of the foils. However, because of the reduced stopping power at the lowest energies and also because the muon may have a dominantly transverse energy before stopping, the electric field between the foils can re-accelerate the muon and give it a sufficient longitudinal energy kick that it escapes stopping in a foil. With its transverse energy rapidly falling down and its longitudinal energy increasing it can reach an energy of 10 keV or less at low divergence and under these conditions, for optimally selected potential difference between the foils, have almost equal energy loss in the foils and energy gain between them. It therefore hops from one foil to next without much change in energy. This results in an accumulation of muons in the low energy region. These muons exit as a low energy muon beam at one side of the stack.

The efficiency of the scheme can be obtained from Monte-Carlo simulations. Preliminary results indicate that for muons entering a stack of 40 foils on its decelerating side, an initial energy distribution extending to 150 keV (with a Lambertian angular distribution) can be converted in a muon beam of energy less than 10 keV with an efficiency of 30%. This corresponds to a PSC factor of 1000.

By increasing the number of foils and letting the muons cross the stack many times (as provided in a trapping configuration) the energy acceptance can be increased to many MeV. This leads to the scheme described in §4 where the accumulation stack is introduced in a second trap.

The general discussion of the full chain of operations is herewith completed. We return now to a more detailed description of each of the successive stages.

### 3. The production bottle

The deuteron beam is obtained from an accelerator of the kind recently proposed for European Spallation Source [8]. It should be able to provide a 5mA average current at a repetition rate of 30/sec. The relatively short deuteron pulses could be significantly lengthened at the advantage of a smaller final transverse beam emittance. This would allow the use of a thinner carbon target rod and reduce the pion and muon losses in the magnetic bottle. With a 30 cm long carbon target more than the half of the deuterons undergo nuclear interaction. In order to insure efficient trapping of the higher energy pion in a compact trap, relatively high magnetic fields should be used. These can be obtained by producing an almost constant high field via a wide shielded superconducting solenoid and using two high field normal conducting coils of small size to provide the mirror action. The resulting configuration with the field distribution is shown in fig. 1.

The carbon target is to be replaced after each pulse. Trapped pions have cyclotron trajectories than return to the axis periodically within about 1 ns. Many of them lose significant energy in the carbon rod itself. Lower energy pions lose a relatively higher energy in the degrader disks so that they rapidly undergo a slight increase of the closest distance to the axis reducing the chance of a stop in the target rod.

Most decays of the trapped pions lead to trapped muons. Due to the momentum kick in the decay a large part of the trapped muons follow trajectories that escape crossing the target. These muons lose their energy in the degrader disks whose thickness is optimized in relation to the extraction voltage in such a way that the probability of escaping before stopping is maximized. Because of multiple scattering, the angle to the axis  $\theta$  increases with slowing down so that already before extraction time most muons have reached the region right to the foil F1. This allows the extraction field to be limited to the region between F1 and the extraction electrode. The electrode is pulsed to a high voltage during the extraction process. For a 1 MV voltage most muons entering the region right of F1 with an energy less than a few MeV will be extracted.

The transverse size of the outcoming beam is controlled by the diameter of the main slowing down disks. Small disk sizes reduce the outcoming beam diameter at the cost of intensity.

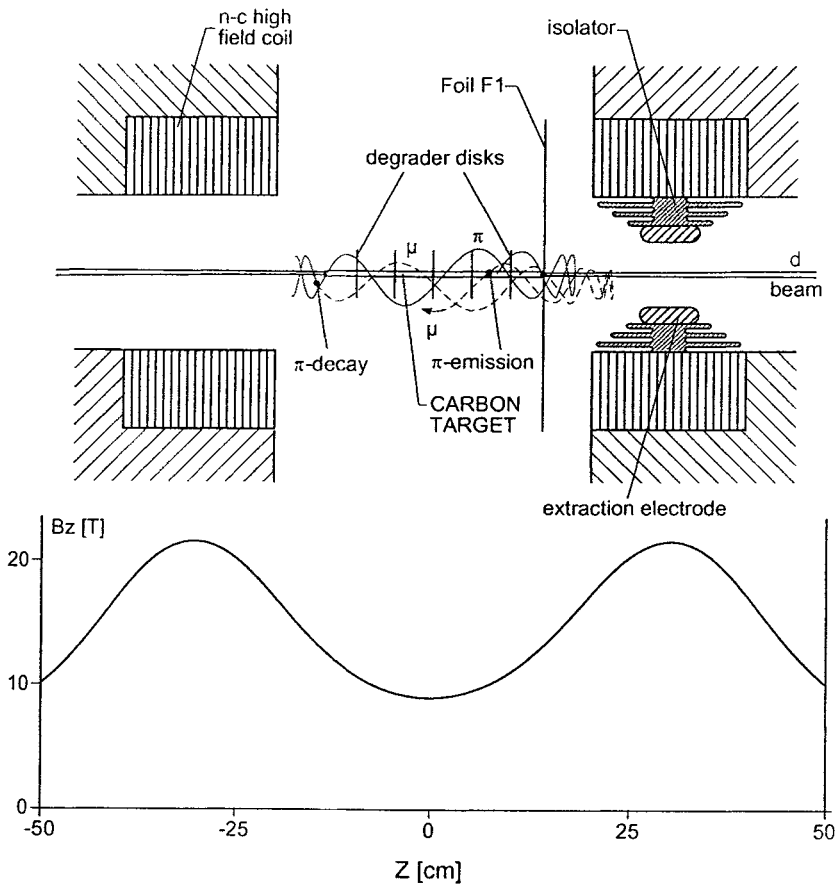


Fig.1: a) A schematic view of the magnetic mirror trap for efficient muon production together with the trajectory of a 120 MeV/c  $\pi$  followed by the decay muon. The shielded large diameter superconducting coils are not shown. b) Axial magnetic field distribution

#### 4. The accumulation trap

A high field curved solenoidal channel transports the muon beam between the production and the accumulation trap. An efficient injection of an external beam in a trap requires the use of some special trapping feature. If, as in the present case, the beam has wide energy, angular and time spreads, the injection operation will necessary lead to a large blow up of the beam size. Living with this fact, we consider off-axis injection followed by beam rotation via the curvature drift. In an axially symmetric magnetic field configuration, the muon beam enters a first high field coil at a distance  $r$  from the axis. It then crosses a low field region where the magnetic field lines curve further away from the axis and then return toward the axis to flow into an intermediate field solenoid. Under adiabatic conditions the guiding center of the trajectory is subjected to the curvature drift which gives rise to a rotation around the axis by an angle  $\theta$ . With the accumulation stack placed in the

intermediate field solenoid and a terminating high field coil acting as a magnetic mirror, a reflected higher energy muon (that lose a limited amount of energy in the stack) crosses again the low field region, rotates further by an angle close to  $\theta$  and reaches the entrance high field coil displaced azimuthally by about  $2r\theta$ . If this quantity is sufficiently greater than the beam diameter  $D$  it is possible to ensure that the muon will remain trapped by introducing an adequate change in electric potential. This is done by having an entrance electrode at a high positive voltage (that sucks the incoming  $\mu^-$  into the trap) and a fast fall to zero of the potential in the azimuthal direction outside the electrode. Optimization of the magnetic and electric field distribution will provide a highly efficient muon trapping. It is possible to improve the design by adding an adequate  $\vec{E} \times \vec{B}$  drift along the muon trajectory. With both  $\vec{E} \times \vec{B}$  and curvature drift the variation of  $\theta$  with muon energy can be significantly reduced.

The allowed number of round trips ( $\sim\pi/\theta$ ) can be adapted to the muon beam energy and the slowing down in the accumulation stack. The downstream outgoing beam of energy around 10 keV (or higher if accelerated) has the shape of an annulus of radius  $r$  and width  $D$  (at maximum field).

A technical difficulty has to be solved before the accumulation process at relatively high muon current can be implemented. Its origin is the very efficient secondary-electron emission by the foils of the stack. These electrons, accelerated by the potential between the foils, will cross them and re-emit more secondary electrons. The resulting significant electron multiplication (observed at PSI in a stack of 10 foils [6]) is too high to be allowed. One way to eliminate this effect is to introduce between any two foil planes a high transparency grid (made of ultra thin wires) at a relatively low potential ( $< 100V$ ) that forces the emitted secondary electrons (of  $\sim 10eV$  energy) to be reflected back to their emitting foil without affecting the transport of the higher energy muons.

Taking in account the various loss processes the efficiency of the accumulation stage will be between 10% and 20% depending of the muon beam energy and the optimization effort. This corresponds to a significant intensity loss but remains the price to pay to allow the mandatory muon cooling to proceed.

## 5. The extraction from the magnetic field

The long-standing problem of extracting a low energy beam from a high magnetic field has recently found two different solutions [9, 10]. We consider here the use of one of them whose practicability has been recently demonstrated with an electron beam [11].

In a magnetic field the true momentum of a particle is  $\vec{P} = \vec{p} - e\vec{A}$  with  $\vec{p}$  being the usual mechanical momentum and  $\vec{A}$  the vector potential which in a solenoid with field  $B$  follows transverse circular lines centered on the axis. On these circles  $|\vec{A}| = Br/2$  ( $r =$  circle radius). At high  $B$  and large  $r$ ,  $|\vec{A}|$  will be much greater than  $p_{\perp}$  (component of  $\vec{p}$  perpendicular to  $\vec{B}$ ) and angular momentum conservation

implies that a plain extraction from the solenoidal field is necessarily associated with an almost uncorrectable transverse momentum kick of the order of the value of  $e\vec{A}(r)$  in the solenoid.

By breaking the field symmetry and inducing  $|\vec{A}|=0$  regions near almost each point of the extraction plane, the amount of transverse kick can be reduced at will. This is done by first adiabatically reducing the magnetic field to a very low value and then terminating the field onto a grid consisting of thin parallel bars made from a high permeability material. Between the bars separated by a distance  $d$  the maximum excursion of the transverse kick is reduced to  $Bd/2$ .

For a muon beam the lowest field that can be reached adiabatically (at an increased radius and a decreased transverse momentum) within a few 100 ns is about 0.05 T. This is low enough to achieve high transparency by using as bars thin longitudinal foils, and for a distance of 5 mm between the foils the transverse kick is distributed between 0 and 0.04 MeV/c which is less than the average adiabatically decreased transverse beam momentum.

At the exit the muon beam has a large area and small divergence. It has to be divided in a large number of separate beams of smaller size so that it can be focused on foils of diameter less than a few cm diameter starting thereby the extended cooling operation.

## 6. The cooling stage

The optimal cooling action is obtained at energies ( $\sim 5$  keV) where straggling and scattering induce relatively large  $\Delta E/E$  and large divergence. These are not adequate conditions for good muon transport. The concept proposed here is to induce a strong acceleration of the muons at their immediate exit from a foil so that a higher energy beam with small  $\Delta E/E$  and divergences is obtained. This allows high quality focusing (electric or magnetic) to be used. Also the accelerating field applied on the conductive foil acts as a high quality cathode lens (or "immersion" lens) that has minimum aberration effects [12]. As the muons are refocused on the next foil a decelerating electric field will reduce the muon energy to the optimally required foil entrance muon energy. This "inverted" cathode lens is also of high quality so the muons are imaged from one foil to the next under optimal conditions. Aberration effects can practically be kept small by using sufficiently strong lenses. Under these conditions the cooling remains unaffected by the beam transport. The immense improvement relative to the standard configuration is however that at each step the cooling can be converted in a decrease of beam radius (for transverse cooling) or a decrease in pulse length (for a bunched beam with a cavity inserted in the beam line) so that the cooling action can be repeated at optimal efficiency from one stage to the next. This leads to introduction of the notion of "PSC factor of a

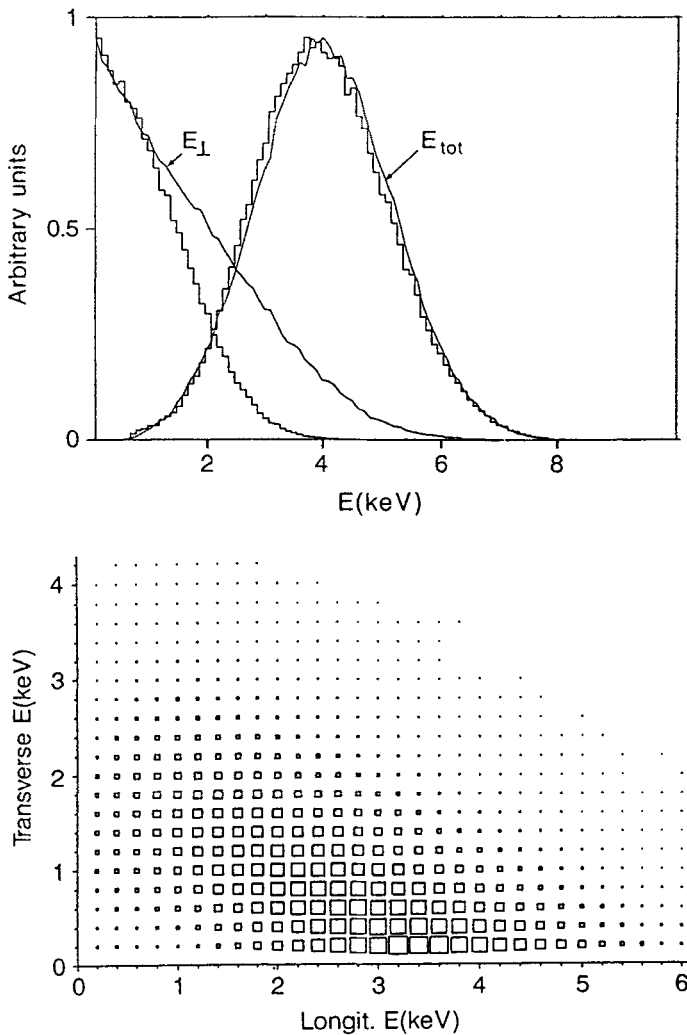


Fig.2: a) Simulation of the conditions for pure transverse cooling with a  $4 \mu\text{g}/\text{cm}^2$  carbon foil. Upper graph: distributions of transverse ( $E_{\perp}$ ) and total energies ( $E_{tot}$ ) before (point to point connected curve) and after (histogram kind curve) foil crossing. Equalization of average  $E_{tot}$  is obtained at a 2.4 kV potential difference. The intensity loss is about 2 %. Lower graph: Transverse versus longitudinal muon energy at the exit of the foil.

single foil" which can be investigated and optimized. It comes out that longitudinal cooling is significantly less efficient than transverse cooling. A simulation of the foil action is presented in Fig. 2 for a beam entering the foil with a large divergence. Under these conditions extensive cooling can be achieved without any beam bunching requirement.

The PSC factor of a single  $4 \mu\text{g}/\text{cm}^2$  carbon foil comes out to be near 0.66 (or a 0.81 decrease in emittance) at the lowest energy for which straggling losses can be



neglected. This gives the relation between the number of stages  $n$  and the PSC factor:

$$f_{\text{PSC}} = (0.66)^n \quad (3)$$

The muon beam cooling will first consist a large number of cooling stages acting in parallel on each of the separate beamlets previously extracted. As the size of the beams becomes sufficiently small (a few mm on the foil) they are merged together transversally (in a few successive steps interspaced by further cooling) until a final single intense beam is obtained. Once transverse cooling has approached its limit, the  $2 \mu\text{s}$  long muon pulse enters a highly efficient buncher which generates a large succession of microbunches of larger energy spread.

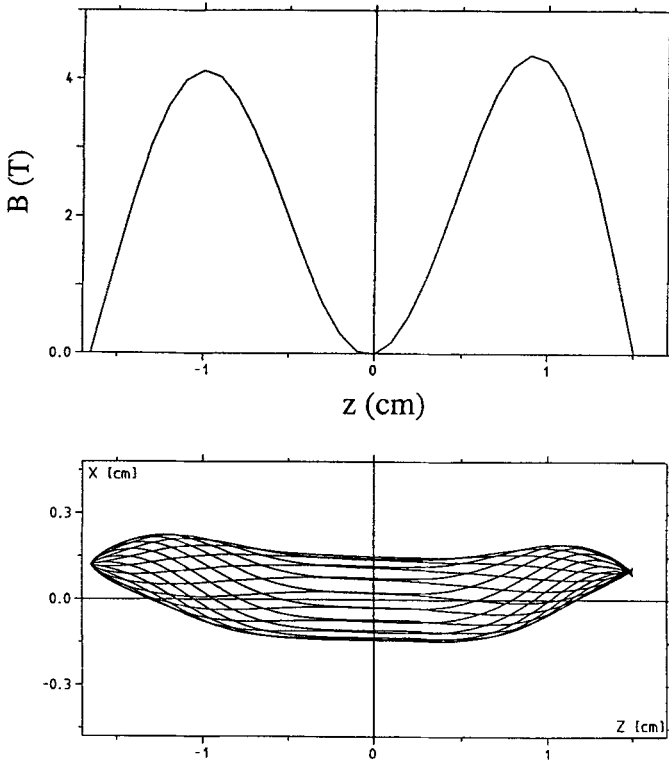


Fig.3:  $\mu^-$  transport between 2 foils in a final cooling stage. a) Axial magnetic field obtained by using small superconducting coils with alternating current excitation. b) Muon trajectories for a point object on the first foil at a radius of 1 mm imaged on a second foil placed at a 3.15 cm distance. The initial muon energy and divergence are  $E = 3 \text{ keV}$  and  $\theta = 45^\circ$  for all beams. The second foil is placed at a 2.4 kV higher potential. Shown is the  $x$  coordinate in function of the axial position  $z$ .

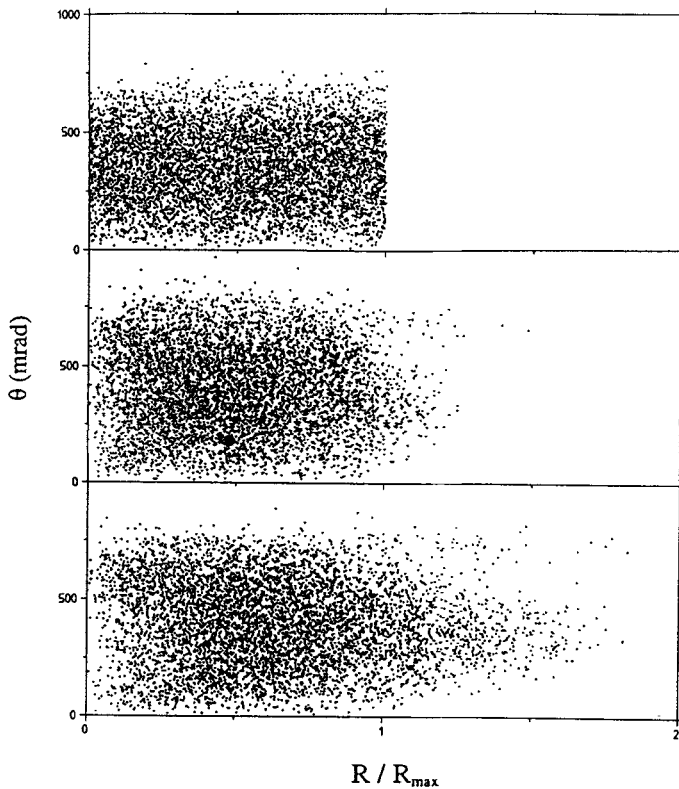


Fig.4: Distribution of divergence angles  $\theta$  at the second foil for muons coming out from the first foil with the energy distribution given in Fig. 2 (lower curve) and a radial distribution taken for illustrative purposes to be homogeneous in the distance  $R$  to the axis up to a distance  $R_{\max}$ . a) Ideal transport with no demagnification; b)c) Transport in the lens of Fig. 3 with slight demagnification ( $\sim 0.9$ ); b)  $R_{\max} = 0.5$  mm; c)  $R_{\max} = 0.25$  mm. At  $R_{\max} = 0.5$  mm, the effect of the aberrations on the radial distribution is small. At  $R_{\max} = 0.25$  mm, the aberrations enlarge appreciably the radial distribution.

An emittance exchange stage [13] (for which no wedges are required but simply some slanted cavities) is then used for converting the enlarged energy spread into an increased beam size which can be further reduced (in cooling stages where from now on RF cavities controls longitudinal phase space transport) until the smallest achievable beam radius is reached.

How small a beam radius can be achieved have been investigated on the basis of a compact high field transport stage where the accelerated muons are focused by magnetic lenses [Fig. 3]. Adopting a 150 kV/cm electric field on the foils and an acceleration of 150 keV the aberration effects can be inspected by looking at the effect of beam transport on the radius and divergence of the muons. It comes out that aberration radii smaller than 0.25 mm are achievable for most of the cooled muons. The effect of the aberrations are illustrated in Fig. 4. A final r.m.s.

radius smaller than 0.5 mm appears to be reachable. With the cooled transverse r.m.s momentum of 0.4 MeV/c this gives a normalized transverse emittance of  $2 \cdot 10^{-6}$  rad m. Longitudinally for the reached r.m.s. energy spread of 1.1 keV and a muon pulse of r.m.s. total time spread of 2  $\mu$ s cut into micropulses with duty cycle  $\delta$  the resulting normalized emittance is  $7 \cdot 10^{-3} \cdot \delta$  rad m. This leads to a final normalized 6-dimensional r.m.s. emittance of:

$$E_6 = 3 \cdot 10^{-14} \cdot \delta \text{ rad}^3 \text{ m}^3 \quad (4)$$

There is however a drastic limitation due to space charge effects. They affect the beam mainly near the foils and their consequence on beam transport has still to be investigated. For the time being it appears that  $E_6 \ll 10^{-14} \text{ rad}^3 \text{ m}^3$  is reachable. This is orders of magnitude smaller than in the standard high energy source concept and leads to the situation where the final collider luminosity will not be limited by the beam quality.

During the acceleration the microbunched beam will be converted in to the required final single bunch. This can be done either by providing increased acceleration to the tail of the pulse or by introducing a compact compressor ring at relatively high energy.

## 7. The $\mu^+$ beam production

At the present time the most powerful  $\mu^+$  cooling technique appears to be slow muonium formation followed by laser ionization [1]. Although the laser power required for the present application exceeds by far what can be achieved nowadays, we can assume that future intense free electron lasers (FEL) will be able to deliver the required amount of UV radiation. This may need very large investments but they will still remain a limited part of the collider costs. The FEL technology is presently advancing in big steps and once an efficient UV laser production scheme will be implemented the production of a large amount of FEL modules can be made quite cost effective.

Under this assumption, the ionization efficiency falls out of the discussion and we can adopt for it a value somewhere between 0.5 and 1. The problem of the production of a sufficient amount of muonium atoms has basically found its solution within the frame of the first two stages considered above. In the first stage efficient  $\pi^+$  production will result from using an intense 1 GeV proton beam. One would even consider the reinjection of the outcoming degraded proton beam and repeat the operation many times with a shorter carbon rod and increase thereby the efficiency of the production trap. Extraction and injection in the second trap can proceed in the same way as described above. Muonium production within this trap is obtained by letting the muons slow down and stop in ultra-thin heated tungsten foils. It is clear that the foil (and trap) configuration have to be arranged in such a way that optimum laser ionization and muon extraction is obtained. Nevertheless in the present context it will be sufficient to assume a plain replacement of the previously considered annular accumulation stack by a large number of ultra-thin radially disposed tungsten foils. For a foil thickness below 1  $\mu$ m, a  $\mu^+$  stopping in

one foil can be re-emitted as a muonium from any side of the foil. It will remain confined in the space between two adjacent foils. The distance between the foils can be made quite small (below 1 cm) as it is expected that a thermal muon hitting the foil will diffuse back into vacuum with an efficiency close to 1. If the foils can be made thin enough the total muonium re-emission efficiency can approach 1. The effective thickness of the tungsten stack being greater than that of the carbon foil stack considered above more efficient slowing down and stopping is achieved. This also allows the injection energy to be increased by using a higher extraction potential in the production trap. Consequently the thickness of the muon slowing down disks in the production trap can be increased so that a higher  $\mu^+$  production efficiency is obtained. Altogether this leads to the accumulation of thermal muonium atoms between the tungsten foils at a rate that is much higher than the rate of low energy  $\mu^-$  production.

The muonium ionization is done by shooting (after about  $2\mu\text{s}$  delay) two laser beams (one for the  $1s - np$  transition and one for the  $np$ -continuum transition) parallel to the solenoid axis between the radial foils. This results in a 0.2 eV plasma of quite appreciable density. Not going into the details of the extraction procedure, we limit ourselves to giving the  $\mu^+$  phase space volume for the case of an ideal extraction. This is the product of the stopping volume (about  $0.02 \text{ m}^3$ ) and the momentum phase space volume associated with the  $\mu^+$  temperature. Expressed in terms of the 6-dimensional r.m.s. normalized emittance this gives  $E_6 < 2 \cdot 10^{-16} \text{ rad}^3 \text{ m}^3$  which is excessively small. In practice it will be significantly increased by space charge effects and by non ideal extraction from the magnetic field (§7).

These very succinct considerations are sufficient to indicate that on the  $\mu^+$  sector a high quality  $\mu^+$  beam of even higher intensity than the  $\mu^-$  beam is in principle achievable. One very important feature of the  $\mu^+$  scheme is the possibility of providing an almost 100% polarized  $\mu^+$  beam without any loss in intensity. This is done by using a longitudinally polarized Lyman- $\alpha$  laser beam and allowing the  $1s - 2p$  transition to take place many times before ionization. This is the standard optical pumping technique whose extension to the  $\mu^-$  case may not be excluded.

## 8. Beam intensity and collider luminosity

The final efficiency of each of the various stages will only be reliably obtained after detailed design optimization. It will also depend on the effort invested in the implementation of the hadron accelerator, the magnetic and electric fields, the foil supports etc. At the present preliminary stage, only a gross estimation of the achievable efficiency will be attempted.

The  $\pi^-$  production spectrum at a 2 GeV deuteron beam energy have been the subject of extended simulations [14] which have been shown to fit reasonably well the existing experimental results. For a carbon target of 30 cm length and 0.8 cm diameter  $\pi^-$  production at a momentum smaller than 200 MeV/c is almost isotropic and the efficiency is close to  $0.1 \pi^-/\text{d}$ . With a production trap providing a high

$B_m/B_0$  and a high extraction field, losses due to pion and muon stopping can be kept quite limited. Muon decay and scattering will lead to an extraction efficiency of the order of  $0.015 \mu^-/d$ .

In the accumulation trap, there are injection loss (which can in principle be reduced to a small value), various stopping losses and muon decay losses. This is expected to lead to an overall efficiency of the order of 10%. This figure may increase somehow after detailed optimization but in order to obtain a significant increase in slowing down efficiency it is necessary to go over to a different and more complex configuration consisting of a large amount of separated accumulation stacks.

Finally the extraction from the magnetic field should be achievable with more than 80% efficiency [11] and with a muon loss per carbon foil between 1% and 2% as obtained from preliminary simulations, the full cooling may be obtained with about 25% efficiency.

With a 5 mA deuteron beam available from a standard accelerator concept [8], this gives  $10^{13} \mu^-/\text{sec}$  as source intensity. Assuming this value to be of the correct order and the corresponding  $\mu^+$  intensity to be 2 to 3 times higher the collider luminosity can be calculated. The very small beam emittances obtained allow such small focused beam sizes that the beam-beam tune shift effects become the limiting factors. Expressed in term of the maximum value of the tune shift parameter  $\xi_m$  the achievable luminosity (for single bunched round beams of equal shapes) will be

$$L = \frac{N f n_s \xi_m \gamma}{r_e \beta^*} \quad (5)$$

where  $N$  is the number of  $\mu^-$  in the (less intense) negatively charged bunch,  $f$  the repetition rate,  $n_s$  the effective number of turns,  $r_e$  the classical electron radius,  $\beta^*$  the transverse amplitude function at the interaction point and  $\gamma$  the beam energy in units of the muon rest energy. While  $\xi_m$  at present colliders is about 0.05 it may be assumed that given the relatively small number of collisions taking place in the muon collider ( $< 1000$ ) a somewhat greater value of  $\xi_m$  will be permitted.

Comparison with the standard high energy muon production scheme [13] where an  $\mu^-$  source intensity of  $15 \cdot 10^{14} \mu^-/\text{sec}$  is assumed we fall short here by a factor of 15. Keeping everything otherwise the same and assuming  $\xi_m = 0.12$  could be achieved, the resulting luminosity comes out to be only 5 times less ( $2 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ ). Moreover, in case recent ideas for reducing greatly tune shift effects (by introducing a sufficient amount of external charges at the I.P.[15]) are successful,  $\xi_m$  could jump up and reduce further the gap between the two concepts.

What should also be mentioned is that the availability of a very small longitudinal emittances will allow to provide a beam of quite small momentum spread ( $\ll 10^{-3}$ ) thereby improving the operation of the collider and the potentials of some experiments. Also the bunch length could be significantly reduced. This allows a smaller  $\beta^*$  further boosting up the luminosity.

One further aspect is that the primary hadron accelerator considered here has the advantage of being a relatively simple and cheap machine. Also the present  $\mu^-$  production concept makes use of compact, easy to duplicate modules. This suggests providing significant intensity increases along various pathways:

- 1) Increased current or duplication of the hadron accelerator
- 2) More muon production traps operated either in serie (by re-utilization of non-stripped deuterons) or in parallel
- 3) Extraction of the muons from both sides of the production trap
- 4) Split the muon beam coming out from the production trap into many beams of variable energy that can be directed to different optimized accumulation traps.

This useful scalability of the source intensity will allow to increase luminosity step by step while keeping a relatively small investment request for a first efficient device. It is to hope that this will help to obtain a relatively early green light for getting things started.

*Acknowledgments:* The author wishes to thank M. Mühlbauer for his muon cooling simulation program and V. Vrankovic for his support in tracking calculations.

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