MERIT Project Review
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MHD Simulation for Free Surface Hg Jet Dispersal at Low Magnetic Reynolds Numbers

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Abstract

- MHD system of equations and approximations
- Front tracking for free surface flows and EB elliptic solver
- Simulations of the MHD processes of mercury jet dispersal
- Conclusion and Future plan

1. MHD system of equations

Full system of MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla P + \mu \Delta \mathbf{u} + \frac{1}{c} (\mathbf{J} \times \mathbf{B})$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) e = -P \nabla \cdot \mathbf{u} + \frac{1}{\sigma} \mathbf{J}^2$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \left(\frac{c^2}{4\pi\sigma} \nabla \times \mathbf{B} \right)}$$

$$P = P(\rho, e), \qquad \nabla \cdot \mathbf{B} = 0$$

Low magnetic Re approximation & charge neutrality

$$\nabla \cdot J + \frac{\partial \rho_e}{\partial t} = 0$$

$$\mathbf{J} = \sigma \left(-\nabla \phi + \frac{1}{c} \mathbf{u} \times \mathbf{B} \right)$$

$$\Delta \phi = \frac{1}{c} \nabla \cdot (\mathbf{u} \times \mathbf{B}),$$

$$\text{with } \left. \frac{\partial \phi}{\partial \mathbf{n}} \right|_{\Gamma} = \frac{1}{c} (\mathbf{u} \times \mathbf{B}) \cdot \mathbf{n}$$

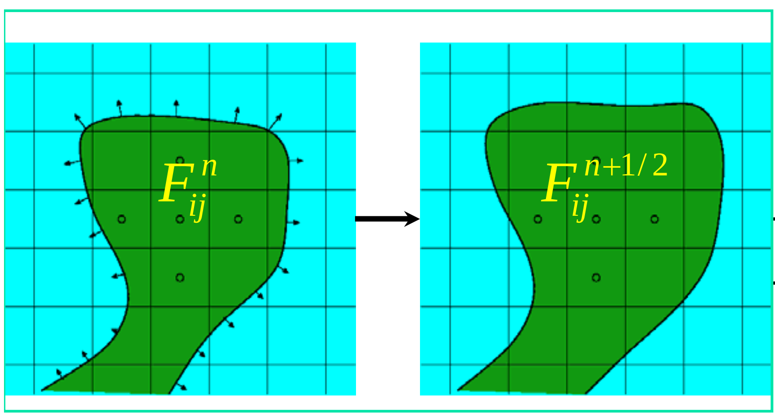
$$\mathbf{B} = \mathbf{B}_{\text{ext}}(x, t), \quad \nabla \cdot \mathbf{B}_{\text{ext}} \equiv 0$$

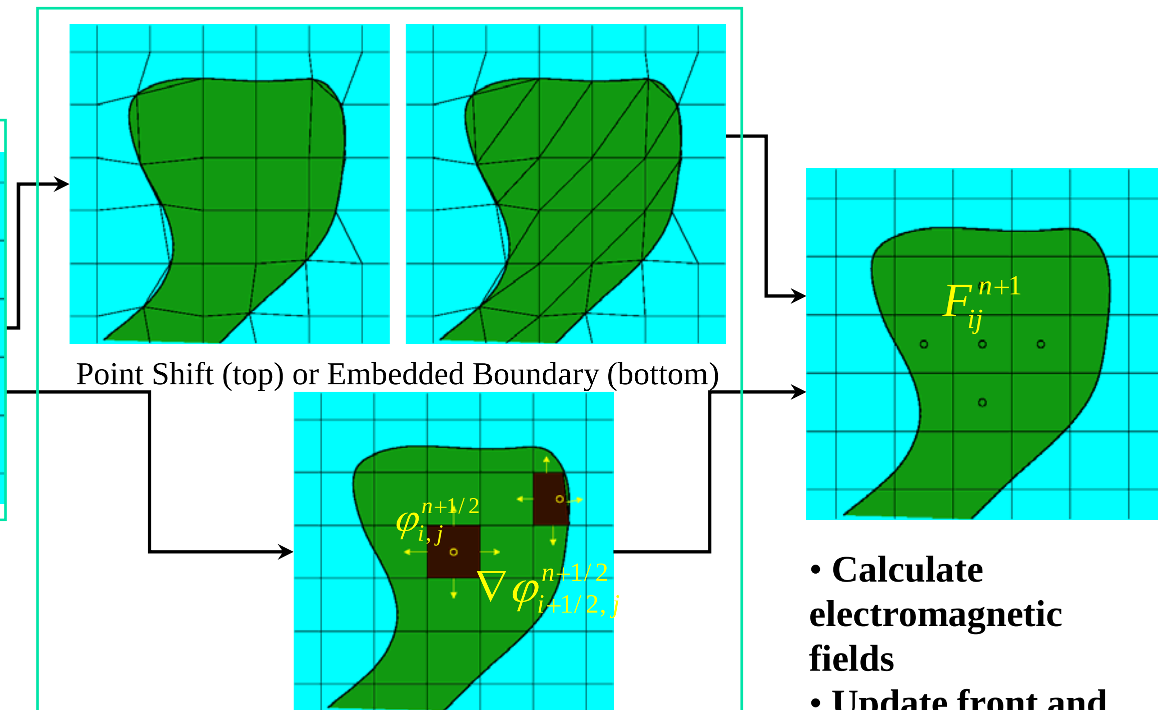
2. Front tracking and EB method

- The low magnetic Re MHD is a coupled hyperbolic/elliptic system. Operator splitting.
- The hyperbolic subsystem is solved on a finite difference grid in both domains separated by the free surface using front tracking numerical techniques.
  - Implemented in FronTier code
  - Riemann problem for interface propagation
  - Complex interfaces with topological changes in 2D and 3D
  - High resolution hyperbolic solvers
  - Realistic EOS models
- The elliptic subsystem is solved in geometrically complex domains
  - Embedded boundary finite volume discretization
  - Fast parallel linear solvers

Elliptic step

Hyperbolic step





- Propagate interface
- Untangle interface
- Update interface states

- Apply hyperbolic solvers
- Update interior hydro states

- Generate finite element grid
- Perform mixed finite element discretization
  or
- Perform finite volume discretization
- Solve linear system using fast Poisson solvers

- Calculate electromagnetic fields
- Update front and interior states

2. EB method

(1) Why EB

> Point-shift grid generation and finite element discretization method

- Second order accurate for gradients
- Compatible with mixed finite element formulation
- Capable of generating grids for vector finite elements
- Not robust (especially in 3D)

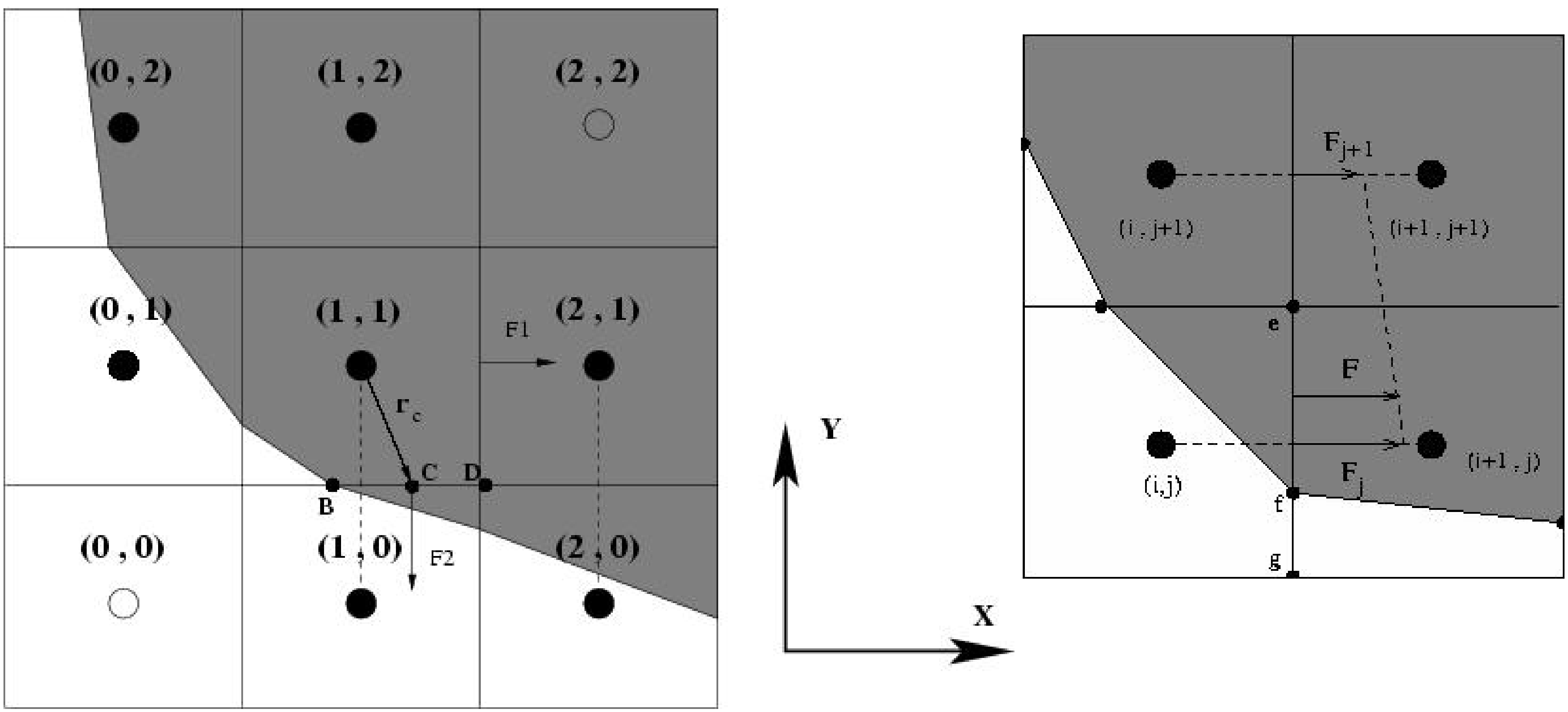
> EB

- Advantages of dealing with complex geometric domains
- second-order accuracy of solution and robust
- Trivial work to implement the algorithm in parallel computing

(2) Main Points

- Based on the finite volume discretizations
- Potential is treated as cell centered value, even if the center is outside the computational domain
- Domain boundary is embedded in the rectangular Cartesian grid, and solution is treated as a cell-centered quantity
- Using finite difference for full cell and linear interpolation for cut cell flux calculation

(3). Stencil Setting



$$\varphi = e^{k_1 X^2 + k_2 Y^2}$$

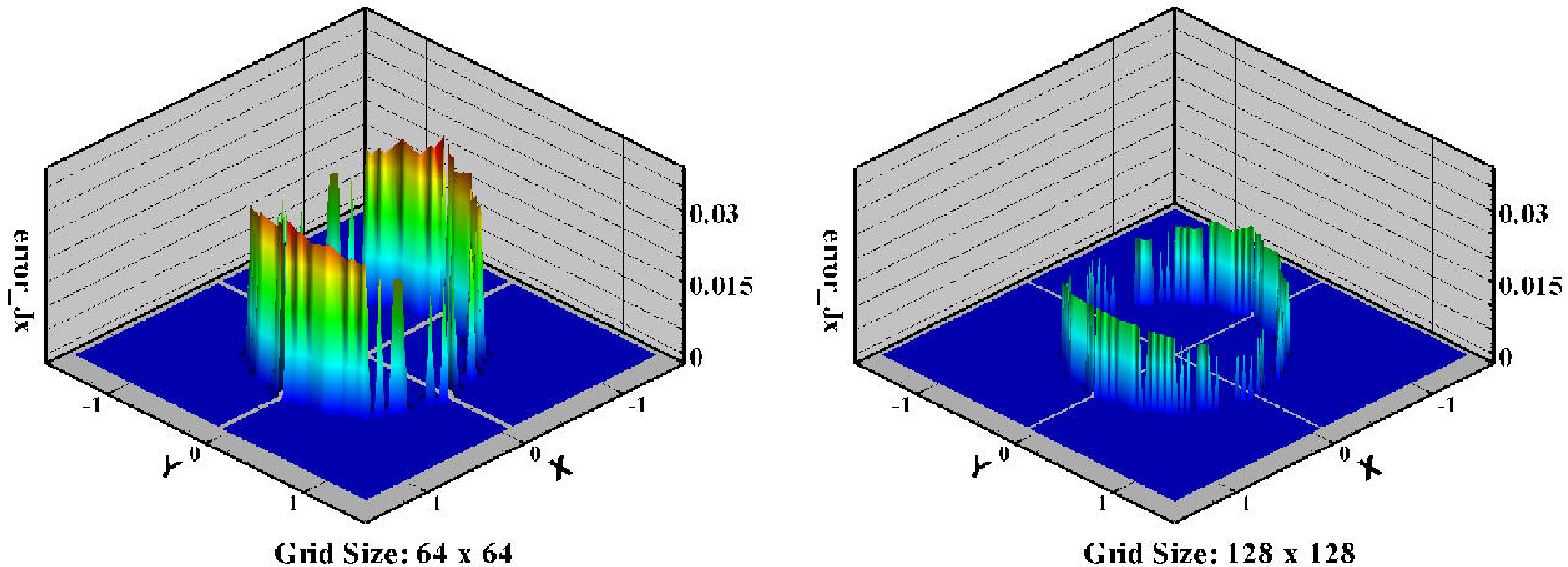
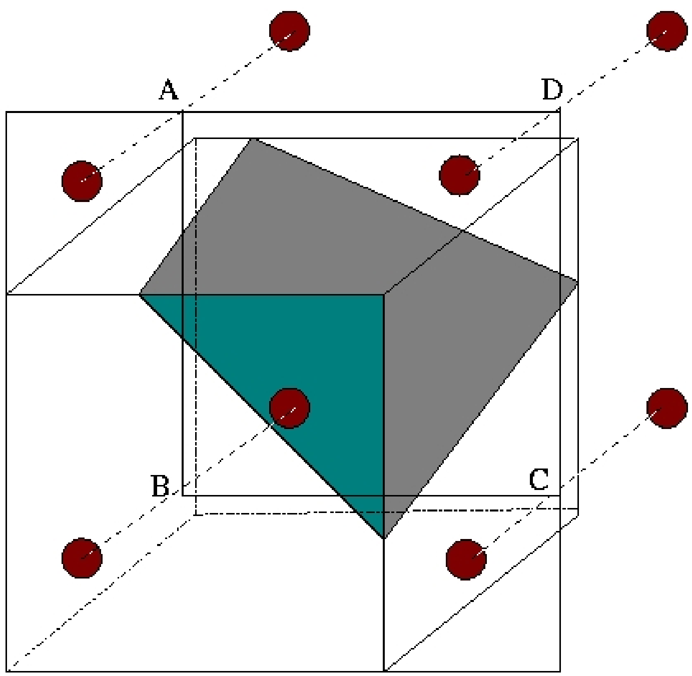


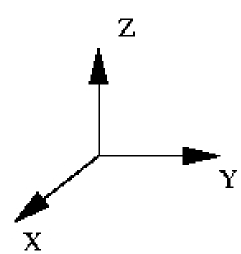
***Figure 5*. Illustration of flux error (X direction)**

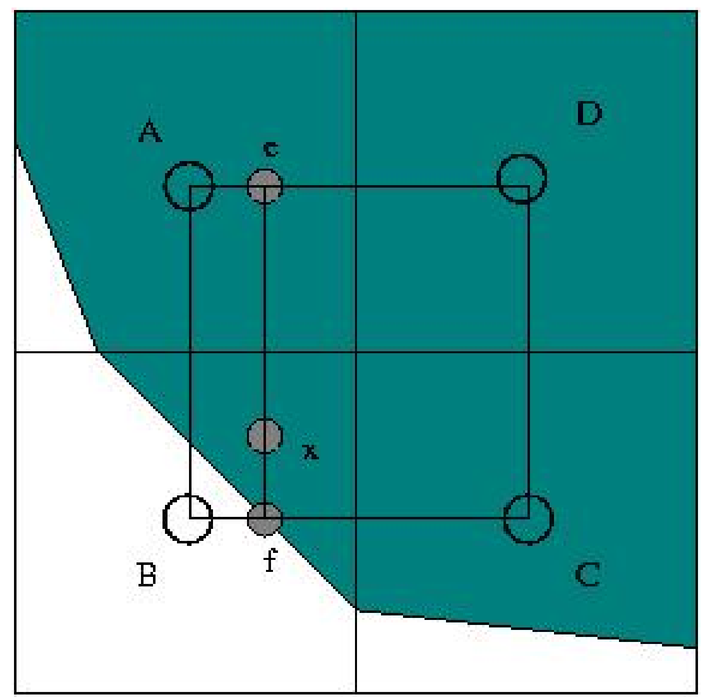
(4). 3D implementation

* Same principle as 2D

* Bilinear interpolation of flux



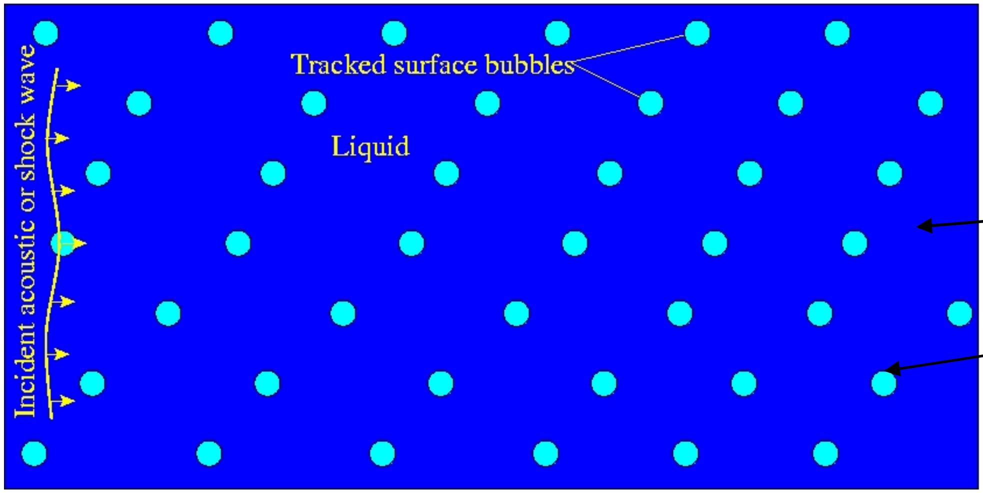




3. MHD Simulations

(1) Introduction of EOS models used

- **Heterogeneous method (Direct Numerical Simulation): Each individual bubble is explicitly resolved using FronTier interface tracking technique.**

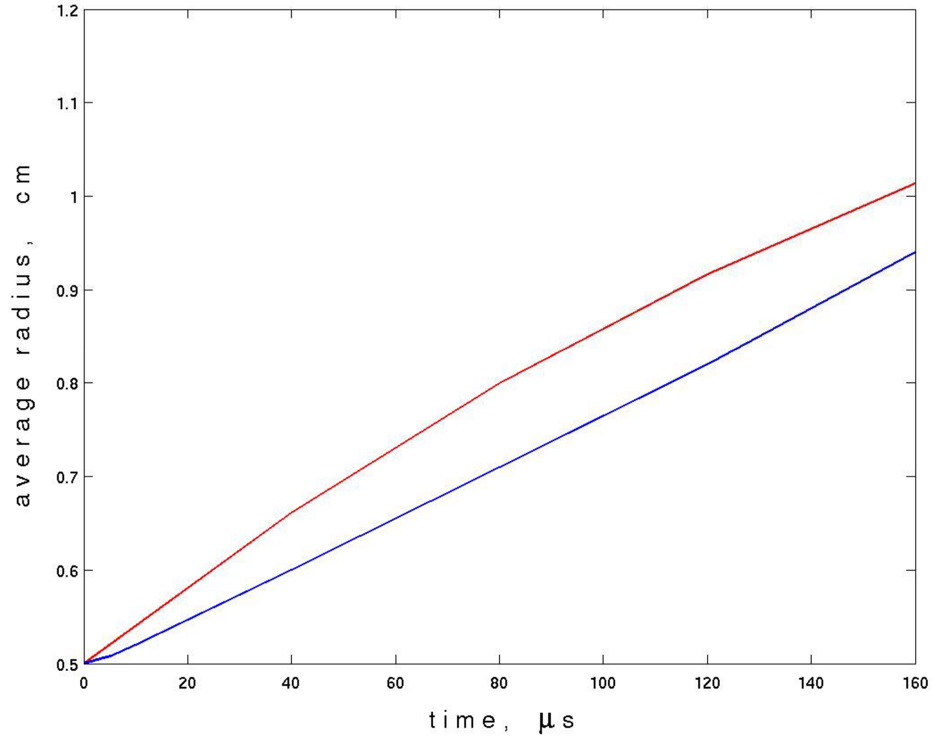


Stiffened Polytropic EOS for liquid

Polytropic EOS for gas (vapor)

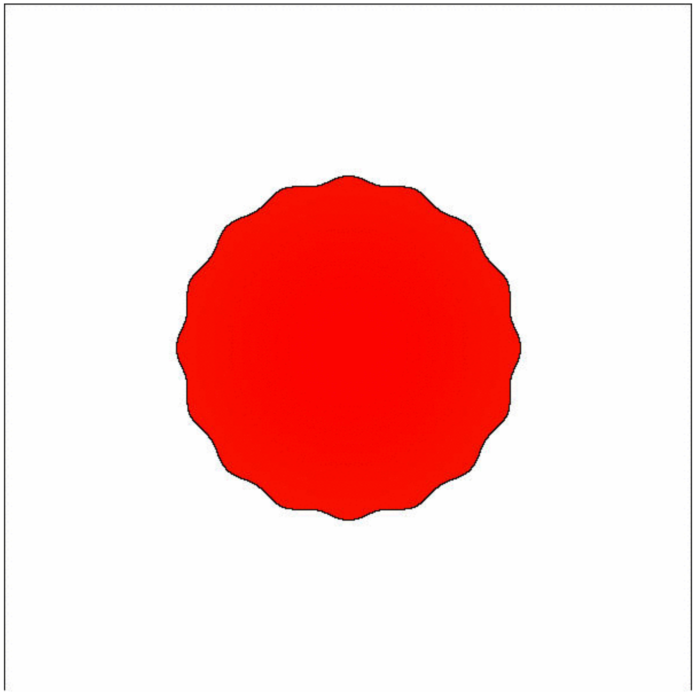
- **Homogeneous EOS model. Suitable average properties are determined and the mixture is treated as a pseudofluid that obeys an equation of single-component flow.**

(2) Comparison of Jet expansion with two EOS models (B = 0)

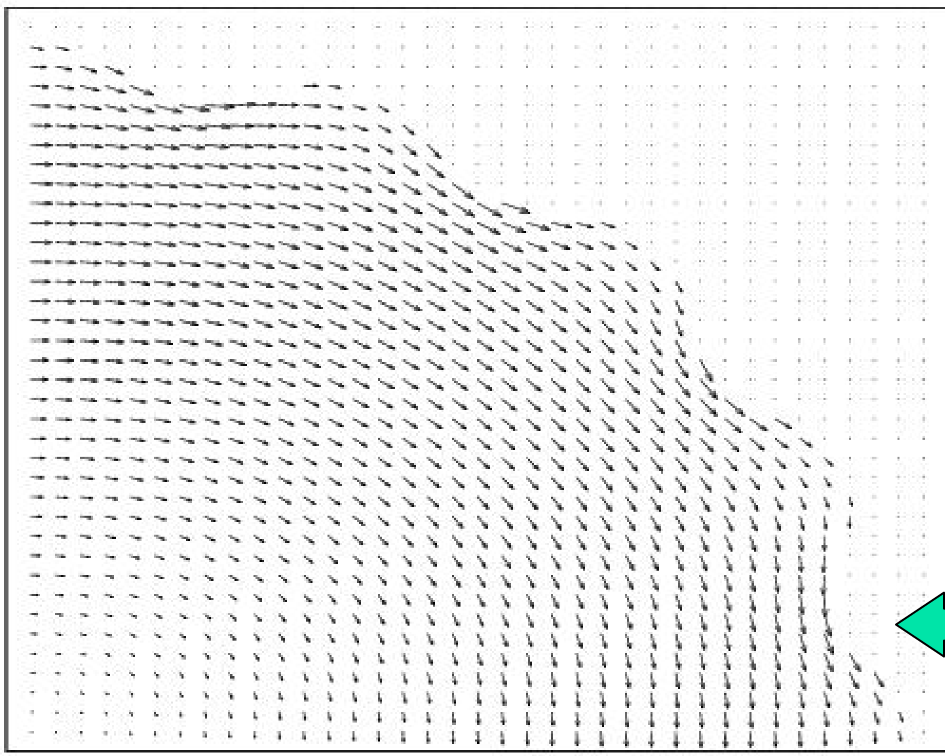


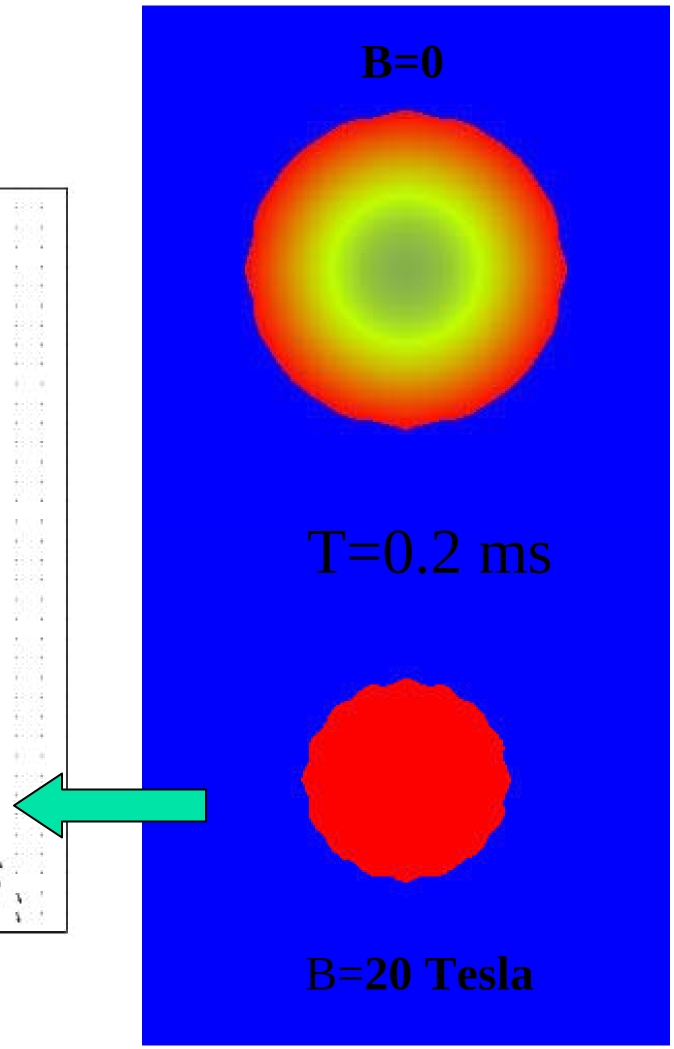
homogeneous

heterogeneous

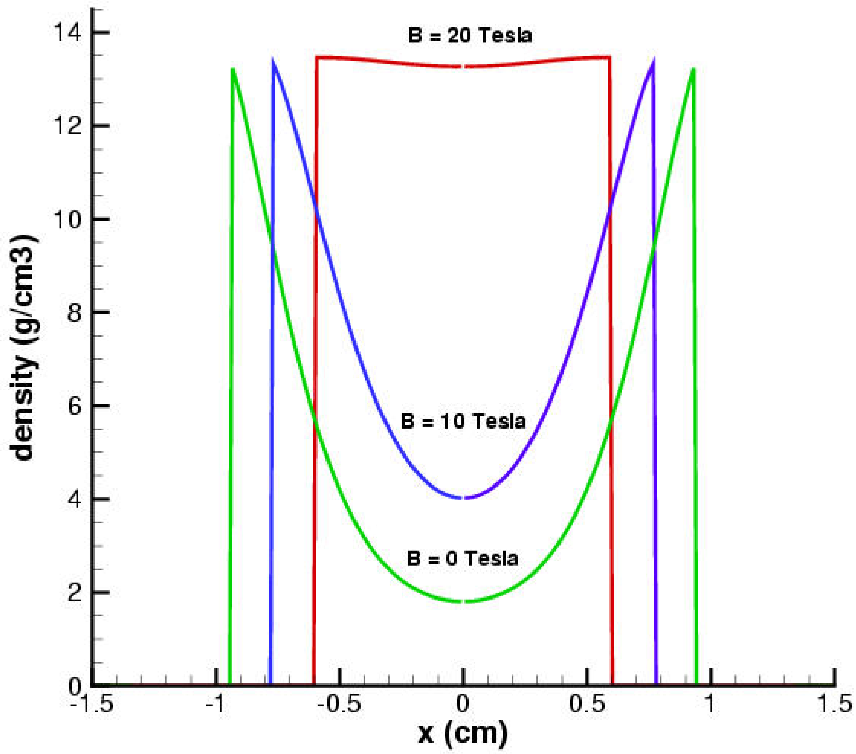


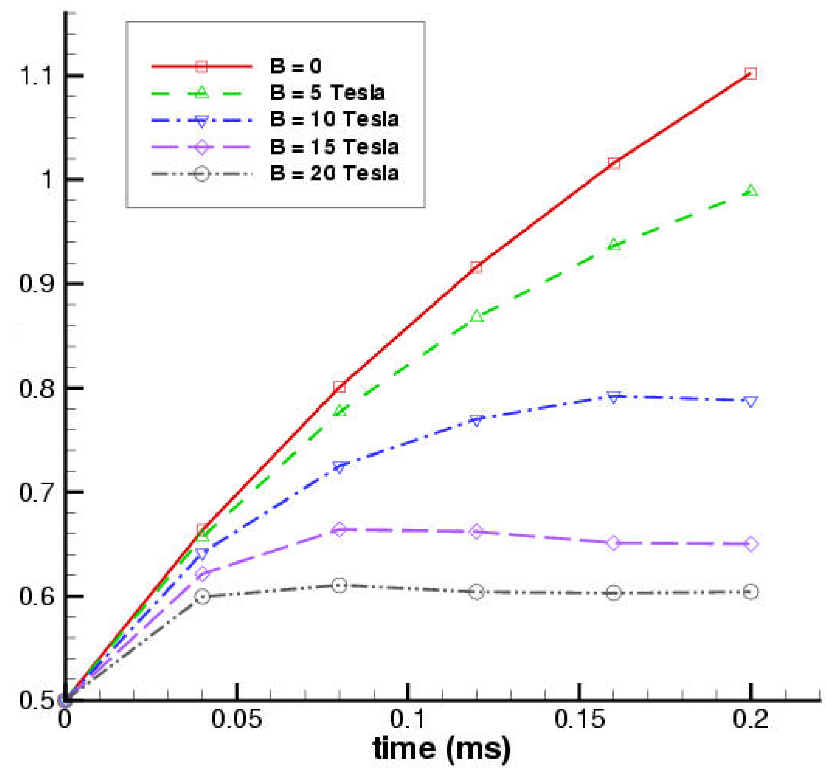
(3) Mercury Jet simulation with S2phase EOS





(4) Mercury Jet Evolution and Density Distribution





4. Conclusions and Future plan

* Embedded Boundary Method for the 2D Neumann boundary elliptic equations are implemented into FronTier code and validated over geometrically complex domain. 3D implementation is finished and under testing.

* Without magnetic field, the two EOS model give similar jet expansion speed. With magnetic field , the growth of the two-phase domain and jet expansion for homogeneous model are strongly restricted by the magnetic field.

* The MHD running for 3D and heterogeneous model is under development.