**Beam Emittance Characterization and Optimization of Target Parameters**

(X. Ding, July 22, 2014; comments by K. McDonald, July 24, 2014)

1. Courant-Snyder Invariant



2. 2-D Transverse Emittance (rms) and Twiss Parameters



 [α = 0 at the beam waist.]

*The 2-d rms, geometric emittance considered above is only appropriate in zero magnetic field. Inside a magnetic field the x- and y-phase subspaces are mixed and the 4-d transverse emittance should have been considered. And, inside a magnetic field, the validity of Twiss parameters derived from 2-d second moments is doubtful.*

3. Gaussian Distribution (Probability Density)

In two dimensional phase space (u,v):



 where u = transverse coordinate (either x or y), v=αu+βu’.

 α, β are the Courant-Synder parameters at the given point along the reference trajectory (central ray).

 In polar coordinates (r, θ):

 u = rcosθ v = rsinθ

 u’ = (v-αu)/β = (rsinθ-αu)/β

4. Distribution Function Method



Random number generator:

θ = 2π\*rndm(-1) r = sqrt(-2\*log(rndm(-1))\*σ

5. Gaussian Distribution (Fraction of Particles)

The fraction of particles that have their motion contained in a circle of radius “a” (emittance ε=πa2/β) is



6. Fraction of particles

|  |  |  |
| --- | --- | --- |
| K = a/σ | εKσ  [Normalized emittance: (βγ)εKσ ] | FGauss |
| 1 | π(σ)2/β | 39.5% |
| 2 | π(2σ)2/β | 86.4% |
| 2.5 | π(2.5σ)2/β or ~6πσ2/β  | 95.6% |

7. Focused Beam

Intersection point (beam waist) (z = 0 cm): α\*= 0, β\*, σ\*

Beam launch point (z = -100 cm): α, β, σ

 L = 100 - 0 = 100 cm = distance from beam waist to the launch point

In zero magnetic field, the Twiss parameters at distance L from the waist would be

 α=L/β\*

 β=β\*+L2/β\*

σ=σ\*sqrt(1+L2/β\*2)

For a nonzero SC field (peak of 20 T at z = 0), we launched the beam at z = -100 cm based on Twiss parameters calculated as follows (beam emittance characterization).

We first launched a beam (typically 1,000,000 particles) at z = 0 cm with α\*= 0, β\*, σ\* at fixed emittance (i.e., 5,10,15,20, … µm).

We tracked all the particles from z = 0 back to z = -100 cm, inside the magnetic field, but with no target or other material.

We computed α, β, σ from the second moments of x and x’ of the beam particles at z = -100 cm, using the expressions given in sec. 2.

Then, we launched a new beam from z = - 100 cm using these Twiss parameters, with the new beam centered in (x,y), and in (x’,y’), on their values for the central ray as backtracked from z = 0 to z = - 100 cm.

*It would have been more correct simply to use the beam particles tracked to*

*z = - 100 cm in the absence of the target, reverse their momenta, and put back the target and other material. This procedure will be used in the future.*

8. Launch Settings with Focused Beam Trajectories in a Magnetic Field

Modeled by the user subroutine BEG1 in m1514.f of MARS code

xv or xh (transverse coordinate: u); xvp or xhp (deflection angle: u’)

XINI=x0+xv DXIN=dcx0+xvp

YINI=y0+xh DYIN=dcy0+xhp

ZINI=z0 DZIN=sqrt(1-DXIN2-DYIN2)

(x0,y0,z0,dcx0,dcy0,dcz0) are the parameters at z=-100 cm for the central ray (that passes through the point x=0, y=0 and z=0).

9. Optimization of Target (or Beam) Radius

1. Fixed beam emittance (εKσ) to π(σ)2/β; Fixed target radius to beam radius (TR/BR=4)
2. Compute (x0,y0,z0,dcx0,dcy0,dcz0) for the central ray at z=-100 cm from single particle (KE=6.75 GeV) having x=0,y=0 and z=0 and specified tilt angle to SC axis at z=0.
3. Vary beam radius σ\*, while vary the β\* at the same time to fix the beam emittance; Launch a beam (1000,000 particles) at z = -100 cm using the procedures given in secs. 7 and 8.
4. Let the beam interact (in the MARS simulation) with the target (and other material), and track the secondary particles to z=50m downstream the target. Consider the yield of “good” particles to be those with KE between 40 and 180 MeV, to find the optimized value of the beam radius.