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PROPERTIES OF AN INTERSECTING-BEAM ACCELERATING SYSTEM

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The possibility of directing two high energy beams of particles against each other so the center of mass of the pair of particles will not recoil has been an essentially vain hope in the past because the number of encounters between particles in the opposing beams was so small that experimental observation would be difficult. However, with the fixed field accelerator, such as a cyclotron or FFAG accelerator, it appears that it should be possible to build up a large circulating beam between the poles of a DC magnet by successive acceleration of additional particles to the energy of the circulating beam. At proton energies well above a billion electron volts the loss of beam is slow enough to allow many frequency modulation cycles to take place, and the disturbance of the high energy beam by the radio-frequency accelerating system which is bringing up new particles is relatively slight.

The important fact is that there is enough phase space in the high energy beam of desired diameter to allow known injectors and reasonable frequency modulation rates

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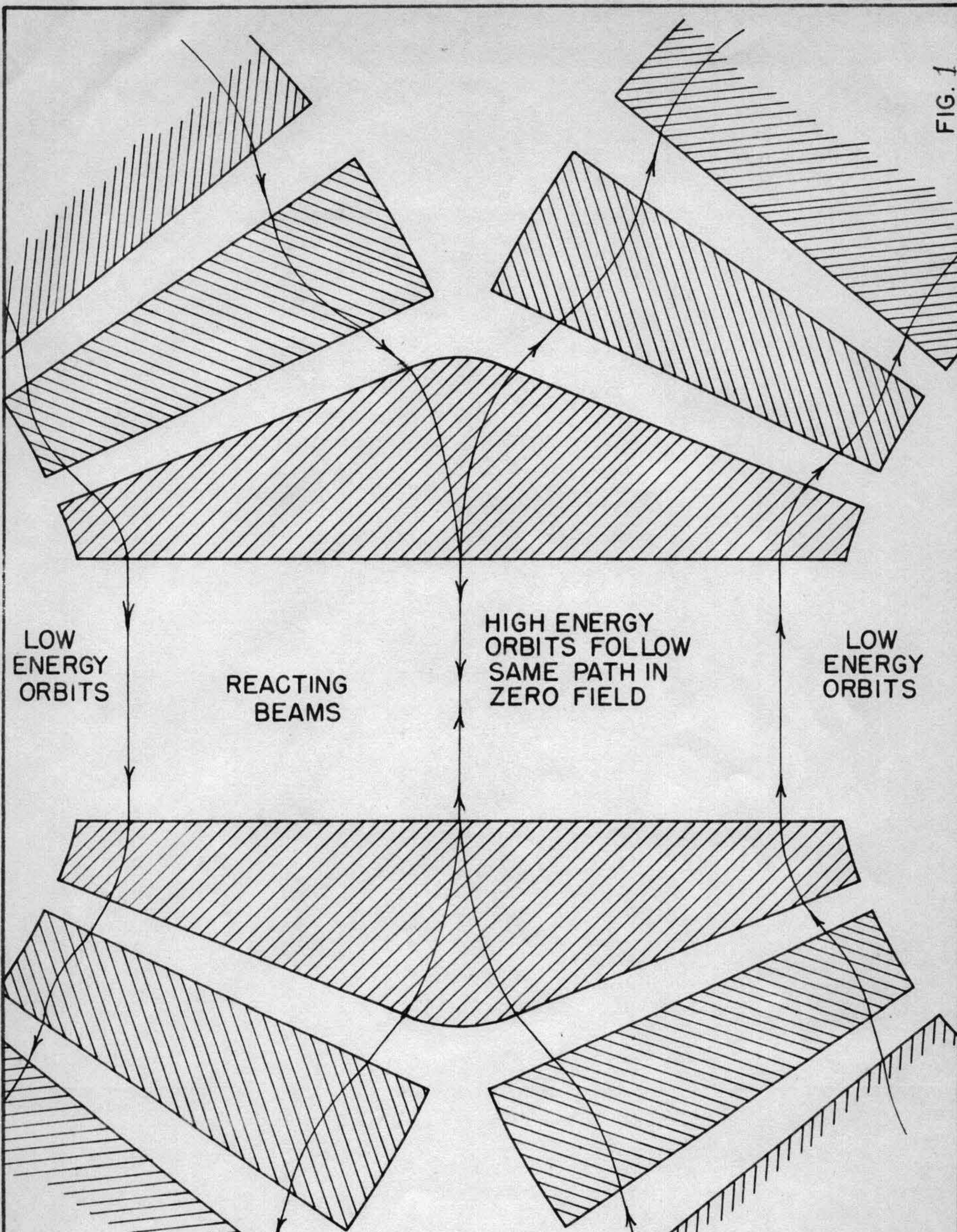
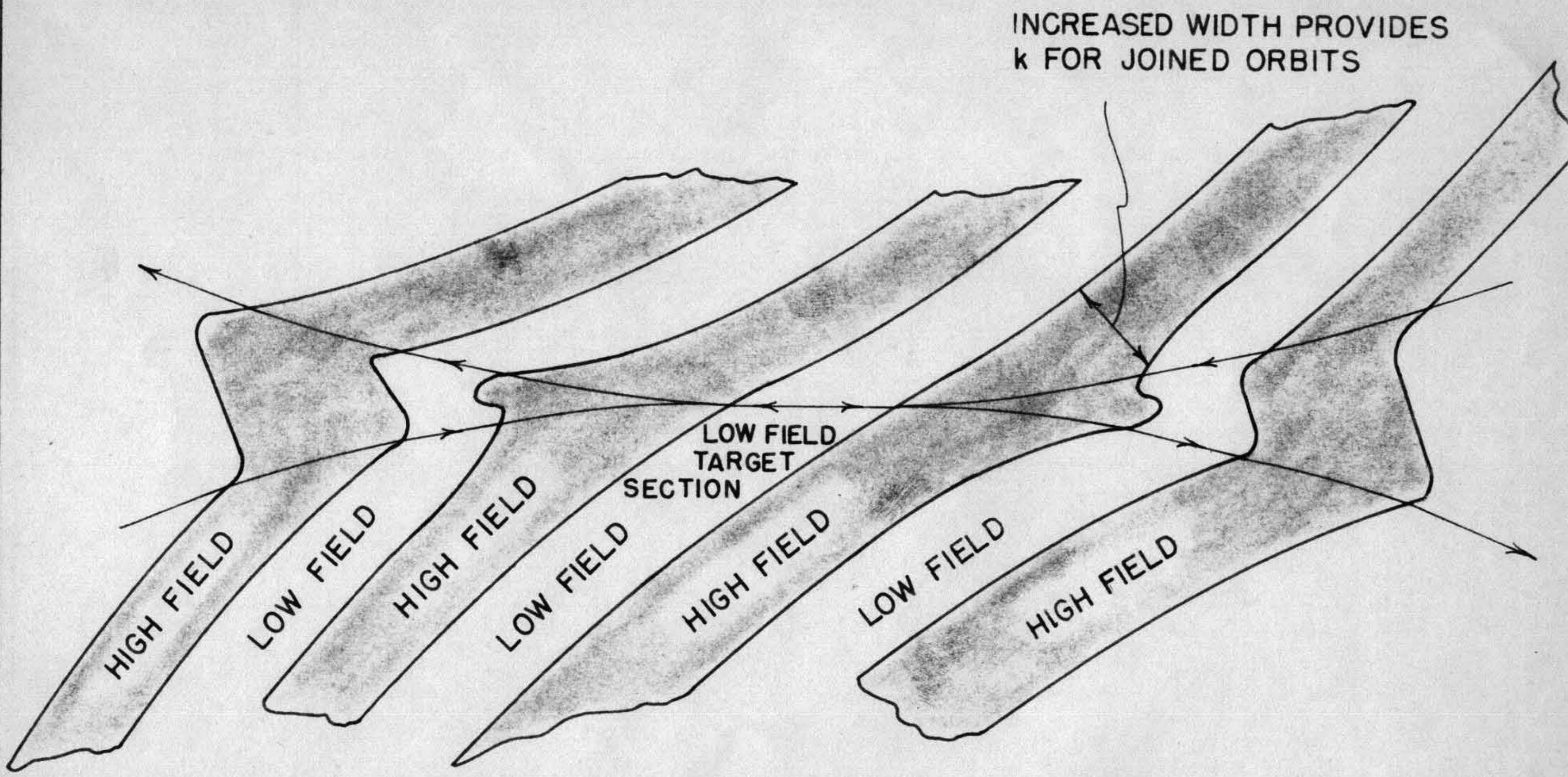
to build up the required circulating current. Analytic and digital computer studies of the interference of the RF system with the circulating beam are giving us an understanding of how to do this.

The geometrical arrangement visualized is two accelerators side by side, with their beams passing in opposite directions through a common straight section. The beams circulate essentially continuously through the same target section, and consequently the same particles traverse this section many times, providing a good probability of interacting with the particles of the other accelerator. Figure 1 shows how such a common straight section could be provided in two adjacent radial sector FFAG accelerators. Figure 2 shows in principle how a pair of spiral sector accelerators can have their beams brought together over the same target section. In the case of spiral sector accelerators it is necessary to use adjacent accelerators. However, it is possible to have radial sector accelerators concentric, the inside accelerator having its high energy beam at its outside radius, and the outside accelerator having its high energy beam at the inside radius. In other terms, the inner accelerator is a positive momentum compaction accelerator, while the outer accelerator is a negative momentum compaction accelerator.



TARGET SECTION OF BEAM
ON BEAM DOUBLE ACCELERATOR



PRINCIPLE FOR CONNECTING TWO
SPIRAL SECTOR ACCELERATORS

FIG. 2

D

Some detailed consideration of the properties of intersecting beam accelerators follow.

a. Energy Relations

If two particles have momentum p_1' and p_2' in the laboratory system, the energy in the center of mass system is

$$E_1 + E_2 = \sqrt{(M_1c^2)^2 + (M_2c^2)^2 + 2\,(E_1' E_2' - c^2 p_2' \cdot p_i)}$$

E is the kinetic plus rest energy of a particle. Mc^2 is the rest energy. If the particles are identical and are going in opposite directions in the lab system, this becomes

$$E^2 = 2 + 2\left(E_2' E_1' + \sqrt{\left[(E_2')^2 - 1\right]\left[(E_1')^2 - 1\right]}\right)$$

using Mc^2 as the unit of energy.

If $(E_2')^2 \gg 1$ and $(E_1')^2 \gg 1$, then

$$E = 2\sqrt{E_1' \; E_2'} \quad \text{when } E^2 \gg 2$$

If we want a certain E in the center of mass system and if we direct beams from two machines of radii $R_1 \propto E_1'$ and $R_2 \propto E_2'$ at one another we have $R_1 R_2 = \text{constant} = R_o^2$. The cost of a machine varies as R^3 so

$$\$ = \left(\frac{R_1}{R_o}\right)^3 + \left(\frac{R_2}{R_o}\right)^3 = \left(\frac{R_1}{R_o}\right)^3 + \left(\frac{R_o}{R_1}\right)^3 .$$ This cost has its

minimum at $R_1 = R_2 = R_o$ so it is most economical to have both accelerators of the same energy. The cost doubles if $R/R_o = 1.5$ for one of the machines and $1/1.5$ for the other machine. That is if one machine is 2 1/4 times bigger than the other, the cost is twice minimum.

The graph is Figure 3 shows that for cases where $(E_2')^2 \gg 1$ the formula $E^2 - 2 = 4E_1' \; E_2'$ overestimates E by no more than 3 1/2% overestimate as soon as the kinetic energy $T_1' \gg Mc^2$.

If only one machine is used bombarding a fixed target then $T_1' = 0$ and

$$E^2 = 2 + 2E_2' \cong 2E'$$
$$E = \sqrt{2E'}$$

If we ask what single machine with energy E' gives the same energy as two machines with energies E_1' and E_2' bombarding each other we have:

$$\sqrt{2E'} = 2\sqrt{E_1' \; E_2'} \qquad \text{or}$$

$E' = 2E_1' \; E_2'$ provided $(E_2')^2 \gg 1$ and $E_1' \gg 2$ that is $T_1' \gg 1$. This means that if number 1 machine gives 1 Mc^2 $(E_1' = 2)$ and number 2 machine gives 9 $Mc^2 (E_2' = 10)$ the equivalent machine would have to be 40 $Mc^2 = E'$. For $T_1' < Mc^2$ the graph should be used. If number 1 were 3 $Mc^2 = T_1'$, we would have 80 $Mc^2 = E'$. While if $E_1' = E_2' = 10\ Mc^2$, we have 200 Mc^2. Mc^2 is .938 Bev.

Calculating accurately, two 15 Bev accelerators would be equivalent to a .54 trillion electron volt (Tev) accelerator bombarding a stationary target but of course they would

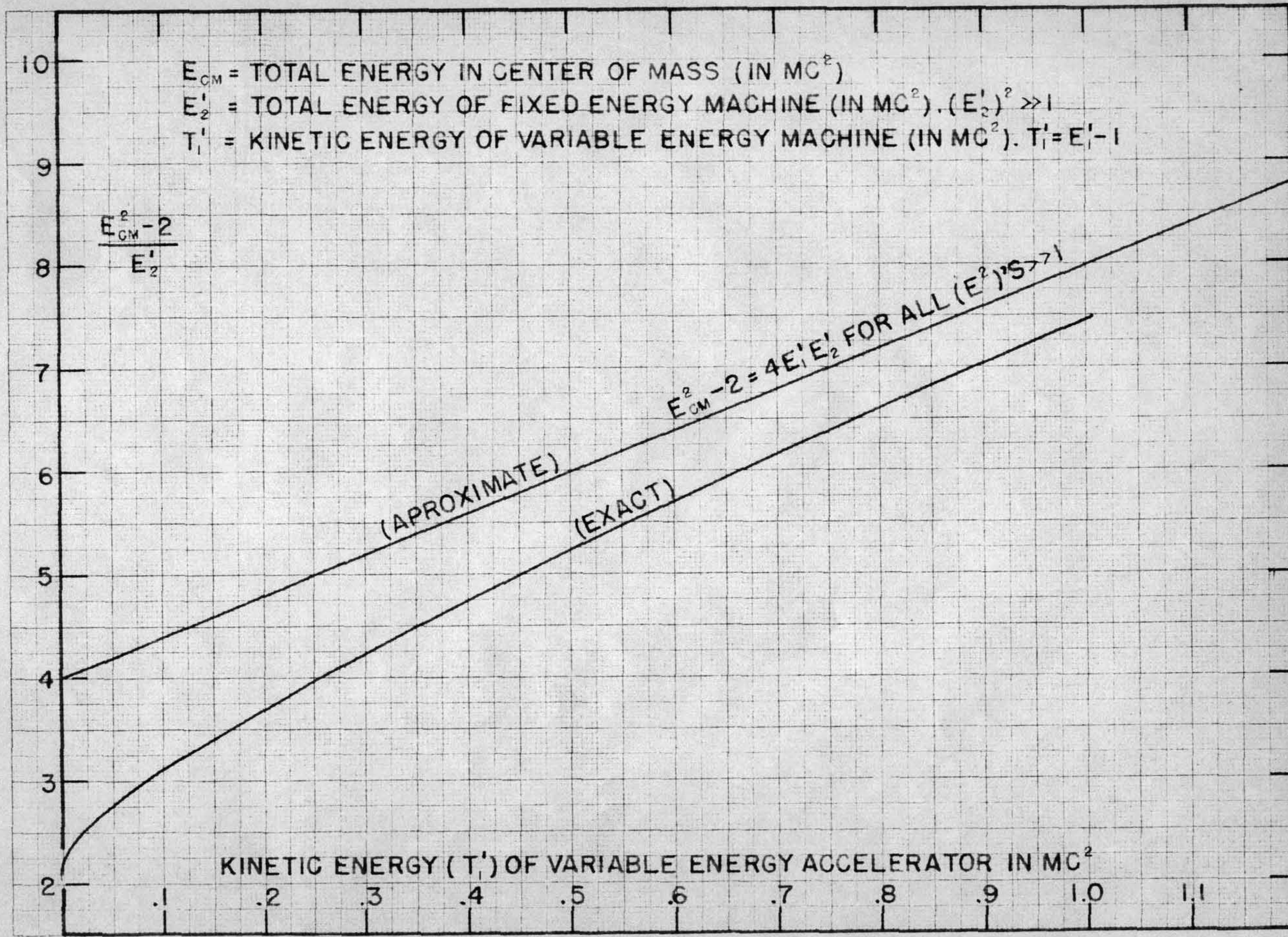

E_{CM} = TOTAL ENERGY IN CENTER OF MASS (IN MC^2)
E_2' = TOTAL ENERGY OF FIXED ENERGY MACHINE (IN MC^2). $(E_2')^2 \gg 1$
T_1' = KINETIC ENERGY OF VARIABLE ENERGY MACHINE (IN MC^2). $T_1' = E_1' - 1$
$\frac{E_{CM}^2 - 2}{E_2'}$
$E_{CM}^2 - 2 = 4E_1'E_2'$ FOR ALL (E^2)'S $\gg 1$
(APROXIMATE)
(EXACT)
KINETIC ENERGY (T_1') OF VARIABLE ENERGY ACCELERATOR IN MC^2
10
9
8
7
6
5
4
3
2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1

FIG. 3
d

give only 30 Bev plus two rest masses in the center of mass system.

It will be important to be able to vary the field strengths of both magnets together so that the energy of the reaction products could be decreased to an energy easily manageable with detecting equipment, provided new thresholds are reached at these high energies. On the other hand, any very short lived particles which do not live long enough to leave the target area near threshold energy might have their lifetimes and consequently their path lengths increased an order of magnitude by high energy bombardment so that they could escape far enough to be detected.

If $E_1' \neq E_2'$ the center of mass is not at rest in the laboratory and the spatial distribution of the reaction products is peaked in the direction of motion of the higher energy beam. This provides a way to get slow particles, although the energy may be far above threshold energy. One uses reaction products emitted in a direction opposite to that of the center of mass motion.

b. Yield.

The number of interactions per second for two azimuthally uniform beams is $n = 2\, N_1\, N_2\, v\, \ell\, \sigma\, A$.

N_1 is the number of particles per square centimeter circulating in the first accelerator per centimeter of circumference, and N_2 is the number percentimeter in the second accelerator, v is the velocity of the particles (about the velocity of light), ℓ is the length of the interaction region of the target section, and A is the cross sectional area of the beam. If we take the geometrical cross section for σ, the interaction cross section, $\sim 5 \times 10^{-26} cm^2$, $A = 1\ cm^2$, R_o the radius of the orbit $\sim 10^4$ cm, N is the total number of particles circulating in each accelerator may be tentatively taken at 10^{10} (as in already existing accelerators), then:

$$n = 2A\ell\sigma\left(\frac{N}{A\ 2\pi R_o}\right)^2 c \cong 6\ \ell \times 10^{-5} \text{ interactions per second.}$$

If the target section is 100 cm. long, we have only $\sim 6 \times 10^{-3}$ interactions per second, but if we now accumulate a large number of groups of circulating particles the reaction rate will go up as the square of the number of groups we can hold in the circulating beam. If we want ~ 100 disintegration particles / cm.^{2} / sec., at 100 cm. distance from the target section, then we need $100 \times 4\pi (100)^2 = 1.2 \times 10^7$ interactions per second. This would require $N \sim 5 \times 10^{14}$ particles in each machine. Thus, for this high a yield we would need thousands of accumulated groups of 10^{10}

particles each. The possibility of reaching this current is discussed under g.

c. Multiple coulomb scattering of the beam.

If we assume the vacuum chamber to contain nitrogen at 10^{-5} mm pressure, what is the angle containing half the beam after scattering has continued for 10^3 seconds?

$$\bar{\Theta}_{1/2} = \frac{Z\sqrt{N}}{2\beta^2 E}$$ where N is the number of moles of gas of atomic number Z per square centimeter. E is the total energy in Mev.

$$\bar{\Theta}_{1/2} = \frac{7\sqrt{3 \times 10^{10} \times 10^3 \text{ sec} \times 10^{-5} / (760 \times 22{,}400)}}{2\ E}$$

$$= \frac{15.}{E} \text{ radians.}$$

At 10 Bev, $\Theta_{1/2} = .0015$ radians.

at 3 Bev, $\Theta_{1/2} = .005$ radians.

For other reasons we should have much lower pressure but at a high pressure of 10^{-5} mm and at 10 Bev, the scattering is less than 1.5 milliradians for half the beam after 1,000 seconds. This gives approximately a two centimeter spread of the beam. At the lower pressures and at shorter times this spread can be brought down to less than one cm.

d. Beam Life.

Nuclear reactions or collisions with the residual gas will consume the beam eventually.

At 10^{-5} mm. of pressure we have 10^{13} nucleons per cubic centimeter. These nucleons are in clusters forming nuclei which provide a certain amount of shielding of each other reducing the cross section per nucleon by approximately 50%. The rate of reaction is then

$n v \sigma = 50\% \times 10^{13}/cm^3 \times 3 \times 10^{10} cm/sec \times 5 \times 10^{-26} cm.^2 \doteq 7.5 \times 10^{-3}$ per sec. So the mean life of the beam is $(1/7.5) \times 10^{+3} = 130$ seconds at 10^{-5} mm.

e. Influence of single scattering on beam life.

Estimation of single coulomb scattering of the beam out of a cone of .0005 radians, which would not spread the beam seriously, shows that at 15 Bev this cross section is about 1/6 the cross section for nuclear collision. Consequently, at this high energy the single scattering would slightly reduce the estimate of beam life time given in "d" to about 110 seconds for the example of 10^{-5} millimeter pressure.

f. Background.

These disintegrations and scatterings estimated in section d cause a background with which the experimenter must contend. If we have 5×10^{12} effective gas nucleons/cc and 5×10^{14} moving protons in a machine or

$$\frac{5 \times 10^{14}}{2\pi \times 10^{4} \text{ cm.}} = 0.8 \times 10^{10}$$

protons/cc. then 600 is the ratio of effective background nucleons to target protons in the target section. But the target protons are moving and hence twice as much volume is effective for producing desired reactions. Thus in the target section the number of interactions with gas at 10^{-5} mm. is 300 times the p-p interactions. We can strive for 10^{-7} mm. pressure in the target section, thereby depressing the number of gas interactions. Modern vacuum techniques make this practicable.

There are two important characteristics of this background: first, the high energy component is expected to be largely confined to the orbital plane and to travel forward, while the reactions of the intersecting beams would produce products traveling in all directions if the center of mass is at rest in the laboratory. There will, however, be several low energy nucleons emitted at an angle like star prongs for each background interaction. Second, troublesome background may originate from the upstream portions of the beam in each machine. To eliminate this a specific type of shielding would be needed. The magnets themselves would help shield, and counting particles above or below the orbital plane would help to avoid the background. Exceptionally high vacuum in the target section is evidently very important.

It is important to note that the yield of disintegrations is proportional to the product of the number of particles circulating in each machine or approximately proportional to the square of the number of accelerated particles, while the background is directly proportional to the number of particles circulating. Thus the ratio of yield to background is proportional to the number of circulating particles. Therefore it is all important to achieve the high currents and small beam diameters.

g. Phase Space Requirements.

We wish to calculate the number of injected pulses which can be stacked in the neighborhood of any orbit. If we neglect coupling between betatron and phase oscillations, and if we assume that the period of phase oscillations is long in comparison with the period of revolution, then, as we have shown in the discussion on radio frequency acceleration, the number of injected pulses that can be accommodated at energy, E_2, within a spread, ΔE_2, is then:

$$(1) \quad n_p = A_2/A_1 = (\Delta E_2/\Delta E_1) \quad (\omega_1/\omega_2)$$

This represents the most efficient stacking possible at E_2. Any less than the optimum acceleration process will result in a mixing of empty phase area with area occupied by particles so that the number of stacked pulses will be less than (1).

In terms of the parameter:

(2) $k \equiv (\partial \, \mathrm{Lnp})/(\partial \, \mathrm{LnR})$

where p is the particle momentum and $2\pi R$ is the orbit circumference, we can express the energy spread, ΔE, in terms of the radial spread δR:

(3) $\Delta E = (k+1)(c^2p^2/E) \quad (\delta R/R)$

We now consider the betatron oscillations. If we consider, just for an example, injection with an electrostatic Van de Graaf accelerator where we know $\psi \cong .001$ and Δx or $\Delta z = 1/2$ cm. then accounting for the damping of the betatron oscillation the number of turns of injection beam at the injection energy which will just fill the betatron phase space available at the high energy is:

$$n_b = p_2^2 \, (\Delta x_2 \Delta z_2)^2 \, \nu_x \nu_z / 4 (R_2{}^2 p_1{}^2 \, \psi_x \psi_z \quad \Delta x_1 \, \Delta z_1 \,)$$

Taking

$\nu_x = 10$, $\nu_z = 5$ and $\Delta x_2 = \Delta z_2 = \Delta x_1 = \Delta z_1 = 0.5$ cm

We find

$n_b = 1{,}250.$

To scan the aperture with 1,250 turn injection from electrostatic accelerator is complex; but the indication is clearly that a high current injector with a much bigger angular spread and a much bigger beam diameter is desirable. Provided the energy spread of an A - 48 type of accelerator is small enough, it might be the right type of injector for just one turn or for a few turns. This is the high current Linac of the University of California which gives .250 amperes.

Going back to the synchrotron phase space ratio according to (1) and (3) we can rewrite (1) as

$$(4) \quad n_p = \frac{(2(k+1)p_2 \delta R)}{(\Delta E_1/T_1)p_1 R} \quad .$$

For δR = 1 cm beam spread at high energy, $\Delta E_1/T_1 = .001$ for an electrostatic generator, $p_2/p_1 = 200$, $k \sim 100$, $R \sim 10^4$ then n_p = 4,000 F.M. cycles possible before the beam is spread more than δR = 1 cm.

It is clear that with n_b = 1,250 and n_p = 4000 and with a one milliampere beam from an electrostatic machine that gives 10^{11} particles for one turn around the machine we could have in principle 10^3 more than the 5×10^{14} particles circulating in the high energy beam which we found previously would give 10^7 disintegrations per second. The conclusion is therefore that we have plenty of phase space even with existing injectors to achieve the yield mentioned.

h. Space charge and space current effects.

If we build up a beam of particles circulating in the accelerator in a perfect vacuum, the space charge repulsion and the space current attraction cancel each other at relativistic speeds; thus, the particles would behave according to the dynamics of single particles, the type of theory we have used in our studies. However, since there is residual gas in the vacuum chamber, some interaction effects are likely to occur. Gas molecules struck by the beam will become ionized, and the positive ions will be repelled by the proton beam while the electrons will be attracted into the proton beam.

The extent to which these ions and electrons can influence the beam is a matter for examination both theoretically and experimentally using a high current FFAG model. The experience in handling such beams will be very important since the beams' currents are large. For the case of 5×10^{14} circulating particles the current is 50 amperes, but it may be possible to greatly exceed this in view of the large amount of available phase space.