Capturing the Rare Decay $K^+ \to \pi^+ \nu \bar{\nu}$

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Abstract

This thesis describes a search for the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$, a Flavor Changing Neutral Current (FCNC) process forbidden to first order in the Standard Model by the GIM mechanism. The decay is allowed at second order, however, and expected to have a branching ratio of approximately $10^{-10}$. Thanks to the presence of an internal top quark in the diagrams for this process, a measurement of the branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ provides a measurement of the Cabibbo-Kobayashi-Maskawa matrix element $V_{td}$. Data collected by Experiment 787 at Brookhaven National Laboratory during the 1995 run were analyzed, and one candidate event surfaced with an expected background of $0.08 \pm 0.03$ events. This first observation of $K^+ \to \pi^+ \nu \bar{\nu}$ yields a branching ratio of $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (2.54^{+5.82}_{-2.16}) \times 10^{-10}$. A simultaneous search for the decay $K^+ \to \pi^+ X^0$, where $X^0$ is any massless, weakly interacting particle, yielded no candidate events, and a 90% confidence level upper limit on the branching ratio is set at $1.81 \times 10^{-10}$. 

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Chapter 1

Introduction

The Standard Model of Particle Physics, which describes our knowledge of the elementary particles and the interactions among them, still contains many ill-measured and not fully understood parameters. These free parameters (such as the particle masses and the elements of the Cabibbo-Kobayashi-Maskawa matrix) are not predicted by theory; however, the Standard Model requires that certain relations exist between them. Thus, by measuring these parameters precisely, we can hope to either confirm or to rule out the Standard Model. That, then, is our task.

1.1 The Standard Model

We assume three generations of leptons and quarks:

\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix}
\begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix}
\begin{pmatrix}
\nu_\tau \\
\tau^-
\end{pmatrix}
\]  

(1.1)
1.1. *The Standard Model*

\[
\begin{pmatrix}
    u \\
    d
\end{pmatrix} \begin{pmatrix}
    c \\
    s
\end{pmatrix} \begin{pmatrix}
    t \\
    b
\end{pmatrix}
\]  

(1.2)

There is no reason to assume *à priori* that these mass eigenstates are also eigenstates of the weak interactions. In general we could have

\[
\begin{pmatrix}
    \nu_e \\
    \nu_\mu \\
    \nu_\tau
\end{pmatrix} = U_0 \begin{pmatrix}
    \nu'_e \\
    \nu'_\mu \\
    \nu'_\tau
\end{pmatrix}, \quad \begin{pmatrix}
    e^- \\
    \mu^- \\
    \tau^-
\end{pmatrix} = U_{-1} \begin{pmatrix}
    e'^- \\
    \mu'^- \\
    \tau'^-
\end{pmatrix},
\]  

(1.3)

\[
\begin{pmatrix}
    u \\
    c \\
    t
\end{pmatrix} = U_{2/3} \begin{pmatrix}
    u' \\
    c' \\
    t'
\end{pmatrix}, \quad \begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix} = U_{-1/3} \begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix},
\]  

(1.4)

where the primes indicate weak eigenstates and the $U_Q$ are rotation matrices. Interactions between the charged and the neutral leptons will contain the term $U_0 U_{-1}^\dagger$; likewise interactions among quarks of different charge will contain the term $U_{2/3} U_{-1/3}^\dagger$.

It will be sufficient, therefore, to define new matrices:

\[
V_{\text{lepton}} = U_0 U_{-1}^\dagger; \quad V_{\text{CKM}} = U_{2/3} U_{-1/3}^\dagger.
\]  

(1.5)

In the lepton sector, a vanishing neutrino mass permits arbitrary rotations between the mass eigenstates and the weak eigenstates, thus allowing $U_0 = U_{-1}$ in which case $V_{\text{lepton}}$ becomes the unit matrix. We will assume the neutrinos are massless. In the quark sector it is sufficient to rotate only the $q_{-1/3}$ quarks using (1.5). Hence the
1.1. The Standard Model

weak eigenstates are:

\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix}, \quad \begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix}, \quad \begin{pmatrix}
\nu_\tau \\
\tau^-
\end{pmatrix}
\]

(1.6)

\[
\begin{pmatrix}
u_u \\
\mu \\
u_t \\
\tau
\end{pmatrix}, \quad \begin{pmatrix}
u_c \\
c \\
u_t \\
t
\end{pmatrix}
\]

(1.7)

and the \(q'_{-1/3}\) states are related to the \(q_{-1/3}\) states through the unitary Cabibbo-Kobayashi-Maskawa matrix [1]:

\[
\begin{pmatrix}
d' \\
s' \\
b'
\end{pmatrix} = V_{\text{CKM}} \begin{pmatrix}
d \\
s \\
b
\end{pmatrix} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \begin{pmatrix}
d \\
s \\
b
\end{pmatrix}.
\]

(1.8)

The quarks interact with each other via the following forces: strong, weak, electromagnetic, gravity. We will concentrate on just the weak interaction, which is described by the Lagrangian

\[
\mathcal{L} = \frac{g_2}{2\sqrt{2}} (J_\mu^+ W^+\mu + J_\mu^- W^-\mu) + \frac{g_2}{2\cos\Theta_W} J_\mu^0 Z^\mu,
\]

(1.9)

where \(g_2\) is the weak coupling constant and \(\Theta_W\) is the Weinberg angle. The first term above describes the charged current interactions, mediated by the \(W^+, W^-\) bosons, and the second term describes the weak neutral current interaction, mediated by the
1.1. The Standard Model

$Z^0$ boson. The charged current is (omitting the $\gamma$-matrices)

$$J^+_{\mu} = \begin{pmatrix} d' \\ \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} d' \\ s' \\ \bar{b} \end{pmatrix}$$

$$= \bar{u} d V_{ud} + \bar{u} s V_{us} + \bar{u} b V_{ub} + \bar{c} d V_{cd} + \bar{c} s V_{cs} + \bar{c} b V_{cb}$$

$$+ \bar{t} d V_{td} + \bar{t} s V_{ts} + \bar{t} b V_{tb},$$

and the weak neutral current is

$$J^0_{\mu} = \begin{pmatrix} u \\ c \\ t \\ d' \\ s' \\ \bar{b} \end{pmatrix}$$

$$= \bar{u} u + \bar{c} c + \bar{t} t$$

$$+ \bar{d} d (|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2)$$

$$+ \bar{d} s (V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts})$$

$$+ \bar{d} b (V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb})$$

$$+ \bar{s} d (V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td})$$

$$+ \bar{s} s (|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2)$$

$$+ \bar{s} b (V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb})$$
1.2. The CKM Matrix

\[ + 5d(V_{ud}^* V_{ud} + V_{cb}^* V_{cd} + V_{ub}^* V_{ud}) \]
\[ + 5s(V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{ub}^* V_{us}) \]
\[ + 5b(|V_{ub}|^2 + |V_{cb}|^2 + |V_{ub}|^2) \]
\[ = \bar{u}u + \bar{c}c + \bar{t}t + \bar{d}d + \bar{s}s + \bar{b}b. \]  

(1.13)

The last equality is a consequence of the unitarity of $V_{\text{CKM}}$, which requires, for example, that

\[ V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{ub}^* V_{ts} = 0. \]  

(1.15)

This cancellation, which results in the absence of flavor changing neutral currents (FCNCs) at tree level, is called the GIM mechanism [2].

1.2 The CKM Matrix

Since each of the entries in $V_{\text{CKM}}$ could be complex, $V_{\text{CKM}}$ contains 18 real parameters. However, unitarity of $V_{\text{CKM}}$ implies that only four of these parameters (three angles and one complex phase) are independent. Making precise measurements of these parameters is an important near-term goal of the High Energy Physics program. Assuming three quark generations and unitarity, the current 90% confidence level
limits on the magnitude of the components are [3]:

\[
\begin{pmatrix}
(0.9745, 0.9757) & (0.219, 0.224) & (0.002, 0.005) \\
(0.218, 0.224) & (0.9736, 0.9750) & (0.036, 0.046) \\
(0.004, 0.014) & (0.034, 0.046) & (0.9989, 0.9993)
\end{pmatrix}.
\]  

The standard parameterization of \(V_{\text{CKM}}\), advocated by [3], is

\[
V_{\text{CKM}} = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13},
\end{pmatrix}
\]  

where \(c_n \equiv \cos \theta_n\), \(s_n \equiv \sin \theta_n\) and the four independent parameters are \(\theta_{12}, \theta_{23}, \theta_{13},\) and \(\delta\). The Wolfenstein parameterization [4] replaces \(\theta_{12}, \theta_{23}, \theta_{13}, \delta\) with \(\lambda, A, \rho, \eta\) using the following definitions [5]:

\[
\lambda = s_{12}, \quad A\lambda^2 = s_{23}, \quad A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}
\]  

We can then approximate \(V_{\text{CKM}}\) as

\[
V_{\text{CKM}} \approx \begin{pmatrix}
  1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
  -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
  A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]  

Thus, the task of measuring the parameters of \(V_{\text{CKM}}\) can be accomplished by measuring \(\lambda, A, \rho, \text{ and } \eta\). \(\lambda\) and \(A\) have been measured using tree level decays. \(\lambda\) has been determined via \(K_{e3}\) decays and via semileptonic hyperon decays to be
1.3. The Unitarity Triangle

$|V_{us}| = \lambda = 0.2205 \pm 0.0018$ [3]. $|V_{cb}|$ can be measured from $B$ meson decays and determines $A$: $A\lambda^2 = |V_{cb}| = 0.040 \pm 0.003$ [6].

Notice that $\rho$ and $\eta$ only appear in the far off diagonal components $V_{ub}$ and $V_{td}$, which are the most difficult to measure due to suppression of $u \leftrightarrow b$ and $d \leftrightarrow t$ transitions. $V_{td}$ in particular is difficult to measure using tree level decays due to the elusiveness of the top quark. We can, however, try to measure $\rho$ and $\eta$ with second order weak processes involving internal $b$ and $t$ quarks.

1.3 The Unitarity Triangle

The unitarity of $V_{CKM}$ requires that

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0,$$  \hspace{1cm} (1.20)

which can be expressed geometrically as a triangle in the complex plane (see Figure 1.1). We can rescale (1.20) as follows:

$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + 1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$  \hspace{1cm} (1.21)

With the use of (1.18) this becomes

$$(-\bar{\rho} - i\bar{\eta}) + 1 + (-1 + \bar{\rho} + i\bar{\eta}) \simeq 0,$$  \hspace{1cm} (1.22)
1.3. The Unitarity Triangle

Figure 1.1: The unitarity triangle.

which can be represented by a triangle in the complex plane with vertices \( A = (\bar{\rho}, \bar{\eta}) \), \( B = (1, 0) \), and \( C = (0, 0) \). The use of \( \bar{\rho} \) and \( \bar{\eta} \), defined as [5]

\[
\bar{\rho} = \rho \left( 1 - \frac{\lambda^2}{2} \right), \quad \bar{\eta} = \eta \left( 1 - \frac{\lambda^2}{2} \right), \tag{1.23}
\]

improves the accuracy of the unitarity triangle to \( \mathcal{O}(\lambda^5) \) over the \( \mathcal{O}(\lambda^3) \) approximation of (1.19).

Measurements of various processes can restrict the allowed values of \( \bar{\rho} \) and \( \bar{\eta} \).

- The measurement of \( |V_{ub}/V_{cb}| \) from \( B \) meson decays determines a circle in the \((\bar{\rho}, \bar{\eta})\) plane centered at \((0, 0)\). The current measurement is \( |V_{ub}/V_{cb}| = 0.08 \pm 0.02 \) [3].

- \( B^0 - \bar{B}^0 \) mixing determines a circle in the \((\bar{\rho}, \bar{\eta})\) plane centered at \((1, 0)\). This is basically a measurement of \( |V_{td}| \) which comes from the \( d \leftrightarrow t \) transition in the box diagram for \( B^0 - \bar{B}^0 \) mixing. The \( c \) and \( u \) quark contributions are highly
1.3. The Unitarity Triangle

suppressed relative to the $t$ quark contribution due to the diagonally dominant nature of $V_{CKM}$ and also due to the large $t$ quark mass.

- A measurement of the indirect $CP$ violation parameter, $\epsilon$, from $K \rightarrow \pi\pi$ decays determines a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane.

- A measurement of the direct $CP$ violation parameter $\epsilon'/\epsilon$ determines $\bar{\eta}$. At the present time, NA31 at CERN measures $Re(\epsilon'/\epsilon) = (23 \pm 7) \times 10^{-4}$ [7], and E731 at Fermilab measures $Re(\epsilon'/\epsilon) = (7.4 \pm 5.9) \times 10^{-4}$ [8]. A measurement of the branching ratio for the directly $CP$ violating process $K_L \rightarrow \pi^0\nu\bar{\nu}$ also determines $\bar{\eta}$. This decay has not yet been observed.

- A measurement of the $K^+ \rightarrow \pi^+\nu\bar{\nu}$ branching ratio determines an ellipse in the $(\bar{\rho}, \bar{\eta})$ plane centered at $(\rho_0, 0)$ with $\rho_0 \approx 1.4$. This has also not yet been measured.

If perfect measurements could be made of these quantities, and if there were no theoretical uncertainties involved in mapping these quantities to the $(\bar{\rho}, \bar{\eta})$ plane, they would determine a set of curves (see Figure 1.2) that would intersect at the point $(\bar{\rho}, \bar{\eta})$. Furthermore, failure of these curves to intersect at a unique point would indicate physics beyond the Standard Model, such as additional generations of quarks, or the existence of new particles such leptoquarks. Thus, each of the items in the list above is an important test of the Standard Model. The remainder of this thesis is
concerned with the last item in the list, the measurement of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio.

![Diagram](image)

Figure 1.2: The unitarity triangle in the $(\bar{\rho}, \eta)$ plane, showing a hypothetical perfect measurement of $\bar{\rho}$ and $\eta$.

### 1.4 The Decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The GIM mechanism forbids first order weak $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays (see Figure 1.3). $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is, however, allowed in highly suppressed second order weak decays (see Figures 1.4 and 1.5), in which the different masses of the internal $u$, $c$, and $t$ quarks spoil the GIM cancellation. In particular, the very large $t$ quark mass increases the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. 
1.4. The Decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Figure 1.3: The first Feynman diagram describes a first order weak $K^+ \rightarrow \pi^0 e^+ \nu_e$ decay, which is allowed in the Standard Model. The second describes a first order weak $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, which is not.

Figure 1.4: The three diagrams that contribute to the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio. All are second order weak.

Figure 1.5: Additional second order weak $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ diagrams. These diagrams are not considered in the branching ratio calculation because they only contribute at about the $10^{-15}$ level [9, 10, 11].
1.4. The Decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The branching ratio calculation for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is relatively clean theoretically. Hadronic uncertainties are removed by normalizing to the branching ratio for the isospin-rotated $K^+ \rightarrow \pi^0 e^+ \nu_e$ mode ($BR = 0.0482 \pm 0.0006$ [3]), for which the hadronic part is very similar to that of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (see Figure 1.3). The branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can be expressed as [5]:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.11 \times 10^{-11} A^4 X^2(x_t) \frac{1}{\sigma} [(\sigma \bar{\eta})^2 + (\rho_0 - \bar{\rho})^2] , \quad (1.24)$$

where

$$\sigma = \left( \frac{1}{1 - \frac{x_t}{2}} \right)^2 \simeq 1.05 , \quad (1.25)$$

and

$$X(x_t) = \frac{x_t}{8} \left( \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln x_t \right) \simeq 0.65 x_t^{0.59} , \quad x_t = \frac{m_t^2}{m_W^2} . \quad (1.26)$$

$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is dominated by the internal $t$ quark contribution; the deviation from unity of $\rho_0$ measures the internal $c$ quark contribution, and the internal $u$ quark contribution is negligible. Equation (1.24) determines an ellipse in the $(\bar{\rho}, \bar{\eta})$ plane centered at $(\rho_0, 0)$. Making use of known constraints, we find [6]

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (0.91 \pm 0.32) \times 10^{-10} , \quad (1.27)$$

where the error is dominated by uncertainties in the $V_{CKM}$ parameters $A$, $\rho$, and $\eta$. Thus, a measurement of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in excess of, say, $2 \times 10^{-10}$ could be a sign of new physics.
1.5 **History of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$**

The data analyzed here are the first to contain experimental evidence for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$; until now, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ had not been experimentally observed. The 90% confidence level upper limit on the branching ratio, the product of Phase I (1988-1991) of E787 at Brookhaven National Laboratory, was $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 2.4 \times 10^{-9}$ [12]. This is over an order of magnitude away from the current Standard Model prediction (see Tables 1.1 and 1.2 and Figure 1.6).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>90% confidence level upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>[13]</td>
<td>1971</td>
<td>$1.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>[14]</td>
<td>1973</td>
<td>$5.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>[15]</td>
<td>1981</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>[16]</td>
<td>1990</td>
<td>$3.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>[17]</td>
<td>1993</td>
<td>$7.5 \times 10^{-9}$</td>
</tr>
<tr>
<td>[12]</td>
<td>1996</td>
<td>$2.4 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 1.1: History of upper limits to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ set by experiment.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>1974</td>
<td>$\sim 10^{-10}$</td>
</tr>
<tr>
<td>[19]</td>
<td>1983</td>
<td>$(4.2 - 33) \times 10^{-10}$</td>
</tr>
<tr>
<td>[20]</td>
<td>1987</td>
<td>$(1 - 8) \times 10^{-10}$</td>
</tr>
<tr>
<td>[21]</td>
<td>1991</td>
<td>$(0.6 - 6) \times 10^{-10}$</td>
</tr>
<tr>
<td>[22]</td>
<td>1994</td>
<td>$(0.7 - 1.5) \times 10^{-10}$</td>
</tr>
<tr>
<td>[6]</td>
<td>1997</td>
<td>$(0.91 \pm .32) \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Table 1.2: History of theoretical ranges for $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. Note that the theoretical prediction from [18] assumed only two generations while the others assume three, hence the jump in predicted branching ratios between 1974 and 1983.

In the years 1992-94, E787 underwent major upgrades to run at higher rates and to obtain better rejection at these higher rates of processes that mimic $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. 

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1.5. *History of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$*

Figure 1.6: Progress in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Notice that the theoretical range for the branching ratio has managed to keep over an order of magnitude away from the sensitivity of concurrent experiments. The experiment points in this plot are 90% confidence level upper limits based on observation of no events.
1.6. The Decay $K^+ \rightarrow \pi^+ X^0$

In 1995, E787 collected 3 times the data used to set the previous limit [12]. The analysis of that data is the subject of this thesis.

1.6. The Decay $K^+ \rightarrow \pi^+ X^0$

We can also conduct a search for $K^+ \rightarrow \pi^+ X^0$ with the E787 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ data set, where $X^0$ is any weakly-interacting massless particle. Various extensions to the Standard Model include such particles, such as the "familon" proposed by Wilczek [23]. Being a two-body decay, $K^+ \rightarrow \pi^+ X^0$ would show up as a monochromatic spike in pion energy at the endpoint of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ spectrum. Although E787 was built primarily to search for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, we can use the data to search for $K^+ \rightarrow \pi^+ X^0$ as well, and we do so in the analysis described here.

1.7. The Decay $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$

There are currently two proposals to search for $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$, E926 at BNL and KAMI at FNAL. $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ is a purely direct $CP$ violating decay, and as such a measurement of the branching ratio would directly measure the Wolfenstein parameter $\eta$. Referring back to Figure 1.2, one can see how a measurement of the $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio nicely compliments a measurement of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio, and together these measurements help to map out the allowed region in the $(\rho, \eta)$ plane.
Chapter 2

The E787 Experiment

The E787 experiment at Brookhaven National Laboratory was designed to search for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. It was proposed in 1983, and a test run in 1988 yielded a factor of 5 improvement in the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ limit [16] over the previous published result [15]. Long data taking runs were conducted in 1989, 1990, and 1991, which further improved the limit [12], but still no $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events were found. This constituted "Phase I" of E787. In 1992 and 1993, the experiment was upgraded to achieve further background suppression with a beam line supplying higher kaon rates in hopes of finally seeing some $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events. "Phase II" of E787 then began with an engineering run in 1994 that produced no useful $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ data but was instrumental in tuning the detector. 1995 saw the first long data-taking run with the new detector, and the analysis of that data set is the subject of this thesis.

The E787 detector is well documented elsewhere [24, 25, 26, 27, 28], thus my review of it here will be brief. In particular, an excellent description of the upgraded
detector can be found in [29].

2.1 E787 Overview

The experimental signature for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is a poor one. Because the neutrinos will pass through the detector without interacting, the search for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is really a search for a $K^+$ decaying to a single $\pi^+$ with no other visible decay products. The three most troublesome background sources are:

- $K^+ \rightarrow \mu^+ \nu \mu$ decays (called $K_{\mu2}$): If the $\mu^+$ is mistaken for a $\pi^+$, this event will appear to be $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

- $K^+ \rightarrow \pi^+ \pi^0$ decays (called $K_{\pi2}$): If both photons from the $\pi^0$ decay are lost, this event will appear to be $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

- $\pi^+$ scattering events (called $\pi-$scats): These are events in which a beam $\pi^+$ scatters into the fiducial region of the detector, which can also appear to be $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

We can also use the measured momentum of the charged track\(^1\) to distinguish between different decay modes. $K_{\mu2}$ and $K_{\pi2}$ are both two-body decays, and thus the charged-track momentum will peak at a single value (205 MeV/c for $K_{\pi2}$'s and 236 MeV/c

---

\(^1\)“Charged track” will refer to the single charged track we see in the detector, which, in the case of a real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, will be a pion. For various backgrounds, however, the charged track will be a muon.
for \(K_{\mu 2}\)'s) in the kaon rest frame\(^2\). Three-body decays however, such as \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\), will have a continuous spectrum of charged-track momenta (see Figure 2.1). The

![Figure 2.1: Spectra of the most common \(K^+\) decay modes, along with the Standard Model spectrum for \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\). The branching ratios of the various decay modes are shown in parentheses.](image)

\(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) spectrum extends up to 227 MeV/c, with about 20% of the spectrum above the \(K_{\pi 2}\) peak at 205 MeV/c. For this analysis, we will restrict the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) search to the region between the \(K_{\pi 2}\) and \(K_{\mu 2}\) peaks, a region of momentum space with little background from these principle decay modes. The \(K^+\) has many other decay modes as well (\(\pi^+ \pi^+ \pi^-, \pi^0 \mu^+ \nu_{\mu}, \pi^+ \pi^0 \pi^0, \mu^+ \nu_{\mu} \gamma, etc\.) but these tend to be easier to reject than \(K_{\mu 2}\) and \(K_{\pi 2}\) due to their low charged-track momenta and/or extra decay products.

\(^2\)E787 uses kaons that have come to rest in the detector, thus the kaon rest frame will be the lab frame.
2.1. E787 Overview

Because the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio is believed to be around $10^{-10}$, we will need at least 10 orders of background rejection, and a very intense $K^+$ beam in order to see this decay. E787 was designed with the following goals in mind:

- High intensity, high purity $K^+$ beam. We will also need good $K^+$ identification, to ensure that we are not fooled by pions in the beam.

- Accurate measurement of the decay product's momentum (essential for adequate background rejection) is needed. To this end, the kaons are required to come to rest before they decay. This requirement also helps to reject the $\pi-$scat background.

- Excellent photon detection is required to reject many of the principle backgrounds, notably $K_{\pi2}$. The detector is surrounded by a $4\pi$-sr dedicated photon veto. In addition, nearly the entire fiducial region of the detector is active, and can thus be used to veto events with photons.

- Redundant kinematic measurement of the charged track helps to reject background. In addition to measuring the momentum of the charged track in a central drift chamber, the kinetic energy and range in scintillator of the charged track are measured.

- Excellent $\pi/\mu$ separation. We can achieved this by requiring observation of the full $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay sequence. Also, the measured charged-track range
and momentum are useful for $\pi/\mu$ separation since $\pi^+$'s and $\mu^+$'s of a given momentum will have different ranges, due to their different masses.

### 2.2 Detector Description

The detector consists of: beam instrumentation, a 3-meter-diameter solenoid that immerses the entire spectrometer in a 1 Tesla magnetic field\(^3\) (for momentum measurements), an active fiber target in which the kaons come to rest, a central drift chamber, a "range stack" of plastic scintillator, and a system of photon vetos. See Figure 2.2.

![Figure 2.2: Side (a) and end (b) views of E787. Only the top half of the detector (which is cylindrically symmetric) is shown.](image)

\(^3\)The B field points in the direction of the kaon beam. Thus, positively charged particles will bend clockwise when viewed looking upstream from a position downstream of the detector.
2.2. Detector Description

2.2.1 Beam

E787 is situated in the Low Energy Separated Beam III (LESB III) at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL), in Upton, New York. During the 1995 run, 790 MeV/c kaons were delivered in 1.6 second "spills", one every 3.6 seconds. Each spill contained roughly $7 \times 10^6 K^+$'s and $2 \times 10^6 \pi^+$'s. About $1.3 \times 10^6$ of these $K^+$'s made it to the stopping target (with the rest decaying in flight, interacting, or scattering out of the beam) per spill.

On their way into the detector, the kaons (and pions) pass through a number of beam counters. The first of these is the Čerenkov counter, which is used to separately identify kaons and pions in the beam. Beam particles pass through a radiator that emits Čerenkov light. This light is directed into 14 "$K^+$ phototubes" for 790 MeV/c particles of the $K^+$ mass, or directed into 14 "$\pi^+$ phototubes" for 790 MeV/c particles of the $\pi^+$ mass. The beam then passes through two beam wire chambers (BWPC's), separated from each other by about 90 cm. The BWPC's can identify multiple beam particles close to each other in space and time. After the second BWPC, the beam enters the degrader, which slows beam particles through ionization energy loss so they can be stopped in the target. The first 15 inches of the degrader is made of Beryllium Oxide, and the last 4 inches is made of Lead Oxide doped glass. This "lead-glass counter" is instrumented with phototubes, and can thus function as a
2.2. Detector Description

detector for beam pions$^4$ or for photons coming from kaon decays in the target. Sandwiched between the lead-glass counter and the target is a hodoscope (the B4 counter) consisting of 2 planes of 8 scintillating fingers each. This counter is used to detect beam particles that make it through the degrader.

2.2.2 Target

The $K^+$ stopping target (see Figure 2.3) is made of 413 5-mm-square plastic scintillating fibers, each 310 cm in length and connected to a phototube. Pulses from the phototubes are fed to ADC's, TDC's, and 500-MHz CCD transient digitizers. The kaons slow down in the target through ionization energy loss, and come to rest roughly in the center of the E787 spectrometer, where they decay. In general, the incoming $K^+$ will travel down the length of the fibers, hitting only a few and depositing $>5$ MeV in each, and the outgoing $\pi^+$ will travel perpendicular to the fibers, depositing $\sim 2$ MeV in each. Thus both time and energy information help to distinguish "$K^+$ fibers" from "$\pi^+$ fibers."

After leaving the target, the charged track passes through an I-counter (see Figure 2.3). The six I-counters are 24.1 cm in length and define the fiducial region in $z$ of the target.

$^4$The lead-glass counter is blind to beam kaons, but will create Čerenkov light when traversed by beam pions.
2.2. Detector Description

Figure 2.3: End and side views of the $K^+$ stopping target, I-counters, and V-counters. $r$ and $z$ are not shown with the same scale in the side view.

2.2.3 Drift Chamber

Next the $\pi^+$ is tracked by the Ultra Thin Chamber (UTC), shown in Figure 2.4. The UTC has 12 layers of drift cells (each with an anode wire) arranged in 3 superlayers and filled with an Argon/Ethane/Ethanol gas mixture. The 3 superlayers are separated by passive buffer regions containing Nitrogen. When the $\pi^+$ passes through the gas, ions are created that drift with a known velocity to the anode wires. The drift times to these wires define isochrones to which a circular track can be fit in the X-Y view for measurement of the track momentum (see Figure 3.1). Six foils (which serve as boundaries for the 3 superlayers) are etched with helical cathode stripes. These stripes provide $z$ measurements that can be used to determine the "dip-angle", the angle of the charged track with respect to the plane normal to the beam, denoted $\theta_{\text{dip}}$. The UTC is described in detail in [30].
2.2. Detector Description

E787 Central Tracking Drift Chamber

Figure 2.4: The Ultra Thin Chamber.

2.2.4 Range Stack

Upon exiting the UTC, the $\pi^+$ enters the range stack (RS), where it will slow down and come to rest through ionization energy loss. The $\pi^+$ can also decay or interact before coming to rest, but E787 will have no acceptance for such events. The range stack consists of 24 azimuthal "sectors" of stacks of plastic scintillator. Each stack has 21 radial layers. The innermost layer (layer 1, also called the T counter) is 0.635 cm thick and 52 cm long, and defines the solid angle acceptance (about 0.5) of the detector. Layers 2-21 are 1.905 cm thick and 182 cm long. Each range stack module is instrumented with a phototube at each end. The pulses from these phototubes are delivered to ADC’s and 500-MHz transient digitizers (TD’s), and they are also
discriminated for use in the trigger. The TD’s are especially useful for observing the \( \pi^+ \rightarrow \mu^+ \rightarrow e^+ \) decay sequence (see Figure 2.5) in the range stack module in which the \( \pi^+ \) comes to rest (called the stopping counter).

There are two layers of straw-tube tracking chambers (RSSC's) embedded in each range stack sector. These are located after range stack layers 10 and 14, and provide measurements of both \( \phi \) and \( z \).

### 2.2.5 Photon Veto

A barrel veto (BV) surrounds the range stack. The barrel veto is made of layers of lead and scintillator stacked into modules, each with a phototube at each end. The lead serves to increase the probability that a photon will convert before exiting the barrel veto, and the scintillator serves to measure the photon energy. The barrel veto is 15 radiation lengths thick, and the fraction of energy deposited in scintillator (the "visible energy") is 0.3. The phototubes are instrumented with ADC's and TDC's.

Endcaps (EC's) made of pure CsI crystals detect photons at large dip-angle. Unlike the barrel veto, this vetoing system is 100% active. The crystals are coupled to phototubes instrumented with 500-MHz CCD transient digitizers.

Photons can also be detected in many other subdetectors, including the lead-glass counter, the target, the I-counters, the V-counters, and the range stack. Since the charged track also passes through many of these subdetectors, some spatial separation
Figure 2.5: Transient digitizer data showing a complete $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ sequence for a pion passing through layer 11 and stopping in layer 12 of the range stack. The muon has an energy of only 4 MeV, so it travels only about 1 mm and stays in the stopping counter. The electron can have up to 52 MeV of energy, so it will generally leave the stopping counter, as in this example where it travels through layers 12, 13, and 14.
2.3. The Trigger

is required to distinguish the photon from the charged track.

2.3 The Trigger

The trigger determines which events\(^5\) will be written to tape for further scrutiny. Fast decisions must be made as the events are coming in, thus the trigger only uses rather crude selection criteria to reject background events. There are a number of different trigger types designed to select different types of events. The \(K^+ \rightarrow \pi^+ \nu \bar{\nu}(1)\) trigger selects candidates for \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) events above the \(K_{\pi 2}\) peak, and the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}(2)\) trigger selects candidates for \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) events below the \(K_{\pi 2}\) peak. It is the analysis of events passing the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}(1)\) trigger that is described here.

The requirements to pass the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}(1)\) trigger are:

- **KBeam**: KBeam is a fourfold coincidence, requiring hits in at least 9 of the 14 Čerenkov counter kaon phototubes, a hit in the B4 counter, energy in the target, and a signal derived from the AGS spill structure that determines the useful part of the beam spill. KBeam tells us that a kaon has entered the target.

- **IC**: An I-counter hit is required, presumably due to the kaon decay product(s).

- **DC**: A "delayed coincidence" is required between the KBeam and IC signals.

\(^5\)Each kaon that enters the detector defines an event.
kaon decayed at rest.

- T·2 : A coincidence is required between the first two layers of the range stack (the hits must be in the same sector). The T counter is thin to avoid triggering on photon conversions. The T·2 sector and the 2 sectors clockwise of the T·2 sector are now defined as the charged track (ct) sectors. Positively charged particles will bend clockwise in the detector, and pions from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events will cross no more than 3 sectors.

- $6_{ct} + 7_{ct}$ : Either range stack layer 6 or layer 7 must be struck by the charged track. This weeds out events with very short tracks.

- $19_{ct} + 20_{ct} + 21_{ct}$ : The charged track is not allowed to enter range stack layers 19-21. This vetos $K_{\mu 2}$'s, which have longer range than $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events. This requirement will often be referred to as the "online $\mu$ veto."

- $BV + EC$ : Events with energy in the barrel or endcap photon detectors are vetoed if this energy is in coincidence with the charged track (hereafter referred to as prompt energy). The barrel veto coincidence window is approximately $\pm 10$ ns, and the threshold for this cut is set at about 5 MeV visible energy (corresponding to a 15 MeV photon). The endcap coincidence window is approximately $\pm 3$ ns, with a threshold for each crystal set at about 20 MeV.

- Refined Range : The $19_{ct} + 20_{ct} + 21_{ct}$ requirement is a loose cut on the total
range of the charged track. $K_{\mu 2}$'s with tracks at large dip-angle, or tracks with a large pathlength in the target, however, will not reach layer 19. The range stack stopping layer and crude measurements of the dip-angle and the target pathlength are fed to a memory lookup that applies a more refined range cut. The refined range masks also cut events which fail to reach range stack layer 11.

- **Hextant Cut**: The 24 sectors of the range stack are grouped into 6 “hextants” of 4 sectors each. Only one hextant is allowed to have prompt energy (presumably the hextant containing the charged track) or two hextants if they are adjacent. This vetos events with photons in the range stack.

- **Online TD Cut**: If the charged track is a pion, the TD data from the stopping counter should show a double pulse from the $\pi^+ \to \mu^+$ decay. The two pulses will most often be joined together due to the short pion lifetime, and result in a single pulse with a lower than normal height-to-area ratio (see, for example, Figure 3.5). The online TD cut passes only events with a low height-to-area ratio or two separated pulses in the stopping counter.

All items except the last one constitute the “Level 0 trigger.” The last item is sometimes referred to as the “Level 1.1 trigger.”

The $K^+ \to \pi^+ \nu \bar{\nu}$ trigger has a rejection of about 14000. Thus, of the $\sim 1.3 \times 10^6$ kaons stopping in the target per spill, about 100 will be written to tape. To get down
to the $10^{-10}$ level, we still need at least 6 orders of magnitude more rejection.

In addition to the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ triggers, there are a number of “monitor” triggers that select non-rare events for purposes of calibration and acceptance calculations. I will give details here of the 3 monitor triggers used in the acceptance calculations of Chapter 5.

- $K_{\mu 2}(1) = K_{Beam} \times T \cdot 2 \times (6_{ct} + 7_{ct}) \times (19_{ct} + 20_{ct} + 21_{ct})$

- $\pi-scatt = Pi_{Beam} \times DC \times IC \times T \cdot 2 \times (6_{ct} + 7_{ct}) \times (20 + 21) \times BV + EC \times HEX$

- $K_{\pi 2}(1) = K_{Beam} \times T \cdot 2 \times (6_{ct} + 7_{ct}) \times (19_{ct} + 20_{ct} + 21_{ct})$

### 2.4 The Data

Digitized information from all subdetectors (including ADC, TDC, TD, and CCD information) is recorded to 8mm tape for each event passing the trigger. During the 1995 run, E787 collected data from $1.53 \times 10^{12}$ kaons entering the detector, about 3 times the amount of data collected during “Phase I” of the experiment. The fraction of these events passing the trigger were written to $\sim 4000$ data tapes.
2.5 Event Simulation

UMC version 5.0 (based on the EGS electromagnetic shower package, version 4) was used to generate Monte Carlo events. UMC includes a simulation of most subsystems\(^6\) of the E787 detector and also a simulation of the trigger. Various detector resolutions are not simulated by UMC and must be inserted by the user if desired. UMC events can be written to tape or disk for further analysis with the event display or with the offline data analysis software, KOFIA. UMC events will be used for some parts of the acceptance calculation (see Chapter 5) and for an estimate of the charge exchange background level (see Chapter 3).

\(^6\)Neither the TD's, CCD's, nor the beam line elements are part of the UMC simulation.
Chapter 3

Offline Analysis

The data stored on tape for each event are scrutinized with software, and events are cut (removed from the data set) if they have features not consistent with $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Events remaining after application of all cuts are considered candidates for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

3.1 Event Reconstruction

The events are reconstructed under the assumption that they are $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, i.e., we only look for a single charged track in the detector, the kinematics of that track are calculated assuming that it is a pion, etc. Reconstruction begins in the range stack, where the rates are lowest. After finding a track in the range stack, the reconstruction code searches for a matching track in the UTC. Next, a pion track that is in line with the UTC track is searched for in the target. Kaon fibers in the target should also appear along UTC track extrapolation, and the kaon time obtained from these fibers is used to identify kaon hits in the various beam line subdetectors. The
3.1. Event Reconstruction

KOFIA software package, developed by E787 for E787, was used. In particular, event reconstruction was performed by the KOFIA routine, SETUP_KINE, which calls routines that reconstruct various pieces of the event and integrates the information. I will now describe each of these steps in more detail. To aid in this discussion, Figure 3.1 shows a reconstructed $K_{\pi 2}$ event, both in the $x - y$ view and in the $r - z$ view\(^1\). Figure 3.2 shows closeup views of the range stack and of the target.

Track recognition in the range stack is performed by the KOFIA routine, RD_TRK. First, RD_TRK attempts to identify the sector that produced the $T \cdot 2$ signal that caused the trigger to notice the event in the first place. This is accomplished by looking for layer T and layer 2 counters with hits close to the detector strobe time (the time of the charged track as recorded by the trigger). Once the $T \cdot 2$ sector has been found, RD_TRK works outward through the range stack, looking for modules with hits at track time until the end of the track is reached. RD_TRK thus provides a list of those range stack counters that are part of the charged track. Next the TRKTIM_RD routine is executed, which computes a single time for this track (called TRS) by averaging together the times of the hits in the various counters. Note that all times are listed in $\text{ns}$, all distances in $\text{cm}$, and all energies, momenta, and masses in $\text{MeV}$.

Now that RD_TRK has determined in which sector the track entered the range

\(^{1}\)The $z$ axis is defined to be along the direction of the kaon beam. $x$ is horizontal, and $y$ is vertical. $r$ points away from the $z = 0$ axis, and $\phi$ is an angle around the $z = 0$ axis.
3.1. Event Reconstruction

Figure 3.1: A typical $K_{\pi 2}$ event, showing the charged track extending from the target to the range stack and two photons in the barrel veto. The $r - z$ view shows $z$ information in the range stack from the RSSC’s and from end-to-end timing in the counters. The thin boxes in the $x - y$ view are range stack counters, and the wide boxes are barrel veto modules. The rectangles in the $r - z$ view and large circles in the $x - y$ view depict the 3 superlayers of the UTC. Isochrones are depicted as the small circles within these superlayers. The event displays in this thesis all show the $x - y$ view looking upstream (thus, the kaon beam and also the B field point out of the page) and the $r - z$ view with positive $z$ pointing from left to right.
3.1. Event Reconstruction

stack, the UTC reconstruction code, UTC\_TRACK, can proceed to search for tracks that point to that sector. UTC\_TRACK fits a circular track to the isochrones of struck UTC wires, and then information from the cathode strips is used to perform a straight line fit in the \( \alpha - z \) plane\(^2\). Thus, a helical track is determined that tells us the track momentum and also the position and direction of the track both when exiting the target and when entering the range stack. If more than one UTC track is found, the track that extrapolates nearest to the first sector crossing in the range stack (see Figure 3.2) will be selected as the UTC track of interest.

\(^2\alpha \text{ is the azimuthal angle along the helix.}\)

Figure 3.2: The first figure shows a close up of the range stack with a fitted track. The numbers shown within modules are hit times in ns. The second figure shows a close up of the \( K^+ \rightarrow \pi^+ \) decay in the target. The kaon hits are at \( \sim 0 \) ns and the pion hits are at \( \sim 14 \) ns.

Now the range routines, TRKRNG, perform a careful accounting of the energy deposited in the range stack by the charged track and of the range of the charged
3.1. Event Reconstruction

track in the range stack. The ADC energies of the range stack modules are summed and, if the TD data shows non-prompt accidental energy along the track, this is corrected for\(^3\). Corrections are also applied for dead material in the range stack (counter wrappings and the RSSC’s) and saturation of the energy-light relation in the scintillator. Charged track range is measured in the individual counters and for the entire track using a track fitter. For each iteration of the fit, a hypothetical track is propagated through the range stack using a simple model for energy loss that approximates the range stack as a solid block of scintillator. The track begins at the range stack entry point (determined by the UTC), and has two free parameters: the initial track momentum and direction in \(x – y\) (the dip-angle is not a free parameter, but is kept fixed at the value determined by the UTC fit). The CERN minimization routine MINUIT [31] varies these parameters to find the track that fits best to \(x – y\) information in the range stack, including sector crossings, RSSC hits, and energy in the stopping layer. Three \(\chi^2\) variables are returned for evaluating the quality of the fit: TF.CHISQ uses only \(x – y\) information, and is in fact the \(\chi^2\) minimized by MINUIT, TF.Z1CHISQ uses only \(z\) information from the RSSC hits, and TF.Z2CHISQ uses only \(z\) information from end-to-end timing in the range stack counters. The best-fit track

\(^3\)The TD pulse height of the hit in each module along the track is compared to ADC energy. Since the shape of the pulses does not change appreciably with pulse area, the pulse height should be proportional to pulse area and (appropriately calibrated) should agree with the ADC energy. If the ADC energy is considerably larger than the energy from pulse height information, this is evidence of some accidental energy in the ADC gate and pulse height information is then used in place of the ADC information.
3.1. *Event Reconstruction*

is then traced through a detailed geometrical description of the range stack (taken from UMC) to determine the range (material traversed) up to the stopping layer. The stopping layer energy is used to estimate the stopping layer range.

After the range routines, the target analysis code, SWATH, is run. SWATH extrapolates the UTC track back into the target to define a 2 cm wide swath on which to look for kaon and pion fibers. Pion fibers should all lie along the swath, and the kaon fibers should be in a cluster that touches the swath. A likelihood function uses energy, time, and distance-from-UTC-extrapolation information to determine if each fiber should be labeled a pion fiber. SWATH also looks for photon hits (called "gamma fibers"). A gamma fiber is any fiber not already assigned as a pion or kaon fiber and that has a hit within ±6 ns of the mean time of the pion fibers. This time must also be closer to the mean pion fiber time than it is to the mean kaon fiber time. SWATH must also determine the kaon decay vertex. The B4 counter is searched for a hit in time with the kaon cluster in the target, and the \( x - y \) location of this hit tells us where the kaon entered the target. The kaon decay vertex is defined as the center of the kaon fiber on the swath and farthest from the target entry point. The range of the charged track in the target (RTGT) is then defined as the distance along the UTC extrapolation from the kaon decay vertex to the edge of the target.

SETUP_KINE combines the information from the above routines, along with estimates of energy loss in the I-counters and UTC, into 3 kinematic quantities: ETUT
3.1. Event Reconstruction

(the kinetic energy of the charged track at the decay vertex), PTOT (the momentum of the charged track at the decay vertex), and RTOT (the total range, converted into equivalent centimeters of scintillator, of the charged track). RTGT is used to correct PTOT for the momentum lost in the target. To facilitate comparisons with Monte Carlo events, ETOT, PTOT, and RTOT are shifted and scaled so the \( K_{\pi 2} \) and \( K_{\mu 2} \) peaks line up at their true values (see Table 3.1). Likewise, the kinematic quantities for UMC events are shifted and scaled to put the \( K_{\pi 2} \) and \( K_{\mu 2} \) peaks where they belong, and they are also smeared to account for detector resolutions (see Table 3.2). Figures 3.3 and 3.4 show the range, energy and momentum peaks for \( K_{\pi 2} \) and \( K_{\mu 2} \) events for data and Monte Carlo with all shifting, scaling, and smearing parameters applied. Good agreement between data and Monte Carlo is necessary for the UMC-based acceptance measurements described in Chapter 5. See [33] for more details of how these parameters were generated.

<table>
<thead>
<tr>
<th></th>
<th>( K^+ \rightarrow \pi^+ \pi^0 )</th>
<th>( K^+ \rightarrow \pi^+ \pi^0 )</th>
<th>( K^+ \rightarrow \mu^+ \nu_\mu )</th>
<th>( K^+ \rightarrow \mu^+ \nu_\mu )</th>
<th>( m )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Value</td>
<td>Measured Value</td>
<td>True Value</td>
<td>Measured Value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RTOT</td>
<td>30.37</td>
<td>30.408</td>
<td>54.34</td>
<td>54.060</td>
<td>1.0134</td>
<td>-0.45</td>
</tr>
<tr>
<td>ETOT</td>
<td>108.55</td>
<td>105.34</td>
<td>152.48</td>
<td>152.45</td>
<td>0.9325</td>
<td>10.32</td>
</tr>
<tr>
<td>PTOT</td>
<td>205.14</td>
<td>205.42</td>
<td>235.53</td>
<td>235.88</td>
<td>0.9977</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 3.1: Shifting and scaling parameters used for data. \( RTOT^{\text{new}} = m \times RTOT + b \) for example.

A number of routines are run to look for the \( \pi^+ \rightarrow \mu^+ \rightarrow e^+ \) signature in the range stack using TD data. The first of these is FITPI, which looks for the \( \pi^+ \rightarrow \mu^+ \) signature by performing single and double pulse fits to the TD data in both ends of the stopping
3.1. *Event Reconstruction*

![Distribution Plots]

**Figure 3.3:** Range, energy and momentum peaks for $K_{\pi 2}$ events. The first column of plots is data and the second is reconstructed Monte Carlo events. The Constant, Mean, and Sigma parameters are shown for Gaussian fits to these distributions.
3.1. Event Reconstruction

Figure 3.4: Range, energy and momentum peaks for $K_{\mu 2}$ events. The first column of plots is data and the second is reconstructed Monte Carlo events. The low-side tail in the ETOT and RTOT distributions is due to muon tracks that penetrate past range stack layer 21; the range and energy deposited outside of the range stack is not accounted for by the reconstruction code.
3.1. Event Reconstruction

<table>
<thead>
<tr>
<th></th>
<th>$m_s$</th>
<th>$b_s$</th>
<th>$m_g$</th>
<th>$b_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTOT</td>
<td>1.00203</td>
<td>0.9316</td>
<td>0.01296</td>
<td>-1.8290</td>
</tr>
<tr>
<td>ETOT</td>
<td>1.00404</td>
<td>2.2290</td>
<td>0.02501</td>
<td>-0.1134</td>
</tr>
<tr>
<td>RTOT</td>
<td>1.02724</td>
<td>-0.6557</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2: Shifting, scaling, and smearing parameters used for UMC events. RTOT\textsuperscript{rew} = ($m_s \times$ RTOT + $b_s$) + $\sigma$($m_g \times$ RTOT + $b_g$) for example, where $\sigma$ is a unit Gaussian random number.

counter (see Figure 3.5). The calibrated pulse shapes for each range stack counter used in the fits are generated by averaging together many single pulses from through-going muon tracks. A number of useful variables come out of this fit (see Table 3.3), which will be used to assess whether the stopping counter contains a $\pi^+ \rightarrow \mu^+$ decay. This pulse fitting technique is necessary for finding those $\pi^+ \rightarrow \mu^+$ decays occurring at early pion lifetimes in which the second pulse may not be visible to a simple leading-edge finding routine. The electron finding routine, ev5, searches for electron hits

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\pi}^i$</td>
<td>Calibrated pion time from double pulse fit for end $i$</td>
</tr>
<tr>
<td>$t_{\mu}^i$</td>
<td>Time of $\pi^+ \rightarrow \mu^+$ decay (relative to $t_{\pi}^i$) for end $i$</td>
</tr>
<tr>
<td>$E_{\pi}^i$</td>
<td>Pion fit energy in TD counts for end $i$</td>
</tr>
<tr>
<td>$E_{\mu}^i$</td>
<td>Muon fit energy in TD counts for end $i$</td>
</tr>
<tr>
<td>$C_{\mu}^i$, $C_{\pi}^i$</td>
<td>Single and double pulse fit quality for end $i$</td>
</tr>
<tr>
<td>PROD</td>
<td>Combined fit quality $C_{\mu}^i/C_{\pi}^i \times C_{\mu}^i/C_{\pi}^i$</td>
</tr>
<tr>
<td>DTPI</td>
<td>$t_{\pi}^i - t_{\mu}^i$</td>
</tr>
<tr>
<td>DTMU</td>
<td>$t_{\mu}^i - t_{\mu}^i$</td>
</tr>
<tr>
<td>TMUAV</td>
<td>$(t_{\mu}^i + t_{\mu}^i)/2$</td>
</tr>
<tr>
<td>EMUT</td>
<td>$\sqrt{E_{\mu}^i \times E_{\mu}^i}$</td>
</tr>
<tr>
<td>EMUMIN</td>
<td>$\min(E_{\mu}^i, E_{\mu}^i)$</td>
</tr>
<tr>
<td>ZPI</td>
<td>$\log(E_{\mu}^i/E_{\mu}^i)$</td>
</tr>
<tr>
<td>ZMU</td>
<td>$\log(E_{\mu}^i/E_{\mu}^i)$</td>
</tr>
</tbody>
</table>

Table 3.3: Some FITPI fit parameters (top) and output variables (bottom).
3.1. Event Reconstruction

Figure 3.5: TD double and single pulse fits for both ends of the stopping counter. In this example, the double pulse clearly fits the data better than a single pulse in each end, so the event passes the test.

from the $\mu^+ \rightarrow e^+$ decay in the range stack. The electron cluster must have hits near the stopping counter, and the sum of the pulse heights must be consistent with the electron energy from a $\mu^+ \rightarrow e^+$ decay. ELVETO5 searches for range stack hits at muon time$^4$ outside of the stopping counter. Since the muon from a $\pi^+ \rightarrow \mu^+$ decay has a very short range, such hits indicate that the second pulse found by FITPI is really an accidental or an electron from a $\mu^+ \rightarrow e^+$ decay. TDFOOL searches for hits at muon time hiding along the charged track by performing double pulse fits in the two range stack counters before the stopping counter on the track.

Photons are searched for with the INTIME routine. INTIME looks for hits occurring

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$^4$"Muon time" is the time at which the pion decayed, which is track time plus the time difference returned by the double pulse fit: TRS + TNUAV.
at pion time in the range stack, barrel veto, endcaps, I-counters, V-counters, collar counters, and micro-collar counter\textsuperscript{5}. Hits in the barrel veto and range stack are required to be seen in both counter ends. A separate routine, INTSE, looks for single-ended hits in the barrel veto and range stack. Photon energy in the target is searched for by SWATH.

3.2 Cuts

Of the $1.53 \times 10^{12}$ kaons entering the detector during the 1995 run, about $1.1 \times 10^8$ events passed the trigger and were written to tape. If the Standard Model is correct, only a few of these will be $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events, and the rest are background. We would like to sift out the signal events, and we do this by “cutting” any event that appears to be background. Each cut in the analysis removes some class of background not attacked by the other cuts; since we need at least 10 orders of magnitude of background rejection, many cuts are required. The cuts are tuned by comparing distributions of various quantities for signal and background events, where the signal distributions necessarily do not come from real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events, but rather from events that look similar to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the variable of interest. For example, suppose we wish to develop a cut on the FITPI quality-of-fit variable PROD (see Table 3.3). For the signal distribution we might use $\pi-$scat events, since they should have a

\textsuperscript{5}The collar and micro-collar counters plug a hole behind the endcaps, close to the beam line.
π⁺ → μ⁺ decay in the stopping counter like K⁺ → π⁺ν¯ν events, and for the background distribution we could use K_{μ2} decays tagged with a cut on the pion momentum. The background studies discussed in Section 3.3 are then used to refine the cuts.

Every cut will have some rejection and some acceptance loss. The rejection is the amount of background removed by the cut. For example, a cut that removes 95% of the background would be said to have a rejection of 95. The acceptance loss is the fraction of K⁺ → π⁺ν¯ν events removed by the cut. We tune each cut by maximizing the rejection for a given acceptance loss, or by minimizing the acceptance loss for a given rejection.

Here I provide a list of the 55 cuts used to select K⁺ → π⁺ν¯ν events. More technical descriptions of some of these cuts can be found in [32].

- **DUPEVENT**: Occasionally some subset of the data will be written to tape more than once (mostly due to tape-handling mistakes at various stages in the analysis). Each event in the data set has a run number and an event number. Check run and event numbers and cut any duplicated events.

- **GOOD_RUN**: Cut events with run numbers identified as “bad” by the BADRUN routine. These correspond to portions of data known to be bad, usually because some part of the detector was not working.

- **ITRG**: Require that the trigger type be K⁺ → π⁺ν¯ν(1).
• **HEXCUT**: Re-apply the online hextant cut. This should have already been
applied by the trigger, but we require that the **HEXCUT** bit is **TRUE** in the data
just to be safe.

• **ONLTD**: Re-apply the online TD cut.

• **ISKCODE**: **SETUP.KINE** was used for event reconstruction. **ISKCODE = 0** requires
good track reconstruction in all subsystems.

• **PNNSTOP**: Cut events with range stack stopping layer less than 11 or greater
than 18.

• **STOP.HEX**: Require that the online and offline stopping hextant and layer agree.

• **DIPANG**: Cut events with \(|\theta_{\text{dip}}| > 30^\circ\).

• **RBOX**: \(33 \text{ cm} < \text{RTOT} < 41 \text{ cm}\).

• **EBOX**: \(115 \text{ MeV} < \text{ETOT} < 138 \text{ MeV}\).

• **PBOX**: \(211 \text{ MeV} < \text{PTOT} < 229 \text{ MeV}\).

• **PFBOX**: This is a separate kinematic box used in the search for \(K^+ \rightarrow \pi^+ X^0\).

PFBOX is *not* applied as a cut in the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) analysis. It is defined as:

- **Range box**: \(35.5 \text{ cm} < \text{RTOT} < 41 \text{ cm}\).

- **Energy box**: \(120 \text{ MeV} < \text{ETOT} < 138 \text{ MeV}\).
3.2. Cuts

- Momentum box: 222 MeV < PTOT < 229 MeV.

- ZTGT: Restrict the kaon decay vertex to the target fiducial region. ZTGT_TP is the z position of the charged track at the target edge and STZ_DC is the z position of the charged track at the $K^+$ decay vertex. Both variables come from extrapolation of the UTC track. The ZTGT cut requires that $-8 \text{ cm} < ZTGT\_TP < 15 \text{ cm}$ and $-8 \text{ cm} < STZ\_DC < 15 \text{ cm}$.

- TGDCVT: The target photon veto uses variables from the SWATH common block: NGAMMA is the number of photon hits and ENERG_SW is the sum of their energies. Events are cut if ENERG_SW > 5 MeV or if ENERG_SW > 2 MeV and NGAMMA > 1.

- RTDIF: The DPLNTH_SW variable from the SWATH common block is used (see Figure 3.6). This is measured as half the length of the portion of the $K^+$ cluster that is within the swath. For any given event, this is no less than half the distance by which the $K^+$ decay vertex could be misreconstructed, thus $2 \times DPLNTH\_SW$ is the maximum uncertainty in RTGT due to vertex mislocation. Cut events with DPLNTH_SW > 1.5 cm.

- TGLIKE: SWATH determines which of the target fibers are the $\pi^+$ fibers by assigning a likelihood to each fiber based on time, distance-from-swath, and energy in that fiber. SWATH then combines the likelihoods of all $\pi^+$ fibers to form two variables. The first of these, LIKE_SW, uses time, distance-from-swath,
and energy information. The second, LIKE2.SW, only uses distance-from-swath information. LIKE2.SW is useful for tagging events with a perfectly good $\pi^+$ track in the target, but a mismeasured UTC track that does not match well with the target $\pi^+$ hits. TGLIKE cuts events in which LIKE.SW $> 3.2$ or LIKE2.SW $> 2.3$.

- **TGB4**: SWATH returns three variables relevant to B4 information: DB4.SW is the distance from the B4 $x – y$ point to the nearest $K^+$ fiber. DB4TIP.SW is the distance from the B4 $x – y$ point to the nearest extreme tip of the $K^+$ cluster. DVXTIP.SW is the distance from the point that SWATH has determined to be the $K^+$ decay vertex to the nearest extreme tip of the $K^+$ cluster. TGB4 cuts events in which DB4.SW $> 2.0$ cm or DB4TIP.SW $> 2.0$ cm or DVXTIP.SW $> 0.7$ cm. All fiber positions are defined to be at the centers of the fibers. See Figure 3.6.

![Figure 3.6: Sketch of target, showing variables relevant to RTDIF, TGB4, and BSCAT cuts.](image-url)
3.2. Cuts

- **TARGF**: Cut events in which the distance between the closest $K^+$ and $\pi^+$ fibers is greater than 0.95 cm.

- **TGDEDX**: Cut events in which $RTGT > 12$ cm or $ETGT > 28$ MeV or $ETGT \times 10.5 > RTGT \times 28$ or $ETGT \times 10 < (RTGT - 2) \times 21.5$. See Figure 3.7. This targets events with a photon hiding along the pion track in the target, and also serves to identify general messiness in the target that could fool the reconstruction.

![Figure 3.7: ETGT vs. RTGT for $K_{\ast2}$ pions, showing the target dE/dX cut.](image)

- **EPIMAX**: Cut events containing a $\pi^+$ fiber with energy greater than 5 MeV. This, like TGDEDX, serves to identify photon energy or accidental energy that could fool the reconstruction.

- **RPIMIN**: This is an attempt to cut events in which the $\pi^+$ exits through the
front face of the target. It finds the $\pi^+$ fiber that is farthest from the center of the target and requires that its distance from the center of the target be greater than 4.7 cm. The target has a radius of 6 cm.

- **NTRIK**: The number of $K^+$ fibers must not exceed 15 and the total $K^+$ energy in the target must be greater than 10 MeV and less than 150 MeV.

- **EIC**: Cut events with I-counter energy greater than 4 MeV, or less than 0.6 MeV. EIC is the sum of the energy in the one or two I-counters that the charged track passed through. This targets events with the $K^+$ stopping in an I-counter, or events with photon energy hiding in an I-counter.

- **KIC**: We would like to cut events in which the $K^+$ comes to rest in an I-counter. EIC gets these if the $K^+$ stopped in the same I-counter that the charged track exited through. The KIC cut is designed to remove the events where the $K^+$ stopped in an I-counter and the charged track exited through an I-counter on the other side of the target. Cut events with an I-counter TDC hit within 3 ns of $K^+$ time if any of the target kaon fibers are within 0.8 cm of that I-counter.

- **PIGAP**: Cut events with gaps greater than 1.2 cm in the target $\pi^+$ track.

- **EKZ**: Require that STZ.DC (the $z$ position of the charged track at the $K^+$ decay vertex) agrees with the $K^+$ energy in the target. EKZ cuts events in which 

$$|4.6 \times \text{STZ.DC} + 73 - E_{K^+}^{Tg_t}| > 40 \text{ MeV}.$$
3.2. Cuts

- **BSCAT:** Look for $\pi^+$ scatters in target. The most CCW\(^6\) $K^+$ fiber that is within 0.8 cm of the most CCW $\pi^+$ fiber is found. The event is cut if this $K^+$ fiber is not CCW of the most CCW $\pi^+$ fiber. For example, the event pictured in Figure 3.6 would pass BSCAT, but it would fail BSCAT if the 3 upper-most kaon fibers were not included in the kaon cluster.

- **PICER:** Look for evidence of a pion in the Čerenkov counter at the time of the charged track in the range stack (TRS). This is one of the more powerful cuts for rejecting $\pi-$scats. We begin by forming clusters in time of hit pion phototubes, where the clusters must contain at least 5 phototubes in coincidence. CPITRS\(_N\) is the average of TDC leading-edge times for hits in the cluster closest to TRS. PICER requires $|\text{CPITRS}_N - \text{TRS} - 0.5 \text{ ns}| \geq 2 \text{ ns}$. The 0.5 ns offset serves to remove an offset between CPITRS\(_N\) and TRS found empirically for $\pi-$scat events.

- **DKNFLT:** Require $|\text{CKTRS}_N - \text{TRS}| \geq 1.5 \text{ ns}$, where CKTRS\(_N\) is defined like CPITRS\(_N\), and it is the time of the kaon cluster closest to TRS. This cut targets kaons that decay in flight, both before and in the target.

- **PISCAT_BW:** This cut targets two beam particles in the BWPC's. Clusters of hits are formed in the BWPC's and we search for a cluster coincident with TRS after excluding the cluster closest in time to the kaon. Require $|\text{BW1TRS} - \text{TRS}|$.

\(^{6}\)Counter-clockwise along the UTC extrapolation.
3.2. Cuts

1.0 ns| \geq 5\, \text{ns} \text{ and } |\text{BW2TRS} - 0.25\, \text{ns}| \geq 5\, \text{ns}. \text{BW1TRS} \text{ is the cluster time minus TRS of the cluster closest to TRS in the first beam wire chamber after excluding the cluster closest to kaon time. BW2TRS is the same quantity for the second beam wire chamber.}

- **PISCAT.B4**: This cut targets two beam particles in the B4 hodoscope. Require $|\text{B4TRS} - 0.5\, \text{ns}| \geq 2.5\, \text{ns and } |\text{B4HRS} - 0.25\, \text{ns}| \geq 2.5\, \text{ns}$. \text{B4TRS} \text{ is the cluster time minus TRS of the B4 cluster closest to TRS, and B4HRS is the single hit time minus TRS of the B4 hit closest to TRS, both found after excluding the cluster and hits closest to kaon time. Also, pulse fitting is performed using B4 TD data (similar to the pulse fitting used for observation of the } \pi^+ \rightarrow \mu^+ \text{ decay in the range stack) and events with double pulses in the B4 are cut. } \chi^2 < 100 \text{ is required for the single pulse fit.}

- **B4DEDX**: \text{B4ABM} > 1.5\, \text{MeV}. \text{This cut targets pions entering B4 at prompt time, which typically leave about 1\, \text{MeV in the B4 counter. Kaons leave about 2.5 MeV in the B4 counter.}

- **PBGLASS**: \text{PBHRS} \leq 1. \text{No more than one lead-glass phototube firing within 5 ns of TRS is allowed. This functions both as a photon veto and as a cut against } \pi^- \text{-scats.}

- **BHTRS**: \text{Located just downstream of the Čerenkov counter, the beam hole}
counter is a piece of scintillator with a hole in it large enough for most beam particles to pass through. To cut stray beam pions, require $|\text{BHTRS}(1)| \geq 5$ ns and $|\text{BHTRS}(2)| \geq 5$ ns. BHTRS(1) is the hit time minus TRS of the hit closest to TRS in the first half-plane of the beam hole counter. BHTRS(2) is the same quantity for the second half-plane of the counter.

- GAMVETP3: INTIME generates sums of prompt energy in the range stack, barrel veto, endcaps, I-counters, V-counters, collar counters, and micro-collar counter. Events are cut if substantial prompt energy is found in any of these subsystems.

A TDC hit in both counter ends and an ADC hit in both counter ends is required in the range stack and barrel veto. See Table 3.4 for time windows and energy thresholds used.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Time offset (ns)</th>
<th>Half window (ns)</th>
<th>Energy threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RD</td>
<td>1.5</td>
<td>3.5</td>
<td>0.2</td>
</tr>
<tr>
<td>RH</td>
<td>1.5</td>
<td>3.5</td>
<td>2.0</td>
</tr>
<tr>
<td>BV</td>
<td>2.25</td>
<td>5.0</td>
<td>0.2</td>
</tr>
<tr>
<td>E2</td>
<td>1.0</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>EU</td>
<td>0.5</td>
<td>2.5</td>
<td>2.0</td>
</tr>
<tr>
<td>IC</td>
<td>-0.5</td>
<td>2.5</td>
<td>0.2</td>
</tr>
<tr>
<td>VC</td>
<td>0.0</td>
<td>5.0</td>
<td>0.2</td>
</tr>
<tr>
<td>CO</td>
<td>0.0</td>
<td>2.0</td>
<td>0.2</td>
</tr>
<tr>
<td>CM</td>
<td>0.0</td>
<td>4.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3.4: Time windows and energy thresholds for GAMVETP3. RD refers to the range stack. RH refers to a special energy sum formed for range stack hits near the charged track. E2 refers to the endcaps, and EU refers to a special energy sum formed for endcap hits in the inner-most ring of crystals in the upstream endcap (where rates are likely to be higher).
3.2. Cuts

- GAMVETSE: Cut events with prompt energy in the range stack or barrel veto that are missing a TDC hit and/or an ADC hit in one end of any counter. Three categories were cut on for both RS and BV hits: SAST (single-ended ADC hit with single-ended TDC hit (not necessarily the same end)), SABT (single-ended ADC hit with both-ended TDC hit), and BAST (both-ended ADC hit with single-ended TDC hit). See Table 3.5 for time windows and energy thresholds used.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Time offset (ns)</th>
<th>Half window (ns)</th>
<th>Energy threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SASTBV</td>
<td>-4.0</td>
<td>8.0</td>
<td>1.0</td>
</tr>
<tr>
<td>SABTBV</td>
<td>0.0</td>
<td>7.0</td>
<td>0.2</td>
</tr>
<tr>
<td>BASTBV</td>
<td>-4.0</td>
<td>8.0</td>
<td>2.0</td>
</tr>
<tr>
<td>SASTRD</td>
<td>0.0</td>
<td>8.0</td>
<td>5.0</td>
</tr>
<tr>
<td>SABTRD</td>
<td>0.0</td>
<td>5.0</td>
<td>1.0</td>
</tr>
<tr>
<td>BASTRD</td>
<td>0.0</td>
<td>8.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 3.5: Time windows and energy thresholds for GAMVETSE.

- NSECRS: The range stack track is not allowed to span more than three sectors.

- DELC: The main delayed coincidence cut requires that $t_\pi - t_K > 2$ ns, where $t_K$ is the average time of the target kaon fibers, and $t_\pi$ is the average time of the target pion fibers. If the event has no target pion fibers, $t_\pi$ is set to TIC, the time of the I-counter hit\(^7\). The DELC cut is made tighter under a number of circumstances (see Table 3.6).

\(^7\)In this analysis, we do not veto events with zero pion fibers if the kaon cluster is at the edge of the target. We do, however, attempt to cut events in which the kaon entered the I-counter.
Table 3.6: DELC requirements. The final column shows the relative frequencies of each of the conditions for normal events. $t_{B4}$ is the TDC-based time of either the hit in the B4 counter closest to and within 5 ns of $t_K$, or if not found, the hit closest to and within 3 ns of TRS.

- TIMCON: Some timing consistency cuts (see Table 3.7).

Table 3.7: TIMCON requirements.

- RNGMOM: The charged track momentum measured in the UTC (PDC) is converted to an expected charged track range. If this expected range disagrees with the measured range stack range (RRS), the event is cut. For a given momentum, muons will have a longer range than pions, thus this is a very powerful cut for events with muon tracks. CHI_RM is defined as $(RRS - \text{Range expected from PDC})/\sigma$, where $\sigma$ is a function of PDC. Require $-2.5 < \text{CHI\_RM} < 2.0$ (see Figure 3.8).
3.2. Cuts

Figure 3.8: The RNGMOM cut. The direct cut on CHI_RM translates into the two-dimensional cut on PDC and RRS depicted in the first plot. The data shown here are \( K^+ \to \pi^+ \nu \bar{\nu} (1) \) triggers after a number of cuts have been applied. The muon band is clearly visible above the pion band in the first plot.

- RSDEDX: The expected energy in each range stack counter along the track is computed from the range measured in that counter by TRKRNG. The event is cut if the expected energy substantially disagrees with the measured energy in any counter. This cut is most useful for vetoing events with photon energy hiding along the charged track, and it also vetos events in which the charged track undergoes a hard scatter in the range stack. Two variables are formed. PROB_RSDEDX is a confidence level based on a \( \chi^2_{RSDEDX} \) and the number of contributions to this \( \chi^2 \), where,

\[
\chi^2_{RSDEDX} = \sum_{i=1}^{STLAY-1} \left( \frac{E_i - E_i^{TRKRNG}}{\sigma_i} \right)^2.
\]
3.2. **Cuts**

**CHIMAX\_RSDEDX** is the largest upward deviation in energy of the counters on the track:

\[
\text{CHIMAX\_RSDEDX} = \left( \frac{E_i - E_i^{\text{TRKING}}}{\sigma_i} \right)_{\text{max}}
\]

the cut is set at \text{PROB\_RSDEDX} > 0.02 and \text{CHIMAX\_RSDEDX} < 5.0.

- **ZDCOW**: \text{ZDCOW} is the \( z \) position of the UTC track extrapolated to the UTC outer wall. Cut events with |\text{ZDCOW}| > 25 cm, since the charged track may have clipped the UTC end plate.

- **UTCQUAL**: Cut events with poor UTC fits. Such events can cause mismeasurement of \text{PTUT}, which is particularly worrisome for the \( K_{\mu2} \) background, for which the upper edge of the \text{PBOX} is the most powerful cut. \text{UTCQUAL} combines a number of variables related to the UTC fit into one "good fit likelihood" variable, \text{UTCQ}. The variables that go into this likelihood are: \text{NPTSZ\_D} (the number of hits used in the \( z \) fit), \text{NPTXY\_D} (the number of hits used in the \( x - y \) fit), \text{NLAYXY\_D} (the number of layers used in the \( x - y \) fit), and the numbers of unused hits within 1.5 cm of the UTC track in each of the three superlayers. **UTCQUAL** cuts events in which \text{UTCQ} < 0.00001.

- **CHIRF**: Veto events in which the charged track scatters in the range stack using the quality-of-fit parameters from the range routines. Cut events with bad fit \( \chi^2 \) using confidence levels \text{PR\_RF} and \text{PR\_RFZ2} that are based on \text{TF\_CHISQ} and
3.2. Cuts

TF.Z2CHISQ. Require PR_RF > 0.01 and PR_RFZ2 > 0.01.

- ZRF: ZSTOP is the $z$ position at which the charged track came to rest, according to the range stack track fit. ZRF cuts events in which $|ZSTOP| > 35$ cm (layers 11 - 12), $|ZSTOP| > 40$ cm (layers 13 - 14), or $|ZSTOP| > 50$ cm (layers 15 - 18). These tracks may have entered the range stack support structure, which will ruin our measurements of ETOT and RTOT. Also the cut is tightened to $|ZSTOP| > 30$ if the charged track stopped in layer 14 and TF.Z2CHISQ indicates a scatter. This is to close a loophole in which tracks scatter only slightly but manage to escape into the support structure by traveling down the space between range stack layers 14 and 15 in which the RSSC's reside.

- LAY14: Cut any event stopping in layer 14 with a prompt RSSC hit in the same sector or one sector clockwise of any layer 14 hit on the RS track. LAY14 cuts events in which the charged track actually stopped in an RSSC.

- FITPI: Require a good double pulse fit in the stopping counter and apply the following cuts (see Table 3.3): $C^i_\mu/C^i_\pi > 1.5$, PROD > 3, $70 < EMUT < 400$, $E^i_\mu > 60$.

- ELVETO: Cut events with range stack hits at muon time outside of the stopping counter.

- ELEC.V5: Require that EV5 find an electron in the range stack.
3.2. Cuts

- TDFOOL: Veto events with good double pulse fits in either of the two range stack counters before the stopping counter on the track.

- RSHEX: The analog pulses from the four counters of each range stack hextant/layer are summed into single TD channels\(^8\). Accidental energy in any of the four counters in the stopping hextant/layer can mimic a \(\pi^+ \rightarrow \mu^+\) decay. RSHEX therefore cuts any event in which more than one of the four counters has more than 0.5 MeV ADC energy.

- TDDFA: There are three ways an event in which the charged track is a muon can fool the TD cuts: an ugly single pulse can fit nicely to a double pulse fit (called TDFLUC), an accidental can occur at “muon time”, or an accidental can occur at “electron time”. TDDFA is trying to attack the TDFLUC background only, so it only operates on events for which TMUAV is less than 10 ns. A Discriminant Function Analysis [27] using the variables \(\log(C^1_\mu C^2_\mu)\), TMUAV, and \(\min(E^1_\mu, E^2_\mu)\) helps veto the TDFLUC part of the muon background. Basically, the DFA determines the plane through the three-dimensional space of the three variables listed above that will maximally separate signal from background. Thus, the DFA has better rejection vs. acceptance than simply applying cuts to the three variables separately. Require:

\[
0.968 \times \log(C^1_\mu C^2_\mu) + 0.246 \times \text{TMUAV} + 0.045 \times \min(E^1_\mu, E^2_\mu) - 11.008 > -1.
\]

\(^8\)It was done this way to save on the number of TD channels needed.
3.2. Cuts

- **TDLIK**: This is really two separate cuts targeting the "muon time" and "electron time" backgrounds mentioned above. TDLIK forms likelihoods, LK.TDL2 and LK.TDL3, based on all quantities relevant to the "muon time" and "electron time" backgrounds respectively. LK.TDL2 is composed of the following variables: DTMU (relative z positions of pion and muon pulses from end-to-end timing), EMUT (energy of muon pulse), ZPI – ZMU (z is measured by comparing pulse heights in the two ends), and TMUAV. LK.TDL3 is composed of: DTELEC (relative z positions of pion and electron pulses), EMUT, TELEC (muon lifetime), TMUAV, and EELECT2 (electron energy). Distributions of the above variables taken from π-scatt events are used as a lookup table of probabilities for a pion to lie at a given value. The appropriate probabilities are multiplied together to form LK.TDL2 and LK.TDL3, and TDLIK cuts events in which LK.TDL2 < 0.005 or LK.TDL3 < 0.0007. A likelihood cut such as this one is essentially identical to a $\chi^2$ cut except in that the distributions of the variables in the cut need not be Gaussian.

- **TMUBV**: ELVETO type cut in the barrel veto. Search the barrel veto within ±4 range stack sectors of the stopping sector and cut events with any hit within 5 ns of muon time.

- **TMUADC**: Require TMUAV < 95 ns.
3.3 Background Studies

If the Standard Model is correct, we will see very few (if any) $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events in the 1995 data set. It is important, therefore, that the cuts listed in the previous section remove all background events. Some recipe was needed for tuning these cuts. We would like the cuts to be as light as possible, since each has some associated acceptance loss for real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events, and yet we need to make them tight enough to remove all background events.

We could begin by applying a number of cuts that seem reasonable and then looking at the events that remain. If any of these events appear to be background, we would then invent cuts to remove those events. However, with an apparatus as complex as E787, even a real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event will tend to have some odd feature that might cause us to suspect it is background. For example, a real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event could have a small splash of prompt accidental energy in, say, the beam hole counter. Even though this is a real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event, and the energy in the beam hole counter is not related to the event, we might worry that in fact this was really a $K_{\pi 2}$ with a photon in the beam hole counter and we would cut the event. Thus, one could argue that a method of tuning cuts based on scrutiny of a few remaining events in the final sample will result in an analysis with zero acceptance for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

Instead we conduct a so-called "blind analysis," in which we set all cuts before looking to see what events remain. Background studies are conducted that allow
us to estimate the level of background events in the final sample without actually looking at the final sample. Cuts are tightened until the background studies indicate background levels well under one event. At that point we freeze the cuts and "open the box" (look at the final sample).

Bifurcated background studies allow us to estimate background levels well under one event. Rejections of two uncorrelated groups of cuts are measured separately, and the product of these rejections is used to estimate the remaining background level. We aim for a total background level of about 0.1 events.

\[
\begin{array}{c|c|c|c|c}
\text{Pass} & \text{C} & \text{D} & \text{D(estimate)} = B^*C/A \\
\hline
\text{Cut1} & & & \\
\hline
\text{Fail} & \text{A} & \text{B} & AD=BC : \text{No correlation} \\
\hline
\text{Cut1} & & & AD>BC : "Bad" correlation \\
\hline
\text{Pass} & & & AD<BC : "Good" correlation \\
\end{array}
\]

Figure 3.9: Generic background study.

Consider the diagram in Figure 3.9. For each background type, two cuts (or groups of cuts), CUT1 and CUT2, are selected that have high rejection for the given background type. CUT1 and CUT2 should be uncorrelated\(^9\). That is, the rejection of CUT1 should not depend on whether CUT2 has been applied and vice versa. To select the background type we wish to study, the entire 1995 data set is analyzed and

\(^9\) The cuts need not be uncorrelated for the \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) analysis itself to work, only for this method of estimating the background to work.
all cuts in the analysis except CUT1 and CUT2 are applied. The remaining events fall into one of four categories, depicted in Figure 3.9, depending on whether they pass or fail CUT1 and CUT2. \( A, B, C, \) and \( D \) are the numbers of events in the four boxes. Since the events that appear in box \( D \) have passed every cut in the analysis, and because we are conducting a blind analysis, we cannot look at \( D \) as these are the events in the final sample. Instead we must try to estimate \( D \) (the number of events of a given background type in the signal region) using \( A, B, \) and \( C \). If the cuts are uncorrelated, \( D_{\text{estimate}} = \frac{C}{A}B \) will be a good estimate of \( D \). Thus \( AD = BC \) is the condition of no correlations. For example, if \( A = 10000, B = 10, \) and \( C = 20, \) we could predict a background level much less than 1 event: \( D_{\text{estimate}} = \frac{20}{10000} \times 10 = 0.02 \).

Anything that results in the condition \( AD > BC \) is a "bad" correlation, that is, we will underestimate our background level using the formula, \( D_{\text{estimate}} = \frac{C}{A}B \). There are two obvious ways the condition \( AD > BC \) can come about: \( A \) can be made too large if the background study is contaminated by a class of events that preferentially fail both CUT1 and CUT2, or \( D \) can be made too large if there exists some class of events that preferentially pass both cuts. This latter class of events is especially bothersome. Extreme care must therefore be taken when designing the background studies to ensure that the estimated background levels are realistic.

Bias can be introduced into the background estimates if we tune cuts based on small samples of events in the background studies in much the same way that bias
can be introduced into the acceptance for \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) if we tune cuts based on a few events in the signal region. Suppose, for example, that box \( B \) in Figure 3.9 contains only 2 events. It would probably not be difficult to invent a cut that rejected one of these events with little acceptance loss, but does this new cut really have a rejection of 2 for this background type? Probably not. To avoid this bias, we tune the cuts listed in the previous section by looking at only one third (sampled evenly throughout the run) of the data set. We then measure the background levels using the full data set.

Separate background studies are performed for the five most troublesome backgrounds: \( K_{\mu 2}, K_{\pi 2}, 1 \)-beam background (\( \pi \)-scats), 2-beam background, and charge exchange background (CEX).

### 3.3.1 \( K^+ \rightarrow \mu^+ \nu_\mu \)

There are two main uncorrelated groups of cuts that attack the \( K_{\mu 2} \) background, TD cuts and kinematic cuts. The basic structure of the \( K_{\mu 2} \) background study is as follows (see Figure 3.10). The entire 1995 data set is subjected to all cuts in the analysis except the TD and kinematic cuts. This leaves a large sample of \( K_{\mu 2} \) events. The number of events passing the TD cuts is \( N \), the normalization. The ratio of the number of events entering the kinematic cuts (\( N_{\text{in}} \)) to the number of events passing the kinematic cuts (\( N_{\text{out}} \)) is the "kinematic rejection", \( R_{\text{kine}} = N_{\text{in}}/N_{\text{out}} \). If the TD
3.3. *Background Studies*

Figure 3.10: $K^+ \rightarrow \mu^+ \nu_\mu$ background study.
3.3. Background Studies

and kinematic cuts are uncorrelated, this rejection applies to the $N$ events passing all TD cuts, and we can estimate the total $K_{\mu2}$ background as $N_{K_{\mu2}} = N/R_{\text{kin}}$. In this study, the TD cuts are ELVETO, ELEC V5, TDFOOLO, RSHEX, TDDFA, TDLIK, TMUBV, and TMUADC. The kinematic cuts are RBOX, EBOX, PBOX, RNGMOM, RSDEDX, ZDCOW, UTCQUAL, and CHIRF.

In reality the situation is more complicated than that, as can be seen from Figure 3.10. The E787 data set is reduced in various stages, or “passes.” In Pass1, loose versions of many cuts were applied: track reconstruction in the range stack and UTC, FITPI, light photon vetoing in the range stack, PDC $< 280$ MeV\textsuperscript{10}, and a light version of DIPANG for tracks stopping in range stack layer 14. At Pass2, the events were divided into different “skim streams” to be used in the different background studies (see Table 3.8). SKIM6 is the data reduction stream, and it is the stream to

<table>
<thead>
<tr>
<th>Cut</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS5</td>
<td>Light versions of B4DEDX, PICER, PISCAT.BW, and PISCAT.B4 (using B4TBS) are applied.</td>
</tr>
<tr>
<td>PS5T1</td>
<td>PS5 and light versions of PISCAT.B4 (using B4TBS) and PBGLASS.</td>
</tr>
<tr>
<td>PSTAG</td>
<td>This requires B4EM $&lt; 1.5$ MeV or activity at pion time in the pion Čerenkov, B4, or BWPC's.</td>
</tr>
<tr>
<td>PV5</td>
<td>Light photon veto in the barrel veto, endcap and range stack.</td>
</tr>
<tr>
<td>PV5T1</td>
<td>Somewhat tighter photon veto in all subdetectors (except target).</td>
</tr>
<tr>
<td>TDC5</td>
<td>Light versions of RSHEX and ELVETO.</td>
</tr>
<tr>
<td>TDC5T1</td>
<td>Light versions of TDLIK, RSHEX, ELVETO, and TDFOOL.</td>
</tr>
<tr>
<td>KIN5</td>
<td>Light versions of RNGMOM and RSDEDX are applied. Also $\text{PDT} &lt; 230$ MeV.</td>
</tr>
<tr>
<td>TGC5</td>
<td>Target reconstruction, a loose version of TIMCON, and a loose DELC ($t_e - t_K &gt; 1.5$).</td>
</tr>
<tr>
<td>TGC5T1</td>
<td>TGC5 and loose versions of EPIMAX, TGEDDX, TARGF, RTDIF, EKZ, TGLIKE, BSCAT, EIC.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SKIM1</th>
<th>PS5 $\times$ TDC5 $\times$ KIN5 $\times$ TGC5</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKIM2</td>
<td>PS5 $\times$ TDC5 $\times$ PV5 $\times$ TGC5 $\times$ (KIN5 $&lt; 50$ cm) $\times$ (Light TGDCTV)</td>
</tr>
<tr>
<td>SKIM3</td>
<td>PSTAG $\times$ TDC5 $\times$ KIN5 $\times$ PV5 $\times$ (Target reconstruction)</td>
</tr>
<tr>
<td>SKIM4</td>
<td>PS5 $\times$ TDC5 $\times$ KIN5 $\times$ PV5</td>
</tr>
<tr>
<td>SKIM5</td>
<td>PS5 $\times$ PV5 $\times$ KIN5 $\times$ TGC5 $\times$ (loose RSHEX)</td>
</tr>
<tr>
<td>SKIM6</td>
<td>PS5 $\times$ TDC5 $\times$ PV5 $\times$ TGC5</td>
</tr>
</tbody>
</table>

Table 3.8: Pass2 skim stream and cut definitions.

\textsuperscript{10}PDC is just the momentum measured in the drift chamber, without the correction for momentum lost in the target.
which the 55 cuts listed in the previous section will be applied in hopes of finding \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \). The other skim streams were designed for various studies. For example, there are more \( K_{\mu 2} \) events in SKIM2 and SKIM5 than there are in SKIM6. Thus, if there are not enough \( K_{\mu 2} \) events in SKIM6 to perform a statistically meaningful \( K_{\mu 2} \) background study, one can use SKIM2 or SKIM5, provided that one is careful to design the study in a way that is compatible with the definitions of the skim streams.

The \( K_{\mu 2} \) background study is statistics limited on the kinematics side, so we must use SKIM2. The TD side, however, has hundreds of events at the bottom so using SKIM6 is fine. The cut of \( RTOT < 48 \text{ cm} \) is necessary because there was a cut of \( RTOT < 50 \text{ cm} \) applied to SKIM2. The cut in this study is moved down to 48 cm because \( RTOT \) was calculated differently in this analysis from how it was calculated at Pass2.

We must be careful not to look at candidate events when conducting these studies. The TD side has hundreds of events at the bottom, so we are not in danger of seeing a few \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) that may be mixed in. The kinematic side, however, has only a few events at the bottom, so we must invert a cut to protect ourselves from seeing candidate events. Thus, TD cuts is applied in this branch, that is, events are required to fail at least one of the TD cuts to appear at the bottom of the kinematics side of the study.

As has been stated, correlations between the TD and kinematic cuts will ruin this
3.3. Background Studies

background study. Two correlations are known, and cuts have been added to the “setup” (the cuts applied in a background study before the bifurcation) to remove the effects of these correlations as much as possible. Consider $K_{\mu 2}$ events in which the $\mu^+ \rightarrow e^+$ decay is picked up by FITPI as the $\pi^+ \rightarrow \mu^+$ decay, and the second pulse energy is too large to be a 4 MeV muon. These events will preferentially fail TDLIK (because the “muon pulse” is too large), and they will preferentially fail EBOX and RBOX because the extra energy in the stopping counter will help push the event up out of the box. This is a correlation of the type mentioned above in which $AD > BC$ because $A$ is contaminated with these events. The solution to this problem is the $\text{EMUT} < 270$ cut in the setup, which removes events with a “muon pulse” much larger than 4 MeV. The second correlation is one between the electron cut and ZSTOP, the $z$ position at which the charged track came to rest. The source of this correlation is currently unknown, but it is observed that events at large $|\text{ZSTOP}|$ preferentially fail ELEC_V5, and they also preferentially fail the kinematic cuts DIPANG, ZFRF, and LAY14. We solve this problem by moving DIPANG, ZFRF, and LAY14 from the kinematic branch to the setup in order to remove these events from the background study. A $K_{x2}$ veto cut is also added to the setup to remove any residual $K_{x2}$’s that could contaminate the study.

The results of the $K_{\mu 2}$ background study are:

$$N_{K_{\mu 2}} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} = 268 \times \frac{7}{114724} = 0.016 \pm 0.006 \text{ events.}$$
3.3. Background Studies

$N$ is from the entire SKIM6 data set, and $N_{\text{out}}, N_{\text{in}}$ are from the entire SKIM2 data set.

There are a great many ways in which this background estimate could be wrong. The normalization could be wrong. The measured kinematic rejection might not apply to the TD survivors. Etc. In order to build confidence that the background estimate is plausible, we perform "outside-the-box" studies. We conduct a $K_{\mu 2}$ background study exactly like the one described above, except that we predict the number of $K_{\mu 2}$ in a region outside the box, and then we count the number of events in that region to see if it is consistent with the predicted number. Two regions are studied for $K_{\mu 2}$; we look outside the kinematic box and outside the TD cuts. For the first study, we redefine PBOX to cover just the $K_{\mu 2}$ peak, $229 \text{ MeV} < \text{PT0T} < 236 \text{ MeV}$. We observe 2 events in this region, which is consistent with the predicted number:

$$N_{K_{\mu 2}}^{229<\text{PT0T}<236} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} = 268 \times \frac{398}{114724} = 0.93 \pm 0.07 \text{ events.}$$

For the second study, we simply remove from the analysis all cuts in the TD branch of the $K_{\mu 2}$ background study. We require, however, that at least one TD cut fail to protect us from seeing candidate events (this is what we mean by outside the box). We observe 1 event in this region, which is consistent with the predicted number:

$$N_{K_{\mu 2}}^{\text{TD}} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} = 9971 \times \frac{7}{114724} = 0.61 \pm 0.23 \text{ events.}$$
3.3.2 \( K^+ \rightarrow \pi^+\pi^0 \)

The two main uncorrelated groups of cuts that attack the \( K_{\pi 2} \) background are the photon veto (GAMVETP3) and the kinematic box (RBOX, EBOX, and PBOX). The basic structure of the \( K_{\pi 2} \) background study is as follows (see Figure 3.11). On the kinematic side, we measure \( N/H_1 \), the ratio of \( K_{\pi 2} \) events below the kinematic box.

\[
\text{Figure 3.11: } K^+ \rightarrow \pi^+\pi^0 \text{ background study.}
\]
3.3. **Background Studies**

To events in the box\textsuperscript{11}. On the normalization side of the study we measure the size of the $K_{\pi 2}$ peak with all cuts applied except the kinematic box, $H_2$. Because the kinematic side of this study is statistics limited, we use SKIM1 (for $N$ and $H_1$). The normalization side uses SKIM6 to get $H_2$. Scaling the size of the $K_{\pi 2}$ peak by the box rejection, we get the background estimate:

$$N_{K_{\pi 2}} = H_2 \times \frac{N}{H_1} = 658.7 \times \frac{0}{27346} = 0.024 \pm 0.024 \text{ events}.$$ 

Here, although we observe $N = 0$ events on the kinematic side, we inflate $N$ to 1, to provide a sort of upper limit for the background estimate, to which we assign 100% error.

Again, we must be careful of correlations. There is one obvious possible correlation in this study. If a photon lands on the charged track, the photon veto becomes less effective (because the photon has a place to hide) and EBOX becomes less effective (because the photon energy will raise the value of ETOT). This is a correlation of the type mentioned above in which $AD > BC$ because $D$ is contaminated with these events. We can hope, however, that RSDEDX and TGDEDX remove this class of events. To investigate this, all cuts in the analysis are applied except the kinematic box and the photon veto. RTOT < 33 cm is applied to protect us from seeing candidate events. The rejection of EBOX is then measured before and after application of the

\textsuperscript{11}$H_1$ and $H_2$ are heights of Gaussian fits to the $K_{\pi 2}$ peak PTOT distribution.
3.3. Background Studies

photon veto:

\[
R_{\text{No PV}} = \frac{N_{E_{\text{TOT}} < 115}}{N_{E_{\text{TOT}} > 115}} = \frac{31951}{421} = 76 \pm 4
\]

\[
R_{\text{With PV}} = \frac{N_{E_{\text{TOT}} < 115}}{N_{E_{\text{TOT}} > 115}} = \frac{3864}{61} = 63 \pm 8
\]

To within statistics, the rejection does not change, indicating that this correlation is not a problem.

As with the \(K_{\mu 2} \) background study, we now perform outside-the-box studies to show that the \(K_{\pi 2} \) background study is working. Six regions outside the kinematic box are inspected. All numbers of observed events are consistent with the numbers predicted (see Table 3.9).

<table>
<thead>
<tr>
<th>Region</th>
<th>( N_{\text{expected}} = H_2 \times \frac{N}{H} )</th>
<th>( N_{\text{observed}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBOX, RBOX, 200 &lt; PTOT &lt; 211</td>
<td>( 658.7 \times \frac{11}{27346} = 0.26 \pm 0.08 )</td>
<td>0</td>
</tr>
<tr>
<td>RBOX, PBOX, 100 &lt; ETOT &lt; 115</td>
<td>( 658.7 \times \frac{66}{27346} = 1.59 \pm 0.20 )</td>
<td>0</td>
</tr>
<tr>
<td>EBOX, PBOX, 28 &lt; RTOT &lt; 33</td>
<td>( 658.7 \times \frac{32}{27346} = 0.77 \pm 0.14 )</td>
<td>0</td>
</tr>
<tr>
<td>PBOX, 28 &lt; RTOT &lt; 33, 100 &lt; ETOT &lt; 115</td>
<td>( 658.7 \times \frac{2180}{27346} = 52.51 \pm 1.12 )</td>
<td>45</td>
</tr>
<tr>
<td>EBOX, 28 &lt; RTOT &lt; 33, 200 &lt; PTOT &lt; 211</td>
<td>( 658.7 \times \frac{1930}{27346} = 46.49 \pm 1.06 )</td>
<td>59</td>
</tr>
<tr>
<td>RBOX, 200 &lt; PTOT &lt; 211, 100 &lt; ETOT &lt; 115</td>
<td>( 658.7 \times \frac{411}{27346} = 9.90 \pm 0.49 )</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.9: Outside-the-box regions for the \(K_{\pi 2} \) background study.

3.3.3 1-beam

In a 1-beam background event\(^\text{12}\), either a single beam pion enters the detector and scatters into the range stack, or a kaon decays in flight. The two main uncorrelated groups of cuts that attack this background are DELC and the beam cuts, PICER and

\(^{12}\)The 1-beam and 2-beam studies for this analysis were carried out by Andrew Bazarko.
3.3. Background Studies

B4DEDX. The basic structure of the 1-beam background study is sketched in Figure 3.12. SKIM6 did not contain enough statistics for either branch of this study due to the inclusion of PSCUT, so we must use SKIM4 for the normalization and SKIM3 to measure the DELC rejection. Target cuts are applied to SKIM3 to remove 2-beam background events. Note that both SKIM3 and SKIM4 have had the same version of the photon veto cuts, PVCUT1, applied — cuts that are different and sometimes tighter than those used for the final analysis. This introduces a complication that can only be definitively resolved by rerunning Pass2 to reproduce SKIM3 and SKIM4 without overly tight cuts. Since this would involve months processing time, we instead conducted a study in which the absolute number of 1-beam background events in SKIM4 and SKIM6 are compared (see [33] for more details of this procedure). This study suggested that the 1-beam background level is a factor of 1.52 higher than that predicted by the background study sketched in Figure 3.12. The final 1-beam background estimate is:

\[ N_{1\text{-beam}} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} \times 1.52 = 20 \times \frac{7}{16618} \times 1.52 = 0.013 \pm 0.006 \text{ events.} \]

Really, this should be corrected for the efficiency of the inverted DELC cut in the normalization branch. However, since the rejection of DELC is about 2500, the efficiency of DELC for 1-beam background events is about 0.9996.

An outside-the-box study is performed to test this background study in which we look outside DELC. The rejection side of the study looks, not in the region \( t_\pi - t_K > 2 \)
3.3. Background Studies

Figure 3.12: 1-beam background study.
3.3. **Background Studies**

ns, but in the region $1 \text{ ns} < t_\pi - t_K < 2 \text{ ns}$\textsuperscript{13}. 1 event is observed in this region (using SKIM4), in good agreement with the prediction:

$$N_{1-\text{beam}}^{\text{DELCA}(1\text{ms})} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} = 20 \times \frac{347}{14963} = 0.46 \pm 0.11 \text{ events.}$$

If we look in this region in the SKIM6 data set, we expect to see $0.46 \times 1.52 = 0.70$ events, and 0 events are observed. This is not really a fair check because $t_\pi - t_K > 1.5$ ns has already been applied to SKIM6 data, but nevertheless it is nice to see that there is no catastrophic loophole in the background study due to differences in the Pass2 skim streams.

### 3.3.4 2-beam

In a 2-beam background event, two beam particles (either two kaons, or a kaon and a pion) conspire to fool the delayed coincidence. The most powerful rejection against this comes from the cuts that look for multiplicity in the beam elements (PICER, DKNFLT, PISCAT.BW, and PISCAT.B4) and the target cuts, notably TARGF. The bifurcation used in this study is between PICER in one branch, and PISCAT.B4 and the target cuts in the other branch. The basic structure of the 2-beam background study is sketched in Figure 3.13. Here "Target cuts" refers to the following cuts: TGLIKE, TGB4, TARGF, TGDEDX, EPIMAX, RPIMIN, NTRIK, KIC, PIGAP, ZTGT, TGDCVT, RTDIF, and EKZ. To gain sufficient statistics, we need to use SKIM3 for

\textsuperscript{13}The other conditions in Table 3.6 are scaled accordingly. \textit{i.e.} $2 < t_\pi - t_K < 4$ is used for the condition $N_{\text{TGB4er}}^{\text{TGB}} = 0$ and so on.
Figure 3.13: 2-beam background study.
both branches of this study. This structure suffers from the same problem with PVCUT1 as the 1-beam study. Studies with SKIM6 [33] suggest that our 2-beam background level is a factor of 1.13 higher than that predicted by the background study sketched in Figure 3.13. The final 2-beam background estimate is:

$$N_{2\text{-beam}} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} \times 1.13 = 14 \times \frac{1}{1728} \times 1.13 = 0.009 \pm 0.009 \text{ events.}$$

Again, a correction for the efficiency loss of the inverted cut in the normalization branch has a small effect here.

To build confidence that the 2-beam background estimate is valid, the background in a region outside the final analysis region is predicted, and the prediction is compared with the background observed. For this look outside-the-box, we are looking in a region outside PISCAT_B4 and four of the target cuts (see Table 3.10). 1 event

<table>
<thead>
<tr>
<th>Cut</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKZ</td>
<td>$40 \text{ MeV} &lt;</td>
</tr>
<tr>
<td>PIGAP</td>
<td>$1.2 \text{ cm} &lt; \text{Gap} &lt; 2.35 \text{ cm}$</td>
</tr>
<tr>
<td>TARGF</td>
<td>$0.95 \text{ cm} &lt; K - \pi \text{ distance} &lt; 2.1 \text{ cm}$</td>
</tr>
<tr>
<td>TGDCVT</td>
<td>$5 \text{ MeV} &lt; \text{ENERG_SW} &lt; 115 \text{ MeV}$</td>
</tr>
<tr>
<td>PISCAT_B4</td>
<td>$0.625 \text{ ns} &lt;</td>
</tr>
</tbody>
</table>

Table 3.10: Outside-the-box region for 2-beam background study.

is observed in this region, in good agreement with the prediction:

$$N_{2\text{-beam}}^{\text{outbox}} = N \times \frac{N_{\text{out}}}{N_{\text{in}}} = 14 \times \frac{78}{1534} = 0.71 \pm 0.21 \text{ events.}$$

If we look in this region in the SKIM6 data set, we expect to see $0.71 \times 1.13 = 0.80$ events, and 0 events are observed. As with the 1-beam study, this is not really a
3.3. *Background Studies*

fair check because light versions of PISCAT_BW and PISCAT_B4 have already been applied to SKIM6 data, but still it is nice to see 0 events in this region.

### 3.3.5 Charge Exchange

After entering the target, a $K^+$ can interact with a neutron in a nucleus to produce a $K^0$ and a proton. The subsequent decay $K^0_L \rightarrow \pi^+ l^- \bar{\nu}$, where $l = e$ or $\mu$, can mimic $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ if the lepton is lost. This is called the charge exchange background (CEX). The main defenses against CEX are cuts that look for a gap between the $K^+$ and $\pi^+$ (TARGF and EKZ) and cuts that could find the lepton hits, such as TGDCVT and other target quality cuts. DELC is also a powerful cut for CEX because the $K^0$ will not slow down in the target, and thus it will only have a few ns in which to decay before leaving the target.

The CEX background study is complicated by the lack of a large sample of CEX events in the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ data set. Instead, we must rely on a sample of CEX events generated by umc for this background study. Thanks to its short lifetime ($\tau = 0.0893$ ns), the $K^0_S$ component is sufficiently rejected by the delayed coincidence cut, so we need only concern ourselves with the $K^0_L$ component. Nine tapes of $K^+ \rightarrow K^0_S \rightarrow \pi^+ \pi^-$ data were collected with a special trigger near the end of the 1997 run, and these data were analyzed and used to measure the momentum and target stopping distributions from real $K^0_S$ CEX events\textsuperscript{14}. The momentum and stopping distributions were then

\textsuperscript{14}Many thanks to Akira Konaka and the crew at TRIUMF for this work.
3.3. Background Studies

used to generate Monte Carlo $K^0_L \rightarrow \pi^+ l^- \bar{\nu}$ events. This sample of CEX events was analyzed with the same KOPIA routines, and subjected to most of the same cuts as were used for the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ triggers in this analysis (Since the beam line elements and range stack TD data are not in UMC events, neither TD nor beam cuts could be applied). Of the 1365343 events in this sample, 50 passed all cuts. We can estimate the CEX background level using the following formula:

$$N_{\text{CEX}} = N_{\text{pass}} \times \frac{N_{\text{1995}}}{N_{\text{UMC}}^{K^+ \text{live}}} \times A_{\text{UMC}}$$

$$= 50 \times \frac{1.53 \times 10^{12}}{5.23 \times 10^{14}} \times 0.149$$

$$= 0.022 \pm 0.022$$

$N_{\text{1995}}$ is the number of kaons entering the detector while the detector was "live" (not busy reading out a previous event) during the 1995 run. $N_{\text{UMC}}^{K^+ \text{live}}$ is the number of $K^+ \text{live}$ corresponding to the 1365343 UMC CEX events analyzed\textsuperscript{15}. The factor, $A_{\text{UMC}}$, accounts for the acceptance loss of the cuts not applied, the TD and beam cuts. The acceptance loss of these cuts should be the same for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and CEX events.

An outside-the-box study was performed that predicted that 0.5 CEX background events would appear with certain cuts relaxed. This region contained 3 CEX candidate events. CEX events are difficult to distinguish from 2-beam background events,

\textsuperscript{15}$N_{\text{UMC}}^{K^+ \text{live}}$ is computed by dividing the number of CEX events generated by the production rate for the $K^+ \rightarrow K^0_S$ reaction. This rate is assumed to be the same as the production rate for $K^+ \rightarrow K^0_S$, which was measured with the data collected.
however, and none of the 3 candidates are obviously CEX and not 2-beam. (0.3-1.4 events of 2-beam background were expected to appear in this same study. See [33].)

3.3.6 Totals

The total estimated background level in the signal region is summarized in Table 3.11.

<table>
<thead>
<tr>
<th>Background type</th>
<th>$N_{\text{Bkg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu 2}$</td>
<td>0.016 ± 0.006</td>
</tr>
<tr>
<td>$K_{\pi 2}$</td>
<td>0.024 ± 0.024</td>
</tr>
<tr>
<td>1-beam</td>
<td>0.013 ± 0.006</td>
</tr>
<tr>
<td>2-beam</td>
<td>0.009 ± 0.009</td>
</tr>
<tr>
<td>CEX</td>
<td>0.022 ± 0.022</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.084 ± 0.035</strong></td>
</tr>
</tbody>
</table>

Table 3.11: Summary of background levels.

3.4 Characterization of Candidate Events

The background studies and outside-the-box studies give us every reason to believe that the background levels in the signal region are low. We could, therefore, freeze the cuts and open the box at this point. Suppose, however, that opening the box revealed a single candidate event right in the lower corner of the kinematic box at, say, $\text{PTOT} = 211.1$, $\text{ETOT} = 115.1$, $\text{RTOT} = 33.1$. Even though the $K_{\pi 2}$ background study suggests a low probability for a $K_{\pi 2}$ to appear in the box, it would be hard to believe that this event was not $K_{\pi 2}$, given its proximity to the $K_{\pi 2}$ peak. Of course,
we have already decided that we cannot simply discard this event; we committed ourselves to keeping every event in the signal region when we decided to do a blind analysis. However, it is useful to be able to assess how deep an event is in the signal region using an \textit{a priori} evaluation.

To this end, we define three sets of tighter cuts (called TIGHT1, TIGHT2, and TIGHT3) before opening the box. Each set of cuts is tighter than the previous set for all known background types. For example, the hypothetical $K_{\pi 2}$ event mentioned above would probably fail the TIGHT1 cuts, increasing our suspicion that it is not really a $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event. Events that pass even the TIGHT3 cuts can be said to lie in a “golden region.”

If there is a candidate event found in the one of the “grey regions”, that is, it passes all standard cuts in the analysis but it fails, say, the TIGHT1 cuts, does this mean the event is unlikely to be $K^+ \rightarrow \pi^+ \nu \bar{\nu}$? Certainly not. The TIGHT1, TIGHT2, and TIGHT3 cuts each have some acceptance loss (see Table 5.17), thus we expect some fraction of real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events in our signal region to lie in a grey region. In fact only about half of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events in our signal region will lie in the golden region. Can we throw away events lying in the grey region after opening the box? If our background studies are correct, the probability of a background event getting into the signal region is quite low, so it’s likely (depending on the true $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio) that these are real $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events. If we throw away any events
in the grey regions, then we do not truly retain the acceptance of the full signal region, and we may as well have defined our signal region to be, say, TIGHT2 or TIGHT3. We are only justified in discarding events under two conditions:

- Based on observation of suspicious events in the signal region, the background studies are revisited and an error is found. For example, if we see what is clearly a $K_{*2}$ event in the box, and there is some feature of this event that would cause it to preferentially pass both the box cut and the photon veto cut (a correlation that would ruin the $K_{*2}$ background study) we could re-run the background study correcting for this error. Presumably the results of this revised background study would predict higher levels of $K_{*2}$ background in the box, and we could discard the event.

- Suspicious events in the signal region suggest a new type of background. If there exist backgrounds that were not considered in the studies listed in the previous section, the levels of these backgrounds are unknown. If an event in the box suggests a new type of background, and subsequent studies of this background type suggest high levels in the box, we can discard the event.

Unless one of these two conditions is met we must claim all events in the box as candidates for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. "Discarding the event" does not mean simply erasing it from the data set and proceeding with the analysis. If events in the signal region need to be "discarded" this means we can not use the results of the analysis to set a
branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (even if there are other very nice events in the golden region), but rather we must set an upper limit to the branching ratio assuming all events in the box are $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

The tighter cuts are defined in Table 3.12. TIGHT1, TIGHT2, and TIGHT3

Table 3.12: Tighter cuts for characterization of candidate events. Where a blank occurs in the “Standard” column of this table, the cut is not part of the standard set of cuts. Where a blank occurs in the “TIGHT1” or “TIGHT2” columns, the cut is the same as that listed in the “Standard” column. CKTBMTN is defined like CKTRS_N (see the description of DKNFLT in Section 3.2) except in that we find the kaon Čerenkov cluster closest in time to kaon time in the B4 counter, not TRS. BW1TRS and BW2TRS are defined like BW1TRS and BW2TRS (see the description of PISCAT_BW in Section 3.2) except in that we use individual wire hits rather than clusters.

were designed to provide factors of roughly 2, 5, and 10 background rejection for each background type. Background studies identical (except where noted) to those presented above were performed with each set of cuts, and the results are summarized below.

The structure of the $K_{\mu2}$ study remains unchanged from that presented above.
3.4. Characterization of Candidate Events

We find that TIGHT1, TIGHT2, and TIGHT3 provide rejections of 2.0, 2.2, and 5.5 (see Table 3.13).

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>$N_{in}$</th>
<th>$N_{out}$</th>
<th>$R_{Relative}$</th>
<th>$N_{K_{\mu 2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>268</td>
<td>114724</td>
<td>7</td>
<td>1.0</td>
<td>0.0164 ± 0.0062</td>
</tr>
<tr>
<td>TIGHT1</td>
<td>223</td>
<td>109151</td>
<td>4</td>
<td>2.0</td>
<td>0.0082 ± 0.0041</td>
</tr>
<tr>
<td>TIGHT2</td>
<td>184</td>
<td>99946</td>
<td>4</td>
<td>2.2</td>
<td>0.0074 ± 0.0037</td>
</tr>
<tr>
<td>TIGHT3</td>
<td>126</td>
<td>85427</td>
<td>2</td>
<td>5.5</td>
<td>0.0029 ± 0.0021</td>
</tr>
</tbody>
</table>

Table 3.13: Additional rejection for $K_{\mu 2}$ background. $R_{Relative}$ is the rejection of the tight cuts relative to that of the standard cuts.

For the $K_{\pi 2}$ study, a second bifurcation on the kinematic rejection side was necessary to gain sufficient statistics. In the standard $K_{\pi 2}$ study, we simply measure the number of events passing EBOX, BOX, and RBOX on the kinematic side, $N$. With this study, we use three separate numbers to estimate $N$. $N_P$ is the number of events passing just BOX, $N_{PE}$ is the number of events passing BOX and EBOX, and $N_{PR}$ is the number of events passing BOX and RBOX. If the rejections of EBOX and RBOX are uncorrelated, we can estimate $N$ as:

$$N = \frac{N_{PE} \times N_{PR}}{N_P}$$

We find that TIGHT1, TIGHT2, and TIGHT3 provide rejections of 3.0, 6.8, and 16 (see Table 3.14).

For the tight 1-beam background estimates, a 1-beam background sample is selected from SKIM3 as described above except that the target cuts are not applied. Not applying the target cuts enhances the prompt sample by about a factor of five,
3.4. Characterization of Candidate Events

<table>
<thead>
<tr>
<th></th>
<th>(N)</th>
<th>(N_P)</th>
<th>(N_{PE})</th>
<th>(N_{PR})</th>
<th>(H_1)</th>
<th>(H_2)</th>
<th>(R_{Relative})</th>
<th>(N_{K_{s2}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>0</td>
<td>2281</td>
<td>32</td>
<td>66</td>
<td>27346</td>
<td>658.7</td>
<td>1.0</td>
<td>0.0241 ± 0.0241</td>
</tr>
<tr>
<td>TIGHT1</td>
<td>0</td>
<td>1990</td>
<td>23</td>
<td>29</td>
<td>24219</td>
<td>582.4</td>
<td>3.0</td>
<td>0.0081 ± 0.0023</td>
</tr>
<tr>
<td>TIGHT2</td>
<td>0</td>
<td>1785</td>
<td>15</td>
<td>20</td>
<td>21732</td>
<td>460.4</td>
<td>6.8</td>
<td>0.0036 ± 0.0012</td>
</tr>
<tr>
<td>TIGHT3</td>
<td>0</td>
<td>1356</td>
<td>13</td>
<td>7</td>
<td>17470</td>
<td>399.6</td>
<td>16</td>
<td>0.0015 ± 0.0007</td>
</tr>
</tbody>
</table>

Table 3.14: Additional rejection for \(K_{s2}\) background. Note that the background study for the standard cuts does not use the second bifurcation (i.e., it does not use \(N_P\), \(N_{PE}\), \(N_{PR}\)), whereas the background studies for the tight cuts do not use \(N\).

but allows a few non-prompt events to remain, which are mostly removed by requiring \(t_{\pi} - t_K < 10\) ns. A sample of 84182 events selected in this way is used to measure the relative rejection of the tightened DELC cut. We find that TIGHT1, TIGHT2, and TIGHT3 provide rejections of 3.0, 6.3, and 40 (see Table 3.15).

<table>
<thead>
<tr>
<th></th>
<th>(N_{out})</th>
<th>(N)</th>
<th>(R_{Relative})</th>
<th>(N_{1-beam})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>40</td>
<td>20</td>
<td>1.0</td>
<td>0.0128 ± 0.0056</td>
</tr>
<tr>
<td>TIGHT1</td>
<td>27</td>
<td>10</td>
<td>3.0</td>
<td>0.0043 ± 0.0021</td>
</tr>
<tr>
<td>TIGHT2</td>
<td>14</td>
<td>9</td>
<td>6.3</td>
<td>0.0020 ± 0.0011</td>
</tr>
<tr>
<td>TIGHT3</td>
<td>10</td>
<td>2</td>
<td>40</td>
<td>0.0003 ± 0.0003</td>
</tr>
</tbody>
</table>

Table 3.15: Additional rejection for 1-beam background.

For the tight 2-beam background estimates, a 2-beam background sample is selected from SKIM3 with inverted PISCAT\_BW, rather than inverted PICER as described above, and neither PISCAT\_BW nor PICER are applied. Also, we remove all target cuts but the four most potent for 2-beam background (PIGAP, TGDCVT, TARGF, and EKZ) to gain statistics. This allows us to measure the relative rejections of the tight cuts. We find that TIGHT1, TIGHT2, and TIGHT3 provide rejections of 2.1, 4.8, and 8.0 (see Table 3.16).
3.4. Characterization of Candidate Events

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{out}}$</th>
<th>$N$</th>
<th>$R_{\text{Relative}}$</th>
<th>$N_{2-\text{beam}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>31</td>
<td>14</td>
<td>1.0</td>
<td>0.0092 ± 0.0094</td>
</tr>
<tr>
<td>TIGHT1</td>
<td>15</td>
<td>14</td>
<td>2.1</td>
<td>0.0044 ± 0.0046</td>
</tr>
<tr>
<td>TIGHT2</td>
<td>13</td>
<td>7</td>
<td>4.8</td>
<td>0.0019 ± 0.0020</td>
</tr>
<tr>
<td>TIGHT3</td>
<td>9</td>
<td>6</td>
<td>8.0</td>
<td>0.0011 ± 0.0014</td>
</tr>
</tbody>
</table>

Table 3.16: Additional rejection for 2-beam background.

For the CEX study, we are left with 21, 11, and 3 events using the cuts in TIGHT1, TIGHT2, and TIGHT3 (see Table 3.17).

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{pass}}$</th>
<th>$R_{\text{Relative}}$</th>
<th>$N_{\text{CEX}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>50</td>
<td>1.0</td>
<td>0.0218 ± 0.0218</td>
</tr>
<tr>
<td>TIGHT1</td>
<td>21</td>
<td>2.4</td>
<td>0.0092 ± 0.0092</td>
</tr>
<tr>
<td>TIGHT2</td>
<td>11</td>
<td>4.5</td>
<td>0.0048 ± 0.0048</td>
</tr>
<tr>
<td>TIGHT3</td>
<td>3</td>
<td>17</td>
<td>0.0013 ± 0.0013</td>
</tr>
</tbody>
</table>

Table 3.17: Additional rejection for CEX background.

Summing together the various background estimates, we find that the tighter cuts provide additional rejections of 2.5, 4.3, and 12 (see Table 3.18).

<table>
<thead>
<tr>
<th>Background type</th>
<th>$N_{\text{Standard Bkg}}$</th>
<th>$N_{\text{TIGHT1 Bkg}}$</th>
<th>$N_{\text{TIGHT2 Bkg}}$</th>
<th>$N_{\text{TIGHT3 Bkg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu 2}$</td>
<td>0.0164 ± 0.0062</td>
<td>0.0082 ± 0.0041</td>
<td>0.0074 ± 0.0037</td>
<td>0.0029 ± 0.0021</td>
</tr>
<tr>
<td>$K_{\tau 2}$</td>
<td>0.0241 ± 0.0241</td>
<td>0.0081 ± 0.0023</td>
<td>0.0036 ± 0.0012</td>
<td>0.0015 ± 0.0007</td>
</tr>
<tr>
<td>1-beam</td>
<td>0.0128 ± 0.0056</td>
<td>0.0043 ± 0.0021</td>
<td>0.0020 ± 0.0011</td>
<td>0.0003 ± 0.0003</td>
</tr>
<tr>
<td>2-beam</td>
<td>0.0092 ± 0.0094</td>
<td>0.0044 ± 0.0046</td>
<td>0.0019 ± 0.0020</td>
<td>0.0011 ± 0.0014</td>
</tr>
<tr>
<td>CEX</td>
<td>0.0218 ± 0.0218</td>
<td>0.0092 ± 0.0092</td>
<td>0.0048 ± 0.0048</td>
<td>0.0013 ± 0.0013</td>
</tr>
<tr>
<td>Total</td>
<td>0.0843 ± 0.0348</td>
<td>0.0342 ± 0.0115</td>
<td>0.0197 ± 0.0066</td>
<td>0.0071 ± 0.0029</td>
</tr>
</tbody>
</table>

Table 3.18: Summary of background levels for standard and tight cuts.
3.5 Opening the Box

The box was opened at 8:00 PM on November 27, 1997, and a single event was found inside\footnote{It should be mentioned that this analysis was conducted in parallel with an analysis effort of the same data set by our collaborators at TRIUMF [34]. The TRIUMF analysis was completed first, and in fact it revealed the same one candidate event. The people involved in the analysis presented here, however, were intentionally shielded from any information about the TRIUMF candidate event until our cuts were frozen. Thus, bias is not an issue. The TRIUMF analysis had a 40\% lower acceptance than this analysis, thus the branching ratio measured here is 40\% lower than the one quoted in [34]. Note, however, that these two results are statistically consistent with each other.}. This event passes even the TIGHT3 cuts, and indeed looks remarkably clean.

The next chapter is devoted to the details of this one event.
Chapter 4

The One Event

The one event passing all cuts was scrutinized in detail. No aspect of this event suggesting that it might be background has been found. It appears to be a genuine \( K^+ \to \pi^+ \nu \bar{\nu} \).

4.1 Information

Some information about the event is given in Table 4.1. Note that the event lies in the standard kinematic box for \( K^+ \to \pi^+ \nu \bar{\nu} \), but it is below the PFBOX defined for \( K^+ \to \pi^+ X^0 \). Thus there are zero events passing the search for \( K^+ \to \pi^+ X^0 \).

4.2 Pictures

Figures 4.1, 4.2, 4.3, 4.4, 4.5, and 4.6 show event displays of the event.
<table>
<thead>
<tr>
<th>Run number</th>
<th>23271</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event number</td>
<td>42251</td>
</tr>
<tr>
<td>Tape number</td>
<td>20699.10.2</td>
</tr>
<tr>
<td>Date on which the event occurred</td>
<td>April 22, 1995</td>
</tr>
<tr>
<td>Logbook</td>
<td># 36, page 1</td>
</tr>
<tr>
<td>Person on shift</td>
<td>Dick Strand</td>
</tr>
<tr>
<td>Pion stopping sector in RS</td>
<td>3</td>
</tr>
<tr>
<td>Pion stopping layer in RS</td>
<td>14</td>
</tr>
<tr>
<td>Kaon time in target ( t_K )</td>
<td>0.4 ns</td>
</tr>
<tr>
<td>Pion time in target ( t_\pi )</td>
<td>24.3 ns</td>
</tr>
<tr>
<td>Kaon lifetime ( \tau_{K^+} = 12.386 \text{ ns} )</td>
<td>23.9 ns</td>
</tr>
<tr>
<td>Pion time in range stack (TRS)</td>
<td>24.5 ns</td>
</tr>
<tr>
<td>Pion time in stopping counter</td>
<td>24.6 ns</td>
</tr>
<tr>
<td>Pion lifetime (TMUAV) ( \tau_{\pi^+} = 26.033 \text{ ns} )</td>
<td>26.9 ns</td>
</tr>
<tr>
<td>Muon time</td>
<td>51.5 ns</td>
</tr>
<tr>
<td>Electron time</td>
<td>3252.6 ns</td>
</tr>
<tr>
<td>Muon lifetime ( \tau_{\mu^+} = 2197.03 \text{ ns} )</td>
<td>3201.1 ns</td>
</tr>
<tr>
<td>( z ) of kaon decay vertex (STZ_DC)</td>
<td>+6.2 cm</td>
</tr>
<tr>
<td>( z ) of pion decay vertex (ZSTOP)</td>
<td>+13.8 cm</td>
</tr>
<tr>
<td>( \theta_{\text{dip}} )</td>
<td>+5.4 degrees</td>
</tr>
<tr>
<td>Kaon energy in target (ENERK_TG)</td>
<td>89.3 MeV</td>
</tr>
<tr>
<td>Pion energy in target (ETGT)</td>
<td>10.3 MeV</td>
</tr>
<tr>
<td>Pion range in target (RTGT)</td>
<td>4.5 cm</td>
</tr>
<tr>
<td>Total range of pion track (RTOT)</td>
<td>34.8 cm</td>
</tr>
<tr>
<td>Total momentum of pion track (PTOT)</td>
<td>218.1 MeV/c</td>
</tr>
<tr>
<td>Total energy of pion track (ETOT)</td>
<td>117.7 MeV</td>
</tr>
<tr>
<td>Total energy of muon (EMUTC)</td>
<td>3.2 MeV</td>
</tr>
<tr>
<td>Total energy of electron (EELECT2C)</td>
<td>50.7 MeV</td>
</tr>
</tbody>
</table>

Table 4.1: Information about the event. EMUTC and ELEEP2C are calibrated versions of EMUT and EELEEP2. It is the uncalibrated quantities that are actually used in the TD cuts FITPI and TDLIK.
4.2. Pictures

Figure 4.1: $x - y$ (left) and $r - z$ (right) views of the event. The UTC and range stack pion track fits are shown. The $r - z$ view shows $z$ information from end-to-end timing in the range stack scintillator and RSSC's. The extra hits shown in the UTC are not in time with the pion track.

Figure 4.2: The UTC track fit. The locations of the target pion fibers exclude the possibility that the UTC fit chose the wrong side of the isochrones in the innermost superlayer.
Figure 4.3: Close up view of the target showing energies (left) and times (right) of the hits. The lower four fibers with times near 0 ns are the kaon fibers. The location of the B4 hit is shown as a circle near the kaon fibers, and indicates the probable entry point of the kaon into the target.

Figure 4.4: Close up view of the range stack track showing energies (left) and times (right) of the hits. The track fit from the range routines is shown.
4.2. Pictures

Figure 4.5: TD data in both ends of the stopping counter, showing the $\pi^+ \rightarrow \mu^+$ decay. The double and single pulse fits are shown.

Figure 4.6: $x-y$ (left) and $\tau-z$ (right) views of range stack activity at electron time. $z$ information is from end-to-end timing in the range stack scintillator and RSSC's. It is clear from the RSSC hits in the $x-y$ view that the electron originated near the pion stopping point as determined by the range routines.
4.3 Plots

Plots of various measured quantities pertaining to the event are shown in Figures 4.7, 4.8, and 4.9. The purpose of these plots is to show that the candidate event lies in regions favored by $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and far away from those favored by background processes. For example, we already know from the results of the $K_{\pi 2}$ background study that the probability of a $K_{\pi 2}$ appearing in the box is very low; nevertheless, it is nice to see in Figure 4.7 that the candidate event is far removed from the $K_{\pi 2}$ peak. A great many plots similar to the ones shown here were inspected for evidence that this event is not a genuine $K^+ \rightarrow \pi^+ \nu \bar{\nu}$; none was found.

Figure 4.7: Spectrum plots of the full 1995 data set. All cuts have been applied except for RBOX, EBOX, PBOX in the first plot, and except for RBOX, PBOX in the second plot. The one candidate event is visible in the box. The events appearing to the left of the box are presumably residual $K_{\pi 2}$ decays that passed the photon veto cuts, and those events appearing to the right of the box are presumably residual $K_{\mu 2}$ decays that passed the TD cuts.
Figure 4.8: The first plot shows target kaon energy vs $z$ of the kaon decay vertex for cleanly reconstructed $K_{\mu 2}$ decays. The second plot shows energy vs range of the pion track in the target for cleanly reconstructed $K_{\pi 2}$ decays. The dark circle in each plot indicates the location of the candidate event.
4.3. Plots

Figure 4.9: Various timing and energy plots. The kaon, pion, and muon lifetime spectra are shown on the left for cleanly reconstructed $K_{S2}$ decays. The pion momentum, muon energy, and electron energy spectra are shown on the right. The pion momentum spectrum comes from UMC events passing the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ trigger, and the muon and electron energy plots were generated with cleanly reconstructed $\pi-$scat events. The arrow in each plot indicates the location of the candidate event.
Chapter 5

The Sensitivity

We would like to convert our observation of one $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ event into a branching ratio for $K^+ \rightarrow \pi^+ \nu\bar{\nu}$. To do this we need to know:

- The number of kaons that entered the detector while it was live (not reading out a previous event) during the 1995 run, $N_{K^{Blive}}$.

- The fraction of those kaons that, had they decayed as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, would have survived the trigger and offline analysis and appeared in our box. This is called the acceptance.

To minimize uncertainties in the acceptance, we normalize the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ branching ratio to the known $K_{\mu2}$ branching ratio [3]. $f_S$, the parameter used for this normalization, is approximately equal to the fraction of kaons entering the detector that come to rest before decaying, the "stopping fraction." All other parts of the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ acceptance are combined into a factor called $A_{\text{tot}}$. The product of $N_{K^{Blive}}$, $f_S$, and
\( A_{\text{tot}} \) is then the effective number of kaons entering the detector during the 1995 run for which we have acceptance if they decay as \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \), and is called the sensitivity:

\[
S_{\text{tot}} = N_{KB\text{live}} \times f_S \times A_{\text{tot}}
\]

The calculation of \( A_{\text{tot}} \) is rather intricate. If we had a large sample of \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) events, we could simply apply all cuts and see how many events survive, and the fraction of survivors would be \( A_{\text{tot}} \). However, we lack a large sample of \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) tagged independently of this analysis. If our Monte Carlo simulation were perfect, we could generate a sample of UMC \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) events and measure \( A_{\text{tot}} \) by applying all cuts to this sample. However, the Monte Carlo is far from perfect, particularly in the area of timing and energy resolutions, so we would like to rely on Monte Carlo as little as possible in the measurement of acceptance.

Fortunately, we can use the monitor triggers collected along with the \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) data for much of the acceptance measurement. \( K_{\mu2} \) decays look like \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \), in that they have a kaon entering the detector and a single charged track penetrating into the range stack. \( K_{\mu2}(1) \) triggers will therefore be used to measure losses due to accidentals and the beam cuts. \( \pi\text{-scat} \) triggers have a single pion track in the range stack, so they will be used to measure losses due to cuts on the quality of the range stack track and losses due to the TD cuts. \( K_{\pi2}(1) \) triggers will be used for measurements requiring a \( K^+ \rightarrow \pi^+ \) decay in the target. UMC is required only to measure the acceptance of cuts that depend on the kinematics of \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \).
5.1. **UMC-based Acceptance Measurements**

It is important to count each acceptance loss once and only once. Correlations between the acceptances of different cuts can easily result in either under-counting or double-counting the losses. Some care is therefore taken to construct acceptance studies that will count each loss exactly once.

We break the acceptance into various factors. Detailed descriptions of the measurements of these factors follow. Acceptance numbers are provided for the standard cuts as well as the TIGHT1, TIGHT2, TIGHT3 cuts used for assessment of candidate events. In addition, separate acceptance numbers are provided for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$ where they are different. A summary is provided in Tables 5.17 and 5.18.

### 5.1 UMC-based Acceptance Measurements

Monte Carlo is used to measure the following:

- **The Level 0 trigger acceptance loss**, $A_{\text{trig}}$. This does not include the delayed coincidence requirement in the trigger, nor does it include the online hextant cut, photon veto cuts, or the online TD cut. These losses are measured with data.

- **The acceptance loss of offline fiducial volume cuts** (including the kinematic box cuts) and the requirement that the pion stop in range stack scintillator, $A_{\text{umc}}$.

- **Losses due to pion nuclear interactions and decay-in-flight**, $A_{\text{nidir}}$. 
5.1. umc-based Acceptance Measurements

An outline of these measurements is sketched in Figure 5.1.

**Figure 5.1:** The umc-based acceptance measurements. $A_{\text{trig}}$ is shown as being approximately equal to $N_{\text{UMC}}/KB$ because certain trigger requirements are not included here.
5.1. umc-based Acceptance Measurements

5.1.1 The Trigger Acceptance Factors, $A_{\text{trig}}$

The Monte Carlo begins with kaons at rest in the target (umc does not include a simulation of the kaon beam line). The kaons are forced to decay as $K^+ \to \pi^+ \nu \bar{\nu}$ (or $K^+ \to \pi^+ X^0$) and the pion is propagated with pion decay and nuclear interactions disabled. The Level 0 trigger is simulated, and those events passing the trigger are written to disk. The trigger acceptance is calculated by counting the numbers of events passing various trigger requirements, and the results are summarized in Table 5.1 for $K^+ \to \pi^+ \nu \bar{\nu}$ and Table 5.2 for $K^+ \to \pi^+ X^0$. These tables also show the results with pion decay\textsuperscript{1} and nuclear interactions enabled, however these numbers are not used in the acceptance calculation.

5.1.2 The Offline Fiducial Volume Acceptance Factors, $A_{\text{umc}}$

The umc events written to disk are read by the KOFIA analysis software and subjected to as many of the offline cuts as the level of simulation permits. Before measuring the fiducial volume acceptance factors, we apply as a selection all cuts whose acceptance is measured with $K_{\pi^2}$ or $K_{\mu^2}$ monitors. This is to avoid double-counting our losses. The cuts measured with $\pi-$scat monitors however are not applied, since we are measuring the box acceptance here, and the box will be applied as a selection in the $\pi-$scat-based acceptance measurement.

\textsuperscript{1}Muon decay is enabled in all studies presented here.
Table 5.1: The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ trigger acceptance factors without and with NIDIF enabled. The acceptance losses of those trigger requirements listed in parentheses will be measured with data, thus they are not counted here. The range stack layer 11 requirement is actually part of the refined range trigger. It is listed separately because it dominates the refined range acceptance loss.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Without NIDIF</th>
<th>With NIDIF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>$KB$</td>
<td>400000</td>
<td>-</td>
</tr>
<tr>
<td>$T \cdot 2$</td>
<td>163945</td>
<td>0.410</td>
</tr>
<tr>
<td>Reach layer 6 or 7</td>
<td>134486</td>
<td>0.820</td>
</tr>
<tr>
<td>(Delayed coincidence)</td>
<td>132489</td>
<td></td>
</tr>
<tr>
<td>Pass $\mu$ veto</td>
<td>132465</td>
<td>1.000</td>
</tr>
<tr>
<td>(Barrel veto)</td>
<td>132465</td>
<td></td>
</tr>
<tr>
<td>(Endcap veto)</td>
<td>132465</td>
<td></td>
</tr>
<tr>
<td>(Hextant veto)</td>
<td>132465</td>
<td></td>
</tr>
<tr>
<td>Reach layer 11</td>
<td>72496</td>
<td>0.547</td>
</tr>
<tr>
<td>Refined Range</td>
<td>71404</td>
<td>0.985</td>
</tr>
<tr>
<td>$A_{\text{trig}}$</td>
<td>0.1812 $\pm$ 0.0006</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: The $K^+ \rightarrow \pi^+ X^0$ trigger acceptance factors without and with NIDIF enabled.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Without NIDIF</th>
<th>With NIDIF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>$KB$</td>
<td>400000</td>
<td>-</td>
</tr>
<tr>
<td>$T \cdot 2$</td>
<td>181255</td>
<td>0.453</td>
</tr>
<tr>
<td>Reach layer 6 or 7</td>
<td>181247</td>
<td>1.000</td>
</tr>
<tr>
<td>(Delayed coincidence)</td>
<td>178587</td>
<td></td>
</tr>
<tr>
<td>Pass $\mu$ veto</td>
<td>177664</td>
<td>0.995</td>
</tr>
<tr>
<td>(Barrel veto)</td>
<td>177664</td>
<td></td>
</tr>
<tr>
<td>(Endcap veto)</td>
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<td></td>
</tr>
<tr>
<td>(Hextant veto)</td>
<td>177664</td>
<td></td>
</tr>
<tr>
<td>Reach layer 11</td>
<td>176814</td>
<td>0.995</td>
</tr>
<tr>
<td>Refined Range</td>
<td>160300</td>
<td>0.907</td>
</tr>
<tr>
<td>$A_{\text{trig}}$</td>
<td>0.4067 $\pm$ 0.0007</td>
<td></td>
</tr>
</tbody>
</table>
5.1. umc-based Acceptance Measurements

Three cuts are applied to umc events that are not applied to data:

- **UFATE**: Require that the pion comes to rest without interacting or decaying. Note that this will cut no events when nuclear interaction and decay-in-flight are disabled.

- **USTMED**: Require that the pion stops in scintillator.

- **USTOP.HEX**: Require that the true range stack stopping layer and hextant agree with the offline stopping layer and hextant.

The point here is to account for the loss that occurs when the TD cuts are applied to the wrong counter or to events in which the pion does not come to rest in scintillator. Since the method of measuring the FITPI acceptance loss will not account for this, we must account for it here. The results are summarized in Table 5.3 for \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) and Table 5.4 for \( K^+ \rightarrow \pi^+ X^0 \).

5.1.3 Nuclear Interactions and Decay-In-Flight, \( A_{\text{nidif}} \)

The entire procedure outlined in the preceding two sections is repeated with pion nuclear interactions and decay-in-flight enabled. \( A_{\text{nidif}} \) is defined as the double ratio:

\[
A_{\text{nidif}} = \frac{(N_{\text{pass}}/N_{KB})_{\text{on}}}{(N_{\text{pass}}/N_{KB})_{\text{off}}}.
\]

At first glance it may seem wrong to simply use the ratio \( N_{\text{pass}}/N_{KB} \) here, since this will include those trigger acceptance factors that we intentionally left out in Tables
5.1. umc-based Acceptance Measurements

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(N_{\text{passed}})</td>
<td>A</td>
<td>(N_{\text{passed}})</td>
<td>A</td>
</tr>
<tr>
<td>Selection</td>
<td>50593</td>
<td>-</td>
<td>50593</td>
<td>-</td>
</tr>
<tr>
<td>UFATE</td>
<td>50593</td>
<td>1.000</td>
<td>50593</td>
<td>1.000</td>
</tr>
<tr>
<td>USTEMED</td>
<td>50299</td>
<td>0.994</td>
<td>50299</td>
<td>0.994</td>
</tr>
<tr>
<td>USTOP_HEX</td>
<td>50126</td>
<td>0.997</td>
<td>50126</td>
<td>0.997</td>
</tr>
<tr>
<td>DIPANG</td>
<td>48369</td>
<td>0.965</td>
<td>48369</td>
<td>0.965</td>
</tr>
<tr>
<td>LAY14</td>
<td>48353</td>
<td>1.000</td>
<td>48353</td>
<td>1.000</td>
</tr>
<tr>
<td>NSECRS</td>
<td>48353</td>
<td>1.000</td>
<td>48353</td>
<td>1.000</td>
</tr>
<tr>
<td>ZFRF</td>
<td>46359</td>
<td>0.959</td>
<td>46359</td>
<td>0.959</td>
</tr>
<tr>
<td>BOX</td>
<td>21158</td>
<td>0.456</td>
<td>20034</td>
<td>0.432</td>
</tr>
<tr>
<td>(A_{\text{umc}})</td>
<td>0.418 ± 0.002</td>
<td>0.396 ± 0.002</td>
<td>0.377 ± 0.002</td>
<td>0.353 ± 0.002</td>
</tr>
</tbody>
</table>

Table 5.3: Fiducial volume acceptance for \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\).

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(N_{\text{passed}})</td>
<td>A</td>
<td>(N_{\text{passed}})</td>
<td>A</td>
</tr>
<tr>
<td>Selection</td>
<td>113820</td>
<td>-</td>
<td>113820</td>
<td>-</td>
</tr>
<tr>
<td>UFATE</td>
<td>113820</td>
<td>1.000</td>
<td>113820</td>
<td>1.000</td>
</tr>
<tr>
<td>USTEMED</td>
<td>112672</td>
<td>0.990</td>
<td>112672</td>
<td>0.990</td>
</tr>
<tr>
<td>USTOP_HEX</td>
<td>112336</td>
<td>0.997</td>
<td>112336</td>
<td>0.997</td>
</tr>
<tr>
<td>DIPANG</td>
<td>106768</td>
<td>0.950</td>
<td>106768</td>
<td>0.950</td>
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<tr>
<td>LAY14</td>
<td>106573</td>
<td>0.998</td>
<td>106573</td>
<td>0.998</td>
</tr>
<tr>
<td>NSECRS</td>
<td>106573</td>
<td>1.000</td>
<td>106573</td>
<td>1.000</td>
</tr>
<tr>
<td>ZFRF</td>
<td>102680</td>
<td>0.963</td>
<td>102680</td>
<td>0.963</td>
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<tr>
<td>PFBBOX</td>
<td>71907</td>
<td>0.700</td>
<td>71907</td>
<td>0.700</td>
</tr>
<tr>
<td>(A_{\text{umc}})</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
</tr>
</tbody>
</table>

Table 5.4: Fiducial volume acceptance for \(K^+ \rightarrow \pi^+ X^0\).
5.2 $K_{\mu2}$-based Acceptance Measurements

5.1 and 5.2 and all of the selection cuts at the top of the $A_{umc}$ study (see Figure 5.1). However, $A_{nisdif}$ is supposed to count all losses associated with nuclear interactions and decay-in-flight. In principle, this could include losses whose efficiencies are not counted by $A_{trig}$ or $A_{umc}$ in the first place. For example, if there are extra splashes of energy in the detector associated with nuclear interactions or decay-in-flight that cause the photon veto to fire, we must count this loss here. It will not be counted in the $K_{\mu2}$-based acceptance measurements because muons do not undergo nuclear interactions and decay-in-flight will be rare. Hence, we take these losses into account by using the double ratio above.

The results of the umc-based nuclear interactions and decay-in-flight analysis are given in Table 5.5 for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and Table 5.6 for $K^+ \rightarrow \pi^+ X^0$. The results of the $A_{nisdif}$ measurement are given in Table 5.7 for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and Table 5.8 for $K^+ \rightarrow \pi^+ X^0$.

Spectrum plots for the nuclear interactions and decay-in-flight enabled samples surviving the complete umc analysis (all cuts listed in Figure 5.1) except for the box cut are shown in Figure 5.2.

5.2 $K_{\mu2}$-based Acceptance Measurements

$K_{\mu2}$ monitor data are used to measure the following:


Table 5.5: umc-based nuclear interactions and decay-in-flight analysis for 

\[ K^+ \rightarrow \pi^+ \nu \bar{\nu}. \]

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N_{passed}</td>
<td>A</td>
<td>N_{passed}</td>
<td>A</td>
</tr>
<tr>
<td>Selection</td>
<td>30509</td>
<td>-</td>
<td>30509</td>
<td>-</td>
</tr>
<tr>
<td>UFATE</td>
<td>28310</td>
<td>0.928</td>
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<td>0.928</td>
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<tr>
<td>USTEMED</td>
<td>28124</td>
<td>0.993</td>
<td>28124</td>
<td>0.993</td>
</tr>
<tr>
<td>USTOP_HEX</td>
<td>27753</td>
<td>0.987</td>
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<tr>
<td>DIPANG</td>
<td>26801</td>
<td>0.966</td>
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<td>0.966</td>
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<td>LAY14</td>
<td>26701</td>
<td>0.996</td>
<td>26701</td>
<td>0.996</td>
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<tr>
<td>NSERCS</td>
<td>26701</td>
<td>1.000</td>
<td>26701</td>
<td>1.000</td>
</tr>
<tr>
<td>ZFRF</td>
<td>25666</td>
<td>0.961</td>
<td>25666</td>
<td>0.961</td>
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<tr>
<td>BOX</td>
<td>10234</td>
<td>0.399</td>
<td>9667</td>
<td>0.377</td>
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</table>

Table 5.6: umc-based nuclear interactions and decay-in-flight analysis for 

\[ K^+ \rightarrow \pi^+ \chi^0. \]

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
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</thead>
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<td>0.951</td>
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<td>PFBOX</td>
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</table>

Table 5.7: \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) nuclear interaction and decay-in-flight acceptance.

<table>
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<th>TIGHT3</th>
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<tr>
<td>off</td>
<td>400000</td>
<td>21158</td>
<td>400000</td>
<td>20034</td>
</tr>
<tr>
<td>on</td>
<td>399860</td>
<td>10234</td>
<td>399860</td>
<td>9667</td>
</tr>
<tr>
<td>( A_{n dif} )</td>
<td>0.484 ± 0.006</td>
<td>0.483 ± 0.006</td>
<td>0.480 ± 0.006</td>
<td>0.478 ± 0.006</td>
</tr>
</tbody>
</table>
5.2. $K_{\mu2}$-based Acceptance Measurements

Figure 5.2: Range vs. Momentum spectra for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$, before and after the EBOX cut, showing the standard RBOX/PBOX cuts, using the nuclear interactions and decay-in-flight enabled sample. The $K^+ \rightarrow \pi^+ X^0$ spectra are shown with the PFBOX.
5.2. $K_{\mu 2}$-based Acceptance Measurements

<table>
<thead>
<tr>
<th>NIDIF</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
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<td>$N_{pass}$</td>
<td>$N_{KB}$</td>
<td>$N_{pass}$</td>
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</tr>
<tr>
<td>on</td>
<td>399955</td>
<td>33050</td>
<td>399955</td>
<td>33050</td>
</tr>
<tr>
<td>$A_{\text{midif}}$</td>
<td>$0.460 \pm 0.003$</td>
<td>$0.460 \pm 0.003$</td>
<td>$0.460 \pm 0.003$</td>
<td>$0.460 \pm 0.003$</td>
</tr>
</tbody>
</table>

Table 5.8: $K^+ \rightarrow \pi^+ X^0$ nuclear interaction and decay-in-flight acceptance.

- Losses due to inefficiencies in the event reconstruction software used to measure the kinematics of the pion track, $A_{\text{recon}}$.

- Accidental losses incurred by the photon veto cuts, the beam cuts, and the target cuts (and a few other cuts as well), $A_{K_{\mu 2}}$.

- Accidental losses incurred by the LAY14 cut, $A_{\text{acc}}^{\text{LAY14}}$. There are actually two components to the LAY14 losses; there are losses due to tracks that penetrate into the outer RSSC, and there are losses due to accidentals that fire the outer RSSC even if the pion comes to rest before exiting layer 14. The former are measured with UMC and the latter are measured here.

- Losses due to the delayed coincidence cuts, $A_{\text{DC}}$.

An outline of these measurements is sketched in Figure 5.3. The layer 21 veto is applied as an initial selection cut by inspection of the online layer 21 trigger bit (thus it does not depend on successful offline event reconstruction). The purpose of this cut is to veto events in which the muon track penetrates into the barrel photon veto.
5.2. $K_{\mu 2}$-based Acceptance Measurements

This is desirable since inclusion of these events in the measurement of the acceptance of the barrel photon veto cuts would result in an erroneously low value of $A_{K_{\mu 2}}$.

5.2.1 The Reconstruction Efficiency, $A_{\text{recon}}$

The ISKCODE cut is required as a selection for all parts of the acceptance calculation. Thus, the ISKCODE efficiency itself must be measured, and we do it here. We begin by requiring the online delayed coincidence trigger bit in order to select kaons decaying at rest. Next we select events that pass the range stack track pattern recognition software, RD\_TRK, and have a range stack track energy greater than 100 MeV. These are assumed to all be good events (they need not actually be $K_{\mu 2}$) and the event reconstruction efficiency is defined as the fraction of these events that pass all other parts of the routine SETUP\_KINE. We find:

$$A'_{\text{recon}} = \frac{N_{\text{out}}}{N_{\text{in}}} = \frac{47736}{49461} = 0.9651 \pm 0.0008.$$  

This must now be corrected for the inefficiency of RD\_TRK itself. For this measurement, events are selected with a momentum box around the $K_{\mu 2}$ peak\(^2\), and we count the fraction of events that pass RD\_TRK and have a range stack track energy greater than 100 MeV. We find:

$$A_{\text{RD\_TRK}} = \frac{N_{\text{out}}}{N_{\text{in}}} = \frac{5319}{5337} = 0.9966 \pm 0.0008.$$

\(^2\)The UTC track finding is run without first running RD\_TRK. To avoid confusion about which UTC track is the track of interest, events with more than one UTC track are cut.
5.2. \( K_{\mu2}\)-based Acceptance Measurements

\[ K_{\mu2} (1) \text{ triggers} \]
\[ \text{Prescaled by } 10 \]

Layer 21 veto

Online DELC

ISKCODE
RNGMOM
\( K_{\mu2} \) BOX

RD_TRK selection

Total momentum selection

Online DELC
DELC
TIMCON
DKNFLT

Photon veto cuts
Beam cuts
Target cuts

Reconstruction efficiency

RD_TRK efficiency

Photon veto cuts
Beam cuts
Target cuts

\[ N_{in} \]

\[ N_{out} \]

\[ A_{\text{recon}} = \frac{N_{out}}{N_{in}} \]
\[ A_{\text{RD_TRK}} = \frac{N_{out}}{N_{in}} \]
\[ A_{K_{\mu2}} = \frac{N_{out}}{N_{in}} \]
\[ A_{DC} = \frac{N_{out}}{N_{in}} \]

\[ A_{\text{recon}} = A_{\text{recon}} \times A_{\text{RD_TRK}} \]

Figure 5.3: The \( K_{\mu2}\)-based acceptance measurements.
5.2. $K_{\mu 2}$-based Acceptance Measurements

The total reconstruction efficiency is then:

$$A_{\text{recon}} = A'_{\text{recon}} \times A_{\text{RD-TRK}} = 0.962 \pm 0.001.$$ 

5.2.2 Photon veto, Beam, Target, and Accidental losses, $A_{K_{\mu 2}}$

To select cleanly reconstructed $K_{\mu 2}$ decays from kaons at rest, an inverted RNGMOM cut and a box in range, energy, and momentum around the $K_{\mu 2}$ peak are applied. To further reject kaon decay-in-flight events, the delayed coincidence cuts are applied. Note that DKNFLT, while perhaps appearing to be a beam cut, has been included with the delayed coincidence cuts in $A_{DC}$. This is because DKNFLT essentially acts as a type of delayed coincidence cut, and is highly correlated with the other delayed coincidence cuts. Now we apply the photon veto, beam, and target cuts and measure their acceptance losses. The results of this study are summarized in Table 5.9. The main offline $\gamma$ veto cut, GAMVETP3, has been broken up by subsystem. The Level 0 $\gamma$ veto is applied offline by looking up the trigger bit. The Level 0 $\mu$ veto (cut events that penetrate to layer 19 in the range stack) is not applied in this manner, however, since it would cut every event in the $K_{\mu 2}(1)$ sample. This acceptance loss is estimated by a separate piece of code that attempts to simulate the Level 0 $\mu$ veto accidental losses by looking at TD data from range stack counters on the other side of the detector. We also have to account for losses due to any cuts at Pass1 or Pass2 that are tighter in some way that the final cuts. Some of these are accounted for here
and some will be accounted for in the TD acceptance measurements.

5.2.3 Layer 14 Cut Accidental losses, $A_{L14}^{acc}$

If the pion from a $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event stops in range stack layer 14 and an accidental happens to fire the RSSC adjacent to the stopping counter, the event will fail the LAY14 cut. To estimate this acceptance loss, the LAY14 code is instructed to look for RSSC hits on the opposite side of the detector for all events passing the $A_{K\mu2}$ measurement. The fraction of events failing is then weighted by the fraction of events stopping in layer 14 of those UMC events passing the $A_{umc}$ analysis with nuclear interactions and decay-in-flight enabled. The results are given in Table 5.10 for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$.

5.2.4 Delayed Coincidence losses, $A_{DC}$

To avoid double-counting our losses, all cuts whose acceptances are measured in $A_{K\mu2}$ are applied as a selection. After this selection and the inverted RNGMOM cut and $K_{\mu2}$Box cut, the remaining events are assumed to be kaon decays at rest. The results are given in Table 5.11.

5.3 $\pi$–scat Acceptance Measurements

$\pi$–scat monitor data are used to measure the following:
5.3. $\pi-$scat Acceptance Measurements

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$N_{\text{passed}}$</td>
<td>$A$</td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>Selection</td>
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<td>-</td>
<td>33398</td>
<td>-</td>
</tr>
<tr>
<td>PICER</td>
<td>33480</td>
<td>0.994</td>
<td>33066</td>
<td>0.990</td>
</tr>
<tr>
<td>PISCAT_BW</td>
<td>31350</td>
<td>0.936</td>
<td>30626</td>
<td>0.926</td>
</tr>
<tr>
<td>PISCAT_B4</td>
<td>30900</td>
<td>0.985</td>
<td>30197</td>
<td>0.986</td>
</tr>
<tr>
<td>B4DEDX</td>
<td>30141</td>
<td>0.976</td>
<td>29457</td>
<td>0.975</td>
</tr>
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<td>PBGLASS</td>
<td>29006</td>
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<td>0.963</td>
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<td>BHTRS</td>
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<td>0.996</td>
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<tr>
<td>GAMVETP3 (RD)</td>
<td>26674</td>
<td>0.924</td>
<td>26125</td>
<td>0.925</td>
</tr>
<tr>
<td>GAMVETP3 (E2)</td>
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<td>25193</td>
<td>0.964</td>
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<td>25497</td>
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<td>24986</td>
<td>0.992</td>
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<td>0.994</td>
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<td>0.997</td>
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<td>GAMVETP3 (CM)</td>
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<td>1.000</td>
<td>24592</td>
<td>1.000</td>
</tr>
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<td>GAMVETP3 (BV)</td>
<td>24352</td>
<td>0.971</td>
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<td>0.971</td>
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<td>GAMVETSE</td>
<td>23475</td>
<td>0.964</td>
<td>23016</td>
<td>0.964</td>
</tr>
<tr>
<td>Level 0 $\gamma$ veto</td>
<td>23049</td>
<td>0.982</td>
<td>22610</td>
<td>0.982</td>
</tr>
<tr>
<td>Level 0 $\mu$ veto</td>
<td>22965</td>
<td>0.996</td>
<td>22527</td>
<td>0.996</td>
</tr>
<tr>
<td>HEXCUT</td>
<td>22269</td>
<td>0.970</td>
<td>21851</td>
<td>0.970</td>
</tr>
<tr>
<td>ZDCOW</td>
<td>21984</td>
<td>0.987</td>
<td>21570</td>
<td>0.987</td>
</tr>
<tr>
<td>UTCQUAL</td>
<td>20983</td>
<td>0.954</td>
<td>20587</td>
<td>0.954</td>
</tr>
<tr>
<td>EIC</td>
<td>20368</td>
<td>0.971</td>
<td>19984</td>
<td>0.971</td>
</tr>
<tr>
<td>ZTGT</td>
<td>20293</td>
<td>0.996</td>
<td>19912</td>
<td>0.996</td>
</tr>
<tr>
<td>TGDCVT</td>
<td>19876</td>
<td>0.979</td>
<td>19039</td>
<td>0.956</td>
</tr>
<tr>
<td>RTDIFF</td>
<td>19627</td>
<td>0.987</td>
<td>18979</td>
<td>0.987</td>
</tr>
<tr>
<td>TGB4</td>
<td>19287</td>
<td>0.983</td>
<td>18484</td>
<td>0.983</td>
</tr>
<tr>
<td>TARGF</td>
<td>19012</td>
<td>0.986</td>
<td>18222</td>
<td>0.986</td>
</tr>
<tr>
<td>RPMIN</td>
<td>18994</td>
<td>0.999</td>
<td>18204</td>
<td>0.999</td>
</tr>
<tr>
<td>NTRIK</td>
<td>18929</td>
<td>0.997</td>
<td>18141</td>
<td>0.997</td>
</tr>
<tr>
<td>KIC</td>
<td>18504</td>
<td>0.978</td>
<td>17731</td>
<td>0.977</td>
</tr>
<tr>
<td>PIGAP</td>
<td>18308</td>
<td>0.989</td>
<td>17548</td>
<td>0.990</td>
</tr>
<tr>
<td>EKZ</td>
<td>18138</td>
<td>0.991</td>
<td>17157</td>
<td>0.978</td>
</tr>
<tr>
<td>BSCAT</td>
<td>18039</td>
<td>0.995</td>
<td>17067</td>
<td>0.995</td>
</tr>
<tr>
<td>Pass1 STOP_HEX</td>
<td>17856</td>
<td>0.990</td>
<td>16901</td>
<td>0.990</td>
</tr>
<tr>
<td>Pass1 $\gamma$ veto</td>
<td>17791</td>
<td>0.996</td>
<td>16840</td>
<td>0.996</td>
</tr>
<tr>
<td>Pass1 UTC/RS recon</td>
<td>17714</td>
<td>0.996</td>
<td>16771</td>
<td>0.996</td>
</tr>
<tr>
<td>Pass2 PVCUT</td>
<td>17652</td>
<td>0.996</td>
<td>16716</td>
<td>0.997</td>
</tr>
<tr>
<td>Pass2 PSCUT</td>
<td>17644</td>
<td>1.000</td>
<td>16708</td>
<td>1.000</td>
</tr>
</tbody>
</table>

\[ A_{K_{\pi2}} = 0.524 \pm 0.003 \]

\[ 0.500 \pm 0.003 \]

\[ 0.460 \pm 0.003 \]

\[ 0.414 \pm 0.003 \]

Table 5.9: The $K_{\pi2}$ acceptance measurement.
### Table 5.10: Accidental losses in the LAY14 cut for $K^+ \to \pi^+ \nu \bar{\nu}$ and $K^+ \to \pi^+ X^0$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{exam}}$</td>
<td>18039</td>
<td>17067</td>
<td>15423</td>
<td>12998</td>
</tr>
<tr>
<td>$N_{\text{passed}}$</td>
<td>17914</td>
<td>16947</td>
<td>15317</td>
<td>12913</td>
</tr>
<tr>
<td>$L14$ fraction ($\pi^+ \nu \bar{\nu}$)</td>
<td>0.2399</td>
<td>0.2420</td>
<td>0.2443</td>
<td>0.2448</td>
</tr>
<tr>
<td>$A_{\text{cc}}^{\text{L14}}$ ($\pi^+ \nu \bar{\nu}$)</td>
<td>0.9983 ± 0.0002</td>
<td>0.9983 ± 0.0002</td>
<td>0.9983 ± 0.0002</td>
<td>0.9984 ± 0.0002</td>
</tr>
<tr>
<td>$L14$ fraction ($\pi^+ X^0$)</td>
<td>0.2074</td>
<td>0.2074</td>
<td>0.2074</td>
<td>0.2074</td>
</tr>
<tr>
<td>$A_{\text{cc}}^{\text{L14}}$ ($\pi^+ X^0$)</td>
<td>0.9986 ± 0.0002</td>
<td>0.9985 ± 0.0002</td>
<td>0.9986 ± 0.0002</td>
<td>0.9986 ± 0.0002</td>
</tr>
</tbody>
</table>

### Table 5.11: The $K_{\mu2}$-based delayed coincidence acceptance measurement.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>Selection</td>
<td>22137</td>
<td>-</td>
<td>21054</td>
<td>-</td>
</tr>
<tr>
<td>Level 0 DELC</td>
<td>19573</td>
<td>0.884</td>
<td>18650</td>
<td>0.886</td>
</tr>
<tr>
<td>DELC</td>
<td>18265</td>
<td>0.933</td>
<td>17271</td>
<td>0.926</td>
</tr>
<tr>
<td>TIMCON</td>
<td>18047</td>
<td>0.919</td>
<td>17173</td>
<td>0.900</td>
</tr>
<tr>
<td>DKNFLT</td>
<td>18039</td>
<td>0.998</td>
<td>17097</td>
<td>0.998</td>
</tr>
<tr>
<td>$A_{\text{DC}}$</td>
<td>0.815 ± 0.003</td>
<td>0.811 ± 0.003</td>
<td>0.801 ± 0.003</td>
<td>0.765 ± 0.003</td>
</tr>
</tbody>
</table>
5.3. \( \pi - \text{scat} \) Acceptance Measurements

- Losses due to the TD cuts, \( A_{\text{TD}} \).

- Losses due to cuts on the quality of the range stack track, \( A_{\pi \text{scat}} \).

An outline of these measurements is sketched in Figure 5.4.

5.3.1 The Efficiency of the TD Cuts, \( A_{\text{TD}} \)

First we must calculate the efficiency of the \( \pi^+ \rightarrow \mu^+ \) double-pulse fit cut, FITPI. Since many of the pions entering the range stack will not come to rest as a pion due to nuclear interactions and decays-in-flight, a simple counting method will underestimate the FITPI acceptance. In fact, such an assessment of the FITPI acceptance would effectively be double-counting our losses, since the losses due to nuclear interaction and decay-in-flight have already been accounted for in the UMC-based acceptance measurements. Instead we measure the FITPI acceptance using the so-called "area method."

Since most of the FITPI losses are due to early \( \pi^+ \rightarrow \mu^+ \) decays in which the double pulse fit is unable to resolve the muon pulse, we begin by fitting an exponential\(^3\) to a plot of TMUAV, the pion lifetime, for those events passing FITPI (see Figure 5.5). The total area under the fitted exponential is assumed to be the number of good stopped pion events before application of FITPI, \( N_A \). We find:

\[ N_A = 17881 \pm 170, \]

---

\(^3\)The slope in this fit is held constant at the accepted value of the pion lifetime, \( \tau_{\pi^+} = 26.033 \text{ ns} \). Only the magnitude of the exponential is allowed to vary.
5.3. $\pi$-scat Acceptance Measurements

$$\Delta N_A = 100 \times 3.144 \times N_H$$

Figure 5.4: The $\pi$-scat acceptance measurements. GAMVETP3 (RS) is just the range stack part of the photon veto cuts, and RNGMOM3 is a tighter version of RNGMOM to ensure good pion selection.
5.3. $\pi$-scat Acceptance Measurements

where the error comes from varying the range of the fit over a variety of different values. This number must be corrected, however, for those FITPI failures that occur uniformly across all pion lifetimes; typically such failures are due to the presence of a third, accidental pulse that confuses the double-pulse fit. This is accomplished by conducting a visual examination of one out of every 100 events failing FITPI.

The number of such events that have a clear $\pi^+ \to \mu^+$ decay signature between one and three pion lifetimes ($26.03 - 78.10$ ns) after the pion pulse is called $N_H$. The correction to the area method is then $\Delta N_A = 100 \times 3.144 \times N_H$, where the factor of 3.144 accounts for the limited portion of the pion lifetime spectrum included in the search. 4 events are found in this sample that have clear $\pi^+ \to \mu^+$ signatures; one of them is shown in Figure 5.6. We do not search for $\pi^+ \to \mu^+$ decays at times less than one lifetime, because these are difficult to see by eye, and we do not search for $\pi^+ \to \mu^+$ decays at times greater than three lifetimes, because this region will start to be dominated by accidentals that mimic $\pi^+ \to \mu^+$. For consistency, the area method exponential fit is limited to the region $26 \text{ ns} < T_MUAV < 78 \text{ ns}$. The area method must also be corrected for the the fraction of muons that escape the stopping counter without leaving enough energy to pass FITPI. This has been estimated to be $\epsilon_{\text{escape}} = 0.982$ in [27]. Putting all these factors in, we find:

$$A_{tdfit} = \frac{N_P}{N_A + \Delta N_A} \times \epsilon_{\text{escape}} = \frac{14057}{17881 + 100 \times 3.144 \times 4} \times 0.982 = 0.721 \pm 0.025.$$
5.3. $\pi-$scat Acceptance Measurements

Figure 5.5: Area method FITPI acceptance plot.

Figure 5.6: Example of TD data showing a clear $\pi^+ \rightarrow \mu^+$ decay, yet FITPI has failed due to the large accidental pulse.
5.3. \( \pi^{--}\text{scat} \) Acceptance Measurements

The events passing FitPI are assumed to all be good stopped pion events, and the acceptance of the remaining TD cuts, \( A_C \), can now be measured using a counting method. We also measure the losses due to the online TD cut, ONLTD, and the losses of those Pass1 and Pass2 TD cuts that were tighter than the final TD cuts here. The results are given in Table 5.12.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( N_{\text{passed}} )</td>
<td>( A )</td>
<td>( N_{\text{passed}} )</td>
<td>( A )</td>
</tr>
<tr>
<td>Selection</td>
<td>14057</td>
<td>-</td>
<td>14057</td>
<td>-</td>
</tr>
<tr>
<td>ELVETO</td>
<td>12687</td>
<td>0.903</td>
<td>12687</td>
<td>0.903</td>
</tr>
<tr>
<td>ELEC_V5</td>
<td>9899</td>
<td>0.780</td>
<td>9899</td>
<td>0.780</td>
</tr>
<tr>
<td>TDFOOL</td>
<td>9835</td>
<td>0.999</td>
<td>9885</td>
<td>0.999</td>
</tr>
<tr>
<td>RSHEX</td>
<td>9476</td>
<td>0.959</td>
<td>9476</td>
<td>0.959</td>
</tr>
<tr>
<td>TDDFA</td>
<td>8965</td>
<td>0.946</td>
<td>8965</td>
<td>0.946</td>
</tr>
<tr>
<td>TDLIK</td>
<td>8023</td>
<td>0.895</td>
<td>7856</td>
<td>0.876</td>
</tr>
<tr>
<td>TMUBV</td>
<td>7846</td>
<td>0.978</td>
<td>7684</td>
<td>0.978</td>
</tr>
<tr>
<td>TMUADC</td>
<td>7826</td>
<td>0.974</td>
<td>7580</td>
<td>0.986</td>
</tr>
<tr>
<td>STOP_hex</td>
<td>7622</td>
<td>0.974</td>
<td>7383</td>
<td>0.974</td>
</tr>
<tr>
<td>ONLTD</td>
<td>6003</td>
<td>0.788</td>
<td>5830</td>
<td>0.790</td>
</tr>
<tr>
<td>Pass1 FitPI</td>
<td>5849</td>
<td>0.974</td>
<td>5677</td>
<td>0.974</td>
</tr>
<tr>
<td>Pass2 TDCUT</td>
<td>5790</td>
<td>0.990</td>
<td>5627</td>
<td>0.991</td>
</tr>
</tbody>
</table>

\[ A_C \approx 0.412 \pm 0.004 \quad 0.400 \pm 0.004 \quad 0.386 \pm 0.004 \quad 0.368 \pm 0.004 \]

\[ A_{\text{tdf}} \approx 0.721 \pm 0.025 \quad 0.721 \pm 0.025 \quad 0.721 \pm 0.025 \quad 0.721 \pm 0.025 \]

\[ A_{\text{TD}} \approx 0.297 \pm 0.011 \quad 0.289 \pm 0.010 \quad 0.278 \pm 0.010 \quad 0.265 \pm 0.010 \]

Table 5.12: Acceptance of the remaining TD cuts.

5.3.2 Tracking Losses in the Range Stack, \( A_{\pi^{--}\text{scat}} \)

We select good \( \pi^{--}\text{scat} \) events by applying all TD cuts to the \( \pi^{--}\text{scat} \) monitors. To avoid double counting losses, the photon veto in the range stack and the kinematic
5.4. $K_{\pi 2}$ Acceptance Measurements

box cut are also applied. We may then proceed to measure $A_{\pi \text{scat}}$ (see Table 5.13). In fact, the tracking losses in the range stack will vary with stopping layer, so we must weight $A_{\pi \text{scat}}$ by the $K^+ \to \pi^+ \nu \bar{\nu}$ and $K^+ \to \pi^+ X^0$ stopping layer distributions using the UMC samples with nuclear interactions and decay-in-flight enabled passing all cuts in the $A_{\text{umc}}$ measurement. Also, $\pi -$scats are quite messy in the target, and this introduces some uncertainty into RTUT, ETUT, and PTUT, which in turn introduces an uncertainty into the box cut. To account for this, we have varied RBOX by $\pm 1$ cm, EBOX by $\pm 3$ MeV, and PBOX by $\pm 3$ MeV/c. The error on $A_{\pi \text{scat}}$ is inferred from these “bigger box” and “smaller box” acceptance measurements. The results are given in Table 5.14 for $K^+ \to \pi^+ \nu \bar{\nu}$ and Table 5.15 for $K^+ \to \pi^+ X^0$.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>Selection</td>
<td>2047</td>
<td>-</td>
<td>1805</td>
<td>-</td>
</tr>
<tr>
<td>RNGMOM</td>
<td>2004</td>
<td>0.979</td>
<td>1766</td>
<td>0.978</td>
</tr>
<tr>
<td>RSDEX</td>
<td>1954</td>
<td>0.975</td>
<td>1725</td>
<td>0.977</td>
</tr>
<tr>
<td>CHIRF</td>
<td>1835</td>
<td>0.939</td>
<td>1616</td>
<td>0.937</td>
</tr>
<tr>
<td>$A_{\pi \text{scat}}$</td>
<td>$0.896 \pm 0.007$</td>
<td>$0.895 \pm 0.007$</td>
<td>$0.903 \pm 0.007$</td>
<td>$0.871 \pm 0.000$</td>
</tr>
</tbody>
</table>

Table 5.13: $A_{\pi \text{scat}}$ assuming stopping layer independence.

5.4 $K_{\pi 2}$ Acceptance Measurements

$K_{\pi 2}$ decays cannot be used to measure the acceptance of those target cuts for which the acceptance may be a function of particle type, TGDEX, EPIMAX, and TGLIKE.
5.4. $K_{\pi 2}$ Acceptance Measurements

<table>
<thead>
<tr>
<th>Description</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal box</td>
<td>0.900</td>
<td>0.899</td>
<td>0.907</td>
<td>0.875</td>
</tr>
<tr>
<td>Bigger box</td>
<td>0.892</td>
<td>0.892</td>
<td>0.895</td>
<td>0.862</td>
</tr>
<tr>
<td>Smaller box</td>
<td>0.910</td>
<td>0.908</td>
<td>0.913</td>
<td>0.872</td>
</tr>
<tr>
<td>$A_{\pi \text{scat}}$</td>
<td>0.900 ± 0.01</td>
<td>0.899 ± 0.01</td>
<td>0.907 ± 0.01</td>
<td>0.875 ± 0.01</td>
</tr>
</tbody>
</table>

Table 5.14: The acceptance of the RS analysis weighted by the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ stopping layer distribution. Error is inferred by varying the box cut used for selection.

<table>
<thead>
<tr>
<th>Description</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal box</td>
<td>0.902</td>
<td>0.903</td>
<td>0.901</td>
<td>0.873</td>
</tr>
<tr>
<td>Bigger box</td>
<td>0.895</td>
<td>0.895</td>
<td>0.894</td>
<td>0.864</td>
</tr>
<tr>
<td>Smaller box</td>
<td>0.932</td>
<td>0.932</td>
<td>0.929</td>
<td>0.926</td>
</tr>
<tr>
<td>$A_{\pi \text{scat}}$</td>
<td>0.902 ± 0.01</td>
<td>0.903 ± 0.01</td>
<td>0.901 ± 0.01</td>
<td>0.873 ± 0.01</td>
</tr>
</tbody>
</table>

Table 5.15: The acceptance of the RS analysis weighted by the $K^+ \rightarrow \pi^+ X^0$ stopping layer distribution. Error is inferred by varying the box cut used for selection.

Instead we must use $K_{\pi 2}$ decays. $K_{\pi 2}(1)$ monitors are used with the following selection cuts:

- Online and offline delayed coincidence cuts.

- All cuts measured in $A_{K_{\pi 2}}$ except the online and offline photon veto cuts in the barrel veto, endcap vetos, and range stack. This is to avoid double-counting our losses.

- All TD cuts. Again, this is to avoid double-counting.

- A $K_{\pi 2}$ box cut in range, energy, and momentum.
5.5. The Total Acceptance for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$

- DIPANG

The results are given in Table 5.16.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
<td>$N_{\text{passed}}$</td>
<td>$A$</td>
</tr>
<tr>
<td>Selection</td>
<td>5575</td>
<td>-</td>
<td>5104</td>
<td>-</td>
</tr>
<tr>
<td>TGDEDX</td>
<td>5369</td>
<td>0.963</td>
<td>4914</td>
<td>0.963</td>
</tr>
<tr>
<td>EPIMAX</td>
<td>-5368</td>
<td>1.000</td>
<td>4913</td>
<td>1.000</td>
</tr>
<tr>
<td>TGLIKE</td>
<td>5283</td>
<td>0.984</td>
<td>4839</td>
<td>0.985</td>
</tr>
<tr>
<td>$A_{K_{e2}}$</td>
<td>0.948 ± 0.003</td>
<td>0.948 ± 0.003</td>
<td>0.949 ± 0.003</td>
<td>0.947 ± 0.004</td>
</tr>
</tbody>
</table>

Table 5.16: $A_{K_{e2}}$ acceptance measurements.

5.5 The Total Acceptance for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$

The total acceptances for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ X^0$ are given in Tables 5.17 and 5.18. We compute total acceptance using the formula:

$$A_{\text{tot}} = A_{\text{trig}} \times A_{\text{umc}} \times A_{\text{midif}} \times A_{\text{recon}} \times A_{K_{e2}} \times A_{L14} \times A_{\text{DC}} \times A_{\text{TD}} \times A_{\pi\text{scat}} \times A_{\text{K_ interactions}}.$$ 

5.6 1995 Kaon Flux, $N_{KBlive}$

The number of KBeams (see the Trigger section of Chapter 2) entering the detector while it is "live" (not reading out a previous event) is recorded for each spill along with the data. These "end-of-spill records" are then copied and carried along with
### Table 5.17: Total acceptance for $K^+ \to \pi^+ \nu \bar{\nu}$.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{trig}}$</td>
<td>0.181 ± 0.001</td>
<td>0.181 ± 0.001</td>
<td>0.181 ± 0.001</td>
<td>0.181 ± 0.001</td>
</tr>
<tr>
<td>$A_{\text{umc}}$</td>
<td>0.418 ± 0.002</td>
<td>0.396 ± 0.002</td>
<td>0.377 ± 0.002</td>
<td>0.353 ± 0.002</td>
</tr>
<tr>
<td>$A_{\text{nidif}}$</td>
<td>0.484 ± 0.006</td>
<td>0.483 ± 0.006</td>
<td>0.480 ± 0.006</td>
<td>0.478 ± 0.006</td>
</tr>
<tr>
<td>$A_{\text{recon}}$</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
</tr>
<tr>
<td>$A_{K_{\mu2}}$</td>
<td>0.524 ± 0.003</td>
<td>0.500 ± 0.003</td>
<td>0.460 ± 0.003</td>
<td>0.414 ± 0.003</td>
</tr>
<tr>
<td>$A_{L14}^{\text{acc}}$</td>
<td>0.9983 ± 0.0002</td>
<td>0.9983 ± 0.0002</td>
<td>0.9983 ± 0.0002</td>
<td>0.9984 ± 0.0002</td>
</tr>
<tr>
<td>$A_{DC}$</td>
<td>0.815 ± 0.003</td>
<td>0.811 ± 0.003</td>
<td>0.801 ± 0.003</td>
<td>0.765 ± 0.003</td>
</tr>
<tr>
<td>$A_{TD}$</td>
<td>0.297 ± 0.011</td>
<td>0.289 ± 0.010</td>
<td>0.278 ± 0.010</td>
<td>0.265 ± 0.010</td>
</tr>
<tr>
<td>$A_{\pi\text{scat}}$</td>
<td>0.900 ± 0.01</td>
<td>0.899 ± 0.01</td>
<td>0.907 ± 0.01</td>
<td>0.875 ± 0.01</td>
</tr>
<tr>
<td>$A_{K_{e2}}$</td>
<td>0.948 ± 0.003</td>
<td>0.948 ± 0.003</td>
<td>0.949 ± 0.003</td>
<td>0.947 ± 0.004</td>
</tr>
<tr>
<td>$A_{\text{tot}}$</td>
<td>0.00381 ± 0.00015</td>
<td>0.00332 ± 0.00014</td>
<td>0.00278 ± 0.00011</td>
<td>0.00204 ± 0.00008</td>
</tr>
</tbody>
</table>

### Table 5.18: Total acceptance for $K^+ \to \pi^+ X^0$.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{trig}}$</td>
<td>0.407 ± 0.001</td>
<td>0.407 ± 0.001</td>
<td>0.407 ± 0.001</td>
<td>0.407 ± 0.001</td>
</tr>
<tr>
<td>$A_{\text{umc}}$</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
<td>0.632 ± 0.001</td>
</tr>
<tr>
<td>$A_{\text{nidif}}$</td>
<td>0.460 ± 0.003</td>
<td>0.460 ± 0.003</td>
<td>0.460 ± 0.003</td>
<td>0.460 ± 0.003</td>
</tr>
<tr>
<td>$A_{\text{recon}}$</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
<td>0.962 ± 0.001</td>
</tr>
<tr>
<td>$A_{K_{\mu2}}$</td>
<td>0.524 ± 0.003</td>
<td>0.500 ± 0.003</td>
<td>0.460 ± 0.003</td>
<td>0.414 ± 0.003</td>
</tr>
<tr>
<td>$A_{L14}^{\text{acc}}$</td>
<td>0.9986 ± 0.0002</td>
<td>0.9985 ± 0.0002</td>
<td>0.9986 ± 0.0002</td>
<td>0.9986 ± 0.0002</td>
</tr>
<tr>
<td>$A_{DC}$</td>
<td>0.815 ± 0.003</td>
<td>0.811 ± 0.003</td>
<td>0.801 ± 0.003</td>
<td>0.765 ± 0.003</td>
</tr>
<tr>
<td>$A_{TD}$</td>
<td>0.297 ± 0.011</td>
<td>0.289 ± 0.010</td>
<td>0.278 ± 0.010</td>
<td>0.265 ± 0.010</td>
</tr>
<tr>
<td>$A_{\pi\text{scat}}$</td>
<td>0.902 ± 0.01</td>
<td>0.903 ± 0.01</td>
<td>0.901 ± 0.01</td>
<td>0.873 ± 0.01</td>
</tr>
<tr>
<td>$A_{K_{e2}}$</td>
<td>0.948 ± 0.003</td>
<td>0.948 ± 0.003</td>
<td>0.949 ± 0.003</td>
<td>0.947 ± 0.004</td>
</tr>
<tr>
<td>$A_{\text{tot}}$</td>
<td>0.01230 ± 0.00049</td>
<td>0.01136 ± 0.00046</td>
<td>0.00996 ± 0.00040</td>
<td>0.00787 ± 0.00032</td>
</tr>
</tbody>
</table>
5.6. 1995 Kaon Flux, $N_{KB\text{live}}$

the data at the Pass1 and Pass2 analysis stages. The KBlive are then summed for
the entire 1995 data set by analyzing the Pass2 SKIM6 output tapes. The result is:

$$N_{KB\text{live}} = 1.556 \times 10^{12}. $$

This number is not quite right, however. Due to various mistakes in tape handling
during Pass1 and Pass2, and due to subtleties in how some of the data was originally
written to tape, much of the data is duplicated on the Pass2 output tapes. i.e.
many of the events, and many of the end-of-spill records, appear more than once
on these tapes. We can look for duplicated events directly by inspection of the run
and event numbers for each event. There is no way, unfortunately, to unambiguously
detect duplicated end-of-spill records. There is evidence (see [33]), however, that
the fraction of duplicated events would be the same as the fraction of duplicated
end-of-spill records, and we can use this fact to correct $N_{KB\text{live}}$.

The SKIM6 tapes contain 1016475 events. If we removed the duplicated events
we are left with 1001473. Thus the level of event duplication is $\frac{1016475}{1001473} = 1.015$ and if
we scale KBlive by this we have:

$$N_{KB\text{live}} = \frac{1.556 \times 10^{12}}{1.015} = 1.53 \times 10^{12}. $$
5.7 The $K_{\mu 2}$ Branching Ratio and $f_S$

With $A_{tot}$ and $N_{K^{0}live}$ taken care of, we now must measure $f_S$ to complete our determination of the sensitivity. $f_S$ was originally meant to be the fraction of the $N_{K^{0}live}$ kaons that came to rest in the target before decaying, the "stopping fraction." Early analyses of E787 data attempted to compute $f_S$ directly. The $K_{\mu 2}$ branching ratio was then measured as a check of $f_S$ and many parts of the acceptance calculation. If there was some discrepancy between the measured $K_{\mu 2}$ branching ratio and the accepted value in the Particle Data Book [3], $f_S$ was adjusted to make the branching ratio come out right. In reality this amounts to normalizing the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio to the $K_{\mu 2}$ branching ratio. This is what we will do here but we will not make any attempt to compute $f_S$ directly. Thus $f_S$ is not really the kaon stopping fraction, but rather a normalization factor that accounts for both the stopping fraction and many inaccuracies in the acceptance calculation. For historical reasons, we still call it $f_S$.

We measure the $K_{\mu 2}$ branching ratio using the full set of $K_{\mu 2}(1)$ monitor data (prescaled by 10). This is the same data set used to measure the $K_{\mu 2}$-based acceptance. It is subjected to the following cuts: Online DELC, ISKCODE, DIPANG, all beam cuts, all $\gamma$-veto cuts except for the barrel veto part, and the offline delayed coincidence cuts. We must avoid the barrel photon veto since many of the muon tracks will penetrate into the barrel. The results are given in Table 5.19. A spectrum plot
after all cuts shows that the sample is free of backgrounds, thus a box cut around the 
$K_{l2}$ peak is not necessary (see Figure 5.7).

<table>
<thead>
<tr>
<th>Cut</th>
<th>$N_{\text{passed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescale</td>
<td>178070</td>
</tr>
<tr>
<td>Level 0 DELC</td>
<td>132518</td>
</tr>
<tr>
<td>ISKCODE</td>
<td>126274</td>
</tr>
<tr>
<td>DIPANG</td>
<td>122498</td>
</tr>
<tr>
<td>PIERC</td>
<td>121470</td>
</tr>
<tr>
<td>PISCAT_BW</td>
<td>111425</td>
</tr>
<tr>
<td>PISCAT_B4</td>
<td>109243</td>
</tr>
<tr>
<td>B4DEDX</td>
<td>106479</td>
</tr>
<tr>
<td>PBGLASS</td>
<td>102042</td>
</tr>
<tr>
<td>BHTRSS</td>
<td>101470</td>
</tr>
<tr>
<td>GAMVETP3 (No BV)</td>
<td>80511</td>
</tr>
<tr>
<td>DELC</td>
<td>73926</td>
</tr>
<tr>
<td>TIMCON</td>
<td>72632</td>
</tr>
<tr>
<td>DKNFLT</td>
<td>72460</td>
</tr>
</tbody>
</table>

Table 5.19: The $K_{l2}$ branching ratio analysis.

As with the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ analysis, we must measure certain acceptance losses with 
Monte Carlo. $K_{l2}$ events generated by UMC are required to measure the trigger 
acceptance losses and losses due to reconstruction and the DIPANG cut. As with 
the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ analysis, those UMC events passing the trigger are written to disk 
and analyzed with KOFIA. The results of the UMC $K_{l2}$ analysis efficiency study are 
presented in Table 5.20.

Lastly we need the effective number of KBlives for the $K_{l2}(1)$ data set. This will 
be the total number of KBlives recorded in the end-of-spill records divided by the 
combined online and offline prescale factor. This is complicated by the fact that the
5.7. The $K_{\mu 2}$ Branching Ratio and $f_S$

Figure 5.7: $K_{\mu 2}$ spectrum after all cuts in the $K_{\mu 2}$ branching ratio analysis.

<table>
<thead>
<tr>
<th>Cut</th>
<th>$N_{\text{passed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$KB$</td>
<td>299997</td>
</tr>
<tr>
<td>Trigger</td>
<td>112219</td>
</tr>
<tr>
<td>ISKCODE</td>
<td>112081</td>
</tr>
<tr>
<td>DIPANG</td>
<td>110037</td>
</tr>
<tr>
<td>$A^{\text{unc}}_{\text{recon}}$</td>
<td>$0.9988 \pm 0.0001$</td>
</tr>
<tr>
<td>$A^{\text{unc}}<em>{K</em>{\mu 2}}$</td>
<td>$0.3668 \pm 0.0009$</td>
</tr>
</tbody>
</table>

Table 5.20: The uncorrected $K_{\mu 2}$ analysis efficiency. $A^{\text{unc}}_{K_{\mu 2}}$ is the total acceptance including all factors in this table. $A^{\text{unc}}_{\text{recon}}$ is the efficiency of just the ISKCODE requirement, and will be used in the $f_S$ calculation.
online prescale factor changed for both the $K_{\mu 2}(1)$ and $K_{e 2}(1)$ triggers just before run number 21688. We compute $N_{KBlive}^{equiv}$ for the two sections of the run separately and sum the results. We find:

$$N_{KBlive}^{equiv} = \frac{KBlive_{before \ 21688}}{prescale_{before \ 21688}} + \frac{KBlive_{after \ 21688}}{prescale_{after \ 21688}} = \frac{0.02104 \times 10^{12}}{819200} + \frac{1.52887 \times 10^{12}}{1998800} = 790554.$$ 

The $K_{\mu 2}$ branching ratio is given by

$$BR_{K_{\mu 2}} = \frac{\frac{\# K_{\mu 2}}{fS N_{KTlive}^{equiv}}}{A_{K_{\mu 2}}^{unc}} \frac{1}{A_{K_{\mu 2}}^{data \ recon}} \frac{1}{A_{K_{\mu 2}}^{DC}} \frac{1}{A_{rest}}.$$ 

Because we would like to use the event reconstruction efficiency measured with data rather than that measured with Monte Carlo, we have divided $A_{K_{\mu 2}}^{unc}$ in the equation above by $A_{K_{\mu 2}}^{data \ recon}$ and multiplied by $A_{K_{\mu 2}}^{data \ recon}$. $A_{DC}$ is the delayed coincidence acceptance already measured, and $A_{rest}$ is the acceptance of the rest of the cuts applied to the $K_{\mu 2}(1)$ sample, the beam and photon veto cuts. $A_{rest}$ can be extracted from Table 5.9:

$$A_{rest} = \frac{25089}{33686} = 0.745 \pm 0.002$$

The above equation for $BR_{K_{\mu 2}}$ can be solved for $fS$

$$fS = \frac{\frac{\# K_{\mu 2}}{BR_{K_{\mu 2}} N_{KTlive}^{equiv}}}{A_{K_{\mu 2}}^{unc}} \frac{1}{A_{K_{\mu 2}}^{data \ recon}} \frac{1}{A_{DC}} \frac{1}{A_{rest}}.$$ 

The result of the $fS$ measurement is given in Table 5.21. Notice that measurement of $A_{recon}$, $A_{DC}$, and $A_{rest}$ are not actually needed to compute the sensitivity for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, since they will all cancel in the product $fS \times A_{tot}$. 

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Table 5.21: The $f_S$ measurement.

5.8 The $K_{\pi 2}$ Branching Ratio

We would now like to test our acceptance measurements by measuring the well known $K_{\pi 2}$ branching ratio. This is a good test of those differences between a $K^+ \rightarrow \mu^+$ decay (which is what we are normalizing to) and a $K^+ \rightarrow \pi^+$ decay (which is what we are trying to measure). In particular it is a good test of the nuclear interaction simulation in the Monte Carlo, and the acceptance of the TD cuts.

The $K_{\pi 2}$ branching ratio is measured with the full set of $K_{\pi 2}(1)$ monitor data (prescaled by 10), the same data set used to measure the $K_{\pi 2}$-based acceptance. These data are subjected to the following cuts: Online DELC, ISKCODE, PNNSTOP, DIPANG, KP2BRBOX, beam cuts, the delayed coincidence cuts, and FITPI. The results are given in Table 5.22. A spectrum plot after all cuts except KP2BRBOX shows some contamination from $K_{\mu 2}$ events, thus the need for the KP2BRBOX cut, a box around the $K_{\pi 2}$ peak in energy, range, and momentum. It is possible that even with
5.8. The $K_{\pi 2}$ Branching Ratio

this box cut there is some residual background contamination from $\pi-$scats. To investigate this, a spectrum plot was generated for both the data and UMC samples after application of all cuts except PNNSTOP and KP2BRBOX (see Figure 5.8). The muon band seen in the data will be removed by the KP2BRBOX cut. We see that the tails extending down from the $K_{\pi 2}$ peak look similar in these plots, thus we need not worry about background in the data sample.\(^4\)

<table>
<thead>
<tr>
<th>Cut</th>
<th>$N_{\text{passed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescale</td>
<td>178271</td>
</tr>
<tr>
<td>Level 0 DELC</td>
<td>111812</td>
</tr>
<tr>
<td>ISKCODE</td>
<td>99885</td>
</tr>
<tr>
<td>PNNSTOP</td>
<td>71116</td>
</tr>
<tr>
<td>DIPANG</td>
<td>63393</td>
</tr>
<tr>
<td>KP2BRBOX</td>
<td>27851</td>
</tr>
<tr>
<td>PICER</td>
<td>27657</td>
</tr>
<tr>
<td>PISCAT_BW</td>
<td>25791</td>
</tr>
<tr>
<td>PISCAT_B4</td>
<td>24944</td>
</tr>
<tr>
<td>B4DEDX</td>
<td>24403</td>
</tr>
<tr>
<td>PBGLASS</td>
<td>22932</td>
</tr>
<tr>
<td>BHTRS</td>
<td>22809</td>
</tr>
<tr>
<td>DELC</td>
<td>20610</td>
</tr>
<tr>
<td>TIMCON</td>
<td>20263</td>
</tr>
<tr>
<td>DKNFLT</td>
<td>20246</td>
</tr>
<tr>
<td>FITPI</td>
<td>13697</td>
</tr>
</tbody>
</table>

Table 5.22: The $K_{\pi 2}$ branching ratio analysis.

As with the $K_{\mu 2}$ branching ratio measurement, some parts of the acceptance are

\(^4\)The question is whether the data outside (and thus also under) the $K_{\pi 2}$ peak comes from $K_{\pi 2}$ events (via, say, nuclear interactions) or not. If not, they shouldn't be counted in the $K_{\pi 2}$ branching ratio. If so, no correction should be made, even if UMC does not properly reproduce them. The main point of this measurement is to test UMC's representation of nuclear interactions.
5.8. The $K_{\pi 2}$ Branching Ratio

Figure 5.8: Spectrum plots for the data and UMC $K_{\pi 2}$ samples after application of all cuts except PNNSTOP and KP2BRBOX. The KP2BRBOX cut is shown.

measured with Monte Carlo. The results of the UMC $K_{\pi 2}$ analysis efficiency study are presented in Table 5.23. As in the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ acceptance measurement, we must apply UFATE, USTMED, and USTOP HEX to UMC events to account for the loss incurred by applying FITPI to pion tracks that do not come to rest in range stack scintillator.

For $N^\text{equiv}_{K\text{Blive}}$, we find:

$$N^\text{equiv}_{K\text{Blive}} = \frac{K\text{Blive}_{\text{before 21688}}}{\text{prescale}_{\text{before 21688}}} + \frac{K\text{Blive}_{\text{after 21688}}}{\text{prescale}_{\text{after 21688}}} = \frac{0.02104 \times 10^{12}}{595680} + \frac{1.52887 \times 10^{12}}{1405760} = 1122906.$$ 

The $K_{\pi 2}$ branching ratio is given by the following equation:

$$BR_{K_{\pi 2}} = \frac{\# K_{\pi 2} \text{anal}}{fS_{K\text{Tlive}}} \frac{1}{A^\text{equiv}_{K_{\pi 2}}} \left( \frac{A^\text{umc}_{\text{recon}}}{A^\text{data}_{\text{recon}}} \right)^{K_{\pi 2}} \frac{1}{A_{DC}} \frac{1}{A_{\text{rest}}} \frac{1}{A_{\text{tdfit}}}.$$
5.8. The $K_{\pi 2}$ Branching Ratio

<table>
<thead>
<tr>
<th>Cut</th>
<th>$N_{\text{passed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KB</td>
<td>233191</td>
</tr>
<tr>
<td>Trigger</td>
<td>84685</td>
</tr>
<tr>
<td>UFATE</td>
<td>72067</td>
</tr>
<tr>
<td>USTMED</td>
<td>70461</td>
</tr>
<tr>
<td>ISKCODE</td>
<td>66484</td>
</tr>
<tr>
<td>PNNSTOP</td>
<td>43703</td>
</tr>
<tr>
<td>USTOP_HEX</td>
<td>42709</td>
</tr>
<tr>
<td>DIPANG</td>
<td>41549</td>
</tr>
<tr>
<td>KP2BRBOX</td>
<td>40318</td>
</tr>
</tbody>
</table>

| $A_{\text{recon}}^{\text{unc}}$ | 0.9436 ± 0.0009 |
| $A_{K_{\pi 2}}^{\text{unc}}$   | 0.1729 ± 0.0008 |

Table 5.23: The umc $K_{\pi 2}$ analysis efficiency.

Substituting in the expression for $f_S$ from the previous section, this becomes:

$$BR_{K_{\pi 2}} = BR_{K_{\mu 2}} \frac{(# K_{\pi 2})_{\text{anal}}}{(# K_{\mu 2})_{\text{anal}}} \frac{N_{K_{\pi 2} \text{live}}^{\text{equiv}}}{N_{K_{\mu 2} \text{live}}^{\text{equiv}}} A_{K_{\mu 2}}^{\text{unc}} A_{K_{\pi 2}}^{\text{unc}} \frac{(A_{\text{recon}}^{\text{unc}}/A_{\text{data}}^{\text{recon}})_{K_{\pi 2}} (A_{\text{rest}})_{K_{\pi 2}}}{(A_{\text{recon}}^{\text{unc}}/A_{\text{data}}^{\text{recon}})_{K_{\mu 2}} (A_{\text{rest}})_{K_{\mu 2}} A_{\text{tdfit}}}$$.  

The factor $(A_{\text{rest}})_{K_{\pi 2}}$ is just the beam cuts, and $(A_{\text{rest}})_{K_{\mu 2}}$ is the beam and photon veto cuts, so the ratio of these quantities is really $(A_{\text{rest}})_{K_{\pi 2}}/(A_{\text{rest}})_{K_{\mu 2}} = A_T$, the efficiency of those photon veto cuts applied in the $K_{\mu 2}$ branching ratio analysis. This can be extracted from Table 5.9:

$$A_T = \frac{25089}{28872} = 0.869 \pm 0.002$$

$A_{\text{tdfit}}$ is the FITPI acceptance already measured.

The double ratio,

$$R = \frac{(A_{\text{recon}}^{\text{unc}}/A_{\text{data}}^{\text{recon}})_{K_{\pi 2}}}{(A_{\text{recon}}^{\text{unc}}/A_{\text{data}}^{\text{recon}})_{K_{\mu 2}}},$$
accounts for differences between the reconstruction efficiencies for $K_{\mu2}$ and $K_{\tau2}$ events, and for differences between the efficiencies for data and Monte Carlo. If we take $A_{\text{recon}}^{\text{data}}$ to be the same for $K_{\mu2}$ and $K_{\tau2}$, we will have $R = (A_{\text{recon}}^{\text{unc}})_{K_{\tau2}} / (A_{\text{recon}}^{\text{unc}})_{K_{\mu2}} \neq 1$. If, however, the presence of photons and nuclear interactions are the reason why $(A_{\text{recon}}^{\text{unc}})_{K_{\tau2}}$ is observed to be lower than $(A_{\text{recon}}^{\text{unc}})_{K_{\mu2}}$, then it may make sense to use $(A_{\text{recon}}^{\text{data}})_{K_{\mu2}}$ for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, since $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ lacks photons and the nuclear interactions efficiency loss is properly built into $A_{\text{ndf}}$. Studies in [28] suggest that this is the case, so the $A_{\text{recon}}$ measured with $K_{\mu2}$ decays should be fine for the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ analysis. For the $K_{\tau2}$ analysis these losses are built into $A_{K_{\tau2}}^{\text{unc}}$ so we will use $R = 1$. The $K_{\tau2}$ branching ratio is then given by:

$$BR_{K_{\tau2}} = BR_{K_{\mu2}} \frac{(# K_{\tau2})_{\text{anal}} (N_{K_{\text{Blues}}})_{K_{\mu2}} A_{K_{\mu2}}^{\text{unc}} A_{\gamma} 1}{(# K_{\mu2})_{\text{anal}} (N_{K_{\text{Blues}}})_{K_{\tau2}} A_{K_{\tau2}}^{\text{unc}} A_{\text{ndf}}}.$$ 

The results of the $K_{\tau2}$ branching ratio measurement are given in Table 5.24. The accepted value [3] is 0.2116 ± 0.0014, so the measurement is about 2% high (a 0.6 σ discrepancy). The $K_{\tau2}$ branching ratio was also measured using slight variations on the method for measuring the acceptance of TD cuts (see [33]). All branching ratio measurements were in rough agreement with accepted value, with one of the measurements coming out 10% too high (the largest disagreement with the accepted value of these measurements). We will for this reason attach at 10% systematic error to the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ acceptance, $f_s \times A_{\text{tot}}$. 

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5.9 The Sensitivity

The total sensitivities for $K^+ \to \pi^+\nu\bar{\nu}$ and $K^+ \to \pi^+X^0$ are given in Table 5.25. The errors listed here are purely statistical, with the exception of $A_{\pi scatter}$ and the FITPI acceptance which include systematic errors as discussed in the text.

<table>
<thead>
<tr>
<th>Standard</th>
<th>TIGHT1</th>
<th>TIGHT2</th>
<th>TIGHT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\pi \nu}^{tot}$</td>
<td>0.00381 ± 0.00015</td>
<td>0.00332 ± 0.00014</td>
<td>0.00278 ± 0.00011</td>
</tr>
<tr>
<td>$A_{\pi X}^{tot}$</td>
<td>0.01230 ± 0.00049</td>
<td>0.01136 ± 0.00046</td>
<td>0.00996 ± 0.00040</td>
</tr>
<tr>
<td>$f_5$</td>
<td>0.673 ± 0.005</td>
<td>0.673 ± 0.005</td>
<td>0.673 ± 0.005</td>
</tr>
<tr>
<td>$N_{KL burst}$</td>
<td>$1.53 \times 10^{12}$</td>
<td>$1.53 \times 10^{12}$</td>
<td>$1.53 \times 10^{12}$</td>
</tr>
<tr>
<td>$S_{\pi \nu}^{tot}$</td>
<td>$(3.93 \pm 0.16) \times 10^{9}$</td>
<td>$(3.43 \pm 0.14) \times 10^{9}$</td>
<td>$(2.87 \pm 0.12) \times 10^{9}$</td>
</tr>
<tr>
<td>$S_{\pi X}^{tot}$</td>
<td>$(12.69 \pm 0.52) \times 10^{9}$</td>
<td>$(11.73 \pm 0.48) \times 10^{9}$</td>
<td>$(10.28 \pm 0.42) \times 10^{9}$</td>
</tr>
</tbody>
</table>

Table 5.25: The sensitivity.
Conclusions

6.1 Establishing Branching Ratio Limits

We would like to convert our observation of one $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event into a range of allowed branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, and to convert our observation of zero $K^+ \rightarrow \pi^+ X^0$ events into an upper limit on the $K^+ \rightarrow \pi^+ X^0$ branching ratio. Because the numbers of events observed (one and zero) are small, we must use Poisson statistics. The Poisson distribution is:

$$P(n; \mu) = \frac{e^{-\mu} \mu^n}{n!},$$

where $\mu$ is the mean number of events expected, and $P(n; \mu)$ is the probability of seeing $n$ events.

For our observation of one $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event, we would like to give 1-$\sigma$ errors so we will construct a 68% confidence level interval. The lower limit of this interval is defined to be the point at which $(1 - 0.68)/2 = 16\%$ of identical experiments would see
6.1. Establishing Branching Ratio Limits

at least 1 event (i.e. 1 event, or 2 events, or ... ∞), and the upper limit is defined to be the point at which 16% of identical experiments would see no more than 1 event (i.e. 0 or 1 events). The interval defined this way is found to be [μ = 0.17435 → 3.28852]. This can be verified by substituting these numbers into the Poisson distribution:

\[ P(1; 0.17435) + P(2; 0.17435) + ... = 1 - P(0; 0.17435) = 1 - e^{-0.17435} = 0.16 \]

\[ P(0; 3.28852) + P(1; 3.28852) = e^{-3.28852} + 3.28852e^{-3.28852} = 0.03731 + 0.12269 = 0.16 \]

For our observation of zero \( K^+ \rightarrow \pi^+ X^0 \) events, we will set a 90% confidence level upper limit, which is defined to be the point at which 10% of identical experiments would see no more than 0 events. We find that \( \mu = 2.30259 \) is the value that will fluctuate down to \( n = 0 \) only 10% of the time:

\[ P(0; 2.30259) = e^{-2.30259} = 0.1 \]

The \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) branching ratio 68% confidence level interval thus obtained is:

\[ BR = \frac{N}{S_{\text{tot}}} = \frac{1^{+2.28852}_{-0.82856}}{3.93 \times 10^9} = (2.54^{+5.82}_{-2.10}) \times 10^{-10}. \]

The \( K^+ \rightarrow \pi^+ X^0 \) branching ratio 90% confidence level upper limit is:

\[ BR \leq \frac{N}{S_{\text{tot}}} = \frac{2.30259}{12.68 \times 10^9} = 1.81 \times 10^{-10}. \]

In computing these limits we have dropped the errors listed in Table 5.25 and the 10% systematic error determined in the \( K^+ \rightarrow \pi^+ \tau^+ \) branching ratio study. We could fold
6.2. Extracting $V_{td}$

these errors on the sensitivity into the branching ratio limits, but the limits would change only by a percent or two. Given the already very large uncertainty in the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio coming from the statistical error on the observation of a single event, it is not very meaningful to add in this additional error on our sensitivity.

6.2 Extracting $V_{td}$

Referring back to Figure 1.2, one can see the relationship between the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio (proportional to the length squared of the thick dashed line in this figure) and $V_{td}$ (proportional to the side of the unitarity triangle extending from (1,0) to $(\bar{\rho}, \bar{\eta})$). If it were not for the charm contribution (parameterized by $\rho_0$), the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio would give a very direct measurement of $V_{td}$.

To provide an independent measure of $V_{td}$, we wish to make use of as few of the current constraints on $\bar{\rho}$ and $\bar{\eta}$ as possible in converting our measurement of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio to $V_{td}$. If we assume that the the entire $(\bar{\rho}, \bar{\eta})$ plane is allowed, $\bar{\rho}$ and $\bar{\eta}$ could lie anywhere along the ellipse centered at $(\rho_0, 0)$. As one can see from the figure, $V_{td}$ will have its maximum value when $\bar{\eta} = 0$ and $\bar{\rho} = \rho_0 + r_0$, where $r_0$ is the length of the line extending $(\rho_0, 0)$ to $(\bar{\rho}, \bar{\eta})$ and is roughly proportional to the square root of the branching ratio. $V_{td}$ will have its minimum value when $\bar{\eta} = 0$ and $\bar{\rho} = \rho_0 - r_0$.

Using this technique, and varying $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in the range $(0.44 \times 10^{-10} \rightarrow$
8.36 \times 10^{-10})$, we obtain the limits\footnote{We have also varied $V_{cb}$, $m_c$, $m_t$, and $\Lambda_{MS}$ in the ranges given in [6]. Many thanks to L. Littenberg for this calculation.}: 

$$0.0042 < |V_{td}| < 0.048.$$ 

This range of values for $|V_{td}|$ is consistent with information on $V_{td}$ from $|V_{us}/V_{cb}|$, $B^0 - \bar{B}^0$ mixing, and $CP$ violation in the $K$ system.

6.3 The Future

Experiment 787 has collected a factor of approximately 1.5 more data over the 1995 sample during the 1996 and 1997 runs. In addition, we are scheduled to run in 1998 and 1999, and this will increase our data set by perhaps a factor of 2. Thus, the total $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ data set from Phase II of E787 is likely to be a factor of 5 larger than the data set used by this analysis. If the Standard Model is right, and the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio is $0.9 \times 10^{-10}$, we expect to see maybe 1 more $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event in this data set. If, however, the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio is really the value we have measured in this analysis, $2.54 \times 10^{-10}$, then we expect to see $4 \pm 2$ more events. Clearly the results of the analysis of this full data set will be very interesting.
Bibliography


