Searching for T-Violation in the Decay $K^+ \rightarrow \mu^+ \nu_\mu \pi^0$

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Abstract

This thesis describes a measurement of the T-violating transverse muon polarization ($P_T$) in $K^+ \rightarrow \mu^+ \nu_\mu \pi^0$ decay. Although the Standard Model prediction for $P_T$ is vanishingly small $\mathcal{O}(10^{-6})$, popular extensions to the Standard Model allow polarizations as high as a few percent. Data taken by experiment E246 at the KEK National Lab between May 1996 and February 1998 were analyzed, and a value of $P_T = 0.1 \pm 4.9(\text{stat}) \pm 1.0(\text{syst}) \times 10^{-3}$ was measured. This measurement is consistent with Standard Model expectations, and can be converted into a limit on new effective scalar or pseudoscalar interaction terms in the Lagrangian: $|\text{Im}(F_s + F_p)| < 1.4 \times 10^{-8} \text{ GeV}^{-2}$ at the 90% confidence level.
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1.1 The Standard Model

The Standard Model (SM) of Particle Physics, which describes interactions between matter (leptons, quarks, Higgs bosons) and gauge fields (photons, W and Z bosons, gluons), has been extremely successful in describing phenomena at subatomic scales. In some cases, predictions from the SM agree with experimentally measured quantities to more than 8 decimal places of accuracy. Despite more than 20 years of research spanning several orders of magnitude in energy, there is no conclusive evidence for any breakdown in the SM.

That's not to say that the Standard Model is perfect. The theory itself has several unpleasant features. In particular, the SM fails to predict the masses of any fundamental particle. It also fails to predict the coupling strength between the matter

\footnote{With the possible exception of neutrino oscillations\cite{1, 2}, although evidence for this process remains sketchy.}
1.1. The Standard Model

particles and the gauge fields. In order to make any predictions with this theory, therefore, one must first measure all of these quantities using experimental data. This is unsatisfactory to say the least. A complete and final theory of particle physics should have no external parameters whatsoever. Clearly, the SM falls somewhat short.

Worse still, the SM fails to predict the large baryon number asymmetry that we observe in the universe. Our galaxy, and most of the rest of the visible universe, is composed predominantly of matter. Balloon-borne experiments designed to search for anti-galaxies and anti-galactic clusters have all have negative results. To evolve from a state of complete matter-antimatter symmetry (The Big Bang) to our present matter dominated universe, there must have been a period in the early universe when there was a large matter-antimatter asymmetry (CP violation)[3]. CP violation does occur in the SM, but at a level that is several orders of magnitude too small to account for the present structure of the universe[4].

Because SM predictions are phenomenally accurate at low energies (< 100 GeV), the SM must be a low energy approximation of some larger unified theory (GUT). The complete theory should incorporate all the features of the SM, should explain the mass hierarchy, predict the coupling strengths between matter and gauge fields, and should contain new physics that has yet to be seen.
1.2 Attacking the Standard Model

There are two approaches that experimentalists can use to search for new physics. First, one can make precision measurements of quantities that can be predicted with a small theoretical uncertainty. New physics effects, which typically enter as one or two-loop corrections to SM predictions, will introduce small differences between the predicted and measured values. These experiments are difficult because a very high precision is needed to conclusively demonstrate the presence of new physics effects. Additionally, experiments of this type are often hindered by theoretical uncertainties in SM predictions. Nonetheless, there have been some impressive efforts; in particular the measurement of the muon gyromagnetic ratio\cite{5}:

\begin{equation}
\gamma^\text{exp}_\mu = 2.0023318460(168) \quad \gamma^\text{SM}_\mu = 2.0023318336(220)
\end{equation}

The second approach is typically much cleaner. One simply looks for a process that is forbidden in the SM. An example of such an experiment is the search for the lepton flavor violating decay $K_L \rightarrow \mu e$. Observation of this decay would be a clear indication of new physics. In addition, because this experiment doesn’t involve measuring a small correction to the SM prediction, it is theoretically very clean.
1.3 \(T\)-Violation as a Probe of new Physics

What we require is an observable with a very small (or null) contribution from the Standard Model. In KEK experiment E246, the observable of choice is the transverse muon polarization in \(K^+ \to \mu^+ \nu_\mu \pi^0 (K_{\mu3})\) decay. The transverse polarization is defined as:

\[
P_T = \frac{\vec{\sigma}_\mu \cdot \vec{P}_\mu \times \vec{P}_{\pi^0}}{|\vec{P}_\mu \times \vec{P}_{\pi^0}|}
\]

(1.2)

\(P_T\) is clearly a \(T\)-odd quantity since both the momentum and spin vectors change sign under \(T\) conjugation. As we will show, a measurement of \(<P_T> \neq 0\) is an indication of direct \(T\) violation.

First, we must show that the Standard Model contribution to \(P_T\) is small. In this case, Standard Model terms are easy to understand. In any local quantum field theory with an appropriate normal ordered Lagrangian, CPT is absolutely conserved. So \(T\) violation is nothing more than CP violation in disguise, and CP violation is the result of interference between two diagrams with different phases. However, at tree level in the SM, there is only one diagram that contributes to \(K^+ \to \mu^+ \nu_\mu \pi^0\) decay. This diagram is the result of a direct \(W\) exchange and is shown in figure 1.1a. Since there is only one diagram, even I can calculate interference term: zero.

Of course, this is not the complete story and matters become slightly murkier when we start to include loop corrections arising from photon and \(W\) exchanges as shown
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in figures 1.1b and 1.1c. We stress that that the electromagnetic diagram is a long-distance effect involving long lived intermediate particles. Phases arising from this class of diagram do not come from point interactions in the Lagrangian, but instead from the propagation of near-mass-shell particles. As a result, even though these diagrams can generate a non-zero \( P_T \), they are not the result of T-violating terms in the SM. The W-exchange diagram, on the other hand, does contain a complex phase from the CKM matrix, and truly violates T.

Fortunately, because of the appearance of oppositely charged \( u \) and \( \bar{u} \) quarks in the final state, one-loop electromagnetic effects cancel. These diagrams could potentially create transverse polarizations as high as \( 10^{-3} \). The largest correction to the \( K^+ \to \mu^+ \nu_\mu \pi^0 \) decay amplitude comes from the diagram shown in figure 1.1b. Because this diagram enters at the two-loop level, it should be suppressed relative to the W exchange diagram by \( \alpha^4_{EM} \). This term has been calculated [6], and its contribution to the transverse polarization was found to be:

\[
P_T \sim 10^{-6} \quad (F.S.I) \tag{1.3}
\]

A back-of-the-envelope calculation shows that the contribution from real T violating terms in the SM is also small, \( P_T < 10^{-6} \) [7]. This is well below the statistical upper-limit of E246, which is \( \delta P_T \sim 10^{-3} \). Therefore, we can ignore SM terms.
1.3. *T-Violation as a Probe of new Physics*

Figure 1.1: Standard Model diagrams for $K^+ \rightarrow \mu^+\nu\pi^0$ Decay. (a) Direct $W$ exchange (b) The lowest order electromagnetic correction that is dominated by a virtual $\pi\pi\gamma$ intermediate state. Note that because of the opposite charge of the $u$ and $\bar{u}$, one-loop photon diagrams do not contribute. (c) The contribution from CP violating one-loop amplitudes in the SM.
1.3. T-Violation as a Probe of new Physics

1.3.1 Making an Unambiguous Measurement

In any measurement, one must be sure that the final result is free from biases caused by the acceptance of the detector. In this sense, $P_T$ is not a good quantity to measure because it depends strongly on the detector acceptance across the Dalitz plot and upon $K_{\mu3}$ decay kinematics. As we will show below, we can relate $P_T$ to a well defined SM quantity, namely $F_-/F_+$, the ratio of the $K^+ \to \pi^0$ decay form factors. This ratio is usually represented by $\xi$. Unlike $P_T$, $Im(\xi)$ is a well-defined, calculable SM parameter.

The SM amplitude for $K^+ \to \mu^+\nu\mu\pi^0$ decay can be written as a hadronic current multiplied by a leptonic current:

$$A_{K_{\mu3}} = J^{\mu\text{(had.)}}J_{\mu\text{(lept.)}}$$

(1.4)

The leptonic piece is the well known V-A current that arises in charged-current electroweak processes. We can construct the most general hadronic current as a arbitrary sum of all available Lorentz four-vectors. In this case, there are two: $P^\mu_x$ and $P^\mu_K$. This leaves us with two arbitrary expansion coefficients, which are typically written:

$$A_{K_{\mu3}} = [F_+(q^2)(p^\mu_K + p^\mu_{\nu\mu}) + F_-(q^2)(p^\mu_K - p^\mu_{\nu\mu})]\bar{\nu}\gamma_\mu \left(\frac{1 - \gamma^5}{2}\right)$$

(1.5)

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Where \( \xi \) is the ratio of the form factors.

\[
\xi = \frac{F_-}{F_+} \quad (1.7)
\]

Written in this form, it is clear that any \( T \) (or \( CP \)) violating amplitude is generated by the interference between the two pieces of the hadronic current. We therefore expect that the transverse polarization should be proportional to the phase difference between the two terms. Indeed, a detailed calculation gives[8]:

\[
P_T = -((n_\mu \times n_\nu) \frac{m_\mu}{m_K} E_{\mu} + \frac{|p_{\mu}|}{m_\mu - m_\nu} I m(\xi) \quad (1.8)
\]

The leading factor of \( m_\mu/m_K \) is the result of helicity suppression. The remaining kinematic factors can be directly measured in our detector and factored out, eliminating effects of detector response and acceptance.

1.4 Previous Experimental Results

There have been several previous experiments to measure \( P_T \) in both \( K_{\mu 3}^+ \) and \( K_{\mu 3}^0 \) decays [9, 10, 11]. The most recent was performed in the early 1980's at the Alternating Gradient Syncotron (AGS) at Brookhaven National Lab (BNL). This experiment used an unseparated in-flight beam along with a lead-glass photon detector; allowing them
to measure $P_T$ from both $K^+ \rightarrow \mu^+ \nu_\mu \pi^0$ and $K^0 \rightarrow \mu^+ \nu_\mu \pi^-$ decays simultaneously. The experiment was statistically limited, and obtained a value of:

$$P_T = -4.2 \pm 6.7 \times 10^{-3} \quad Im(\xi) = -1.6 \pm 2.5 \times 10^{-2}$$

for the charged mode[11, 12]. In this work, we achieve a final sensitivity of $Im(\xi) = (0.2 \pm 1.6 \pm 0.3) \times 10^{-2}$. The E246 experiment will continue taking data until the year 2001, at which point we hope to reduce the errors on $Im(\xi)$ below $10^{-2}$.

1.5 Looking beyond the SM

Currently, there are nearly as many extensions to the Standard Model as there are theorists. To avoid unnecessary model dependence, we will attempt to discuss the general features of a T-violation search without specifying any particular model. First, we explain what new physics processes E246 can detect, and then we will determine what constraints already exist from other experimental data.

T-Violation in $K^+ \rightarrow \mu^+ \nu_\mu \pi^0$ decay is sensitive to any effective scalar or pseudoscalar interaction in the Lagrangian. The general low energy effective Lagrangian can be written[13]:

$$\mathcal{L}' = F_s \bar{s}u\bar{\nu} + F_p \bar{s}u\bar{\nu}\gamma^5\nu$$

(1.10)

There are many ways that such an interaction can appear. New scalar particles
1.5. *Looking beyond the SM*

can be introduced that will generate competing Feynman diagrams at tree level. New gauge particles or extra quark/lepton generations can mimic scalar interactions at one loop order. Because the SM diagram contains both vector and axial-vector (V-A) components, it is impossible to distinguish between scalar and pseudoscalar interactions in this experiment. In fact, a detailed calculation shows[14]:

\[ P_T \propto \text{Im}(F_{\gamma} + F_{\rho}) \]  

(1.11)

Limits on new scalar or pseudoscalar interactions from previous kaon T-violation searches are shown in figure 1.2 [9, 10].

In a large number of extensions to the SM, effective scalar or pseudoscalar interactions are proportional to the lepton mass. This is the direct result of helicity suppression. Although this is not the case in general, using heavier leptons typically means a higher sensitivity to new physics. This is the principal motivation for looking at $K_{\mu 3}$ rather than $K_{e3}$ decays.

1.5.1 $d_n$, $\epsilon$, $\epsilon'$ and other Constraints

Apart from previous kaon T-violation searches, one might expect that the strongest constraint on $P_T$ would come from measurements of other CP violating observables; in particular the neutron electric dipole moment $d_n$ and the kaon CP violation parameters $\epsilon$ and $\epsilon'$. Unfortunately, the literature is extremely murky on this point.
1.5. Looking beyond the SM

Figure 1.2: Limits on new scalar or pseudoscalar interactions as a function of year. All experiments up to the present have involved kaon T-violation measurements. B-factory and LHC limits have been estimated from $B \to D\tau\nu$ decays. The coupling constants $F_s$ and $F_p$ play similar roles in the Lagrangian as $G_f$, and hence have dimensions of $g^2/M_{\phi}^2 =$GeV$^{-2}$. 

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Most authors generally assume some particular model, or make assumptions about the relative strength of coupling constants across generations. Depending on which model is chosen, limits on $P_T$ can be as strong as $P_T = 0$ or as weak as $P_T < 7\%$ [14]. The more “popular” extensions to the Standard Model, such as SUSY or multi-Higgs models, typically give polarizations on the order of a few tenths of a percent. However, by discarding some of the assumptions in these models, many of these constraints can be made to vanish.

For example, a strong constraint on $P_T$ comes from the T-violating neutron electric dipole moment. Contributions to $d_n$ from effective scalar interactions arise from the scalar penguins shown in figure 1.3 [13]. Following Garisto and Kane, and using the experimentally measured value $d_n < 5.6 \times 10^{-12}$ GeV$^{-1}$ [5] we can calculate an upper limit on $Im(F_s + F_p)$ of $1.7 \times 10^{-9}$ GeV$^{-2}$. This is an extremely strong limit, implying $P_T < 10^{-3}$. However, it relies on the assumption that the scalar coupling is identical for quarks and leptons. It is possible to introduce models in which CP violation occurs only in the lepton sector, while the quark sector coupling is completely real. This eliminates the $d_n$ constraint altogether. Using similar arguments, and considering both theoretical and experimental uncertainties in $\varepsilon$ and $\varepsilon'$, these constraints can be discarded as well.

So can a model-independent constraint on $P_T$ be constructed? In order to eliminate assumptions about coupling strengths between different generations, we must
1.5. *Looking beyond the SM*

Figure 1.3: Diagrams contributing to $d_n$ from an effective scalar interaction in the Lagrangian.

study a decay amplitude that has exactly the same terms as equation 1.10. The obvious choice is $K^+ \rightarrow \mu^+\nu_\mu$ decay. The diagram for this decay, shown in figure 1.4, contains the same weak components as figure 1.1a except in this case the $u$ and $\bar{u}$ quarks annihilate rather than hadronize into a $\pi^0$.

In the SM, $W$ bosons couple only to right-handed anti-muons. Therefore, all anti-muons from $K_{\mu2}$ decay should be longitudinally polarized with $P_L = +1.0$. Effective scalar or pseudoscalar interaction terms can couple to left-handed anti-muons. Therefore, if $F_s + F_p \neq 0$, $P_L$ should be less than unity. So $Im(F_s + F_p)$ can be constrained using the best measurement of $P_L[15]$:

$$P_L > 0.99\% \ (90\% \text{ C.L.}) \quad (1.12)$$

After a detailed calculation (see Appendix A), we find:
1.6. Techniques

\[ \text{Im}(F_s + F_p) \leq 7 \times 10^{-8} \text{GeV}^{-2} \] (1.13)

This constraint is much weaker than any previous experimental constraint, and implies that E246 may observe a polarization as high as \( P_T \sim 4.2\% \).

![Figure 1.4: The SM diagram for K_{\mu 2} decay. The diagram is very similar to the K_{\mu 3} diagram, shown in figure 1.1a.](image)

1.6 Techniques

Given the lack of strong theoretical bounds, there is a large window for discovery. In this section, I will attempt to evaluate various experimental techniques using kaons. The possibility of searching for the same physics at the upcoming B-factories or at the LHC will also be discussed.
1.6.1 Kaons: Decays in-flight

All previous T-violation searches using kaons have been done with an in-flight beam. The main advantages are rate and cost. Typically, because the decay products are boosted forward, the detector can be smaller and can use cheaper detector elements such as lead-glass instead of CsI for photon detection[11]. Higher rates translate into a better statistical sensitivity. This can be quite a large effect, as high energy (2 GeV/c) kaon beams can be produced with a much higher intensity than low energy (700 MeV/c) beams. For example, a comparison of beamlines at the KEK-PS shows that a 2 GeV/c beamline (K6) can provide 7 times more kaons than a 660 MeV/c beamline using equivalent proton intensities[8]. For a 5 meter long decay tank, decay fractions of better than 30% are possible. This is comparable to the kaon stopping rate in the E246 target (40%). The overall rate difference translates into nearly a threefold improvement in sensitivity for the same run time.

Another potential advantage of the in-flight technique is the ability to search for T-violating muon polarization in $K_L^0 \rightarrow \mu^+\pi^-\nu_\mu$ decays. In principle, this mode is sensitive to the same physics as $K^+ \rightarrow \mu^+\nu_\mu\pi^0$. However, because of the presence of two charged particles in the final state, long distance electromagnetic effects are large and produce a transverse polarization of $P_T \sim 10^{-3}$. In order to separate the physics polarization from F.S.I., a comparison between the $K^0$ and $\bar{K}^0$ modes must be made. However, because of depolarization caused by atomic orbital capture, it is nearly
impossible to measure the polarization of a negative muon. Therefore $P_T = 10^{-3}$ is a practical limit on this measurement. The most recent experiment to search for T-violation in $K_L^0$ decays was performed at the AGS [11, 12], and found:

$$P_T = 1.7 \pm 5.6 \times 10^{-3} \quad Im(\xi) = 0.9 \pm 3.0 \times 10^{-2}$$

(1.14)

There are also definite disadvantages to the in-flight method. Because of the large boost, the polarimeter components must be placed very close to the beam line. This results in a large beam-induced background in the polarimeter, as well as problems from activation of the polarimeter aluminum to form $^{28}$Al. Signal to noise in previous in-flight experiments was typically no better than 3:1[8]. Additionally, the in-flight technique is susceptible to systematic errors from the beam structure itself. Since this is difficult to measure and may change over time, control of systematics may be problematic.

1.6.2 Kaons: Decays at Rest

E246 is the first experiment to use stopped kaons for a T-violation search. The initial motivations were to reduce beam backgrounds, and to gain better control over systematics. Because the polarimeter counters are nearly 1 meter off the beam line, backgrounds are usually 10:1, an impressive improvement over any previous in-flight experiment. Additionally, because the kaon is stopped, and the kaon decay vertex is
completely reconstructed, allowing us to reduce beam introduced systematics.

Rate is a major stumbling block for stopped kaon experiments. In addition to lower beam rates, the desire to place the polarimeter components far from the beam line typically means a lower acceptance. In E246, for instance, the acceptance of the toroidal spectrometer is at most 4%. The acceptance of the entire detector is also small: $5 \times 10^{-5} K_{\mu 3}/K^+$. In the current phase of E246, we are statistically limited.

In order to eliminate bias asymmetries, E246 measures the difference in asymmetry between events with forward moving $\pi^0$s and events with backward moving $\pi^0$s. As we will see in chapter 4, most potential systematics are canceled well below the $P_T < 4 \times 10^{-4}$ level. However, because forward and backward going samples have different polarizations, some potential bias asymmetries do not cancel at all. This means that extremely precise detector alignment is needed to eliminate systematics. Given the typical alignments achieved in E246, it is unlikely that systematics in this type of experiment can be controlled below $P_T^{\text{syst}} \sim 7 \times 10^{-4}$.

1.6.3 T-violation in D and B decays

Kaon T-violation physics experiments now appear to be reaching the level where systematics can play a major role. Coupled to the fact that a large sample of kaons is needed to make future progress, strong consideration should be given to other decay channels that might be sensitive to the same physics. Future prospects for
the production of large $B$ and $D$ samples at BaBar, BELLE, HERA-B, CDF, and LHC opens up a wealth of new modes. Additionally, since some models predict polarizations as high as 30\% in $B \rightarrow D\tau\nu$ decays, the time has come to give serious thought to what experiments can be done without kaons.

The first logical "step up" from $K_{\mu3}$ decay is $D^+ \rightarrow K^0_L\mu^+\nu_\mu$ [16]. Although this mode is potentially interesting, it has a large number of practical problems. First, because it is necessary to reconstruct the flight direction of the decay positron to measure the muon polarization, it is necessary to stop the muon. This is virtually impossible given the energies at a typical $B$-factory or hadron machine. Second, it is impossible to do this experiment in a solenoidal collider detector since the large magnetic field will quickly wash out the muon spin. In order to contemplate a $T$-violation measurement with $D$'s, a dedicated charm fixed-target experiment would be required. As many models predict a higher polarization for $B \rightarrow D\tau\nu$ decays, future efforts should be focused there.

I will focus on two potentially interesting modes, $B \rightarrow D\tau\nu$ and $B \rightarrow D^*\tau\nu$. In the latter mode, both the $\tau$ and $D^*$ polarizations can be used to separate contributions from scalar and pseudoscalar interactions [17]. Because both the $\tau$ and $D^*$ lifetimes are 1000 times shorter than the precession times even in large (1 Tesla) magnetic fields, we don't have to worry about polarization dilution or field induced systematics. Even though these modes may have contributions from F.S.I., they are expected to be small.
1.6. Techniques

compared to the achievable statistical precision attainable in the near future.

In order to make a measurement of \( \tau \) polarization, we must use modes in which the \( \tau \) decays to hadrons, and then measure the average hadronic momentum along the \( \tau \) flight direction. This can theoretically give an extremely large analysing power of 0.4 [18]. In order to reconstruct the decay plane while keeping combinatorial background to a minimum, we need vertex information on both the decaying hadron and the \( \tau \). In order to reconstruct the \( \tau \) vertex and perform the polarization measurement, we must study modes in which the \( \tau \) decays to more than one hadron.

We can estimate the future sensitivity for these modes by scaling up results from CLEO II and CDF runs Ia and Ib [19, 20] and by using a crude estimate of the vertexing efficiency. The B-factories should be able to collect several thousand events, and can most likely measure \( P_T \) to a precision of 7%. Assuming that coupling constants are generation independent, this is comparable to the present E246 sensitivity to \( Im(F_s + F_p) \). Due to the enhanced B production cross section, LHC can potentially obtain tens of thousands of events. Limits from future experiments are plotted in figure 1.2. These limits were constructed assuming \( Im(F_s + F_p) \) is independent of generation. In reality, most models typically predict larger coupling constants for B quarks. This makes T-violation searches in the B system particularly attractive.
1.7 Future Prospects for T-violation

Future prospects for T-violation experiments are extremely good. With the existing data from kaon experiments, we can already constrain the existence of new scalar particles at the $M_\phi = 1.5$ TeV scale\textsuperscript{2}. Although LHC will be able to search for these particles directly, the large number of $B$ mesons produced allow sensitivity at the 20 TeV scale. This is much higher than a direct search will permit.

\textsuperscript{2}Assuming $g_\phi = g_2$. 
Chapter 2

The E246 Detector

The E246 detector was designed specifically for $P_T$ measurements in both the $K_{\mu 3}$ and $K_{\mu 2\gamma}$ decay modes. This requires the ability to reconstruct the momenta of charged particles, detect low energy photons with good efficiency and accuracy, and requires extremely precise detector alignment. This chapter provides an overview of the detector elements essential to the $K_{\mu 3}$ analysis.

2.1 Beamline Optics and Kaon Tagging

The E246 detector is located at the end of the K5 beamline of the 12 GeV Proton Syncotron (PS) at the Japanese Lab for High Energy Physics (KEK\(^1\)). Protons are extracted from the accelerator ring and delivered onto a production target, which is a 6 cm platinum rod located just upstream of the K5 beamline optics. Typically the PS is able to produce $2.5 \times 10^{12}$ protons per 0.7 second spill. It then takes 2 seconds

\(^1\)KEK:Ko Enerugi butsuri gaku Kenkyujo, High Energy Physics Research Institute
2.2. \textit{K$^+$ Stopping Target}

Proton collisions with the production target result in the production of numerous secondary particles including pions and kaons. The K5 beamline optics, shown in figure 2.1, are designed to select kaons with a momentum of $660 \pm 30$ MeV/c, and to reduce contamination from charged pions. Kaons are selected using a one-stage electrostatic separator, a mass selection magnet and slit, and a momentum slit. A lead-tungsten collimator located immediately after the final focus quadrupole (Q5) is used to reduce contamination from charged pions in the beam halo.

The final result is a beam of approximately $2 \times 10^5$ K$^+$ per spill. Residual pion contamination is large, with typical $\pi^+ : K^+$ ratios around 7:1. A lucite "Fitch-type" Čerenkov counter with $n=1.49$ is located at the entrance to the detector to do the final $\pi/K$ separation. In lucite, Čerenkov light emitted from pions is contained within the radiator due to total internal reflection, while light from kaons, which has a smaller Čerenkov angle, escapes. This separates light from each source, which can then be directed with mirrors into separate rings of photomultiplier tubes (PMTs). A schematic of the Čerenkov detector is shown in figure 2.2.

2.2 \textit{K$^+$ Stopping Target}

After passing through the Čerenkov counter, kaons pass through a cylindrical BeO-degrader where they lose most of their energy. Kaons emerging from the degrader
2.2. $K^+$ Stopping Target

typically have less than 30 MeV of kinetic energy, and come to rest in a completely active target. A typical $K_{\mu 3}$ event, showing a kaon stopping and then decaying into a muon and a $\pi^0$ is illustrated in figure 2.4, which provides a cross-sectional view of the entire detector.

The target itself, shown in figure 2.3, is made of two-hundred-fifty-six $5 \times 5$ mm$^2$ square scintillating fibers. Each fiber is viewed by a Hamamatsu H1635 photomultiplier tube (PMT), which allows us to measure the energy deposited by the kaon as well the decay time. Using this information, we can extract the $x/y$ stopping point of the kaon over minimum ionizing backgrounds.

The target fibers are surrounded by a set of 12 fiducial counters. Each counter corresponds to one magnet “gap” and is moulded to the irregular shape of the target fibers to simplify the $dE/dx$ correction. The fiducial counters have a sensitive region of 20 cm ($-10 < z < 10$ cm) that defines the active region of the target. These counters are used to select events in which kaons stop in the central region of the target. They also provide a fast trigger signal and are used to generate a “start” timing signal for a time-of-flight (TOF) measurement.

Surrounding the fiducial counters are a set of 32 scintillating rings that provide information on the $z$ position of the $\mu^+$ upon exiting the target. Each ring is 6 mm thick. The entire assembly completely covers the active region of the target. There is a wavelength shifting fiber (WLS) embedded in each ring to collect the scintillation.
light. The light is then transferred to a clear fiber through an epoxy interface and transported to a PMT located downstream of the target. Information from the ring counters ($z$), combined with information from the target fibers ($x,y$) and gap wire chambers is used to reconstruct the kaon stopping position to $\sim 1$ cm in $x/y$ and $0.6$ cm in $z$.

### 2.3 Kaon Decays

After coming to rest in the target, the kaon will decay with a mean lifetime of 12.4 ns. There are numerous final states available, so one important job of the detector is to pick out $K_{\mu 3}$ events from all the other kaon decay modes. For reference, the major decay channels and branching ratios are tabulated in table 2.1.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Nomenclature</th>
<th>Branching Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \to \mu^+ \nu_\mu$</td>
<td>$K_{\mu 2}$</td>
<td>63.5%</td>
</tr>
<tr>
<td>$K^+ \to \pi^+ \pi^0$</td>
<td>$K_{\pi 2}$</td>
<td>21.2%</td>
</tr>
<tr>
<td>$K^+ \to \pi^+ \pi^+ \pi^-$</td>
<td>$K_{\pi 3 \pi}$</td>
<td>5.6%</td>
</tr>
<tr>
<td>$K^+ \to e^+ \nu_e \pi^0$</td>
<td>$K_{e 3}$</td>
<td>4.8%</td>
</tr>
<tr>
<td>$K^+ \to \mu^+ \nu_\mu \pi^0$</td>
<td>$K_{\mu 3}$</td>
<td>3.2%</td>
</tr>
<tr>
<td>$K^+ \to \pi^+ \pi^0 \pi^0$</td>
<td>$K_{\pi 3n}$</td>
<td>1.7%</td>
</tr>
<tr>
<td>$K^+ \to \mu^+ \nu \gamma$</td>
<td>$K_{\mu 2 \gamma}$</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table 2.1: Branching ratios for the major kaon decay modes.

Fortunately, the charged particle momentum spectra are quite different for the
Figure 2.1: A bird’s-eye view of the K5 beam line.
2.3. Kaon Decays

Figure 2.2: A schematic of the Čerenkov Counter. Light emitted by pions is contained within the lucite radiator, and collected by the 14-PMTs in the $\pi^+$ collector ring. Light emitted by kaons escapes the radiator and is directed with mirrors into the $K^+$ PMT ring.

Figure 2.3: An exploded view of the active stopping target.
2.3. Kaon Decays

Superconducting Toroidal Magnet

Figure 2.4: A cross-section of the E246 detector. A typical event is illustrated showing a $K^+$ stopping in the target and decaying. The two photons from the $\pi^0$ are detected by the CsI(Tl) array, while the muon passes through a gap in the CsI, through the analysis magnet, and comes to rest in the muon polarimeter. The decay positron is finally detected by a set of positron counters located adjacent to the muon stopper.
2.4. $\pi^0$ Reconstruction

various decay channels, as shown in figure 2.5, providing a strong background suppression method. Additional suppression of $K_{\pi2}$ backgrounds can be achieved by reconstructing the $\pi^0$, and eliminating events in which the $\pi^0$ and charged particle are nearly back-to-back. $K_{e3}$ decays can be eliminated both by using a TOF cut, and by detecting the decay positron from the muon Michel decay. More details of the event selection process will be given in chapter 3.

![Figure 2.5: Typical charged particle momentum spectra for various kaon decay modes (in the kaon rest frame).](image)

2.4 $\pi^0$ Reconstruction

Following the $K^+ \rightarrow \mu^+\nu_\mu\pi^0$ decay in the target, the $\pi^0$ decays almost instantly into two photons. In order to reconstruct the photons, the target is surrounded by a
CsI(Tl) barrel that measures both the energy and direction of the $\gamma$'s. The electromagnetic calorimeter is made up of 768 crystals each with a length of 25 cm ($13.5 \times X_0$) and is segmented into 20 divisions in $\theta$ and 48 divisions in $\phi$. The crystals are trapezoidal prisms that increase in size as one moves from the center (inner length = 2.9 cm, outer length = 6.2 cm) to the edge of the barrel (i.l. = 4.9 cm, o.l. = 8.2 cm). There are 12 gaps in the CsI detector to allow charged particles to enter the magnetic spectrometer without passing through or scattering in one of the crystals. Each gap covers $\Delta \phi = 15^\circ$ and $67.5 < \theta < 112.5^\circ$.

Each CsI(Tl) crystal is read out using a silicon PIN photodiode coupled to a charge sensitive preamplifier. Signals from the preamp are shaped using a Clear Pulse multistage shaping amplifier with a 1.2 $\mu$s shaping time. Both amplitude and timing information are obtained from each crystal. Additionally, all CsI channels are digitized in order to reject pulse pileup from beam halo backgrounds. The readout chain is shown in figure 2.6.

Initial energy calibration of the CsI array was performed with an $^{241}$Am source. Once data taking began, a more precise calibration was done with beam data by studying events in which a mono-energetic muon from a $K_{\mu2}$ decay stops in a single crystal. Because the calibration is time consuming, it is important to be able to monitor gain drifts over time. In order to accomplish this, we used a Xe ARC lamp calibration system. Light from the Xe lamp is distributed throughout the barrel.
using fiber optics, and can be used to provide a gain reference as well as providing a useful tool to search for dead channels. Gain drift was less than 3% over 1.5 years of operation.

In order to reconstruct the $\pi^0$, we must first take a set of hits in individual crystals, and group them into photon clusters. This is a three step process. First, we select a set of "seed" crystals. A "seed" crystal is any crystal with a in-time TDC hit. Then, we take each seed crystal and look at its nearest neighbors. If any of those crystals has an in-time TDC hit, or its energy is below the TDC threshold of 5 MeV, it is included in the cluster. We then loop over all the nearest neighbors that have been included in the cluster, and look at their nearest neighbors. This process continues until the cluster has reached a critical size (3 × 3), or until we have run out of crystals. A typical reconstructed $\pi^0$ event is shown in figure 2.7. The large number of hits in the ADCs without corresponding TDC matches are the result of out of time events piling up. These are typically the result of scattered halo pions. Despite large backgrounds, we are typically able to reconstruct the $\pi^0$ direction with a resolution of 6°.
2.4. $\pi^0$ Reconstruction

Figure 2.6: Readout chain for the CsI(Tl) calorimeter. Light from the crystals is detected using a silicon PIN diode. Signals then pass through a charge sensitive preamplifier and a multistage shaper. After the shaper, there are two separate readout chains. The timing chain consists of constant fraction discriminators (CFDs) followed by TKO TDCs, which have an intrinsic resolution of 0.7 ns. The analog chain consists of a series of transient digitizers (TDs) daisy chained with TKO ADCs.
Run 9606026
Event/Spill 18/3
Sector 8

CsI TDC

CsI ADC

chi^2 = 0.0122, Pfit = 184.12 MeV/c
Pmu = 201.43 MeV/c, Ppi = 205.04 MeV/c
E1* = 122.37 MeV, E2* = 97.89 MeV
OpAng(mu-phi0) = 173.82 deg., Pphi0 = 183.11 MeV/c
OpAng(g1-g2) = 68.01 deg.

Mass = 122.42
Esum = 220.26
Nclust = 2

Muon Hit
Photon Cluster
In Time Hit

50.0--  MeV
20.0--50.0 MeV
10.0--20.0 MeV
5.0--10.0 MeV
2.5--  5.0 MeV
0.6--  2.5 MeV

Figure 2.7: An unwrapped view of the CsI(Tl) calorimeter showing two reconstructed γ clusters from a π^0. The large number of ADC hits are from out of time events, involving for the most part scattered halo pions.
2.4.1 CsI Transient Digitizers

Because of the 1.2 μs shaping time of the CsI(Tl) crystals, pulses that occur earlier in time can “pileup” with real signal pulses. This has two effects. First, the measured energy of the real pulse is too large. Second, in-time pulses with large pileup may not have a TDC hit, or may have a hit that is shifted out of time. As a result, they will not be included in the photon cluster.

In order to suppress this background, the shaped waveform from all CsI crystals is digitized by a set of switched-capacitor-array transient-digitizers (SCA/TD). Each digitized channel contains 32 points with a 600 ns sampling time. Zero suppression is done at the hardware level with an onboard DSP processor. The large time window (20 μs) allows us to clearly resolve multiple pulse events. At the offline level, pileup events are separated by fitting digitized waveforms to a double pulse hypothesis, and then comparing the $\chi^2$ of the fit to a single pulse fit. The pulse fitting technique allows us to extract both amplitude and timing information. A typical event showing a pileup pulse and fit is shown in figure 2.8. Instructions for programming the TD and a description of the TD software can be found in appendix C.

2.5 Charged Particle Tracking

The CsI(Tl) barrel is surrounded by a superconducting toroidal magnet, which is divided into 12 gaps. Because the magnet windings are not continuous, the actual field
2.5. Charged Particle Tracking

A typical double pulse event showing both the recorded waveform (solid line) and the double pulse fit (dashed line).

is non-homogeneous, and can be roughly described by a superposition of a toroidal \((1/r)\) and a dipole field. The field strength between the poles is 0.9 T, and was chosen for optimal coverage of the \(K_{\mu3}\) Dalitz space.

Three multi-wire proportional chambers (MWPCs) are used to track particles through the magnet and measure the particle momentum. Each chamber uses a two-dimensional cathode strip readout that provides both \(x\) and \(y\) information. The strips are made of 18 \(\mu\)m of copper deposited on 25 \(\mu\)m of Kapton, and have a constant pitch of 1.0 cm (9 mm of Cu with a 1 mm gap between strips). The MWPCs are filled with a 50:50 mixture of Argon and Ethane. The gas is continuously recycled and impurities are removed by first filtering the return gas and then bubbling it through ethanol at \(-15^\circ\text{C}\).

As charged particles pass through the chamber, they leave an ionization trail.
2.5. *Charged Particle Tracking*

Liberated electrons drift towards the anode wires building up gain through secondary ionization. The image charge of this avalanche then appears on the cathode strips. The induced charge is amplified using a fast preamp, and signals from six different chambers are then summed before going into an ADC. By measuring the ratio of charges on three adjacent strips, we can reconstruct the position where the particle passed through the chamber with a resolution of 200 $\mu$m (for cathode strips perpendicular to the anode wire axis). Resolution for strips parallel to the anode wires is determined by the wire pitch, which is 2 mm.

As shown in figure 2.4, one chamber (C2) is located at the entrance to the magnet gap, while the other two (C3, C4) are located at the exit. C2 is positioned so that its long (high resolution) axis runs parallel to the beam axis. The chamber is located directly above the CsI(Tl) gap. Both C3 and C4 are positioned so their long axes run along the radial direction. This provides the best possible momentum resolution, since charged particles moving through the toroid tend to be momentum dispersed in $r$. Because the muons tend to spread as they pass through the gap, the chambers increase in size with C2 being the smallest and C4 being the largest. The exact dimensions are C2: $16 \times 56$ cm$^2$, C3: $20 \times 64$ cm$^2$, and C4: $20 \times 72$ cm$^2$. In each chamber, the cathode strip pitch remains constant, so the number of strips also increases in each chamber.

In order to track particles through the non-homogeneous field present in the
toroidal magnet, a field map was calculated using the magnetic field mapping program TOSCA[21]. Charged-particle trajectories can be constructed by numerically integrating the equations of motion. After information from the target has been used to check the track quality, and to correct for $dE/dx$ losses, we achieve a momentum resolution of 2.5 MeV/c for $K_{\pi^2}$ events.

### 2.6 $\mu^+$ Polarimeter

The muon polarimeter is by far the most critical component of the detector. Its purpose, as the name suggests, is to measure the polarization of the muon through the decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$. This measurement makes use of the fact that positrons from muon Michel decay are emitted preferentially along the muon spin direction. Specifically, the angular distribution of positrons follows:

\[
f(x, \cos \theta) = x^2[(3 - 2x) + (2x - 1)P_\mu \cos(\theta)]; \quad x = \frac{2E_e}{m_\mu}
\]

Equation 2.1 implies that the polarization is proportional to the asymmetry between the number of clockwise (CW) going positrons and the number of counterclockwise (CCW) going positrons:
2.6. $\mu^+$ Polarimeter

\[ P_T = \frac{1}{\alpha} \frac{N_{e^+} - N_{e^+}}{N_{e^+} + N_{e^+}} \]  

(2.2)

The constant of proportionality, $\alpha$, is called the analysing power and will be discussed further in chapter 3.

The polarimeter itself, shown in figure 2.10, consists of three components: the copper muon degrader, the aluminum muon stopper, and the positron counters. As the muon exits the magnetic spectrometer, it first passes through the degrader in which it loses a large fraction of its energy through ionization loss. The degrader is a copper wedge, 57 cm long and 20 cm wide with a pitch of 2.7°. The degrader was designed (and later optimized) so that high energy muons, which arrive at larger radii, will lose more energy than low energy muons. This tends to eliminate the radial dependence of the muon stopping distribution. After the degrader was constructed, a 1-cm-thick aluminum plate was added in order to improve the overall muon stopping efficiency, which is 80%.

After passing through the degrader, muons come to rest in the aluminum stopper. The stopper is made up of 8 Al blocks, 16 cm wide, 55 cm long, and 6 mm thick. The blocks are made of 99.99% pure aluminum, which is required to maintain the muon spin\(^2\). The stopper is segmented in order to reduce the effective density $\rho_{\text{stopper}} = 0.4\rho_{\text{Al}}$.

\(^2\)Any pure metal will do for this purpose. The basic requirement is that there can be no stray electric fields in the stopper material.
2.6. $\mu^+$ Polarimeter

Positrons from $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay are detected by two scintillation counters located on either side of the Al stopper. Each positron counter is divided into three strips along the beam axis. Each strip is read out by two finemesh PMTs located on either end of the scintillators. This allows us to measure the $e^+$ hit position to within $\Delta r < 10$ cm and $\Delta z = 10$ cm. The positron counters are arranged so that the CW counter in one gap is also the CCW counter in the neighboring gap, as shown in figure 2.9. This tends to cancel many potential systematic errors. There are two large counters (coincidence counters) located on either side of each positron counter. These counters are used at trigger level to suppress backgrounds from neutral particles in the beam; in particular backgrounds caused by $(n,p)$ reactions in the scintillators$^3$.

Although the Al stopper is located outside of the magnet, there is a large $\vec{B}$ field on the order of 150G throughout the polarimeter. The field points along the azimuthal direction, which is also the direction of the transverse polarization. After the muon comes to rest in the stopper, the in-plane components of the spin tend to precess, while the transverse spin is preserved. This design is essential to prevent depolarization of $P_T$ from stray magnetic fields or from the magnetic field of the Earth.

Because the stopper is located outside of the magnet, the $\vec{B}$ field actually fringes.

$^3$When neutrons come into contact with the scintillator, nuclear interactions can produce charged particles (primarily protons). These charged particles produce scintillation light, but typically have such low kinetic energy that they stop within one counter. Thus they can be rejected by requiring a coincidence between multiple counters.
2.6. $\mu^+$ Polarimeter

As non-azimuthal components in the field will depolarize $P_T$, iron field-trimming plates were attached to all of the magnet poles. These plates extend the effective area of the magnet coil, and reduce the non-azimuthal components of the field to below 30G.

To minimize counter inefficiencies, trigger thresholds are set extremely low ($E_{\text{thresh}} < 100$ keV). In order to avoid possible systematics arising from phototube gain variations, the positron counters are calibrated before each run using cosmic rays. Gain drifts are continuously monitored and corrected during the run using data. Drifts are typically less than 5%.

2.6.1 Polarimeter Veto Counters

In order to suppress beam background in the polarimeter, a set of veto counters were installed shortly after the April 1996 run. These counters are designed to either veto beam particles that hit the positron counters, or veto muons that don't stop in the Al stopper. There are four independent veto systems, which are shown in figure 2.10.

The first two counters are designed to reject events in which muons do not stop in the muon stopper. A wedge shaped POLVETO counter is located directly behind the Al stopper to reject events in which the muons stop in the polarimeter stand. A rectangular SUPERVETO counter is located directly above the stopper to complete the coverage. In order to reject beam background caused by halo particles that scatter
Figure 2.9: An endview of the E246 detector showing the gap structure of the magnet and the polarimeter.
2.7. **The Trigger**

or interact in the target frame, the entire target assembly is surrounded by a set of 13 **BEAMVETO** counters. Beam particles can also be vetoed using the **B0** beam hodoscope counters, which are located just upstream of the Čerenkov counter. In order to improve the rejection of stopped pions, and also to reject scattered particles that punch through the CsI crystals, a set of **URA** veto counters were positioned behind each positron counter.

In general, application of the veto counters coupled with veto information from the TOF and POLTRIG counters can reduce backgrounds by as much as 40%. This reduces the ambient background level (which is typically 18%) to 11%.

### 2.7 The Trigger

The E246 trigger system is designed for high acceptance of the two main T-violating modes $K_{\mu3}$ and $K_{\mu2\gamma}$, and as such is relatively inflexible. It consists of three components, a level 0 piece that arrives in coincidence with the $\mu^+$, a level 1 photon trigger that arrives 150 ns later, and a level 2 positron trigger that arrives 22 $\mu$s after the prompt.

The fast trigger (level 0) consists of 3 separate elements. We require a hit in the Čerenkov kaon ring with at least 4 struck phototubes ($\tilde{C}_k$). We require that one fiducial counter be hit (FID). And finally, we require a charged particle within ±
2.7. The Trigger

Figure 2.10: The polarimeter assembly showing the muon stopper, the positron counters, and the veto counters.

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one gap of the hit fiducial counter. The presence of a charged particle is determined by requiring that both gap counters (TOF and POLTRIG) are hit. The POLTRIG counter is a thin scintillator located directly behind the Cu degrader. Its main purpose is to suppress $K_{\pi2}$ decays, which tend to interact and stop in the Cu wedge. The TOF counter is used to generate a TOF stop signal, and is located directly in front of the muon degrader. The gap trigger counters are shown in figure 2.10. The prompt trigger logic can be summed up as follows:

$$L_0 = C_k \cdot (\text{FID}_{n-1} + \text{FID}_n + \text{FID}_{n+1}) \cdot \text{TOF}_n \cdot \text{POLTRIG}_n$$  \hspace{1cm} (2.3)

The level 1 trigger from the calorimeter arrives 150 ns after the level 0 trigger, which is due to the long shaping time of the crystals. A photon trigger is issued if any crystal has more than 5 MeV of energy deposited within a 150 ns window. In order to remove events in which the charged particle hits the CsI before entering the magnet, the photon trigger is vetoed if any crystal around the hit gap has more than 5 MeV of deposited energy within the trigger timing window. As a background monitor, the photon trigger is eliminated (prescaled) every 50 triggers. The level 1 logic can be summed up:

$$L_1 = (1 \gamma \cdot \overline{\text{GAPVETO}}) + \frac{1}{50} \text{PRESCALE}$$  \hspace{1cm} (2.4)

The level 2 trigger requires that either positron counter in the hit gap is struck,
2.8. The DAQ

along with both of its coincidence counters. Because of the long (2.2 $\mu$s) muon lifetime and the need to measure beam background levels in the counters, the level 2 delay is 22 $\mu$s. During this time, all additional prompt triggers are vetoed. The level 2 trigger logic can be summed up as follows:

$$L_2 = (e^+ \cdot CC_L \cdot CC_R)_{CW} + (e^+ \cdot CC_L \cdot CC_R)_{CCW}$$

(2.5)

The final trigger $T = L_0 \cdot L_1 \cdot L_2$ rate is roughly 60 Hz. Contributions from each stage are broken down in table 2.2.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>Typical Rate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_0$</td>
<td>2000</td>
</tr>
<tr>
<td>$L_1$</td>
<td>250</td>
</tr>
<tr>
<td>$L_2$</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 2.2: Typical trigger rates after each trigger stage.

2.8 The DAQ

The purpose of the data acquisition system (DAQ) is to convert analog information from the detector into digital data, and then write the data to tape for future analysis. The DAQ contains three different types of hardware. FASTBUS, which houses the polarimeter and target time-to-digital converters (TDCs) and the CsI(Tl) transient digitizers (TDs). CAMAC, which houses the spill-monitor scalers. And TKO, which
2.8. The DAQ

houses the TOF and CsI TDCs, and all of the analog-to-digital converters (ADCs). Both FASTBUS and CAMAC are considered “standard” equipment in high energy physics. TKO (TRISTAN-KEK-Online) is a standard developed at KEK for the TRISTAN experiments.

Once the trigger is asserted, all modules begin converting their data. This process takes 500 μs and is dominated by the conversion time of the TKO TDCs. Under optimum beam conditions, the readout dead time is determined by the TKO conversion time and 22 μs-level 2 delay. This gives us:

\[ D.T. = \frac{250 \cdot 22 \, \mu s + 60 \cdot 500 \, \mu s}{0.7 \, s} = 5\% \]  

(2.6)

In reality, RF instabilities at the KEK-PS typically bunch the beam, which reduces the effective spill length. Average dead time is 10%, although depending on beam conditions has been as low as 5% and as high as 30%.

After conversion is finished, all crates (with the exception of the TDs) transfer their data to a VME memory unit. Because the TD data is extremely large (approximately 50% of the entire data volume), each TD unit will buffer the data for an entire spill. Once the spill has ended, data from the VME memory units and from the TDs are transferred to an HP-RT VME single board computer (node:tvhppt.kek.jp) which then writes the data to a digital linear tape (DLT). The entire readout requires roughly 1.5 seconds to complete. The DAQ system uses UNIDAQ software, which is a distributed,
2.8. The DAQ

UNIX based code system originally developed for the Superconducting Supercollider. Data written to tape is stored in raw UNIDAQ binary format. An overview of the DAQ is shown in figure 2.11.

Figure 2.11: An overview of the DAQ system.
2.9 Coordinate Systems

There are two different coordinate systems that are used throughout this analysis. First there is the global system, typically referred to as toroidal coordinates. In this system the \( z \) axis points along the beam direction, the \( y \) axis points towards the 12 o'clock magnet gap, and the \( x \) axis completes a right-handed coordinate system. The toroidal system is commonly used when discussing the target or CsI assembly.

The second system is the local, or gap, coordinate system that is frequently used when discussing the polarimeter. Here, the \( z \) axis still points along the beam. The \( x(r) \) axis points in the radial direction, and the \( y \) axis completes a right handed system. In gap coordinates, the \( y \) axis is equivalent to the vector that connects the center points of the CW and CCW positron counters and is sometimes referred to as the "polarimeter axis" or the "azimuthal axis." The gap system is shown in figure 2.10.

When discussing MWPCs, the \( x \) coordinate runs along the long (high resolution) axis of the chamber while the \( y \) axis runs along the short (low resolution) axis. For C3 and C4, these coordinates are equivalent to the gap coordinates.
2.10 Alignment

As we will see in chapter 4, systematic errors in E246 are dominated by the alignment of the various detector elements. In order to minimize these systematics, all detector elements were aligned precisely with respect to the magnet poles. The MWPCs and muon stopper were installed with a special jigs to ensure reproducibility. The alignment precision for each subsystem is listed in table 2.3.

<table>
<thead>
<tr>
<th>Location</th>
<th>Component</th>
<th>Direction</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECAL</td>
<td>CsI Crystals</td>
<td>(\phi)</td>
<td>7 mm</td>
</tr>
<tr>
<td>ECAL</td>
<td>CsI Crystals</td>
<td>(\theta)</td>
<td>7 mm</td>
</tr>
<tr>
<td>GAP</td>
<td>MWPCs</td>
<td>(x, y)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>GAP</td>
<td>Magnet Shim Plates</td>
<td>(y)</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>GAP</td>
<td>Magnet Shim Plates</td>
<td>(z)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>Al Stopper</td>
<td>(x)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>Al Stopper</td>
<td>(y)</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>Al Stopper</td>
<td>(z)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>(e^+) Counter</td>
<td>(y)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>(e^+) Counter</td>
<td>(z)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Polarimeter</td>
<td>(e^+) Counter</td>
<td>(x)</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

Table 2.3: Initial alignment accuracy of each subsystem.

Despite the precision of the initial alignment, there is a clear indication that each component has moved slightly over time. In particular, a resurvey of positron counter and MWPC positions performed 17 months after installation shows position variations in the MWPCs of more than 1 mm in the azimuthal direction[22]. Similar position offsets have been seen in the \(e^+\) counters. Perhaps even worse, radial variations in
the position of the positron counters as large as 9 mm were observed. Given these somewhat disturbing facts, it is worthwhile to multiply all position resolutions by a factor of 2 or 3 when studying systematic errors.
Chapter 3

Sensitivity to $P_T$ and $Im(\xi)$

The remainder of this thesis deals with the details of the measurement, including the selection and optimization of the analysis cuts, an estimate of the background levels, and an analysis of the systematic errors. This chapter focuses on the former; namely the optimization of analysis cuts and a discussion of the statistical sensitivity.

3.1 An Overview of the Measurement

In E246, we measure the transverse component of the muon polarization as a counting asymmetry between CW and CCW positron counters. The contribution to $P_T$ from a set of $K_{\mu 3}$ events can be written:

$$P_T = \frac{1}{\alpha \cdot \cos(\theta_{PT})} \frac{N_{CW} - N_{CCW}}{N_{CW} + N_{CCW}}$$

(3.1)

where $\theta_{PT}$ is the angle between the decay plane normal vector $(\vec{P}_{\mu} \times \vec{P}_{\pi^0})$ and the
3.2 Data Analysis

polarimeter axis, and \( \alpha \) is the analysing power. The decay plane angle \( \theta_{PT} \) can be measured directly in the detector. The analysing power \( \alpha \) can also be measured using data and will be discussed further in section 3.5.

The statistical error associated with equation 3.1 is obtained from simple counting statistics, and can be written\(^1\):

\[
\delta P_T = \frac{1}{\alpha \cdot \cos(\theta_{PT}) \sqrt{N_{CW} + N_{CCW}}} \prod 1 + \beta_i \cdot \frac{B_i}{S} \quad (3.2)
\]

Where \( B_i/S \) is the background to signal ratio for each class of background, and \( \beta_i \) is the attenuation factor associated with that particular class of background (See appendix B for a discussion of \( \beta_i \)). The goal of this chapter is to estimate the statistical error by measuring the residual background contamination after analysis cuts have been applied, the analysing power, and \( < \cos(\theta_{PT}) > \).

3.2 Data Analysis

The analysis of E246 data is performed in two steps. First, raw UNIDAQ data written by the online HP-RT computer is analyzed in a PASS1 process. During PASS1, data from each spill is sorted into events and converted from raw UNIDAQ format into YBOS format. In addition to simple event building, pulse shape fitting is done on the digitized TD pulses, and the raw pulse shape data is discarded. Tracking analysis

\(^1\)See appendix B for more details.
3.2. Data Analysis

is also performed at this step, and the track fit results are stored in a YBOS bank.

Data from PASS1 is written to three separate analysis streams: events with one photon cluster, events with two or more photon clusters, and events in which the PRESCALE (0γ) trigger was active. Events in which a track cannot be reconstructed or that contain no photon clusters\(^2\) are discarded. Because PASS1 uses crude timing calibrations, we use loose timing cuts on the CsI TDCs and TD modules in order to prevent accidental data loss.

Once PASS1 is finished, the sorted YBOS data is analyzed again using tighter timing cuts. This analysis is referred to as PASS2. Because the CPU intensive jobs (track fitting, TD fitting) have been done in PASS1, PASS2 analysis is approximately a factor of six faster than PASS1. The PASS2 process writes output data as HBOOK N-TUPLES, which can then be further analyzed using UWFUNC user scripts or FORTRAN calls to CERNLIB.

YBOS is a standard developed for data storage by Fermilab for use in their large collider experiments. An N-TUPLE is a data structure used to store many events that each have many variables. It was developed at CERN and is supported in the CERNLIB distribution.

\(^2\)All events with no in-time clusters are discarded except if the PRESCALE bit is active.
3.3 Analysis cuts

The cuts used in this analysis are designed to reject backgrounds from other kaon decay modes as well as from the $\pi^+$ beam halo. Cuts can be divided into three categories: $\mu^+$ tracking cuts, $\pi^0$ reconstruction cuts, and counter timing and energy cuts. In general, the majority of cuts in this analysis were optimized to minimize the statistical error.

3.3.1 Tracking Cuts

In order to separate $K_{\mu3}$ events from all other kaon decay modes, we must select muons which lie in a narrow momentum range $100 < P_\mu < 190$ MeV/c. Furthermore, we must apply cuts to ensure high quality reconstruction of the track, and correct for $dE/dx$ ionization loss in the target. The tracking analysis is a four step process:

- Calculate the $K^+$ stopping position.
- Calculate the charged particle momentum in the gap using C2,C3,C4.
- Correct for $dE/dx$ ionization loss in the target by range.
- Apply quality cuts to ensure good tracking and to eliminate scattering.

The stopping position of the kaon in the $x-y$ plane is tagged with the scintillating fiber target. Because incident kaons typically deposit more than 20 MeV in a single target fiber, while minimum ionizing particles deposit less than 2 MeV, the kaon
3.3. Analysis cuts

vertex can be chosen by selecting the highest energy fiber. This simple algorithm can find the correct kaon vertex to within ± 1 fiber (0.5 mm) with an efficiency better than 98%. A typical $K_{\mu 3}$ event is plotted in figure 3.1, showing a high energy kaon fiber, a set of low energy fibers from a minimum ionizing track, and one struck fiducial (trigger) counter.

In order to remove backgrounds, timing cuts are applied to the kaon stop fiber ($2 < T_K < 50$ ns) as shown in figure 3.2. This rejects both beam related backgrounds from $\pi^+$ scatters, as well as kaon decay-in-flight. In order to eliminate events in which the kaon fails to stop in the target, we require that the kaon fiber has more than 5 MeV of deposited energy.

The $z$ position of the stopped kaon is reconstructed using the hit position on C2, target, and the ring counters. Tight timing cuts are applied to reject backgrounds in the ring system ($|t_{\text{ring}}| < 15$ ns). Since the track fit algorithm uses neither target nor ring counter information, tracking and vertex reconstruction can be performed independently. Thus, in the case where multiple rings are struck, we can usually select the correct counter by choosing the ring that is closest to the MWPC track.

Track fitting in the magnet is performed using only the MWPC information; beginning with a hit on C4 and extrapolating backwards through the magnet to C2. Hits are required in all 6 MWPC layers, which means that fit has 1 degree of freedom. Gain calibration is applied to each cathode strip, giving a position resolution
3.3. Analysis cuts

of $\sim 200 \mu m$ in $x$ and $2 mm$ in $y$. Once the charged track has been reconstructed, we seek to cut events in which the momentum has been mismeasured either due to scattering or pion decay in flight. In particular, there are five processes which can lead to a faulty momentum reconstruction:

- $K_{\pi 2}$ decay in the target, followed by $\pi^+ \rightarrow \mu^+$ decay in flight (DIF).
- Scattering the target, fiducial, or ring counters.
- Scattering in the aluminum CsI supports.
- Scattering in the CsI ($< 5$ MeV deposited in a crystal).
- Scattering in the magnet pole face.

To eliminate pion DIF and scattering before the magnetic spectrometer, we require that the distance between the track and the closest hit ring counter is less than 2.5 cm (RING) and that the distance of closest approach between the track and the kaon stopping fiber is less than 2.0 cm (TARG). Both of these cuts were optimized using Monte Carlo by maximizing the figure of merit:

$$F.O.M. = \sqrt{\frac{N}{1 + \frac{N_{x2}}{N_{\mu3}}}}$$ (3.3)

As shown in figure 3.3, these cuts strongly suppress below-the-peak $K_{\pi 2}$ background. Pole-face scattered events are suppressed by requiring that the fit $\chi^2$ be less than
10. Pole-face scatters are potentially dangerous as muons passing through iron can develop a spurious polarization. Furthermore, energy loss in the pole face and small angle scattering will lead to a momentum mismeasurement, which will increase the background contamination. As shown in figure 3.4, the majority of pole face scattered events have $\chi^2 > 10$. A comparison between data and Monte Carlo$^3$ (which does not include pole face effects) shows that the residual contamination from pole scattered events after applying the $\chi^2$ cut is less than 1.3%.

The momentum spectrum after all cuts have been applied is shown in figure 3.5. The resolution, determined from the width of the $K_{e2}$ peak, is 2.5 MeV/c.

3.3.2 $\pi^0$ Reconstruction

$\pi^0$ is detected through its decay chain $\pi^0 \rightarrow \gamma \gamma$, which has a branching ratio of 98.8%[5]. The majority of recorded events contain both photons, which allows us to fully reconstruct the $\pi^0$ direction, energy, and invariant mass. However because of the loose photon requirement at the trigger level ($\geq 1\gamma$), we also collect events in which one photon misses the calorimeter. Since high energy photons tend to travel along the flight direction of the $\pi^0$, these events can potentially be utilized in the

---

$^3$The E246 full simulation uses GEANT 3.1 and contains a complete trigger and background model. It is referred to as GMC (General Monte Carlo).
3.3. Analysis cuts

Figure 3.1: A typical $K_{\mu3}$ event shown in the $x-y$ plane. The event display shows the kaon stopping point (light hashed fiber), the $\mu^+$ track through the target (dark hashed fibers) and #9 fiducial counter, and the track from the MWPCs.
3.3. Analysis cuts

Figure 3.2: (a) Maximum energy deposited in a single target fiber showing both the kaon peak and a bump at $E_K \sim 3$ MeV from mistagged minimum ionizing events. The spikes which appear above $E_K > 20$ MeV are the result of ADC saturation in individual target channels. (b) Decay time spectrum of the highest energy fiber. $T_0$ timing is extracted from the fiducial counters, so this plot is actually $T_\mu - T_K$. From an exponential fit, $\tau_k = 12.7$ ns.

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3.3. Analysis cuts

Figure 3.3: (a) Events which are rejected by cuts on the target and ring. The majority of events with $P_\mu < 190$ MeV/c are $K_{\pi2}$ decays in which the pion has scattered or decayed in flight to a muon. (b) Angle between the charged particle and the $\pi^0$ for rejected events with $P_\mu < 190$ MeV/c. The peak at $\theta \sim 180^\circ$ is consistent with large $K_{\pi2}$ contamination.
3.3. Analysis cuts

Figure 3.4: In order to estimate contamination from pole scattered events, we extrapolate the charged track from C4/C3 back to C2 and calculate the \( y \) intercept of the track on C2. Plotted here is the fraction of events with \(|C2Y| > 10\) cm. Solid circles are data and empty circles are GMC. Both data and GMC are normalized using the \( \chi^2 < 1 \) bin. The excess in the data for larger \( \chi^2 \) is caused by magnet pole scattering, which is not included in GMC.

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3.3. Analysis cuts

Figure 3.5: (a) Momentum spectrum after all cuts have been applied. (b) The distance between the track and the closest hit ring counter (c) The distance of closest approach between the track and the struck target fiber.

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### 3.3. Analysis cuts

<table>
<thead>
<tr>
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<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>EHOLE</td>
<td>$E_{\text{hole}} &lt; 5 \text{ MeV}$</td>
<td>Reject $\mu^+$ hits in the ECAL</td>
</tr>
<tr>
<td>PMU</td>
<td>$100 &lt; P_\mu &lt; 190$</td>
<td>Reject $K_{\mu 2}, K_{\pi 2}$</td>
</tr>
<tr>
<td>CHI2</td>
<td>$\chi^2 &lt; 10$</td>
<td>Reject scatters, $K_{\pi 2}$ in-flight decay</td>
</tr>
<tr>
<td>TARG</td>
<td>$</td>
<td>\vec{X}<em>{\text{targ}} - \vec{X}</em>{\text{track}}</td>
</tr>
<tr>
<td>RING</td>
<td>$</td>
<td>Z_{\text{ring}} - Z_{\text{track}}</td>
</tr>
<tr>
<td>ENK</td>
<td>$E_{K^+} &gt; 5 \text{ MeV}$</td>
<td>Target vertex selection</td>
</tr>
<tr>
<td>TIMEK</td>
<td>$2 &lt; T_{\text{Ber}} &lt; 50 \text{ ns}$</td>
<td>Target vertex selection</td>
</tr>
<tr>
<td>$M_{\pi^0}$</td>
<td>$50 &lt; M_{\pi^0} &lt; 180 \text{ MeV/c}^2$</td>
<td>$2\gamma \pi^0$ selection</td>
</tr>
<tr>
<td>$MM^2$</td>
<td>$MM^2 &gt; -30000 \text{ MeV}^2/c^4$</td>
<td>$2\gamma$ kinematic cut</td>
</tr>
<tr>
<td>$E_{\text{low}}$</td>
<td>$70 &lt; E_\gamma \text{ MeV}$</td>
<td>$1\gamma \pi^0$ selection</td>
</tr>
<tr>
<td>$E_{\text{sign}}$</td>
<td>$E_\gamma &lt; 210 - 0.94 \times</td>
<td>90 - \theta_{\mu\pi}</td>
</tr>
<tr>
<td>PROMPT</td>
<td>$t_\mu &gt; 15 \text{ ns}$</td>
<td>Reject $K_{e3}$ Decays</td>
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<tr>
<td>TMUON</td>
<td>$t_\mu &lt; 8 \mu$s</td>
<td>Reject Beam Backgrounds</td>
</tr>
<tr>
<td>TOPPID</td>
<td>$m^2 &gt; 3500 \text{ MeV}^2/c^4$</td>
<td>Reject $K_{e3}$ Decays</td>
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</tbody>
</table>

Table 3.1: Analysis cuts used in this thesis.

physics analysis. The one and two photon streams must be analyzed separately, and will have different sensitivities and background levels.

Before photon reconstruction is attempted, we must first reject events in which the muon strikes a CsI crystal. These events are problematic because they will contain a "fake" photon cluster, and will also have very poor track resolution. To eliminate these events, we reject any event in which a CsI crystal adjacent to the hit gap has an in-time hit ($E_{\text{hole}} > 5 \text{ MeV}$). Since these events comprise a large fraction of the trigger rate (~ 40%) this cut was eventually implemented in hardware to reduce DAQ dead time.

Two photon events contain the most information and make up the bulk of the data.
3.3. Analysis cuts

sample. Events in this category often have more than two photons in the calorimeter because of backgrounds from scattered beam halo particles that strike the CsI. In order to keep acceptance for $K_{\mu3}$ events high, we accept all events with NCLUST ≥ 2. For events with three or more $\gamma$ clusters, we select the pair that gives the best invariant mass. Invariant mass resolution is limited by leakage of shower particles around the muon holes in the CsI calorimeter. This produces a long tail on the low mass side, and means that we cannot cut too strongly on $M_{\pi^0}$ without throwing away most of the events.

In order to optimize the $\pi^0$ mass cut, we maximize the quantity:

$$F.O.M. = \frac{\sqrt{\mathcal{N}} \cdot \langle \cos(\theta_{PT}) \rangle}{\sqrt{1 + \frac{B}{S}}}$$

(3.4)

where $B$ is the background contamination from $K_{\pi2}$ events. As shown in figure 3.6, both the high and low end mass cuts exhibit broad maxima. We choose cuts well into the flat regions, $50 < M_{\pi^0} < 180$ MeV$/c^2$. As shown in figure 3.7(a), this cut eliminates very few events. In order to ensure the quality of the reconstructed event, an additional cut is placed on the event kinematics, $MM^2 > -30000$ MeV$^2/c^4$. This cut eliminates most of the non-physical region, and is used to reduce beam backgrounds as well as events containing a kaon decay-in-flight.

Cuts for the one photon sample are much simpler because there is only one useful variable: the photon energy. We are forced to cut low energy photons because they do
3.3. *Analysis cuts*

not accurately tag the $\pi^0$ direction. We can optimize the energy cut using a similar optimization technique that was employed for the $2\gamma$ sample. As shown in figure 3.8, there is a clear maximum at $E_{\gamma} > 70$ MeV. In order to reject beam accidentals as well as $K^+$ DIF we use a two dimensional energy cut: $E_{\gamma} < 210 - 0.94 * |\theta_{\mu\pi} - 90^\circ|$ MeV. Both this cut and the two photon $MM^2$ cut will be discussed further in section 3.4.5.

![Figure 3.6: The normalized Figure Of Merit as defined in equation 3.4. Both the low and high mass sides exhibit broad plateaus, indicating that the particular choice of $\pi^0$ mass cut is not critical.](image-url)
3.3. Analysis cuts

![Figure 3.7: Cuts on the $\pi^0$ mass (a) and event Missing Mass Squared (b). The $MM^2$ is calculated assuming kaon decay at rest.](image)
3.3. Analysis cuts

Figure 3.8: (a) $\theta_\pi - \theta_\gamma$. Only high energy photons accurately tag the $\pi^0$ direction. (b) Normalized Figure of Merit, indicating that the optimal cut is $E_\gamma > 70$ MeV.
3.4 Backgrounds

Having reconstructed both the muon and pion kinematics as well as the kaon stopping position, we are now prepared to tackle the thorny issue of backgrounds. There are two types of backgrounds: physics backgrounds and beam backgrounds. Physics backgrounds are events in which a kaon decays into something besides a muon, pion and neutrino, but yet is mis-classified as a $K_{\mu 3}$. Beam backgrounds are events in which scattered beam halo particles (mainly $\pi^+$s) interact in the detector. These can occur either in coincidence with a real $K_{\mu 3}$ decay (partial mis-reconstruction), or can themselves produce enough secondary particles to mimic a $K_{\mu 3}$ event (total mis-reconstruction).

3.4.1 Physics Backgrounds

There are three major physics backgrounds that must be dealt with: $K^+ \rightarrow \pi^+\pi^0$, $K^+ \rightarrow \pi^+\nu_e\pi^0$, and $K^+ \rightarrow \pi^+\pi^0\pi^0$ decays. Physics backgrounds can generally be reduced to less than 10% after analysis cuts are applied. More importantly, none of these backgrounds is polarized, so at worst they simply dilute the statistical sensitivity by a factor of 1+B/S. As we will argue in Chapter 4, they do not affect the systematic error.
3.4. Backgrounds

\[ K^+ \rightarrow e^+ \nu_e \pi^0 \]

\( K_{e3} \) decays are kinematically very similar to \( K_{\mu3} \) decays, and as a result cannot be separated by momentum or photon cuts. However, the positron counter timing spectrum is very distinctive for \( K_{e3} \) events, which allows us to virtually eliminate them. In fact, there are only two ways that a \( K_{e3} \) event can satisfy the E246 trigger. First, if the positron initiates an electromagnetic shower in the muon degrader or muon stopper and a secondary particle hits one of the positron counters. This occurs in coincidence with the kaon decay as shown in figure 3.9a, and is eliminated by the PROMPT cut on the muon lifetime: \( t_\mu > 15 \text{ ns} \). Second, even if the position does not hit one of the positron counters, a beam accidental can hit one of the counters up to 22 \( \mu s \) after the kaon decay.

In order to reject the second class of events, a TOF measurement is made in the spectrometer, as shown in figure 3.9b. Positron events can be rejected by requiring \( m^2 > 3500 \text{ MeV}^2/c^4 \). This cut rejects 48\% of the flat background in the positron counters, which is the result of a real \( K_{e3} \) decay in coincidence with a beam accidental. The remaining \( K_{e3} \) contamination can be removed on a statistical basis by fitting the muon decay spectrum to an exponential plus a constant. After applying all cuts and fitting, the unaccounted for \( K_{e3} \) contamination is less than \( 10^{-3} \), which is negligible compared to other background sources.
Figure 3.9: (a) Timing in the positron counters. The prompt peak at $t=0$ is the result of positron generated showers from $K_{\ell 3}$ decays. (b) TOF $m^2$ spectrum for events with momentum $100 < P < 190$ MeV/c clearly showing positron and muon peaks. (c) Events rejected by application of the TOF $m^2$ cut showing the PROMPT peak and flat background.
3.4. Backgrounds

\[ K^+ \rightarrow \pi^+ \pi^0 \pi^0 \]

\( K_{\pi 3} \) decays are kinematically quite different from \( K_{\mu 3} \) events. First, there is an additional \( \pi^0 \), so one might imagine that these events can be vetoed based on the presence of extra photons. In practice, however, this method is extremely costly in statistics, so we instead choose to accept all events with \( \text{NCLUST} \geq 2 \). Worse still, because there are four photons, the invariant mass spectrum is actually sharper for \( K_{\pi 3} \) events as shown in figure 3.10b. Therefore the \( M_{\pi^0} \) cut provides little suppression.

The best defense against \( K_{\pi 3} \) decay is simply the acceptance of the spectrometer. The charged pion from \( K_{\pi 3} \) events has a kinematic endpoint of 133 MeV/c. After ionization loss in the target, most pions do not have a high enough momentum to pass through the spectrometer. Those that do typically undergo nuclear interaction or simply stop from ionization loss in the copper muon degrader. Because the POLTRIG is used at the trigger level to select the hit-gap, these events fail to satisfy the trigger requirement. As a result, the estimated background contamination is less than 0.5%.

\[ K^+ \rightarrow \pi^+ \pi^0 \]

\( K_{\pi 2} \) events are by far the most dangerous of all the physics backgrounds. They have both a good acceptance in the detector and a high branching ratio of 21%. Our main defenses are the muon momentum cut and a cut on the opening angle between the charged particle and the neutral pion.
3.4. Backgrounds

![Diagram](image)

**Figure 3.10:** (a) GMC generated momentum spectrum for accepted $K_{\pi 3}$ events. (b) Invariant mass spectrum for $K_{\pi 3}$ events. Because there are two $\pi^0$s and four photons, the invariant mass distribution is actually sharper for $K_{\pi 3}$ events as compared to $K_{\mu 3}$ decays because we select the pair that minimizes $|M_{\pi^0} - 134 \text{ MeV}/c^2|$. (c) Number of reconstructed clusters for $K_{\pi 3}$ decays. Because the detector is non-hermetic, the probability of missing one photon is nearly 30%.
3.4. Backgrounds

Because $K^+ \rightarrow \pi^+\pi^0$ decays involve only two bodies in the final state, they produce a mono-energetic $\pi^+$ with momentum of 205 MeV/c. The majority of $K_{\pi 2}$ events are therefore eliminated by the muon momentum cut. However, it is possible to mismeasure the $\pi^+$ momentum, typically as the result of scattering or after a $\pi \rightarrow \mu$ decay in flight. Both processes tend to push the pion momentum into the $K_{\mu 3}$ signal range, as shown in figure 3.11a. After applying momentum and track quality cuts, we can reduce the background level to 13%.

In order to conserve total momentum, the $\pi^+$ and $\pi^0$ must be back-to-back. Additional background suppression can be achieved by cutting on the opening angle of the $\pi^+$ and $\pi^0$, as shown in figure 3.11b. Hemisphere cuts on $\theta_{\mu\pi}$ to select forward or backward events require that the opening angle be less than 160° (see section 3.6). The final result is a background contamination of roughly 4% in the $2\gamma$ stream. Because $\pi^0$ direction resolution is worse in the $1\gamma$ sample, this cut is not as effective, and as a result background contamination is typically much worse. Monte Carlo simulations predict 9% $K_{\pi 2}$ contamination in the one photon sample.

3.4.2 Beam Backgrounds: CsI

Backgrounds from scattered beam halo particles appear both in the CsI calorimeter as well as the positron counters. Because the $\pi : K$ ratio at the entrance to the

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Figure 3.11: The (a) momentum and (b) opening angle distributions for $K_{n3}$ (dashed histogram) and $K_{n2}$ events (solid histogram). The $K_{n2}$ spectra were scaled by a factor of 6.6 in order to account for the difference in branching ratios. Both plots were made using GMC 4.3.
3.4. Backgrounds

Čerenkov counter is around 7:1, these backgrounds are quite large and can have a serious impact on the final result. We will consider each subsystem individually.

In the CsI calorimeter, accidentals can have two effects. First, in-time accidentals will appear as "extra" or "fake" photons that will fool the $\pi^0$ reconstruction software. Reconstructing a $\pi^0$ with one real and one fake photon results in a mismeasurement of both the energy and flight direction of the pion. Second, accidentals can "pile up" with real photon clusters, which will lead to a mismeasurement of the photon energy. Both of these effects can be expressed as a reduction in $\langle \cos(\theta_{PT}) \rangle$. In order to understand how these two effects will alter the final result, we must consider the one and two photon streams separately.

**One Photon Stream**

For events with one photon cluster, beam backgrounds will only be problematic when we miss both photons from the $\pi^0$ and reconstruct one beam accidental in the CsI. The rate for accidental reconstructed photons can be estimated by eliminating the photon requirement in the trigger, and then studying $K_{\mu2}$ decays. Because any photons associated with a $K_{\mu2}$ decay in the target must be caused entirely by beam accidentals, we can directly measure both the accidental rate and energy spectrum$^4$.

Analysis of prescaled data shows that there is a 13% probability of having an

$^4$In reality, there is roughly a 0.2% contamination of real photon events from $K_{\mu2}$ decays in our background sample.
3.4. Backgrounds

accidental photon cluster in any given event. Fortunately, since the timing cuts on the CsI can be made very tight, a large fraction of the accidental energy usually lies outside the CsI timing window. The result is that "fake" photons have a very soft spectrum as shown in figure 3.12b. In contrast, the signal region shown in figure 3.12a is much broader and tends to peak at $E_\gamma \sim 70$ MeV. Beam backgrounds are therefore strongly suppressed by the $E_\gamma^{\text{low}}$ cut that is employed to select hard photons.

In order to estimate the residual contamination, both the rate and photon energy spectrum from accidentals were used as inputs for GMC. We then calculate the reduction in $<\cos(\theta_{PT})>$ after applying our standard kinematic cuts. We find an 11% reduction in $<\cos(\theta_{PT})>$ as a result of CsI accidentals. Although we also mismeasure the photon energy, this does not dramatically effect the final result as only the $\pi^0$ direction matters.

**Two Photon Stream**

Effects of beam backgrounds in the $2\gamma$ stream are nearly as bad, despite the additional constraints. In the two photon sample, we do not place a energy cutoff on the energy of a single photon. We only require that $E_{\gamma_1} + E_{\gamma_2} > 100$ MeV. Therefore, we accept more accidentals, which we then attempt to eliminate using $M_{\pi^0}$ and $MM^2$ cuts. Furthermore, since we reconstruct the $\pi^0$ using two photons, mismeasuring the energy of one photon translates into a mismeasurement of the $\pi^0$ direction. This was
3.4. Backgrounds

not the case in the $1\gamma$ stream.

We can simulate background effects for the $2\gamma$ stream by feeding raw ADC and TDC spectra from prescaled $K_{\mu2}$ events directly into GMC. Both the $\pi^0$ mass and momentum spectra generated by GMC are in good agreement with raw data. As shown in figure 3.13, the angular resolution of the $\pi^0$ becomes much worse once beam accidentals are added. With accidentals included, we find a reduction of 6.7% in $<\cos(\theta_{PT})>$. This is more severe than the attenuation factor from shower leakage, which is estimated at 5%.

3.4.3 Beam Background: Polarimeter

In addition to beam related backgrounds in the CsI calorimeter, we also observe a large beam induced background in the positron counters. The background is approximately flat in time, and can be extracted by fitting the muon decay spectrum. As long as this background is flat\(^5\), the background level and the true number of signal events can be measured. The overall effect is a reduction of the statistical power of the measurement by:

\[
N_{\text{eff}} = \frac{S}{1 + 2 \cdot B/S} \tag{3.5}
\]

\(^5\)Non-flat backgrounds will be discussed in Chapter 4.
3.4. Backgrounds

Figure 3.12: (a) Single photon energy in the signal region, $100 < P < 190$ MeV/c. (b) Single photon energy for $K_{\mu 2}$ events. Because of tight timing cuts on the CsI calorimeter, fake photons tend to be very soft and are strongly suppressed by requiring $E_\gamma > 70$ MeV.
3.4. Backgrounds

Figure 3.13: The angular resolution of the electromagnetic calorimeter: $\theta_{\text{M.C.}} - \theta_{\text{Reconstructed}}$. (a) Only including effects of shower leakage. (b) Including both leakage and beam accidentals. Beam accidentals decrease the angular resolution by a factor of 1.7.
3.4. Backgrounds

We can optimize the cut on the muon lifetime by maximizing $N_{\text{eff}}$ and find

$15 < t_\mu < 8000$ ns.

Backgrounds can be strongly suppressed by the application of polarimeter veto cuts. Specifically, we construct a three fold veto cut. First, we reject $K_{e3}$ events using the TOFPID cut. This rejects 48% of the beam background in the polarimeter. Second, we reject events in which the TOF counter has a hit in coincidence (10 ns) with the positron counter. This eliminates events in which scattered charged particles pass through the magnet gap and hit the positron counters after the kaon decay. Finally, we reject events that have a coincidence between the positron counter and beam veto counter within 15 ns. As shown in figure 3.14, these cuts reduce the background from 18% to 11%.

In addition to individual hit vetoes, we also veto entire events if the muon fails to stop in the Al stopper. These events are problematic because the fringing magnetic field is not well controlled or measured outside of the stopper. Additionally, while the muon stopper is aligned well enough to prevent spurious geometrical systematics, other objects surrounding the stopper (Al polarimeter supports, etc.) are not. In order to veto this class of events, we reject events in which either the PV, SV, or UV counters has a hit within 10 ns of the kaon decay. Finally, we veto any event if either $e^+$ or coincidence counter TDC in the hit gap has overflowed$^6$.

$^6$We use multi-hit TDCs for all veto, coincidence, and $e^+$ counters. These TDCs (LeCroy 1877) have a depth of 16 hits.

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3.4. Backgrounds

Figure 3.14: Polarimeter backgrounds with no vetoes applied (a) compared to backgrounds with TOF and BEAMVETO cuts applied (b). Both histograms are fit to an exponential plus a flat background. The small bump at $t \sim 600$ ns is the result of $\mu$SR oscillations in the polarimeter fringing field. In both plots, the TOFPID cut has already been applied, which reduces background by 50%.

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3.4.4 Beam Background: $\pi^+$ Scatters

Until now, we have only considered backgrounds in which a $K_{\mu3}$ decay occurs in coincidence with a beam $\pi^+$. However, beam halo pions can also produce another type of background. Specifically, pions passing through the BeO degrader or the fiber target can undergo a nuclear interaction producing enough secondary particles to "mimic" a real $K_{\mu3}$ event. As illustrated in figure 3.15, these events are likely to be boosted forward, since the charged pion must have $P_z > 0$ when it interacts.

In order to study such events, we took 20 hours of data using a pion Čerenkov trigger:

$$\text{TRIG} = C_{\pi} \cdot \text{FID} \cdot \text{TOF} \cdot \text{POLTRIG} \cdot 3e^+ \cdot 1\gamma \cdot \frac{1}{50} \text{PRS}$$  (3.6)

As indicated in figures 3.16 and 3.17, these events tend to peak in the forward direction, have non-descript momentum distributions, and do not exhibit an exponential lifetime in the target as do real kaon decays. After applying the standard delayed coincidence ($T_K > 2$ ns) and kinematic cuts, as well as the momentum reconstruction requirement, this class of background is strongly suppressed. Not including suppression from the kaon Čerenkov counter in the trigger, we find background contaminations of:
The kaon Čerenkov counter, which has a timing gate of 100 ns\textsuperscript{7}, should suppress the 1 MHz pion background by an additional factor of ten. These levels are therefore completely negligible.

\begin{equation}
2\gamma : \frac{B}{S} < 5 \times 10^{-4} \quad (3.7)
\end{equation}

\begin{equation}
1\gamma : \frac{B}{S} < 5 \times 10^{-3} \quad (3.8)
\end{equation}

\textsuperscript{7}This is the trigger timing gate. If necessary, a tighter cut can be applied at the offline level.

Figure 3.15: A cartoon of a typical $\pi^+$ nuclear interaction in the target. Since the pion is moving in the forward direction ($P_z > 0$), the interaction products will be boosted forward.
3.4. Backgrounds

Figure 3.16: (a) The charged particle momentum spectrum after requiring no in-time hits in the kaon Čerenkov ring. The spectrum is broad and relatively featureless. (b) Target timing spectrum for background events. Almost all events lie in the prompt region. (c) Maximum energy deposited in a single target fiber.
3.4. Backgrounds

Figure 3.17: Distribution of $\pi^0$ polar angle for background events. The majority of reconstructed $\pi^0$'s are in the forward direction, which is consistent with pions interacting in the target or degrader, producing secondaries (or scattering), and hitting the CsI(Tl) in the forward hemisphere.

3.4.5 Beam Background: $K^+$ Decay-in-Flight

The last major background that we must consider is $K^+$ decay-in-flight (DIF). Kaon DIF is particularly dangerous for several reasons:

- Because of the boost, we mismeasure $P_\mu \times P_\pi$
- We also mismeasure OPENANG, leading to increased $K_{\pi2}$ contamination.
- Systematics cancelation relies on forward/backward symmetry, which is broken by DIF.

It is therefore preferable to reduce kaon DIF as much as possible.

There are two quantities that we can use to reject DIF events. First, we expect that DIF events will tend to have a prompt hit in the target. As shown in figure 3.18, the charged particle momentum spectrum is severely distorted for prompt kaons, which is the result of the $K^+$ Lorentz boost. Secondly, we can reconstruct the missing mass.
of the event, assuming a kaon decay at rest:

\[ M M^2 = (P_K^\mu - P_\mu - P_\pi^\mu)^2 \]  

\[ P_K^\mu = (M_K, 0, 0, 0) \]  

For decays-at-rest, this should be the neutrino mass (zero). However, if the kaon has an initial boost, then the \( M M^2 \) will be shifted to negative values, as shown in figure 3.19. To reject \( K^+ \) DIF, we require \( T_K > 2 \) ns and \( M M^2 > -30000 \) MeV/c\(^4\).

Decay in flight is harder to reject in the one photon stream because incomplete kinematic reconstruction of the \( \pi^0 \) prevents us from applying a missing mass cut. The equivalent cut in this case is \( E_\gamma^{\text{high}} \):

\[ E_\gamma^{\text{high}} : E_\gamma < 210 - 0.94 \times |\theta_{\mu\pi} - 90^\circ| \text{ MeV} \]  

This cut eliminates hard photons from the extreme forward (and backward) regions. As shown in figure 3.20, decay-in-flight events are boosted forward and have elevated photon energies.

In order to estimate residual DIF contamination, we measure the numerical asymmetry between the forward and backward samples:

\[ A = \frac{N_F - N_B}{N_F + N_B} \]
3.5. The Analysing Power

for three different regions in $T_K$ ($-2 < T_K < 2$ ns, $2 < T_K < 6$ ns, $6 < T_K < 50$ ns).

The later time region is used to normalize the F/B acceptance difference. We find a contamination of 2.3% for the $1\gamma$ sample and 1.9% for the $2\gamma$ sample. The rejection power of each DIF cut is listed in table 3.2.

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<th>Rejection Power</th>
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<td>$2 &lt; T_K &lt; 50$ ns</td>
<td>0.44</td>
</tr>
<tr>
<td>$4 &lt; T_K &lt; 50$ ns</td>
<td>0.87</td>
</tr>
<tr>
<td>$MM^2$</td>
<td>0.51</td>
</tr>
<tr>
<td>$E_{\gamma}^{\text{high}}$</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 3.2: The rejection power of each DIF cut. Rejection for a particular cut is defined as $R = 1 - f_{\text{cut}}/f_{\text{uncut}}$ where $f$ is the fraction of DIF contamination.

3.5 The Analysing Power

The analysing power, $\alpha$, is a constant of proportionality that relates the measured asymmetry to the true polarization. In principle, we can calculate $\alpha$ exactly from the $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ decay kinematics, the geometry of the polarimeter, and the components of the fringing field in the muon stopper. However, in order to eliminate Monte Carlo dependence, a better approach is to measure $\alpha$ directly using $K_{\mu3}$ events.

The basic idea behind this measurement is to select events in which the decay plane, $\vec{P}_\mu \times \vec{P}_\pi$, lies in the plane of the polarimeter. The in-plane components of the
3.5. The Analysing Power

Figure 3.18: The charged particle momentum spectrum for different kaon decay times. The prompt regions ($T_K < 2$ ns) show clear distortion of the spectrum as evidenced by the broadening of the $K_{\pi 2}$ peak. This is the result of the boost from $K^+$ DIF contamination.
Figure 3.19: The Missing Mass distribution for different kaon decay times. The long negative $MM^2$ tail at prompt times is the result of $K^+$ DIF.
3.5. The Analysing Power

Figure 3.20: $E_\gamma^{\text{high}}$ cut for events with only one photon. As shown, the early time regions have a large contamination from $K^+$ DIF events, which tend to be boosted into the forward direction and have higher photon energies. The GMC plot contains Monte Carlo $K_{\mu3}$+accidentals events, and does not simulate kaon decay-in-flight.
3.5. The Analysing Power

polarization, in particular the normal polarization $P_N$, can then be directly measured. By comparing the observed counting asymmetry with the expected polarization calculated using GMC, we can extract the analysing power.

In order to select a $P_N$ enriched sample for this measurement, we use the "IN-CONE" cut, which is constructed using a three step procedure. First, rotate coordinates about an axis perpendicular to both $\vec{P}_\mu$ and $\hat{z}$ and so that $P_\mu^x = 0$. Then, rotate coordinates about $\hat{z}$ so that $P_\mu^z = 0$. In the new coordinate system, we select pions which lie within a cone centered along the $x$ axis with half-angle $\theta_c$, as shown in figure 3.22. In $K_{\mu3}$ decay, the normal component of the muon polarization typically points anti-parallel to the pion flight direction. Therefore, the INCOME cut selects events with a non-zero normal polarization which points along the polarimeter axis. The polarization can be flipped by rotating the cone about the $z$ axis by 180 degrees. As shown in figure 3.21, the measured asymmetry for the left handed cone is equal and opposite to the measured asymmetry for the right handed cone.

The results are shown in figure 3.23. Measurements of both the asymmetry and the polarization are made for various cone angles. Figure 3.23(a) shows the polarization as a function of $\theta_c$, while 3.23(b) shows the measured asymmetry. The polarization simulations were done with GMC 4.4 and TRIVIA 2.6.2, and include the effects of $K_{\pi2}$ and $K_{\pi3}$ backgrounds. The ratio of the asymmetry to the polarization is the analysing power, and is plotted in figure 3.23(c). We find an averaged value of:
3.5. The Analysing Power

\[ \alpha = 0.198 \pm 0.003 \text{(stat)} \pm 0.004 \text{(Modeling)} \pm 0.004 (Re(\xi)) \]  \hspace{1cm} (3.13)

The modeling error is an estimate of how well we can predict the dilution from CsI accidentals using GMC. The dominant error is from the uncertainty in \( Re(\xi) \), which determines the orientation of the in-plane polarization. In this analysis, we take the particle data group number: \( Re(\xi(0)) = -0.35 \pm 0.15[5] \).

Finally, we must determine whether or not the analysing power measured using this technique is the same as the analysing power that we need to apply when calculating \( P_T \). In particular, the only possible difference is the muon stopping distribution. If the distributions are different, then we would have incorrectly taken into account the contributions of the fringing field, and hence our \( \alpha \) measurement would have to be modified. To eliminate the asymmetry caused by the polarization component in the INCOME sample, we compare a sample constructed by requiring the OR of both cone cuts \( (\theta_c < 70^\circ \ + \ \theta_c > 110^\circ) \) with a sample constructed by requiring a \( \pi^0 \) either in the forward or backward hemispheres. A detailed comparison of the stopping distributions, shown in figure 3.24, indicates that they are nearly identical. In particular, both the mean \( y \) and \( z \) stopping distributions agree to better than 1 mm. The \( x \) stopping distribution agrees to approximately 1 mm. The shapes of each distribution are extremely similar. We can estimate the error in \( \alpha \) from a \( \sim 1 \) mm shift using Monte Carlo to calculate the spin precession effect. We find:
3.5. The Analysing Power

\[ \frac{\delta \alpha}{\alpha} = 2 \times 10^{-4} \]  

(3.14)

Which is negligible compared to the other measurement errors given in equation 3.13.

![Figure 3.21: Measured asymmetry for different cone angles. As expected, \( A_{\text{left}} = -A_{\text{right}} \) to within statistical fluctuations. Using a 70° cone cut, for instance, we find \( A_r = 11.2 \pm 0.1\% \) and \( A_l = -11.2 \pm 0.1\% \). The error bars are shown, but are too small to discern.](image-url)
3.6 Putting it all Together

In order to construct the final result, we separate events with a forward going $\pi^0$ from events with a backward going $\pi^0$. We then measure the asymmetry for each sample separately. The true physics asymmetry should be proportional to the difference $(A_f - A_b)/2$.

In order to select events either in the forward or backward hemispheres, we apply an additional cut on $\theta_{\mu\pi}$. This is polar angle of the $\pi^0$ corrected for the dip angle of the muon. It is constructed by first rotating coordinates so that $P^x_\mu = 0$, then rotating coordinates so that $P^z_\mu = 0^9$. In this coordinate system, $\theta_{\mu\pi} = \cos^{-1}(\theta_\pi)$. We adopt this convention to eliminate differences in the forward and backward samples which are caused by the kaon stopping profile in the target. This cut can be optimized by

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8Identical to the INCOME system which is used in the $\alpha$ measurement

---

Figure 3.22: A schematic of the INCOME cut used for the analysing power measurement. In this figure, the beam direction is perpendicular to the cone.
3.6. Putting it all Together

Figure 3.23: The results of the $\alpha$ measurement showing (a) the predicted polarization, (b) the measured asymmetry, and (c) the ratio of the asymmetry to the polarization.
3.6. Putting it all Together

Figure 3.24: A comparison of the stopping distribution between the forward-backward sample (left side), and the INCONE sample used for the $\alpha$ measurement (right side).
maximizing the figure of merit:

\[
F.O.M. = \langle \cos(\theta_{PT}) \rangle \times \sqrt{N} \times \frac{P_T}{Im(\xi) \cdot \sqrt{1 + \frac{B}{S}}}
\]  
(3.15)

The results are shown in figure 3.25, which indicates that the optimum cut for both one and two photon samples is \( \theta_{\mu\pi} < 70^\circ \) (forward) and \( \theta_{\mu\pi} > 110^\circ \) (backward).

Once the cut point has been established, we integrate over all kinematical variables to find \( \langle \cos(\theta_{PT}) \rangle \) and \( \langle P_T/Im(\xi) \rangle \). The results are different for the one and two photon samples, and are shown in table 3.3. The final sensitivity is:

\[
\begin{align*}
\langle \cos(\theta_{PT}) \rangle &\geq 0.733 \ (2\gamma) \quad \langle \cos(\theta_{PT}) \rangle \geq 0.637 \ (1\gamma) \\
\langle P_T/Im(\xi) \rangle &\geq 0.314 \ (2\gamma) \quad \langle P_T/Im(\xi) \rangle \geq 0.284 \ (1\gamma)
\end{align*}
\]  
(3.16)

Table 3.3: The average values of all important kinematic variables for both the one and two photon samples. In each case, the "Ideal" parameters refer to the maximum possible value with no backgrounds or detector resolution effects included.
3.6. Putting it all Together

Figure 3.25: The Figure of Merit as a function of the $\theta_{\mu\pi}$ cut. The F.O.M. is normalized so that its maximum value is 1. For both one and two photon samples, the maximum F.O.M. occurs at $\theta_{\mu\pi} < 70^\circ$.

3.6.1 Reliability of GMC

Because we rely on GMC to calculate both $P_T/Im(\xi)$ and $<\cos(\theta_{PT})>$, we must understand how well the Monte Carlo can reproduce the data.

E246 uses two independent Monte Carlo simulations, GMC (General Monte Carlo) and TMC (Toy Monte Carlo). TMC is designed for fast event generation for systematics study, and will be described in detail in chapter 4. GMC is a full Geant detector simulation and includes an online trigger simulation. Digitized events from GMC are written into YBOS banks that can then be analyzed using the same offline code that is used to analyze data. In addition to the raw hit information, GMC also writes

---

9 Although these quantities can potentially be estimated using data, in practice shower leakage and beam accidentals makes this impractical.
3.6. *Putting it all Together*

YBOS banks containing the primordial hit and true kinematic information for later comparison with reconstructed values.

Beam accidental are simulated at the offline level by superimposing the measured ADC and TDC spectra from pre-scaled $K_{\mu2}$ events on top of the GMC generated event. As shown in figure 3.26, this provides excellent agreement between data and Monte Carlo.

By comparing the GMC kinematic distributions with measured distributions, we conclude that Monte Carlo can simulate the acceptance of the detector (in particular the acceptance of the calorimeter) quite well. The largest Monte Carlo uncertainty comes from understanding background effects. In order to understand errors from Monte Carlo simulation, we compare measurements made with and without background effects included, and measurements made under different background conditions.

$P_T/Im(\xi)$ depends primarily on the detector acceptance for both charged particles and photons, and changes by only a few percent with radically different background conditions. By comparing GMC results with and without background effects included, we conclude that we can measure $P_T/Im(\xi)$ to within 1.5% of itself. $<\cos(\theta_{PT})>$ decreases by 7% once background effects are included. In order to estimate systematics coming from the Monte Carlo, we measure $<\cos(\theta_{PT})>$ using different background conditions and in different cycles. Taking the differences as an upper limit on the
error, we find that \(< \cos(\theta_{PT}) > \) is accurate to within 2%.

### 3.7 Final Statistical Sensitivity

The statistical error can be readily calculated from the discussion in section 3.6. In general, the statistical error has the form:

\[
\delta \text{Im}(\xi) = \frac{1}{\alpha \cdot < \cos(\theta_{PT}) > \cdot P_T/\text{Im}(\xi)} \frac{1}{\sqrt{N_f + N_b}} \prod \sqrt{1 + \beta_i \cdot \frac{B_i}{S}}
\]  

(3.18)

where \(N_f\) is the number of events with \(\theta_{\mu\pi} < 70^\circ\), \(N_b\) is the number of events with \(\theta_{\mu\pi} > 110^\circ\), and \(P_T/\text{Im}(\xi)\) is the kinematic factor given in equation 1.8. Plugging in the appropriate numbers, we find:

\[
1 \gamma : \delta \text{Im}(\xi) = \frac{32.4}{\sqrt{N_f + N_b}}
\]

(3.19)

\[
2 \gamma : \delta \text{Im}(\xi) = \frac{24.7}{\sqrt{N_f + N_b}}
\]

(3.20)
Figure 3.26: (a) $M_{\tau\phi}$ for data (dashed histogram) and "pure GMC" (solid histogram). (b) $M_{\tau\phi}$ for data (dashed histogram) and "GMC+accidentals" (solid histogram).
Chapter 4

Systematic Errors

If you really understood your systematic errors, they wouldn't be errors...

In E246, we measure a counting asymmetry between two scintillators. Anything that might generate a spurious asymmetry has to be considered as a potential systematic error. Systematic errors are dominated entirely by imperfections in the detector alignment and by geometrical offsets in the $K^+$ beam. In this sense, E246 is unique as most other kaon experiments suffer from very different problems. Rare kaon decay experiments, for instance, suffer mainly from physics and beam backgrounds. In the direct CP violation experiment ($\epsilon'/\epsilon$) errors are dominated by the need to understand the acceptance for different modes. None of these effects will pose a serious problem for E246.

We are studying a mode with a relatively high branching ratio (3.2%), so we expect both physics and beam related backgrounds to be reasonably small. More
importantly, the majority of backgrounds are unpolarized, and as a result they do not generate a spurious asymmetry but instead only dilute the statistical sensitivity to $\text{Im}(\xi)$. Real physics processes, such as multiple scattering, will decrease the overall detector resolution. However, while they might play a major role in rare decay experiments, in E246 they only lead to a slight increase in the background rate. As we will argue, they do not create a spurious asymmetry.

So how is it possible to generate a false asymmetry? Any geometrical flaw in the detector can lead to an asymmetric counting rate. For instance, if one of the positron counters is positioned too close to the Al muon stopper, the increase in solid angle means that the probability of a positron to hit that counter is higher than the probability for a positron to hit the opposite counter. The result is a counting asymmetry that we interpret as a physical polarization. It is extremely difficult, and in some cases impossible, to eliminate these asymmetries in offline analysis. So we must answer two questions. First, how large is the spurious asymmetry in the detector? And second, how well can it be suppressed in the final analysis?

4.1 Bias Cancelation

There are two main techniques for suppressing bias asymmetries in E246. First, the detector has a 12-fold rotational symmetry. In other words, the CW $e^+$ counter in one gap is also CCW $e^+$ counter in the neighboring gap, and each gap is designed
to be identical to all of the others. This tends to cancel contributions from counter
inefficiencies and some counter positioning errors. Rotational symmetry also tends
to cancel contributions from asymmetric photon detection efficiency, and eliminates
some beam induced asymmetries.

We can measure the residual bias level after summing over all 12 gaps \( A_\Sigma \) di-
rectly from data. We select \( K^+ \rightarrow \mu^+ \nu_\mu \pi^0 \) events using our standard kinematic cuts
but without tagging the \( \pi^0 \) direction. The physics contribution to the asymmetry
(from a non-zero \( P_T \)) will change sign depending on whether the \( \pi^0 \) is moving for-
ward \( (\theta_{\mu\pi} < 70^\circ) \) or backward \( (\theta_{\mu\pi} > 110^\circ) \) in the detector. Integrating over all
\( \pi^0 \) directions eliminates the true physics contribution, and will leave only the bias
asymmetry. We will study \( A_\Sigma \), the so-called “null asymmetry,” in section 4.1.1.
Furthermore, we can break down the contribution from each individual sub-detector
separately by introducing “false” asymmetries using asymmetric cuts on the data, and
then measuring how well these bias asymmetries cancel by summing over all gaps.

In addition to the detector’s 12-fold rotational symmetry, systematics will be
further reduced after tagging the \( \pi^0 \) direction because most bias asymmetries in the
polarimeter are independent of \( \pi^0 \) direction. Since \( P_T \) changes sign with \( \pi^0 \) direction
while most spurious asymmetries do not, we can extract the true physics asymmetry
even in the presence of a residual detector bias.
4.1. Bias Cancelation

4.1.1 $A_\Sigma$, the "Null Asymmetry"

The first step in understanding the systematic error is to measure the residual detector bias before and after summing over all magnet gaps. As shown in figure 4.1, there are large variations in the measured asymmetry as a function of gap, with peak-to-peak fluctuations of approximately 3%. The gap to gap fluctuations in asymmetry ($A_n$) have a very distinctive pattern that is indicative of a non-screw asymmetry. Non-screw asymmetries (see section 4.4) should be strongly suppressed by summing over all gaps. Indeed, we find a very small residual asymmetry:

\[
2\gamma : A_\Sigma = -5.9 \pm 5.9 \times 10^{-4} \quad (4.1)
\]
\[
1\gamma : A_\Sigma = -9.9 \pm 9.1 \times 10^{-4} \quad (4.2)
\]

Combined : $|A_\Sigma| < 1.4 \times 10^{-3}$ (90% C.L.) \quad (4.3)

The large asymmetries in each gap are consistent with shifts in the muon stopping position. A shift in the muon stopping distribution can have many causes, such as offsets in the position of the Al muon stopper, inefficiencies in individual MWPCs, and so forth. In this case, however, the main cause is a geometrical shift in the target kaon stopping distribution. As shown in figure 4.2, shifts in the target stopping distribution create an oscillatory pattern of asymmetries in each gap.
4.1. Bias Cancelation

In order to quantify this statement, we can use data from C4 to measure the position of muons entering the polarimeter. From the data, we measure the asymmetry for different slices in C4Y as shown in figure 4.3. This serves as an effective acceptance curve that provides the relative probability for positrons to strike a particular counter as a function of the muon \(y\) (azimuthal) position. We can then convolve this distribution with the measured C4Y distribution in each gap to predict \(A_n\). As shown in figure 4.1, the predicted asymmetry agrees extremely well with the measurement, with the exception of gap 9, in which we always measure a lower asymmetry than is predicted. It was originally suggested that this was due to a large background effect. However, this does not appear to be the case, as the measured asymmetry does not change with the application of veto cuts or by changing TMUON as shown in table 4.1. It is more likely that this is due to an alignment problem in that particular gap.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15 &lt; t_\mu &lt; 4000) ns + VETO</td>
<td>(-11.7 \pm 2.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(15 &lt; t_\mu &lt; 8000) ns + VETO</td>
<td>(-10.7 \pm 1.9 \times 10^{-3})</td>
</tr>
<tr>
<td>(15 &lt; t_\mu &lt; 8000) ns</td>
<td>(-9.8 \pm 2.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(200 &lt; t_\mu &lt; 8000) ns</td>
<td>(-10.6 \pm 2.1 \times 10^{-3})</td>
</tr>
<tr>
<td>(15 &lt; t_\mu &lt; 12000) ns + VETO</td>
<td>(-10.4 \pm 2.0 \times 10^{-3})</td>
</tr>
</tbody>
</table>

Table 4.1: Changes in the gap 9 asymmetry with different cuts.

4.1.2 Bias Cancelation from Forward/Backward Tagging

In general, the physics contribution to the asymmetry can be expressed as:
4.1. Bias Cancelation

Figure 4.1: The measured asymmetry (before tagging the $\pi^0$ direction) in each gap. The oscillatory pattern is caused by offsets in the muon stopping distribution, and can be predicted using MWPC (C4) measurements of the $y$ (azimuthal) stopping distribution of muons in the polarimeter.
Figure 4.2: A cartoon drawing indicating the effect of a small shift in the target kaon stopping distribution. Because of the narrow angular acceptance of the CsI hole, this type of displacement leads to an oscillatory shift of the muon stopping distribution from gap to gap which in turn creates an oscillatory pattern of asymmetries.
Figure 4.3: The measured asymmetry as a function of the muon azimuthal position (C4Y). A symmetric, parameterized version of this curve is convolved with the measured C4Y distribution in each gap in order to predict the geometric contribution to the counting asymmetry.
4.1. Bias Cancelation

\[ A_{\text{phys}} = \frac{N_{\text{cw}} \pi^0_{\text{f}} - N_{\text{ccw}} \pi^0_{\text{f}} + N_{\text{ccw}} \pi^0_{\text{b}} - N_{\text{cw}} \pi^0_{\text{b}}}{N_{\text{cw}} \pi^0_{\text{f}} + N_{\text{ccw}} \pi^0_{\text{f}} + N_{\text{ccw}} \pi^0_{\text{b}} + N_{\text{cw}} \pi^0_{\text{b}}} \quad (4.4) \]

since the decay plane normal vector \((\vec{P}_\pi \times \vec{P}_\mu)\) changes sign depending on whether the \(\pi^0\) is moving forward or backward in the detector. By taking the F/B difference, we expect any residual bias to cancel. This cancelation is limited by three factors.

First, by simply taking the ratio defined in equation 4.4, bias cancelation is limited by the numerical difference in forward and backward going events. The exact value of:

\[ \frac{N(\pi^0_{\text{forward}}) - N(\pi^0_{\text{backward}})}{N(\pi^0_{\text{forward}}) + N(\pi^0_{\text{backward}})} \quad (4.5) \]

depends strongly on the choice of cuts and upon the \(K^+\) stopping distribution in the target. The target stopping distribution has the largest effect because of the narrow geometrical acceptance of the CsI gap. We can eliminate this dependence by correcting the \(\pi^0\) polar angle by the muon dip angle as described in section 3.6. After the dip angle correction is applied, the difference in the number of forward and backward going events is less than 1.5% for the two photon sample and less than 4% for the one photon sample.

The second limitation comes from the difference in \(\mu^+\) stopping distributions between forward and backward going events. In general, both the \(z\) and \(y^1\) stopping

\(^1\)These coordinates are “gap coordinates,” where \(z\) points along the beam axis, \(y\) points in the azimuthal direction (the polarimeter \(P_T\) axis), and \(x\) points in the radial direction.
distributions agree extremely well:

\[ <x^f_{\text{stop}} - x^b_{\text{stop}} > \leq 1 \text{ mm} \]  
\[ <y^f_{\text{stop}} - x^b_{\text{stop}} > \leq 0.1 \text{ mm} \]  

However, events with backward going \( \pi^0 \)s come to rest lower in the stopper than events with forward going \( \pi^0 \)s, as shown in figure 4.4. Typical differences are small, with \( <x^f_{\text{stop}} - x^b_{\text{stop}} > \sim 0.5 \text{ cm} \). This is the result of slightly different kinematics for forward and backward going events.

Finally, in addition to the differences in \( x_{\text{stop}} \), the components of in-plane polarization are different for forward and backward going events. Both forward and backward going events have a large component of normal polarization, typically around 57\%. \( P_N \) tends to point anti-parallel to the \( \pi^0 \) flight direction, which implies that \( P_N \) has a different sign for forward and backward events. After \( \mu \text{SR} \) in the analysis magnet\(^2\), \( P_N \) tends to point along the radial direction. This leads to a radial polarization imbalance between forward and backward events, as shown in figure 4.5. This asymmetry can couple critically to other detector asymmetries, such as \( \vec{B} \) field rotations, and significantly reduce the rejection power of forward/backward tagging. It will be further discussed in sections 4.3.3 and 4.6.

The rejection power of forward/backward tagging can be estimated by introducing

\(^2\)While passing through the analysis magnet, the muon spin will rotate at the same rate as the muon momentum (up to a factor of \( g - 2 \)).
Figure 4.4: The radial stopping distribution for events with forward going $\pi^0$s (solid curve) compared to events with backward going $\pi^0$s (dashed curve). Note that there are more forward going events at higher $x$ values. This creates a difference in the mean radial stopping position of 5 mm.

false asymmetries into the data, and then directly measuring how well they cancel after tagging the $\pi^0$ direction. In order to maintain a blind analysis, we restrict ourselves to only a small fraction of the data (less than 2%). In general, this should allow us to either directly measure, or at least set limits on the rejection power of the F/B tag for each type of bias asymmetry.

4.2 E246 Fast Simulator

In general, we hope to rely as much as possible on data to constrain potential systematic errors. However, there are times when this is not possible. In these cases, we
Figure 4.5: The residual radial polarization for forward and backward going events. These plots were generated using GMC 4.3 (since we cannot measure the radial polarization directly from data). These distributions are created after $\mu$SR in the analysis magnet, but before $\mu$SR in the fringing field.
must rely on Monte Carlo to set limits on systematics and to estimate how critical certain geometrical effects might be.

Any simulation of detector systematics requires the generation of a large sample of events; at least on the order of the statistical sensitivity of the experiment ($\sim 10^7$ events). Although we have a full Geant Monte Carlo (GMC) it is sorely limited by speed. The typical throughput from GMC is around .01 events/CPU-sec$^3$, which means that a detailed simulation would take months of CPU time. For that reason, we rely on a simplified "Toy Monte Carlo" (TMC) that uses a cruder tracking algorithm and simpler detector geometry, and ignores "higher order" physics effects such as $\delta$-ray production, bremsstrahlung, and so forth. In general, since systematic errors in E246 are dominated by geometrical effects rather than subtle physics effects, TMC provides an accurate estimate of the ultimate systematic error. In sacrificing some simulation accuracy, one gains an enormous amount in speed. Typical TMC event rates are approximately 70 events/second, nearly 4 orders of magnitude higher than GMC.

In order to check the reliability of the fast simulator, TMC results were compared to data whenever possible, and to GMC when an exact comparison with data wasn't available. There is good agreement between TMC and data for all kinematical variables including $P_\mu$, $P_\pi$, and $\theta_{\mu\pi}$. The $K^+$ and $\mu^+$ stopping distributions are in decent

---

8On an SGI Challenge-L with four-200 MHz R4400 processors and two-150 MHz R4400 processors.
agreement with both data and GMC. Because TMC uses constant field tracking, the average $x_{\text{stop}}^{\mu}$ distribution in TMC is 5 cm lower than the data distribution calculated using MWPC information. Additionally, TMC $\mu^+$ stopping distributions are more tightly bunched in the $y$ (azimuthal) direction than the data. In general, this will cause TMC to underestimate certain effects, particularly magnetic field effects. In order to account for this, a modified version of TMC was used to study $\vec{B}$ field effects. This will be discussed further in section 4.3.3.

Having now laid out the general tools, we begin by studying the two main classes of systematic error: direct-screw asymmetries and non-screw asymmetries. For each class, we will estimate the rejection power both from summing over all gaps, and from tagging the $\pi^0$ direction.

## 4.3 Direct-Screw Asymmetries

Direct-screw asymmetries are asymmetries that don't completely cancel after the sum over 12 gaps. One example is a position shift in one of the $e^+$ counters as depicted in figure 4.6. If the CW positron counter in any gap is shifted closer to the Al muon stopper, then a positive counting asymmetry appears in that gap. However, since the CW counter in one gap is also the CCW counter in the neighboring gap, an asymmetry also appears in the neighboring gap. In this case, the CCW counter in the neighboring gap is shifted further away from the Al stopper, also generating
4.3. Direct-Screw Asymmetries

a positive asymmetry. At best, assuming a random distribution of errors, we can expect cancelation of only $1/\sqrt{12}$ after summing over all gaps. In reality, however, we believe that most shifts are likely to be correlated, so we must consider the worst case scenario. In any case, however, these asymmetries are expected to be suppressed by tagging the $\pi^0$ direction.

Direct-screw asymmetries are typically related to alignment problems within the polarimeter. We can set limits on each type of asymmetry using the positioning errors of the polarimeter components, which have been determined by multiple surveys of the counters. Positioning errors in the positron counters are typically difficult to simulate using data, so we must rely on TMC. Positioning errors in gap trigger counters or in the Al $\mu^+$ stopper can be simulated by applying fiducial cuts on the muon stopping position using data. We will discuss each separately.

4.3.1 $e^+$ Counter Position Offsets

The Al positron counters supports in the polarimeter were initially positioned relative to the magnet poles with an accuracy of 500 $\mu$m. As discussed before, small offsets in the counter position will change the acceptance of a given counter, leading to a false asymmetry.

TMC simulations were run for several different offsets. Azimuthal counter shifts typically result in a 0.25% asymmetry per mm shift$^4$. Effects of radial or $z$ shifts

$^4$This is the asymmetry in a single gap caused by shifting one counter closer to the stopper.
4.3. Direct-Screw Asymmetries

Figure 4.6: Typical $e^+$ counter configurations that can lead to false asymmetries in the polarimeter. (a) A rotation in the plane of the polarimeter, which results in the inclusion of some fraction of either the normal or longitudinal spin component and which also changes the effective solid angle of the counter. (b) A position offset, causing a difference in solid angle between CW and CCW counters.
are much smaller than the effect of an azimuthal shift. To be precise, shifts in the radial direction lead to an asymmetry of $5 \times 10^{-4}$ per mm shift, while shifts in the $z$ direction lead to an asymmetry of $2 \times 10^{-4}$ per mm shift.

Although the positron counters were initially positioned to 500 $\mu$m accuracy, there is clear evidence that counter positions changed during the course of running\cite{22}. In particular, measurements performed 16 months after installation indicate variations in both the azimuthal and radial counter positions. Although most counters did satisfy the 500 $\mu$m positioning requirement in the azimuthal direction, variations as large as 1.5 mm were observed. Even worse, variations in the radial direction of up to 9 mm were seen.

These large position shifts imply that $A_{\Sigma}$ could be as high as 1%. We can actually constrain the position of the counters by using the measured value of $A_{\Sigma} = -7 \pm 5 \times 10^{-4}$ (see section 4.1.1). Since $|A_{\Sigma}| < 0.14\%$, we conclude that the average azimuthal shift must be $|\Delta y| < 600\mu$m.

It can be argued that we can correct counter induced asymmetries with careful position measurements in each gap. However, this is a very risky endeavor since it is clear that these problems have developed slowly over time. Some position shifts are clearly caused by gravity; but others are less certain and may have been caused by counters being bumped or jostled when the polarimeter stand was removed. At any rate, since it is not clear when these problems developed, it is hard to calculate
the exact correction factor to apply. The best solution is to allow these asymmetries to cancel after forward/backward tagging, and assume the worst case scenario in the systematic error estimate.

4.3.2 $\text{e}^+$ Counter Rotations

Rotation of a positron counter can occur in two different axes ($\vec{V_r}, \vec{V_z}$) as shown in figure 4.6a. Similar to a position offset, counter rotations will change the effective solid angle and generate a false counting asymmetry. In addition to a purely geometric effect, we must also worry about picking up either the normal or longitudinal component of the muon spin. In $K_{\mu3}$ decay, muons tend to be polarized along their flight direction with $P_L \sim 65\%$. However, after selecting events with either forward or backward going $\pi^0$s, we obtain a sample of events with both a large $P_L$ and $P_N$. Therefore, both $\vec{V_r}$ and $\vec{V_z}$ can potentially pick up components of the in-plane spin.

Rotations about both $\vec{V_r}$ and $\vec{V_z}$ have been simulated with TMC. We can estimate the angular alignment of each counter using the following simple analysis. The width of the positron counter is 30 cm. If we allow the end of the counter to move by 1 mm (roughly the azimuthal positioning alignment), we obtain a positioning accuracy of $0.2^\circ$ about $\vec{V_r}$. The length of the counter is 84 cm, which implies a resolution of better than $0.1^\circ$ for rotations about $\vec{V_z}$.

We will begin first by estimating the effect caused only by the change in solid
4.3. Direct-Screw Asymmetries

angle. For rotations about $\vec{V}_z$ the maximum asymmetry generated in a given gap is $9 \times 10^{-5}$. For rotations about $\vec{V}_r$, the effect is comparable; appearing at the level of $1.5 \times 10^{-4}$. Both of these asymmetries will be further reduced by forward/backward tagging, and are therefore negligible.

Even though the geometrical effect is small, we might worry that a rotation will push a piece of either $P_L$ or $P_N$ into the $P_T$ direction. This is especially dangerous since forward and backward going events have a different polarization in the Al stopper. Fortunately, $\mu$SR washes out most of the in-plane polarization. The residual in-plane polarization at the time that the $\mu^+$ decays is only a few percent. Bias asymmetries from the in-plane polarization are no worse than $2 \times 10^{-5}$. Even though reduction from F/B tagging is small (TMC predicts approximately 2:1), this effect is also negligible.

4.3.3 $\vec{B}$ Field Screw Asymmetries

The polarimeter stopper is immersed in an azimuthal fringing field of 150G. This field is necessary to preserve the transverse spin component, to wash out the in-plane polarization, and to eliminate $\mu$SR effects from the $\vec{B}$ field of the Earth. The non-azimuthal components of the magnetic field were reduced using a iron shim plates positioned on either side of the exit to the magnet gap. The field was mapped with a temperature controlled Hall probe. Typical field asymmetries between the left and right halves of the Al stopper are less than 0.5G.
4.3. Direct-Screw Asymmetries

Azimuthal asymmetries in the magnetic field are particularly dangerous for two reasons. First, the non-azimuthal components of the field will cause both $P_N$ and $P_L$ to precess into the decay plane of the polarimeter. If the precession rate is different between the left and right halves of the stopper, this will appear as a counting asymmetry. Second, the $\vec{B}$ field asymmetries are determined entirely by the design and alignment of the magnet poles and shim plates. Since these are identical from gap to gap, one might worry that design or alignment defects are coherently repeated. Therefore, spurious asymmetries may be correlated and may not cancel after summing over all gaps.

In order to obtain the best possible estimate, several modifications were made to TMC. In standard TMC, we begin by placing a kaon in the target, simulating a $K_{\mu3}$ decay, and subsequently tracking all of the daughter particles through the detector. Our modified version of TMC begins by placing a stopped muon with a specified polarization in the muon stopper. The muon stopping distribution is obtained from data using MWPC hit information and then estimating energy loss in the both the degrader and stopper by range. Spin distributions were obtained from GMC simulations. Separate stopping and spin distributions were used depending on whether the $\pi^0$ is moving forward or backward in the detector. By measuring the residual transverse polarization after precession, we obtain an estimate of the systematic error.
4.3. Direct-Screw Asymmetries

We must consider two different types of screw asymmetries. The first is a translation of the entire field along the $y$ (azimuthal) direction. This is parameterized by one number ($e$) that, from a fit to the measured field map, is constrained to be $\leq 0.2$ mm. The second is a rotation in the field either about the radial or $z$ axes. Rotations are parameterized by a rotation angle ($\delta_x$, $\delta_z$) each of which, according to the field measurement, is less than 1.0 mr.

For $e$ skews in the $\vec{B}$ field, we find asymmetries of $4.4 \times 10^{-4}$ for shifts of approximately 1 mm. TMC indicates that this asymmetry is reduced by a factor of 1:10 after forward/backward tagging. Using a similar analysis for rotations about the radial direction ($\delta_x$) we find an induced asymmetry of $5.3 \times 10^{-4}$ for every 10 mr rotation. This asymmetry is reduced by more than 1:20 after F/B tagging. Even after multiplying the errors in the field map by a factor of two as a safety margin and to account for possible Monte Carlo errors, the expected residual error from both sources is less than $10^{-4}$.

The situation is somewhat different for a field rotation about the $z$ axis ($\delta_z$). The residual asymmetry from this rotation is $6.9 \times 10^{-4}$ for every 10 mr rotation. However, unlike the case of a rotation about the radial axis, this time we obtain no cancelation from forward/backward tagging. The reason is illustrated in figure 4.7. Because the polarization in the $\hat{r}$ direction is different for forward and backward going events, they precess in opposite directions. The bias asymmetry therefore appears with opposite
4.3. Direct-Screw Asymmetries

signs in the forward and backward samples, just like the true physics asymmetry. Because this rotation does not cancel after forward/backward tagging, it is expected to dominate the systematic error.

Analysis of the measured $\vec{B}$ field map indicates that $\delta_z \leq 1.0$ mr in all gaps. The field in each magnet gap was mapped independently using the same apparatus. The average “offset” of the field was obtained by fitting the measured map. The results are shown in figure 4.8. Summing over all gaps, we find $<\delta_z> = 0.2$ mr.

Figure 4.7: Effects of a rotation in the $\vec{B}$ field about the $\hat{z}$ axis. Although the true field is curved, we have drawn a constant field for illustration purposes. As shown above, a sample with 100% polarization in the radial direction will develop a positive asymmetry, while a sample with -100% polarization in the radial direction will develop a negative asymmetry.
4.3. Direct-Screw Asymmetries

![Graph showing rotation angle vs gap number with φ=0.2 mr]

Figure 4.8: The average rotation angle $\delta_z$ of the fringing magnetic field in each gap. Typical offsets are less than 1 mr with $<\delta_z>=0.2$ mr. Peak to peak fluctuations are on the order of 1.6 mr. Errors are dominated by the alignment accuracy of the probe ($\leq0.1$ mm). The overall measurement error for each gap is $\Delta\delta_z=0.3$ mr.

4.3.4 MWPC Inefficiencies

Inefficiencies in a proportional chamber, or more specifically a difference in the efficiency between the left and right halves of a chamber, can also lead to a counting asymmetry. This effect can be easily simulated using data by introducing an efficiency gradient across C4. A linear dependence was introduced into C4 at the offline level of the form:

$$\epsilon = 1.0 - \delta \frac{10.0 - y_\mu}{20.0}$$

(4.8)

where $y_\mu$ is the hit position of the muon in the $y$ direction (measured in cm) and $\delta$ is the efficiency reduction parameter. Losses of 50\% and 100\% over the length of the chamber were simulated. An asymmetry of $2 \times 10^{-3}$ is created for each 1\% drop in chamber efficiency. These false asymmetries cancel at 1:40 after F/B tagging (90\% C.L.).

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4.3. Direct-Screw Asymmetries

Because the azimuthal and $z$ stopping distributions for forward and backward going events are nearly identical, while the radial distributions are different, we might expect that asymmetries that don't depend on $x$ will cancel to an extremely high order. As shown in figure 4.4, differences in $x_{\text{stop}}$ are largest for $x > 115$ cm. In order to exaggerate the F/B difference, we introduce an efficiency gradient in C4 of the form:

$$
\begin{align*}
\epsilon &= 1.0 \quad x < 115 \text{ cm} \\
\epsilon &= 1.0 - \frac{10.0 - y_{\mu}}{20.0} \quad x > 115 \text{ cm}
\end{align*}
$$

This is the worst possible case scenario. In this case, we find a residual asymmetry of $5 \times 10^{-4}$ is created for each 1% drop in efficiency. Cancellation from F/B tagging is better than 1 : 8. Even though this measurement is statistically limited, we can already constrain the possible contribution from this source to less than $10^{-4}$.

4.3.5 Trigger Counter Position Offsets and Rotations

Offsets in the gap trigger counters (TOF and POLTRIG can generate a shift in the muon stopping distribution, leading to a false asymmetry. Position offset and a rotations in the POLTRIG counter were simulated both using TMC and by making asymmetric cuts on the data. Because the dimensions of the POLTRIG counters ($18 \times 58 \text{ cm}^2$) are larger than the Al muon stopper ($16 \times 55 \text{ cm}^2$), small shifts are
4.4 Non-Screw Asymmetries

almost negligible. For $K_{\mu 3}$ events, which tend to peak near the center of the counter, a trigger counter rotation or offset creates an asymmetry smaller than $10^{-4}$ before F/B tagging. These systematics are clearly negligible.

4.4 Non-Screw Asymmetries

The term "non-screw asymmetry" refers to any systematic that affects all gaps simultaneously. Typically, these are related to problems either in the target or in the calorimeter. Because they do not define a screw sense in the detector, they are expected to cancel exactly after summing over all gaps. Of course, this is only strictly true as long as all other detector elements are perfect.

We have already seen an example of a non-screw asymmetry in section 4.1.1, where a small shift in the kaon stopping distribution led to large asymmetries in individual gaps. In this case, rotational symmetry ensures that $A_3 = -A_9$, $A_6 = -A_{12}$, and so forth. However, if there are differences between each gap, the rotational symmetry is broken and the cancelation scheme is ruined. We must understand how non-screw asymmetries can couple to other detector imperfections and thereby produce a residual detector bias.

The approach that we will take in studying non-screw asymmetries is similar in spirit to the study of direct-screw asymmetries. We begin by looking at data, making asymmetric cuts, and measuring the change in $A_{\Sigma}$. Studying these effects using
Monte Carlo is slightly more complicated because residual asymmetries will only appear in an imperfect detector. For TMC simulations, therefore, we will introduce a non-screw asymmetry, and then smear all other detector elements using reasonable estimates of their alignment precision. By generating an ensemble of such detectors, we can constrain the level of possible systematics.

4.4.1 Target Shifts

Because target shifts strongly affect the measured asymmetry in each gap, we will begin our studies here. As shown in figure 4.9, the \( K^+ \) stopping distribution is centered to within 1 mm after summing over all gaps. We can estimate the effect of “shifting” the \( K^+ \) stopping distribution by making asymmetric cuts on \( x^{\text{targ}} \). For instance, by requiring that \( x^{\text{targ}} > -1 \text{ cm} \), the mean stopping position shifts by +1 cm. The effect of this cut is dramatic, and as indicated in figure 4.10, has a very distinctive pattern that is typical of non-screw asymmetries.

From figure 4.10, we conclude that the maximum asymmetry induced in an individual gap is approximately 3% per cm shift. In order to estimate the target contribution to \( A_\Sigma \), we measure total asymmetry after shifting the target distribution in many different directions. Since we expect non-screw asymmetries to cancel exactly unless they couple to some other detector imperfection, it is important to shift the distribution in as many different directions as possible to obtain the best.
4.4. Non-Screw Asymmetries

upper bound. $A_{\Sigma}$ is consistent with zero for most target shifts. A non-zero asymmetry only appears in one case; namely a shift along the $-\hat{y}$ axis. We can take this as the worst case scenario, and use this situation to constrain the overall systematic contribution from the target. As shown in figure 4.11, $A_{\Sigma} = 0.5\%$ per cm shift. By using the mean value of the target stopping position ($|y_{\text{targ}}| < 1 \text{ mm}$), we obtain $A_{\Sigma}^{\text{targ}} < 6 \times 10^{-4}$. Note that this asymmetry is calculated before accounting for suppression from forward/backward $\pi^0$ tagging.

In reality, we do not have a sharp cutoff in the target stopping distribution, but instead a smooth shift caused by beam mis-targeting. This effect of a smooth shift should be smaller than what was estimated using sharp cuts in the data, and can be calculated using TMC. Monte Carlo studies indicate that the asymmetry introduced from a smooth shift is a factor of three lower than what was measured using data. This implies a target contribution of $A_{\Sigma}^{\text{targ}} < 2 \times 10^{-4}$.

4.4.2 Target Skew

Because kaons pass through a dipole magnet before entering the detector, the beam is momentum dispersed along $x$. This creates a 25° skew in the $x-z$ beam profile in the target, as shown in figure 4.12. When coupled with the narrow acceptance of each gap, this skew helps to create the offsets that we previously studied. It also serves to break the rotational symmetry of the detector, and its contribution to $A_{\Sigma}$ must be
4.4. Non-Screw Asymmetries

Figure 4.9: $K^+$ stopping distributions measured using the reconstructed target vertex. Plotted here is the profile along the “Gap-$y$” axis after summing over all gaps. The structure is caused by the finite size of the target fibers (5 mm). The distribution is centered to better than 0.3 mm, although given the size of the fibers, it is unlikely that we can measure more accurately than 1.4 mm.

Figure 4.10: The measured asymmetry in each gap after requiring $y_{\text{targ}} > 1$ cm. This cut, which is equivalent to a 2 cm shift in the mean stopping position, creates a very distinctive pattern. In this case, we select kaons which stop in the upper section of the target, and as a result gaps 1-5 have a positive asymmetry, gaps 7-11 have a negative asymmetry, and gaps 6 and 12 have nearly a zero asymmetry.
4.4. Non-Screw Asymmetries

Figure 4.11: After applying asymmetric cuts in the target, $A_{\Sigma}$ is consistent with zero in all cases except for a shift along the $-\hat{y}$ axis. (a) $A_{\Sigma}$ from data after two successive shifts along $-\hat{y}$. Also shown is a prediction from TMC using the same sharp cut that has been applied to the data. (b) TMC results for a smooth shift in the target distribution. In this simulation, the position of all polarimeter components were smeared. Gaussian tails were truncated at $\pm 4\sigma$. In each plot, the best straight line fit to the data is shown (note the logarithmic scale).
understood.

This effect was simulated in TMC by rotating the initial kaon stopping distribution to match the data. Because the data target profile depends on the acceptance of the detector, this was an iterative process. There was no significant change in the results of the previous section after including the skew in the MC simulation.

Additionally, we applied three different cuts on the data in order to suppress the effects of the skew: a $z$-fiducial cut ($|z| < 3\,\text{cm}$), a box cut ($|z| < 3\,\text{cm}$, $|x| < 2\,\text{cm}$), and a cylinder cut ($\sqrt{x^2 + y^2} < 3\,\text{cm}$). None of these cuts altered $A_\Sigma$, and perhaps more importantly, none of these cuts changed the structure of the gap structure of $A_n$. We conclude that the symmetry breaking effect of the skew is negligible.

![Figure 4.12: The $x$ vs. $z$ distribution of stopped kaons in the target. The skew is the result of a momentum gradient along $x$. Higher momentum kaons near gap 9 (negative $x$) will penetrate deeper into the target than lower momentum kaons near gap 3 (positive $x$).](Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.)
4.4. Non-Screw Asymmetries

4.4.3 Polarimeter Beam Backgrounds

Beam backgrounds appear both in the CsI and in the positron counters. CsI backgrounds will be covered in detail later. Backgrounds in the positron counters can be eliminated by fitting the muon decay spectrum to the function:

\[ S = Ae^{-t/\tau_{\mu}} + B \]  (4.11)

As long as \( B \) has no time dependence, this technique introduces no systematic error. Since \( e^+ \) counter background comes from the beam, there are only two ways to introduce a non-trivial time dependence into the background. First, if there are large variations in the instantaneous beam intensity, then a large number of halo \( \pi^+ \)'s might arrive in coincidence with a real \( K^+ \). If secondary positrons from the pion decay (\( \pi \rightarrow \mu \rightarrow e \)) strike one of the polarimeter counters, then we would see a component of the background with lifetime \( \tau_{\mu} \).

Second, if there is a fast time-dependent component in the proton intensity from the Main Ring, it might appear in the \( \mu^+ \) spectrum. In fact, the typical beam structure at the KEK-PS shows large spikes in the instantaneous intensity. There is typically a 6 MHz substructure in the beam, which is fast enough compared to the muon lifetime not to pose a problem. More seriously, we have also observed beam oscillations at 500 kHz (2 \( \mu s \)), which could present a problem to our fitting procedure.

In order to measure the contribution from “non-flat” backgrounds, we study the
Figure 4.13: The results of the beam background study. The top plot shows the time spectrum of the hit gap, the middle plot shows the time spectrum of the two nearest adjacent gaps, while the bottom plot shows the sum of all other gaps. The bottom plot shows only a small time-dependent piece in the background, and a high frequency piece that is the result of the 6 MHz beam structure at the KEK-PS.
4.4. Non-Screw Asymmetries

Muon decay spectrum in empty-gaps. In other words, we select events with a good track in one gap, and then look at $e^+$ counter hits in neighboring gaps. Typical results are shown in figure 4.13. The hit-gap shows a clear exponential decay with lifetime $\tau_\mu$. The two adjacent-gaps show a similar spectrum that is the result of a positron punching from one gap to the next. The signal summed over all other gaps is mainly background, and has only a very small time-dependent piece with the muon lifetime.

We can estimate the background systematic by fitting the empty gap time spectrum using equation 4.11. The ratio:

$$ R = \frac{B \times 8\mu s}{A \times 2.2\mu s} $$

is less than 2% at the 90% confidence level. Since background contamination in the hit gap is 11% once veto cuts have been applied, the total contribution to the individual gap asymmetry is less than $0.11 \times 0.02 = 2.0 \times 10^{-3}$. Even if we assume minimal reduction from summing over all gaps, after $F/B \pi^0$ tagging systematic contributions from beam background will be negligible.

4.4.4 $\phi$ Dependent $\gamma$ Detection Efficiency

The last non-screw asymmetry that we must consider is an inefficiency in the photon detector. This can potentially generate large asymmetries in each gap because the components of the muon normal polarization directed along the polarimeter axis only
cancel completely if there is perfect azimuthal symmetry in the CsI calorimeter.

For the two-photon sample, it is easier to study the $\pi^0$ rather than the single photon detection efficiency, so we will focus our attention there. We can search for inefficiencies in the electromagnetic calorimeter by looking at the $\phi$ distribution of the reconstructed $\pi^0$s, as shown in figures 4.14a and 4.14b. Even in the forward and backward regions (figure 4.14b), some gap structure is apparent that is the result of kinematic correlations between the $\pi^0$ and its decay photons. By measuring differences between the height of each peak, shown in figure 4.14c, we conclude that the average detection efficiency fluctuates by 3% as a function of azimuthal angle. The magnitude of these fluctuations does not change if we tighten the $\pi^0$ mass cut, implying that this is not simply a resolution effect.

In order to simulate an efficiency loss in the calorimeter, we introduce a 50% loss at the offline level in one hemisphere of the calorimeter ($0 < \phi < 180^\circ$). The gap to gap structure of the asymmetry is, as one might expect, similar to the patterns that result from asymmetric target cuts. A 50% drop in calorimeter efficiency induces a 4% (maximum) asymmetry in an individual gap. These asymmetries are found to cancel at better than 4:1 after summing over all gaps (this is a statistically limited measurement). We can use TMC to estimate the rejection from forward/backward tagging, which predicts a cancelation of at least 1:17. As a result, we can set an upper limit on spurious asymmetries caused by calorimeter inefficiencies of $5 \times 10^{-4}$. 

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4.5 Bias in the Offline Analysis

Results are similar for the one photon sample.

As a cross check, we can directly measure the angle that the normal polarization vector makes with the polarimeter. In a perfect detector, this should be zero. This analysis can actually provide a much tighter bound, and will be discussed further in section 4.6.

4.5 Bias in the Offline Analysis

Even if the detector itself is perfect, it is still possible to generate spurious asymmetries during the offline analysis. As an example, consider what might happen if we use incorrect pedestals on one set of MWPC cathode strips. Tracks that pass through this region of the chamber will have both their momentum and position mis-measured, and will naturally have larger fit residuals. When we apply tracking cuts (PMU, CHI2, TARG, RING), we will introduce a geometrical systematic. Cuts will no longer be uniform across the chamber, but instead will depend on position in a complicated manner.

This type of bias is not specific to the tracking analysis, and in general can effect any detector subsystem. The best defense is simply to check for gain and pedestal shifts as often as possible. Typically in E246, we take pedestal runs daily and re-calibrate most sub-detectors during each beam cycle.

We can study this effect in detail by measuring $A_\Sigma$ with different cuts applied.
4.5. Bias in the Offline Analysis

Figure 4.14: The $\phi$ distribution of reconstructed $\pi^0$s. (a) All events. The large spikes are caused by the CsI magnet gaps. (b) After F/B tag. (c) The average fluctuations from the mean in plot (b). Some gap structure is still apparent. From the peak to peak fluctuations, we can constrain possible changes in detection efficiency to be less than 3%.
4.5. Bias in the Offline Analysis

This is a somewhat murky business because the detector isn't perfect, and kinematic cuts can couple to real geometrical flaws in a manner that is typically hard to disentangle. For instance, the POLTRIG counter in gap 12 was slightly rotated for several months of running. High momentum events such as $K^+ \rightarrow \pi^+\pi^0$, which tend to hit near the top of the POLTRIG are very susceptible to a geometrical bias from this rotation, while $K^+ \rightarrow \mu^+\nu_\mu\pi^0$ events that peak in the center of the counter are not. Cutting high momentum events, therefore, changes the measured asymmetry. Correlated systematics are unavoidable, and we must proceed as best we can.

We will begin by analysing a set of data with the fewest possible cuts applied; namely a set of PASS1 data containing events with two or more photons that have passed EHOLE. As cuts are applied, it becomes clear that neither target, $\pi^0$ reconstruction, nor kinematic cuts have a dramatic effect on $A_n$. Muon momentum cuts and the associated track quality cuts eliminate the large bias asymmetry in gap 12, which was the result of the POLTRIG shift. Changes in other gaps are small, although it is evident that $A_1$, $A_2$, and $A_5$, also change slightly. Unlike the situation in gap 12, however, the asymmetry in these gaps becomes worse. The results are summarized in figure 4.15.

Because the changes are small, statistical fluctuations are difficult to rule out. However, in each gap, we note that the asymmetry only changes when we apply momentum and track quality cuts, and is unaffected by all other cuts. This suggests
that we are seeing a real effect. If so, we are left with two possibilities. Either these shifts are an artifact of the tracking analysis or the MWPC calibration. Or, the tracking cuts exaggerate geometrical flaws in the detector, as they did in gap 12. In particular, since tracking cuts eliminate $K_{\pi^2}$ events, they change the effective target stopping distribution. Since we already know that the kaon stopping distribution creates a large geometrical systematic in each gap, it is difficult to separate what contribution comes from changes in the acceptance, and what contribution comes from biases in the offline code.

In order to estimate the magnitude of this effect, we assume that all changes in $A_n$ are due to offline biases (the worst case scenario). From figure 4.15, we note that the RMS of the $A_n^{\text{cuts}} - A_n^{\text{nocuts}}$ distribution (excluding gap 12) is 0.7%. Assuming a random distribution of errors, we expect that $A_{\sigma} < 0.7\%/\sqrt{12} = 2 \times 10^{-3}$. This agrees with the direct measurement $A_{\sigma}^{\text{cuts}} - A_{\sigma}^{\text{nocuts}} = 0.9 \pm 2.3 \times 10^{-3}$. This implies $P_{T}^{\text{syst}} < 1 \times 10^{-2}$ (using $\alpha = 0.198$). Again we stress that this error is calculated before accounting for rejection from F/B tagging.

4.6 Rotations in the Decay Plane $\vec{P}_\mu \times \vec{P}_\pi$

As we mentioned earlier, the radial polarization difference between forward and backward going samples tends to be washed out by $\mu$SR in the fringing field. Polarization asymmetries caused by rotations in the positron counters, therefore, will tend to be
Rotations in the Decay Plane $\bar{P}_\mu \times \bar{P}_\pi$

Figure 4.15: $A_{\text{nocuts}} - A_{\text{cuts}}$, i.e. the difference between the measured asymmetry with no cuts applied, and with all analysis cuts applied. Changes are small, and in most gaps consistent with statistical fluctuations. The large change in gap 12 is most likely due to a shift in the POLTRIG counter in that gap that was present for several months of running before being corrected. The changes in gaps 1, 2 and 5 are not completely understood, but are likely created as cuts generate biases in the target stopping distribution.
4.6. *Rotations in the Decay Plane* $\vec{P}_\mu \times \vec{P}_\pi$

vanishingly small. However, both rotations in the magnetic field and rotations of the
decay plane with respect to the magnet poles will lead to a systematic error that fails
to cancel after F/B tagging. In this section, we study the latter effect. In particular,
a rotation of the decay plane about the beam ($z$) axis will have a similar effect to a
$\delta_z$ rotation in the magnetic field. Rotations about $x$, which lead to inclusion of some
fraction of $P_L$ into the $P_T$ sample (equivalent to a $\delta_x$ rotation in the $\vec{B}$ field), will
also be studied. Unlike a $\delta_z$ rotation, $\delta_x$ rotations are expected to cancel after F/B
tagging.

In order to convert a measured rotation angle into a systematic error in the polar-
ization, we note that Monte Carlo predicts $P_N \sim 50\%$ and $P_L \sim 60\%$. Furthermore,
GMC predicts roughly a 20\% attenuation of $P_T$ due to the non-uniform fringing field.
This gives us:

$$P_T^{\text{sys}} = 0.6 \times 0.8 \times \delta_z + 0.5 \times 0.8 \times \delta_z$$  \hspace{1cm} (4.13)

In order to measure the decay plane angle, we begin by reconstructing the decay
plane normal vector:

$$\hat{n} = \frac{\vec{P}_\mu \times \vec{P}_\pi}{|\vec{P}_\mu \times \vec{P}_\pi|}$$  \hspace{1cm} (4.14)

We then calculate $\cos(\delta_z)$ (or $\cos(\delta_x)$) converting $\hat{n}$ into gap coordinates, projecting it
into the $x - y$ ($y - z$) plane, and then taking the dot product with a unit vector.
4.6. *Rotations in the Decay Plane $\vec{P}_\mu \times \vec{P}_\pi$*

connecting the center of the polarimeter counters. Looking separately at the forward and backward samples, we find:

$$1\gamma: \frac{\delta^f - \delta^b}{2} = 1.01 \pm 0.55 \text{ mr} \quad (4.15)$$

$$2\gamma: \frac{\delta^f - \delta^b}{2} = 0.06 \pm 0.35 \text{ mr} \quad (4.16)$$

$$1\gamma: \frac{\delta^f + \delta^b}{2} = 0.45 \pm 0.15 \text{ mr} \quad (4.17)$$

$$2\gamma: \frac{\delta^f + \delta^b}{2} = 0.58 \pm 0.10 \text{ mr} \quad (4.18)$$

As shown in figure 4.16, the gap structure of $\delta_z$ strongly resembles the structure observed in $A_n$, which follows naturally since both arise from offsets in the $\mu^+$ stopping distribution.

Although these numbers look promising, we must understand how reliable a measurement we can make. Precision on $\delta_z$ is determined by how well we know the position of the MWPCs. From a recent C4 position survey[23], we know that position fluctuations (in+out)/2 as large a 1 mm have been observed. Assuming that these are random fluctuations, and taking the average flight length to be ~ 120 cm, we find:

$$\delta^{\text{syst}}_z = \frac{1 \text{ mm}}{\sqrt{12}} \times \frac{1}{1200 \text{ mm}} = 0.24 \text{ mr} \quad (4.19)$$
4.6. Rotations in the Decay Plane $\vec{P}_\mu \times \vec{P}_\pi$

This implies:

$$1\gamma : \delta_z < 0.9 \text{ mr (90\% C.L.)} \quad (4.20)$$

$$2\gamma : \delta_z < 1.1 \text{ mr (90\% C.L.)} \quad (4.21)$$

The precision on $\delta_z$ is in principle much worse since it depends on how well we know the global alignment of the CsI barrel. However, since we only need to measure the difference ($\delta_z^I - \delta_z^b$), most errors cancel. The overall precision on $\delta_z^I - \delta_z^b$ is better than 0.1 mr.

These rotations add coherently. Taking into account the different weights for the one and two photon samples, we find a systematic of $\delta P_T^{\text{sys}} \sim 6 \times 10^{-4}$.

4.6.1 Rotations in the CsI

Both $\delta_z$ and $\delta_x$ have been calculated assuming perfect CsI efficiency and perfect knowledge of the crystal positions. We claimed that these effects do not strongly affect $\delta_z$ because this particular angle depends mainly on the muon, and not the pion flight direction. However, since decay plane rotations dominate the $P_T$ systematics, we should be very careful and attempt to constrain systematics coming from CsI positioning errors as well as possible.

A global rotation of the CsI with respect to the magnet poles can cause a rotation of the decay plane. There are two types of rotations that we can perform; a rotation
4.6. Rotations in the Decay Plane $\vec{P}_\mu \times \vec{P}_\pi$

$\delta_x$ (mm) vs. Gap

Figure 4.16: The decay plane angle $\delta_x$ as a function of gap. Variations similar to the ones observed in figure 4.1 are apparent.

about the beam axis ($\phi$) or a rotation about the $x$ or $y$ axes ($\theta$ in toroidal coordinates). Rotations about the $x$ or $y$ axes are expected to cancel after summing over 12 gaps. More importantly, rotations of the CsI about $\theta$ create $\delta_x$ decay plane rotations, which cancel after the F/B tag. TMC simulations show that the maximum asymmetry in any gap is $6 \times 10^{-4}$ for every 10 mr rotation in the CsI. This cancels to at least 1:15 after F/B tagging. Since it is also expected to cancel after summing over all gaps, it will have a negligible contribution to the overall systematic error.

Rotations about the beam axis are much more dangerous since these can translate directly into $\delta_z$ type rotations. A global rotation of the calorimeter is not expected to cancel after summing over all gaps nor after F/B tagging. Global CsI misalignment

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4.6. Rotations in the Decay Plane $\vec{P}_\mu \times \vec{P}_\pi$

is potentially an extremely serious problem.

Despite the weight of each crystal and the difficulty involved in accurately mounting the barrel, we were able to position the CsI to within 4 mm. If we take 25 cm as the average radius, we obtain a position accuracy of 15 mr. By introducing a rotation into the CsI during the data analysis, we find that the decay plane rotates by roughly 2 mr for every 100 mr rotation in the CsI. This implies an systematic asymmetry of $2 \times 10^{-5}$ from the CsI. Even though this effect is already small, we stress that 15 mr should be treated as an upper limit for crystal positioning errors. We can actually constrain the average rotation angle much better using $K_{\pi 2}$ events (see figure 4.17):

$$\phi = \sin^{-1}(n_{\mu}^{\text{proj}} \times n_{\pi}^{\text{proj}}) = 0.2 \pm 0.3 \text{ mr} \quad (4.22)$$

Where $n^{\text{proj}}$ refers to the unit direction vector projected into the $x/y$ plane. For a perfectly aligned detector, $\phi$ should be zero.

4.6.2 $\phi$ Dependent $\gamma$ Detection Efficiency

An asymmetric photon detection efficiency can also lead to a average rotation in the decay plane. In the spirit of section 4.4.4, we can study this effect by introducing a 50% efficiency drop in one hemisphere of the calorimeter. This introduces a maximum 2 mr $\delta_z$ rotation in an individual gap. Since this is a non-screw asymmetry, we expect cancelation from summing over all magnet gaps. Indeed, we find that for a 50%
4.7 Estimating Forward/Backward Rejection

Figure 4.17: $\phi$ for $K_{e2}$ events. $\phi$ is defined in equation 4.22, and should be zero if the CsI is perfectly aligned.

inefficiency in the CsI, the change in $\delta_z$ after summing over all gaps is less than 0.3 mr. In reality, the fluctuations in $\pi^0$ detection efficiency with $\phi$ are only a few percent, so after summing over all gaps we expect $\Delta\delta_z < 0.02$ mr. This is completely negligible.

4.7 Estimating Forward/Backward Rejection

With the exception of magnetic field and decay plane asymmetries, which have been discussed in detail in sections 4.3.3 and 4.6, all potential systematics can be divided into two categories. First, there is anything that creates a shift in the muon stopping distribution in the aluminum stopper. This covers numerous sources including errors in the MWPC position, shifts in the Al stopper, offsets in the target stopping
distribution, and so forth. Second, there is anything which effectively changes the acceptance of one of the position counters. Again, this covers many sources including counter position offsets and inefficiencies.

We can estimate the forward/backward rejection for both cases by introducing false asymmetries in the data, and then determining how well they cancel. In each case, we will consider asymmetries that are independent of $x_{\text{stop}}$, and asymmetries which depend on $x_{\text{stop}}$. The latter type are designed to exaggerate the difference in muon stopping distributions between the forward and backward samples, and can therefore be considered as "the worst case scenario." Because of rapid precession in the fringing field, differences in the in-plane spin distributions only couple to magnetic field imperfections or rotations in the decay plane $\vec{B}_\mu \times \vec{P}_\pi$, which have already been discussed. What we wish to measure here is the cancelation of geometrical, rather than polarization, asymmetries.

Shifts in the muon stopping distribution can be easily generated by making fiducial cuts on the muon stop point in the Al stopper. The simplest cut that we can introduce is $y_{\text{stop}} < 0$ cm (i.e. eliminate one-half of the stopper). This introduces an asymmetry of 44%, which cancels to better than 1:55 at the 90% confidence level. In order to create a radial dependence to the asymmetry, we can cut the stopper into quarters (i.e. $y_{\text{stop}} < 0$ cm and $x_{\text{stop}} > 115$ cm). These cuts introduce asymmetries of the same magnitude, and cancel to better than 1:25 at the 90% confidence level. The
lower cancelation is not simply the result of limited statistics. The upper region, $x_{\text{stop}} > 115 \text{ cm}$, typically gives poorer F/B cancelation.

Changes in the acceptance of a counter are also easy to simulate. The simplest thing to do is introduce a software inefficiency in the CW counter in each gap. This type of asymmetry is found to cancel at better than 1:50 at the 90\% confidence level. A radial dependence can be introduced in the asymmetry by cutting on TDIF, the time difference between the upper and lower PMTs. For instance, we can require that TDIF < 0 for the CW counter in each gap. Effectively, this is equivalent to a radial shift in the counter, which we know is a serious problem in our detector [22]. These asymmetries are found to cancel at better than 1:40 at the 90\% confidence level.

To summarize, most geometrical systematics cancel at better than 1:25 after forward/backward tagging. Since shifts in the muon stopping position are responsible for most of the structure in $A_n$, and cancel to better than $|A^\Sigma| < 0.13\%$ after summing over all gaps, this systematic will be reduced to $|P^\text{syst}| < 2 \times 10^{-4}$ after F/B tagging.

As an independent check, we can measure the geometrical bias separately for the forward and backward samples. In section 4.1.1, we found that we could reproduce the structure of $A_n$ using the measured hit distribution from C4 convoluted with the acceptance curve shown in figure 4.3. Doing this for forward and backward samples, we find:
4.8. Summary

A summary of all systematic errors is given in table 4.2. The rejection power from F/B tagging for each mode was either calculated specifically for that mode, or estimated using the results of section 4.7. It should be clear that the effect of a $\delta_z$ rotation in the decay plane dominates the systematic error.

We can estimate the effect on the final result by using $\alpha = 0.198$ and $P_T/Im(\xi) = 0.32$. We obtain a final systematic error of:

$$\delta P_T = 1.0 \times 10^{-3}$$

$$\delta Im(\xi) = 3.1 \times 10^{-3}$$

This is a factor of five lower than our final statistical error. With that said, we are now ready to measure $P_T$. 

---

$$1\gamma : \frac{A_f^{C^4} - A_B^{C^4}}{2} = -2.9 \pm 4.4 \times 10^{-4} \tag{4.23}$$

$$2\gamma : \frac{A_f^{C^4} - A_B^{C^4}}{2} = 2.1 \pm 3.2 \times 10^{-4} \tag{4.24}$$

This is consistent with a high order of systematics cancelation.
### 4.8. Summary

Table 4.2: Contribution to the systematic error from each source. The first column shows the residual $P_T$ expected after summing over all magnet gaps, and the second column shows the reduction after forward/backward tagging. Asymmetries were converted into polarization errors using $\alpha = 0.198$. Most numbers in this table are upper limits on the systematic contribution from each source.

<table>
<thead>
<tr>
<th>Source of Error</th>
<th>$\delta P_T$ after $\sum_{ij}$</th>
<th>$F/B$ tagging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Shift</td>
<td>$&lt; 7 \times 10^{-3}$</td>
<td>$&lt; 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$e^+$ Counter Rotation $V_x$</td>
<td>$8 \times 10^{-4}$</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$e^+$ Counter Rotation $V_z$</td>
<td>$5 \times 10^{-4}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$e^+$ Counter y Offset</td>
<td>$&lt; 7 \times 10^{-3}$</td>
<td>$&lt; 2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$e^+$ Counter x Offset</td>
<td>$1 \times 10^{-3}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$e^+$ Counter z Offset</td>
<td>$&lt; 1 \times 10^{-3}$</td>
<td>$&lt; 3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\vec{B} \epsilon$ Screw Asymmetry</td>
<td>$9 \times 10^{-4}$</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\vec{B} \delta_z$ Screw Asymmetry</td>
<td>$8 \times 10^{-4}$</td>
<td>$4 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\vec{B} \delta_z$ Screw Asymmetry</td>
<td>$&lt; 5.0 \times 10^{-4}$</td>
<td>$&lt; 5.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>Trigger Counter Rotation</td>
<td>$&lt; 10^{-4}$</td>
<td>$&lt; 10^{-5}$</td>
</tr>
<tr>
<td>MWPC Efficiency Gradient</td>
<td>$&lt; 2 \times 10^{-3}$</td>
<td>$&lt; 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Bias induced in Offline Analysis</td>
<td>$&lt; 1 \times 10^{-2}$</td>
<td>$&lt; 4 \times 10^{-4}$</td>
</tr>
<tr>
<td>Beam Background</td>
<td>$&lt; 1 \times 10^{-3}$</td>
<td>$&lt; 4 \times 10^{-5}$</td>
</tr>
<tr>
<td>Decay Plane Rotation</td>
<td>$6 \times 10^{-4}$</td>
<td>$6 \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>$&lt; 1.4 \times 10^{-2}$</td>
<td>$&lt; 1.0 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Chapter 5

Conclusions

5.1 Analysis Results

We analyzed data taken from May 1996 up to and including February 1998. The final sample contains 1.93M $2\gamma$ events and 867K $1\gamma$ events in the forward and backward hemispheres. We find residual asymmetries of:

\begin{align*}
1\gamma: \frac{A_f - A_B}{2} &= -1.7 \pm 1.2 \times 10^{-3} \\
2\gamma: \frac{A_f - A_B}{2} &= 6.8 \pm 8.1 \times 10^{-4}
\end{align*}

Both the one and two photon results are consistent with zero at the 90% confidence limit. Combining results from both streams, we find:

\begin{align*}
P_T &= 0.1 \pm 4.9\text{(stat)} \pm 1.0\text{(syst)} \times 10^{-3} \\
Im(\xi) &= 0.2 \pm 1.6\text{(stat)} \pm 0.3\text{(syst)} \times 10^{-2}
\end{align*}
5.2. Gap Dependence

Finally, expressing the result as a limit on new effective scalar or pseudoscalar interactions:

\[ |\text{Im}(F_s + F_p)| < 1.4 \times 10^{-8} \text{ GeV}^{-2} \ (90\% \ C.L.) \]

5.2 Gap Dependence

Although every gap is connected to its neighbor (the CW $e^+$ counter in one gap is the CCW counter in the neighboring gap), each gap also contains many independent elements. For instance, each gap has its own MWPCs, its own stopper, and each Al stopper has a slightly different fringing field configuration. For that reason, it is worthwhile to check if any particular gap displays a large systematic error. The physics asymmetry $(A_f - A_b)/2$ in each gap is shown in figure 5.1.

In general, rejection from F/B cancelation seems to be very high for both the one and two photon streams. The variations in $A_n$, which were as high as 3.0%, vanish after taking the forward/backward difference. There is no residual gap structure either in the data or in the C4Y prediction.
5.2. Gap Dependence

Figure 5.1: The physics asymmetry for both one and two photon samples along with the predictions from the forward/backward C4Y distributions. The large gap variations seen in the null asymmetry measurement vanish after taking the forward/backward difference.
5.3 Cut Dependence

In order to check for possible systematic errors that may have been missed, track quality, $\pi^0$ reconstruction, and kaon timing cuts were varied to determine if there is any particular anomalous region. In particular, the two primary concerns were:

- Asymmetric $K_{\pi2}$ scattering at small dip angles.
- Systematic contributions from $K^+$ DIF or beam backgrounds (particularly in the $1\gamma$ stream).

In order to check for possible systematics coming from scattered $K_{\pi2}$ events, we measure the asymmetry as a function of $\theta_{\mu\pi}$. As shown in figure 5.2, there is no strange behavior of the asymmetry near $\theta_{\mu\pi} \sim 90^\circ$.

In order to determine the effect of backgrounds in the $1\gamma$ sample, we measure the physics asymmetry $(A_f - A_b)/2$ for different slices of photon energy and $T_K$. Beam background as well as $K^+$ DIF events will tend to cluster at higher photon energies and near $T_K = 0$. As shown in figure 5.3, there is no significant variation of $(A_f - A_b)/2$ in the large background regions.

No dependence of $P_T$ on cuts was observed. We therefore conclude that there is no evidence for an unpredicted systematic error.\footnote{It was previously noticed\cite{24} that calculating the kaon stopping position uses only MWPC information causes some scattered $K_{\pi2}$ events to be mismeasured into the backward hemisphere. This problem is eliminated by reconstructing the kaon vertex using only C2, target, and ring counter information.}

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5.3. Cut Dependence

Figure 5.2: Asymmetry vs. $\theta_{\mu\pi}$. There is no evidence of an asymmetric contribution from $K_{\pi 2}$ scatters, which would appear near $\theta_{\mu\pi} \sim 90^\circ$. 

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5.3. Cut Dependence

Figure 5.3: Physics asymmetry for different slices in $T_K$ and $E_\gamma$. For the bottom plot, both $E_\gamma^{\text{high}}$ and $E_\gamma^{\text{low}}$ cuts were removed. There is no clear dependence of $(A_f - A_b)/2$ on backgrounds, consistent with the systematic error estimate presented in chapter 4.
Appendix A

Constraints on $Im(F_s + F_p)$ from $P_L$

in $K_{\mu 2}$ Decay

In this section, we demonstrate how to calculate a constraint on $Im(F_s + F_p)$ using the measured value of $P_L$ in $K_{\mu 2}$ decay. We begin with the Lagrangian:

$$L = G_f \bar{s} \gamma_\mu \left( \frac{1 - \gamma^5}{2} \right) u \bar{\mu} \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \nu + F_s \bar{s} u \bar{\mu} \nu + F_p \bar{s} u \bar{\mu} \gamma^5 \nu$$  \hspace{1cm} (A.1)

Which includes both SM and non-SM terms. Simple algebra shows that the non-SM terms can be written:

$$L^{\text{NON-SM}} = (F_s + F_p) \cdot \bar{s} u \bar{\mu} \gamma^\mu \left( \frac{1 + \gamma^5}{2} \right) \nu + (F_s - F_p) \cdot \bar{s} u \bar{\mu} \left( \frac{1 - \gamma^5}{2} \right) \nu$$  \hspace{1cm} (A.2)

The first term couples to left-handed anti-muons while the second term couples to right-handed anti-muons. Since the second term will produce muons with the same polarization as the SM term, we discard it. That leaves us with:
Computing the matrix element for $K_{\mu 2}$ decay, we find:

$$<\mu\nu|\mathcal{L}|K> = G_F \cdot F^V_K \cdot q_\mu \cdot \bar{\mu} \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \nu + (F_s + F_p) \cdot F^S_K \cdot \bar{\mu} \left(\frac{1+\gamma^5}{2}\right) \nu$$ \hspace{1cm} (A.4)

Where $q_\mu$ is the four-momentum transfer, and $F^V_K$ and $F^S_K$ are the vector and scalar kaon-decay-constants, which are defined as:

$$q_\mu \cdot F^V_K = <0|\bar{s}\gamma_\mu \left(\frac{1-\gamma^5}{2}\right) u|K>$$

$$F^S_K = <0|\bar{s}u|K>$$ \hspace{1cm} (A.5, A.6)

Applying the Dirac equation, we obtain:

$$<\mu\nu|\mathcal{L}|K> = G_F \cdot F^V_K \cdot m_\mu \cdot \bar{\mu} \left(\frac{1-\gamma^5}{2}\right) \nu + (F_s + F_p) \cdot F^S_K \cdot \bar{\mu} \left(\frac{1+\gamma^5}{2}\right) \nu$$ \hspace{1cm} (A.7)

This amplitude contains two terms $\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{NON-SM}}$. The first term is the familiar SM term, which produces right-handed anti-muons. The second term is new and produces left-handed anti-muons. These two terms cannot interfere\(^1\). The longitudinal polarization can therefore be read off directly from the Lagrangian:

\(^1\)The interference term goes as $(1 + \gamma^5) \cdot (1 - \gamma^5) = 0$
\[ P_L = \frac{\mathcal{L}_{\text{SM}}^2 - \mathcal{L}_{\text{NON-SM}}^2}{\mathcal{L}_{\text{SM}}^2 + \mathcal{L}_{\text{NON-SM}}^2} \]  

Substituting from equation A.7 and rearranging terms:

\[ |F_s + F_p| < \frac{G_f \cdot F_K^V \cdot M_K \cdot \frac{M_e}{M_K} \sqrt{1 - P_L}}{F_K^S} \]  

The Standard Model (vector) kaon-decay-constant can be extracted from the \(K_{\mu 2}\) branching ratio and has a value of \(F_K^V = 160\) MeV[5]. The scalar kaon-decay-constant can, in principle, be estimated from lattice QCD. To our knowledge no such calculation exists. In order to construct a bound, we assume that \(F_K^S \sim M_K^2\).

Assuming the weakest possible constraint \(Re(F_s + F_p) = 0\), and using the best measurement of the longitudinal muon polarization[15]:

\[ P_{\mu}^{\text{long}} > 0.99 \quad (90\% \text{ C.L.}) \]  

we find:

\[ |Im(F_s + F_p)| \leq 7 \times 10^{-8} \text{ GeV}^{-2} \quad (90\% \text{ C.L.}) \]
Appendix B

Calculation of the Statistical Sensitivity

In this section, we calculate the impact of backgrounds to the statistical sensitivity. We wish to measure the signal asymmetry between two counters, which we will call left and right:

\[ A = \frac{L - R}{L + R} \]  \hspace{1cm} (B.1)

In the presence of background, what we actually measure is the sum of the signal \((L,R)\) and the background \((B_L,B_R)\) counts. In order to obtain the correct asymmetry, we must estimate the background in each counter \((B'_L,B'_R)\), and subtract it:

\[ A = \frac{L + B_L - B'_L - R - B_R + B'_R}{L + R + B_L - B'_L + B_R - B'_R} \]  \hspace{1cm} (B.2)

Taking the derivative, we find:
\[ \delta A = \frac{T_1 - T_2}{T_3} \] (B.3)

\[ T_1 = (L + R + B_L + B_R - B'_L - B'_R) \cdot (\delta L - \delta R + \delta B_L - \delta B_R - \delta B_L + \delta B_R) \] (B.4)

\[ T_2 = (L - R + B_L - B_R - B'_L + B'_R) \cdot (\delta L + \delta R + \delta B_L + \delta B_R - \delta B'_L - \delta B'_R) \] (B.5)

\[ T_3 = (L + R + B_L - B'_L + B_R - B'_R)^2 \] (B.6)

To simplify notation, we write:

\[ S = L + R \] (B.7)

\[ B = B_L + B_R \] (B.8)

\[ B' = B'_L + B'_R \] (B.9)

We also assume that we can accurately estimate the true background fraction, i.e. \(< B > = < B' >\). This gives us:

\[ T_1 = S \cdot (\delta L - \delta R + \delta B_L - \delta B_R - \delta B_L + \delta B_R) \] (B.10)

\[ T_2 = (L - R + B_L - B_R - B'_L + B'_R)(\delta L + \delta R + \delta B_L + \delta B_R - \delta B'_L - \delta B'_R) \] (B.11)

\[ T_3 = S \] (B.12)

After squaring and ensemble averaging, we find:
\[(\delta A)^2 = \frac{\delta L^2 + \delta R^2 + \delta B^2 + \delta B'^2}{S^2} + \mathcal{O}(A^2)\]

using Poisson statistics \((\delta n^2 = n)\), and dropping higher order terms:

\[(\delta A)^2 = \frac{L + R + B + \delta B'^2}{S^2}\]

\[\delta A = \frac{1}{\sqrt{S}} \cdot \sqrt{1 + \frac{B + \delta B'^2}{S}}\]

We now derive specific attenuation factors for two different types of background:

- Flat background in the positron counters.
- \(K_{\pi 2}\) backgrounds.

To estimate the background level in the positron counters, we measure the number of counts in a delayed time window \((12 < T_\mu < 20 \, \mu s)\). The uncertainty in the background estimate comes from counting statistics, so \(\delta B'^2 = B'\). Since the background time window is the same size as the signal time window, \(B = B'\). This leaves us with:

\[\delta A = \frac{1}{\sqrt{S}} \cdot \sqrt{1 + \frac{2 \cdot B}{S}}\]

\(K_{\pi 2}\) background estimates are made using GMC simulations, and are typically accurate to within a few percent. Therefore, \(\delta B'^2 \ll B\). This implies that:

\[\delta A = \frac{1}{\sqrt{S}} \cdot \sqrt{1 + \frac{B}{S}}\]
Appendix C

SCA/TD Operating Manual

This section is intended to act as a brief summary of both the online and offline software used by the TD system. It contains a complete listing of TD software and a set of instructions explaining how to boot the system at KEK. In addition, it briefly summarizes the offline analysis, and presents typical results from the TD pulse fitting.

C.1 Programming

In order to start the SCA/TD system, three separate modules must be programmed. The first is the FASTBUS XILINX\(^1\) controller on the SCA board. This chip is typically programmed on power-up by an onboard PROM. Source code for this PROM can be found on node:pupcad1.princeton.edu in `/u/chris/xilinx/fastbus_e246_1997`

Secondly, the onboard SCA controller XILINX must be programmed. The XILINX\(^1\) chips are field-programmable-gate arrays, which can be programmed either from a PROM, or from a computer download.

\(^1\)XILINX chips are field-programmable-gate arrays, which can be programmed either from a PROM, or from a computer download.
source can be found on node:pupcad1.princeton.edu in /u/chris/xilinx sca_e246_1997. This software must be downloaded to each SCA/TD board using the FPI FASTBUS interface.

All FPI download and control software is located on node:tvhprt.kek.jp. The binary FPI executable is called boottd5, and can be downloaded using the command loadveri5. The software to download the XILINX bit-stream to the FPI is called xlxboot5. All modules in FB02 and FB03 are programmed automatically after executing the shell script /home/tvhunter/run/starttd5. This script calls xlxboot5 for each TD module and also downloads boottd5 to each FPI.

Once the TD modules have been programmed, it is necessary to download the zero suppression code and pedestal tables to the DSP modules. Once the DSPs have been reset (by pushing the white button on the back on the board), both the program code and pedestal data is transferred to the DSP via the RS232 interface on node k5ws1.kek.jp. There are two scripts available:

k5ws1:/home/tvhunter/run/startdsp: Loads DSP with 5 MeV threshold
k5ws1:/home/tvhunter/run/startdsp_ped: Loads DSP with 0 MeV threshold

The latter is used for taking TD pedestal data.
C.1.1 Rebuilding On-line pedestal Data

From time to time, boards break and need to be replaced, so we need a system to reconstruct the online pedestals. TD pedestals can be built automatically from pedestal runs using TRIVIA by issuing the command:

CHEST/DRAWER/SET TD_ANALTYPE C 'PED'

This automatically builds two pedestal files: fb02_ped.00001 and fb03_ped.00001 which can be directly imported to CFM. On-line pedestal files are identical to off-line files except that they do not contain a header. Before rebuilding the online DSP software using these files, the header must be removed by hand. On-line pedestal files are stored on k5ws1:

/home/tvhunter/work/sca-td/dsp/pro/pedestals/FB02
/home/tvhunter/work/sca-td/dsp/pro/pedestals/FB03

In order to rebuild the DSP pedestal list:

cd /home/tvhunter/work/sca-td/dsp/pro/
makeall

This will generate DSP files with thresholds of 1.5, 2.5, 3.0, 5.0, 7.5 and 10.0 MeV using the FB02 and FB03 pedestal files.
C.1.2 TD Calibration

TD energy calibration is performed by comparing TD fit results with data from the TKO ADCs. TRIVIA contains a built-in calibration routine which can be executed by issuing the command:

CHEST/DRAWER/SET TD_ANALTYPE C 'CAL'

The calibration analysis begin by fitting all TD pulses using a single pulse hypothesis. Good single pulses are selected by requiring a good fit $\chi^2$, and by requiring that the fit time agree with the time from the TKO TDC. The calibration constant for each channel is obtained by dividing the fit pulse amplitude by the energy measured in the TKO ADC. The results are placed in two TD histograms (one for each crate) which can be analyzed using PAW. These histograms are only created if 'CAL' mode is selected, and are not booked during standard analysis.

C.2 Timing

The TD system requires three external signals which provide both trigger and timing information. They are an external 20 MHz clock, the experimental trigger with encoded fast clear information, and a ramp for timing calibration. The 20 MHz clock signal is fanned to all modules using a NIM fanout. The internal clock divider on each TD module is synchronized each time a trigger or fast clear is issued. In order
C.2. **Timing**

to ensure that the clock signals are synchronized at the start of a spill, a "fake" fast clear signal is sent to all the TD modules from a CAMAC output register during pre-spill initialization. Clock signals are divided to provide a sampling time of 600 ns while logic decisions are performed at 20 MHz. The trigger signal is fanned out in the same way. The ramp signal, which is generated by an incoming trigger, is fed into the first channel of the first module.

The TD trigger is derived from the same source that generates the ADC gates for the CsI TKO ADCs. This signal is only issued if both a prompt and level 1 (photon) trigger is received. Once a trigger is asserted, the TD modules will stop processing data, and will wait 22 μs in order to check for a level 2 trigger. If there are no positron hits at level 2 (e+ fast clear is asserted), the trigger signal is de-asserted, and the TD will continue collecting data. If the trigger signal remains high at the end of 20 μs, the TD boards immediately begin converting data using onboard ADCs. This process takes 200 μs. The trigger logic is shown in figure C.1.

Once conversion has begun, the onboard DSP will issue a prompt trigger veto, which is converted into a NIM signal and returned to the master experiment trigger veto. Once conversion has finished, the DSP modules will scan all channels in order to check for above threshold hits, and then buffer the data in its onboard memory. The zero suppression algorithm, which checks for any point above 5 MeV in a 2.4 μs window, takes 50 μs to complete. It takes approximately 200 μs to completely buffer
all the data in the DSP memory. Readout then occurs while the PS Main Ring is refilling for the next spill (2.0 seconds).

Figure C.1: Trigger logic diagram for the TD system. This section of the trigger is implemented in fast NIM logic. Trigger/fast clear processing is done in an onboard XILINX 4005A chip.

C.3 Offline Analysis and Typical Results

The TD unpacking and analysis routine is called automatically by anacsi.f. This software takes blocks of data from crates FB02 and FB03 and divides them into individual events. Data consistency checks are performed by comparing the number of events recorded by the TD modules with the number of events in the TKO crates.

Once the event build is completed, pulse shape analysis commences. This is essentially a six step process. First, the ramp pulse is fit in order to extract the
C.3. Offline Analysis and Typical Results

correct $t_0$ of the event. This eliminates the 600 ns jitter caused by the TD sampling time. Second, all pulses are pedestal subtracted and checked for glitches coming from bad SCA capacitor cells. Any glitches are smoothed using a simple interpolation routine. Then, the pulse area is compared to the peak amplitude. For good single pulse events, the area and amplitude should be directly proportional and should satisfy the empirical relationship:

$$\text{AREA} = 6.85 \cdot \text{AMP} \quad (C.1)$$

Events which fall on this line are tagged as good single pulse events.

Events which fail the AREA/AMP cut and which lie near a CsI cluster are fit assuming a single pulse hypothesis. If the scaled $\chi^2$:

$$\chi^2_{\text{scal}} = 500.0 \cdot \frac{\chi^2_{\text{fit}}}{(\text{AMP} + 200 \text{ ch})} \quad (C.2)$$

is less than 5.0, then the pulse is also flagged as a good single pulse, and no further analysis is needed. If not, the pulse shape is fit assuming a double pulse hypothesis. This allows us to extract both the time and amplitude of the signal pulse.

Once pulse fitting is finished, pulse height, time, and $\chi^2$ information for both single and double pulse fits are stored in YBOS banks TDSR and TDPR respectively. Raw TD waveforms are discarded during the PASS 1 analysis.

Typical timing plots are shown in figure C.2. We usually obtain timing resolutions
of $\sigma = 45$ ns for single pulse fits. The resolution for double pulse events is approximately 50% worse. As a result, we use loose TD timing cuts in PASS2 ($|t| < 150$ ns). Most importantly, the TD analysis is able to recover 95% of all pulses containing a pileup. The absolute pileup rate depends strongly on beam conditions, intensity, and structure and has typically varied from less than 1% to as high as 15%.

Figure C.2: Timing plots for TD fits after PASS1.
Appendix D

Toy Monte Carlo

TMC, short for "Toy Monte Carlo," is a simplified Monte Carlo which has been extensively used for systematics studies. The code was originally developed on SGI although it has recently been ported to HIUX, which is the system used for analysis at KEK. This section is intended to be a brief summary of TMC features and commands.

TMC files are stored on node:viper.princeton.edu in $TMC\_SOURCE$. The run script, called tm, defines all the necessary environment variables, sets up pointers to the lookup table directory ($TMC\_TABLES$), and starts the tmc executable. There are several user defined environment parameters, which are typically set up in tm.

$TMC\_CTRL$: Control file for geometry definition.

$GEN\_POLPOS$: 'YES'/\'NO'
'YES' runs the setup for $\bar{B}$ field studies. In this mode, the user must specify files containing stopping distributions for forward and backward going events: $\$POL\_FORWFIL$E and $\$POL\_BACKFILE$. The user must also specify files containing spin distributions for forward and backward going events: $\$POL\_FORWSPIN$ and $\$POL\_BACKSPIN$.

$\$HBOOK$: HBOOK file name for histograms and n-tuples.

$\$RES\_FILE$: Results file showing the counting asymmetry in each gap.

$\$DETECTOR\_PERFECTION$: 'PERFECT'/ 'IMPERFECT'

Setup for the study of non-screw-asymmetries. If set to 'IMPERFECT', the position of all polarimeter elements are smeared using estimates of their positioning accuracy. The exact positions are written to a file: $\$DETECTOR\_GEOM$. The geometry control file ($\$TMC\_CTRL$) allows the user to easily introduce position shifts and inefficiencies into many different subsystems and specifies both the initial random number seeds and the total number of events to generate. It has the following format:

```
# ISEEDs
44812904
5128992
```
# TMC Vers.

1.2

# Number of events to generate

500000

# Target Shift (Physical translation of the target)

0.0 0.0 0.0

# K+ Stopping distribution shift (Translation of the beam)

0.0 0.0 0.0

# E+ counter efficiencies (gaps #1-12)

1.0 1.0 1.0 1.0 1.0 1.0

1.0 1.0 1.0 1.0 1.0 1.0

# E+ counter rotation V_z

0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0

# E+ counter rotation V_r

0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0

# E+ counter position offsets (Y,X,Z)

0.0 0.0 0.0

0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0

# B Field, X(G) Y(G) Z(G) delta(x) delta(z) epsilon
0.0 0.0 0.0 0.0 0.0 0.0

# SIGMA, used to set random imperfections in the polarimeter
1.0

# Pol Trig Shifts+z Rotation z-\textdegree X,Y,Theta(deg)
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
0.0 0.0 0.0
The first three magnetic field parameters $x(G), y(G)$, and $z(G)$ add a constant magnetic field along the specified direction. SIGMA is a multiplier used for the study of non-screw-asymmetries (the positions of polarimeter elements are smeared by SIGMA $\times$ Resolution). The first parameter used to setup an efficiency gradient in C4 determines the minimum radius at which to introduce the efficiency loss. Other parameters should be self-explanatory.
Bibliography


[8] J. Imazato et al., “Search for T-violating Muon Polarization in $K^+ \rightarrow \mu^+\nu_\mu \pi^0$ Decay using Stopped Kaons,” KEK Report 91-8


[21] TOSCA is a commercial electromagnetic field mapping package distributed by Vector Fields inc. (www.vectorfields.com).
