Space Charge in Ionization Detectors

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1 Problem

The effect on the operation of a vacuum diode of a spatially varying distribution of free electrons, commonly called space charge, was first considered by Child [1] and Langmuir [2]. In that example electrons are released from a hot cathode at low velocity, and then drift freely under the influence of the local electric field as modified by the drifting electrons.[1]

In this problem, deduce the steady-state electric field distribution $\mathbf{E} = E(x)\,\hat{\mathbf{x}}$ in an ionization detector (whose nominal field, $E_0\,\hat{\mathbf{x}}$, is uniform over the length D of the detector) as affected by the positive-ion space-charge density $\rho(x)$. Suppose that ionization is created uniformly throughout the volume of the detector (by cosmic rays or by secondary particles from a particle accelerator) at a volume rate of K ion pairs/volume/sec. The ionization electrons are quickly collected at the anode and the associated electron space-charge density can be ignored. That is, you may assume the positive-ion space-charge density is not so great as to prevent the ionization electrons from reaching the anode. The positive ions drift slowly to the cathode with a velocity given by

$$\mathbf{v} = \mu\mathbf{E}, \tag{1}$$

where the positive-ion mobility μ is independent of the electric field strength.[2]

2 Solution

The solution follows [5]. The related problem of charging of insulators in the ionization medium is discussed in [6].

This problem is technically somewhat simpler than the case of the vacuum diode because the positive-ion velocity, rather than acceleration, is related to the electric field strength by eq. (1).

Although total charge is conserved, the assumption of rapid clearing of ionization electrons out of the volume of the ionization detector onto its anode leads to an apparent creation of positive charge density ρ according to a modified form of the continuity equation,

$$\boldsymbol{\nabla}\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = \boldsymbol{\nabla}\cdot\rho\mathbf{v} + \frac{\partial\rho}{\partial t} = K, \tag{2}$$

[1]For a related example of a laser-driven vacuum photodiode, see [3].

[2]The velocity (1) of the positive ions can be less than the velocity of convective motion due to thermal gradients in the case of liquid-noble-gas ionization detectors [4].

where $\mathbf{J} = \rho\mathbf{v}$ is the current density of the positive ions, and K is the volume rate of creation of ion pairs.

We consider an ionization detector whose geometry is that of a parallel-plate capacitor with electrode planes perpendicular to the x axis with separation D between anode and cathode. We seek a steady-state solution, in the approximation that the electric field and the ion velocity are in the x direction,[3] so that eq. (2) simplifies to

$$\frac{d\rho v}{dx} = K, \tag{3}$$

whose solution can be written

$$\rho v = Kx, \tag{4}$$

taking the anode to be at $x = 0$ and the cathode at $x = D$. This result has the interpretation that the steady-state ion density $\rho = Kx/v$ in a volume element at position x is equal to the positive ion density created in that volume element during time x/v, which is the time during which positive ions have drifted from the anode to their present position. Ions created at an earlier time within the ionization detector ($0 < x < D$) will have already drifted past position x.

We can eliminate the velocity $\mathbf{v} = v\,\hat{\mathbf{x}}$ from eq. (4) in favor of the electric field $\mathbf{E} = E\,\hat{\mathbf{x}}$ using eq. (1) for the ion mobility μ, which leads to

$$\rho = \frac{Kx}{\mu E}. \tag{5}$$

Another relation between the charge density ρ and the electric field $\mathbf{E}$ is the Maxwell equation (in MKS units)

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{dE}{dx} = \frac{\rho}{\epsilon}, \tag{6}$$

supposing that the electric field points only in the x direction, and where $\epsilon = \epsilon_{\rm relative}\,\epsilon_0$ is the (relative) dielectric constant of the ionization medium. Combining eqs. (5) and (6) we find

$$\frac{dE^2}{dx} = \frac{2Kx}{\epsilon\mu}, \tag{7}$$

whose solution is

$$E(x) = \sqrt{E_A^2 + \frac{Kx^2}{\epsilon\mu}} = E_0\sqrt{\left(\frac{E_A}{E_0}\right)^2 + \frac{Kx^2}{\epsilon\mu E_0^2}} \equiv E_0\sqrt{\left(\frac{E_A}{E_0}\right)^2 + \alpha^2\frac{x^2}{D^2}} \approx E_A\left[1 + \frac{1}{2}\left(\frac{\alpha E_0 x}{E_A D}\right)^2\right], \tag{8}$$

where $E_A = E(x = 0)$ is the electric field at the anode, $E_0 = V/D$ is the nominal electric field in the absence of space charge, V is the voltage difference between the anode and cathode, and α is the dimensionless parameter

$$\alpha = \frac{D}{E_0}\sqrt{\frac{K}{\epsilon\mu}}, \tag{9}$$

[3]In general, the steady state of eq. (2) is that $\boldsymbol{\nabla}\cdot\rho\mathbf{v} = \boldsymbol{\nabla}\cdot\rho\mu\mathbf{E} = K$, using eq. (1). We can eliminate ρ in favor of $\mathbf{E}$ using eq. (6) to find $\boldsymbol{\nabla}\cdot[\mathbf{E}(\boldsymbol{\nabla}\cdot\mathbf{E})] = K/\epsilon\mu$. If $\mathbf{E} = E(x)\,\hat{\mathbf{x}}$, this simplifies to $d^2E^2/dx^2 = 2K\ \epsilon\mu$, which integrates to eq. (7).

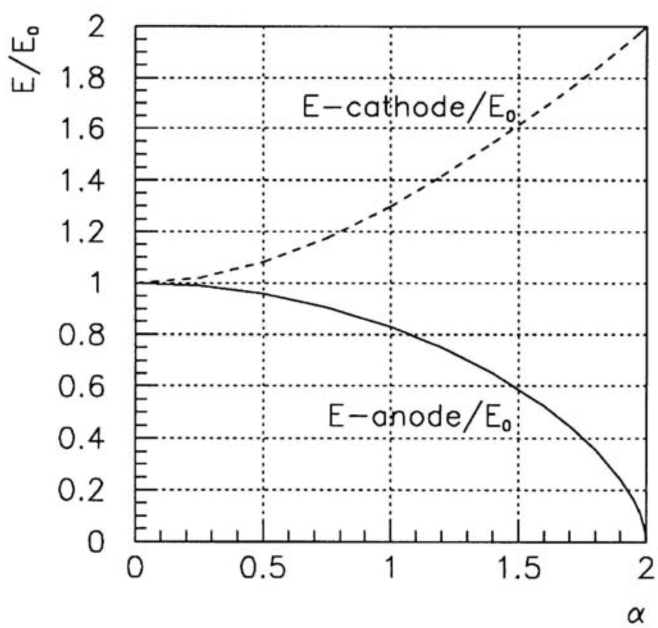
and the approximation holds when $\alpha E_0 x / E_A D \ll 1$, which requires $\alpha \ll 1$.

An analytic expression for the field E_A can be obtained from eq. (8) by calculating the voltage between the anode and cathode,

$$\begin{aligned} V &= E_0 D = \int_0^D E\,dx = \frac{\alpha E_0}{D} \int_0^D \sqrt{x^2 + \left(\frac{D E_A}{\alpha E_0}\right)^2}\,dx \\ &= \frac{\alpha E_0 D}{2}\left[\sqrt{1 + \left(\frac{E_A}{\alpha E_0}\right)^2} + \left(\frac{E_A}{\alpha E_0}\right)^2 \ln\left(\frac{\alpha E_0}{E_A} + \sqrt{1 + \left(\frac{\alpha E_0}{E_A}\right)^2}\right)\right]. \end{aligned} \tag{10}$$

This expression is cumbersome, but we see that if $\alpha = 2$ then the quantity in brackets must be 1, which implies that $E_A = 0$. If the field at the anode goes to zero, the electrons are no longer collected there, and we cannot ignore space charge due to electrons. Thus, the present solution is valid only for $\alpha < 2$, outside of which region the ionization detector would not function effectively.

A plot of the ratio $E_A/E_0 = E\text{-anode}/E_0$ as a function of α is shown below [5].[4]



Also shown on the plot is the ratio $E_C/E_0 = E\text{-cathode}/E_0$ of the field at the cathode to the nominal field. The cathode is at $x = D$, so from eq. (8) we have,

$$\frac{E_C}{E_0} = \sqrt{\alpha^2 + \left(\frac{E_A}{E_0}\right)^2}. \tag{11}$$

Using eqs. (5) and (8), the positive-ion charge density $\rho(x)$ can be written as,

$$\rho(x) = \frac{Kx}{\mu E_0 \sqrt{\left(\frac{E_A}{E_0}\right)^2 + \alpha^2 \frac{x^2}{D^2}}} \approx \frac{Kx}{\mu E_A}\left(1 - \frac{\alpha^2}{2}\frac{x^2}{D^2}\frac{E_0^2}{E_A^2}\right) \approx \frac{Kx}{\mu E_0}\left(1 + \frac{\alpha^2}{6} - \frac{\alpha^2}{2}\frac{x^2}{D^2}\right). \tag{12}$$

[4]From the approximation to eq. (8) for small α, we find that $E_A/E_0 \approx 1 - \alpha^2/6$.

For further discussion of positive-ion space charge in ionization chambers at particle accelerators, see [5].

The above discussion assumed that the electric field was entirely along the x-direction, and that the potential $V(x)$ could have nonlinear dependence. However, in most drift chambers the potential is constrained to have a linear dependence on x along the edges of the drift volume due to field-shaping electrodes there. In such devices, the field distortions are less than those found here, which therefore represent only an upper limit to what will be observed in practice.

However, the field distortions due to space charge in a 3-dimensional detector result in the electric field lines becoming curved, particularly near the cathode plane, such that straight tracks will appear to be curved in a reconstruction that does not include corrections for the space charge. The need for these corrections in the track-reconstruction algorithms will be the main practical impact of space charge on the physics output of the detector.

Example: Space Charge in a Large Liquid Argon Detector at the Earth's Surface

Following the successful commissioning of the ICARUS T600 liquid-Argon time projection chamber in Pavia, Italy in 2001 [7], very large liquid Argon detectors have been considered both underground [8, 9] and at the Earth's surface [10, 11]. For detectors with drifts longer than 1.5 m (as used in the ICARUS T600), space-charge effects from ionization by cosmic-ray muons become important if the detector is at the Earth's surface [9].

We consider liquid-argon detectors operated at a nominal electric field of $E_0 = 500$ V/cm. The positive-ion mobility is roughly $\mu = 0.0016$ cm^2/V/s $= 1.6 \times 10^{-7}$ m^2/V/s [12], so the nominal drift velocity is $v_0 = 0.8$ cm/s. That is, the positive ions take 125 s to drift 1 m, so the accumulation of positive-ion charge density can be significant.

The relative dielectric constant of liquid argon is 1.6 [12], so $\epsilon = 1.4 \times 10^{-11}$ MKS units.

The flux of cosmic rays at the Earth's surface is roughly (see eq. (5) of [14], which is based on [13]),

$$\frac{dR}{d\cos\theta} \approx \begin{cases} 0.05\cos^2\theta/\text{cm}^2/\text{s} & (0 < \theta < \pi/2), \\ 0 & (\pi/2 < \theta < \pi), \end{cases} \tag{13}$$

which implies that the total path length in cm of cosmic-ray muons in one second inside a cube of volume 1 cm^3 is [14]

$$\int_0^1 \frac{1}{\cos\theta}\frac{dR}{d\cos\theta}\, d\cos\theta \approx 0.025. \tag{14}$$

The effective number of ion pairs created by cosmic rays muons is about 50,000/cm [7], so the rate of creation of positive-ion density is 1,250/cm^3/s $= 1.25 \times 10^9$/m^3/s. Expressed in Coulombs, this is $K = 2 \times 10^{-10}$ C/m^3/s.[5]

At a nominal electric field $E_0 = 500$ V/cm $= 50{,}000$ V/m, the parameter α for liquid argon at the Earth's surface is, for distance D in meters,

$$\alpha = \frac{D}{E_0}\sqrt{\frac{K}{\epsilon\mu}} = 0.18D, \tag{15}$$

[5]This implies, for example, that the positive-ion current in a volume $6 \times 6 \times 6$ m^3, as for the CERN ProtoDUNE detectors, will be ≈ 40 nA (with an equal electron current), ignoring recombination.

for detector thickness D in meters. A detector at the Earth's surface with $D = 12$ m would be rendered nonfunctional by the positive-ion space charge, and, referring to the figure on p. 3, even for $D = 2.5$ m (as in the μBooNE detector) effects of cosmic-ray-induced space charge will be noticeable.

The actual situation in a large liquid Argon detector will be complicated by thermal-induced convective motion, for which the liquid flow velocity has been estimated [15, 16] to be an order of magnitude larger than the positive-ion drift velocity of 0.8 cm/s.

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