Can Magnetic Field Lines Break and Reconnect?

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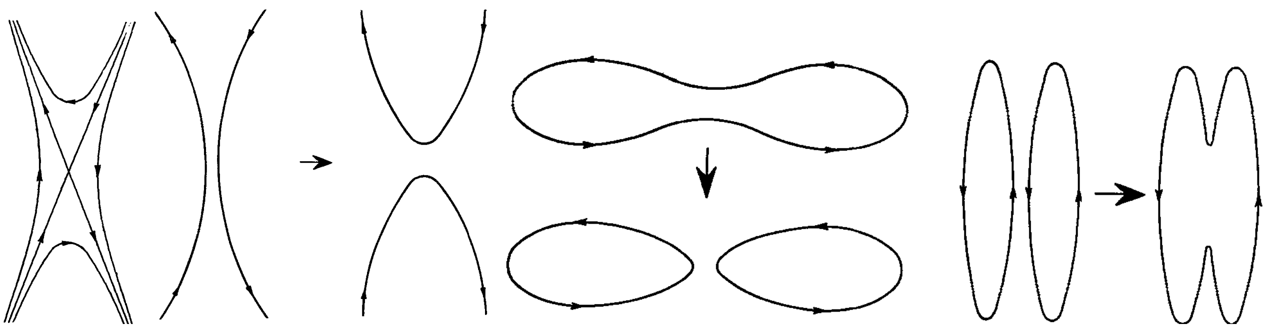
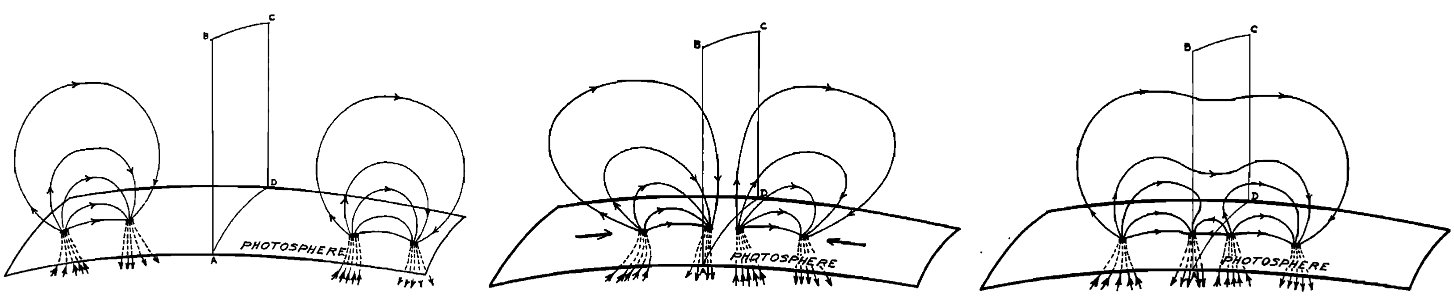
1 Problem

Following discussion by Giovanelli [1]-[3] and Hoyle [4] of flares near "null points" in the solar magnetic field,[1] Dungey (1953) [6] argued that in a region where time-dependent magnetic fields "collide", field lines near the null point can be said to "break" and "reconnect".



Although the magnetic field energy is negligible very close to a null point, Dungey argued that if the separatrices do not intersect at a right angle, as illustrated in the above left figure, the field energy density is greater in the region between where the angle between the separatrices is obtuse, rather than acute, such that "breaking and reconnection" as in the right figures above is associated with the release of magnetic field energy.

This theme was pursued by Sweet [7, 8], and then by Parker [9, 10],[2] who considered the figure below in [9].



Discuss "breaking and reconnecting" in a simplified version of Parker's example, taking this to be a pair of collinear, identical, parallel, "point" magnetic dipoles, $\mathbf{m} = m\,\hat{\mathbf{z}}$ located at $(x, y, z) = (0, 0, \pm a)$ in vacuum.[3]

[1] The magnetic field is zero at a null point. Such a point in a static electric field is a point of (unstable) equilibrium for electric charges, as considered by Maxwell in arts. 118-121 of his *Treatise*, and illustrated in his Figs. I-IV at the end of Vol. 1 [5].

[2] Sweet and Parker used the term "neutral point" to mean what is called a "null point" here.

[3] Solar flares take place in a plasma, not vacuum, in which the magnetic fields are "frozen in" to a first approximation [11, 12], as further discussed in footnote 8 below. The left two figures of Parker are more appropriate for "frozen-in" fields of a plasma than for vacuum (where some field lines from the left dipole

2 Solution

2.1 Collinear, Identical, Parallel, Point Dipoles in Vacuum

The magnetic field of two "point" dipoles $\mathbf{m} = m\,\hat{\mathbf{z}}$ can be written (in Gaussian units) as,

$$\frac{\mathbf{B}(\mathbf{r})}{m} = \frac{3((\mathbf{r}-\mathbf{a})\cdot\mathbf{z})(\mathbf{r}-\mathbf{a})}{|\mathbf{r}-\mathbf{a}|^5} - \frac{\hat{\mathbf{z}}}{|\mathbf{r}-\mathbf{a}|^3} + \frac{3((\mathbf{r}+\mathbf{a})\cdot\mathbf{z})(\mathbf{r}+\mathbf{a})}{|\mathbf{r}+\mathbf{a}|^5} - \frac{\hat{\mathbf{z}}}{|\mathbf{r}+\mathbf{a}|^3}\,, \tag{1}$$

when they are at positions $\pm\mathbf{a} = (0, 0, \pm a)$. At a point $\mathbf{r} = (x, 0, 0)$ on the midplane $z = 0$ between the two dipoles, the magnetic field is,

$$\begin{aligned}\frac{\mathbf{B}(x,0,0)}{m} &= -\frac{3a(x\,\hat{\mathbf{x}} - a\,\hat{\mathbf{z}})}{(x^2+a^2)^{5/2}} - \frac{\hat{\mathbf{z}}}{(x^2+a^2)^{3/2}} + \frac{3a(x\,\hat{\mathbf{x}} + a\,\hat{\mathbf{z}})}{(x^2+a^2)^{5/2}} - \frac{\hat{\mathbf{z}}}{(x^2+a^2)^{3/2}} \\ &= \frac{4a^2 - 2x^2}{(x^2+a^2)^{5/2}}\,\hat{\mathbf{z}}. \end{aligned} \tag{2}$$

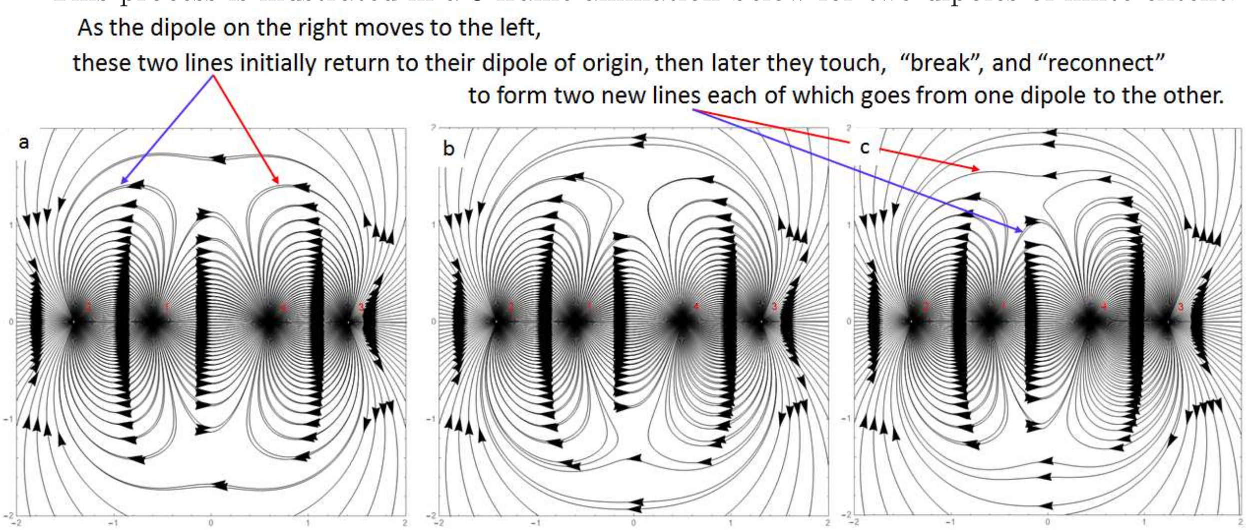
The magnetic field is zero on the ring $x^2 + y^2 = 2a^2$ of radius $r_0 = \sqrt{2}a$ in the midplane, $z = 0$.

The magnetic flux Φ through the null ring is, using Dwight 201.05 and 203.05 [13],

$$\begin{aligned}\Phi &= 2\pi\int_0^{r_0} B_z(r,0,0)\,r\,dr = 4\pi m\int_0^{\sqrt{2}a} r\,dr\,\frac{2a^2 - r^2}{(r^2+a^2)^{5/2}} \\ &= 4\pi m\left[\frac{1}{(r^2+a^2)^{1/2}} - \frac{a^2}{(r^2+a^2)^{3/2}}\right]_0^{\sqrt{2}a} = \frac{8\sqrt{3}\pi m}{9a}\,. \end{aligned} \tag{3}$$

The flux through the null ring is always nonzero, but is small for large separation $2a$ between the two dipoles. As the dipoles move towards one another, field lines that initially started and ended on the same dipole approach the null point, at which they "break and reconnect" with a "mirror" line from the other dipole, to form two new lines.

This process is illustrated in a 3-frame animation below for two dipoles of finite extent.[4]



always reach the right dipole, as in the rightmost figure of Parker).

[4]The figure was generated by the Wolfram CDF applet at
http://demonstrations.wolfram.com/ElectricFieldLinesDueToACollectionOfPointCharges/

2.2 Energetics

The notion of "breaking and reconnection" arose in consideration of solar flares, which seem to involve spectacular release of energy. In the view that sunspots are a kind of "magnetic storm", one supposes that the energy released is from that stored in the magnetic field.

In the model of flares as resulting from the collision/interaction of two giant magnetic dipoles, we note that the interaction magnetic field energy of dipoles $\mathbf{m}_1$ and $\mathbf{m}_2$ is,

$$U_{\rm int} = -\mathbf{m}_1 \cdot \mathbf{B}_2 = -\mathbf{m}_2 \cdot \mathbf{B}_1 = -\frac{3(\mathbf{m}_1 \cdot \hat{\mathbf{r}}_{12})(\mathbf{m}_2 \cdot \hat{\mathbf{r}}_{12}) - \mathbf{m}_1 \cdot \mathbf{m}_2}{r_{12}^3}\,, \tag{4}$$

where $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ is the distance vector between the dipoles. For the case of parallel, identical dipoles $\mathbf{m}$ aligned along $\mathbf{r}_{12}$, the interaction energy is $U_{\rm int} = -2m^2/r_{12}^3$. This negative interaction energy is associated with an attractive force, and as the dipoles approach one another the magnetic energy decreases while some other form of energy, presumably kinetic, increases.[5]

There is no need to invoke "breaking and reconnection" (which occurs in all dipole-dipole interactions whether noticed or not) to understand the release of magnetic energy when the configuration of the dipoles changes.[6] In particular, there is no additional release of magnetic energy associated with "breaking and reconnection" beyond that accounted for in the relation $\Delta U = U_{\rm initial} - U_{\rm final}$ [$= 2m(1/r_{12,\rm final}^3 - 1/r_{12,\rm initial}^3)$ for collinear, parallel dipoles]. The "breaking and reconnection" occurs at points of zero magnetic field strength, so the local density of magnetic field energy is negligible at such points, and there is no energy cost specifically associated with "breaking and reconnection".[7]

Of course, "breaking and reconnection" of field lines only occurs when the sources of the fields are in motion, in which case the field energy changes. If the motion is due only to the magnetic interaction of the sources, starting from rest, then the magnetic field energy must decrease to provide the kinetic energy of the resulting motion.[8]

[5] For discussion of magnetic energy conversion in a science toy based on a set of steel balls and permanent magnets, see [14].

[6] See also [15].

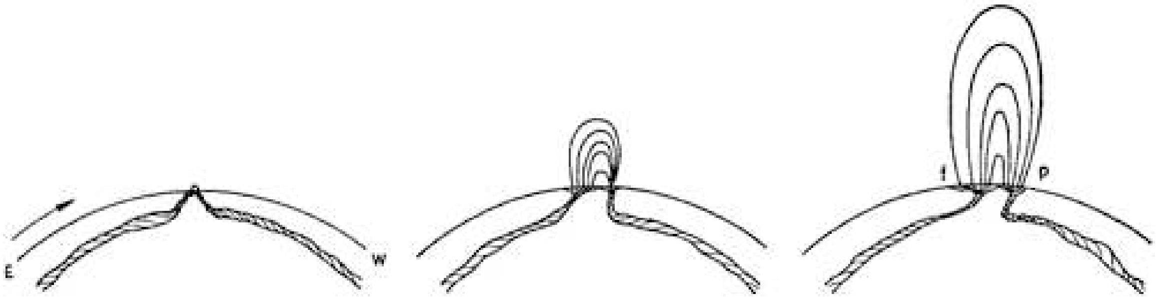
[7] This theme was anticipated by Slepian [16] prior to the nominal invention of the concept of "breaking and reconnection". He advocated that any field "line" can be thought of as being "broken" into segments at any point, with no consequence to the physics:

> No observable electromagnetic phenomenon can exist which involves two points in space, and which depends upon there being a *continuous* line of force joining these points. Such a phenomenon would contradict our postulate of the complete sufficiency of the local *vector* fields for describing local phenomena.

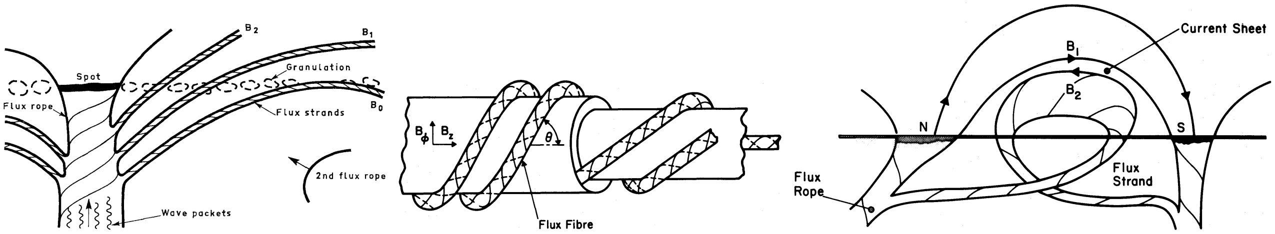
[8] The solar chromosphere is a plasma, which supports electrical currents. As noted by Alfvén [11] (building on Art. 654 of Maxwell's *Treatise* [17]; see also [18]), to a good approximation magnetic field lines are "frozen in" with respect to the highly conductive plasma, at least in regions where the field strength is large. Hence, the fields of interacting magnetic dipoles in a plasma can change significantly only in weak-field regions, particularly those that contain null points/lines/surfaces. This gives greater prominence to such null points in plasmas compared to vacuum, as noted by Sweet and Parker. However, the "frozen-in" behavior of field line in most of the plasma makes it difficult to account for larger changes in magnetic field energy, which can only occur in weak-field regions. This has lead to various elaborations of the original model of Sweet and Parker, in which currents in the vicinity of null points/surfaces play a special role; see, for example, [19, 20, 21].

2.3 Collinear, Nested, Antiparallel Dipoles

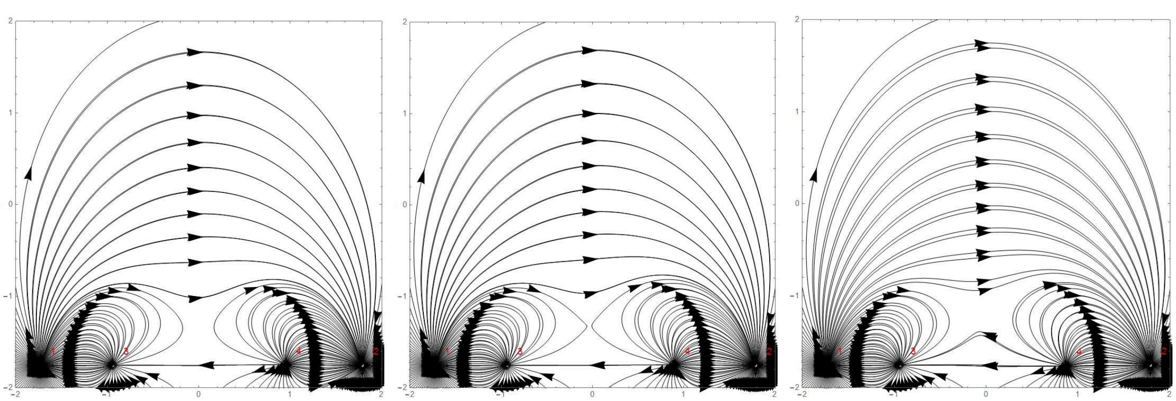
In the solar model of Sweet and Parker, the magnetic field above the surface of the solar photosphere was described by two magnetic dipoles, each being a pair of equal and opposite (fictitious) magnetic poles that were on the surface, supported by some kind of half-circular solenoids current below the surface. This type of model was elaborated by Babcock [22], who discussed helical flux tubes below the surface of the photosphere,[9] as illustrated in the figure below.



Further elaborations by Piddington [23, 24] included the possibility that the magnetic flux below a sunspot branches into multiple flux "ropes", and that two such flux-ropes might lead to nearby magnetic fluxes in opposite directions in the chromosphere.



A nested pair of flux-ropes of different strengths can exhibit "breaking and reconnection", as illustrated in the 3-frame animation below. The outer two poles belong to one flux-rope(in vacuum), of strength three times that of the other (which corresponds to the inner two poles). In the animation, the third pole from the left moves to the left while the other three poles are fixed. Two opposing, nearly vertical flux lines in the lower center of the figure approach one another, touch, break, reconnect, and move apart as nearly horizontal field lines.

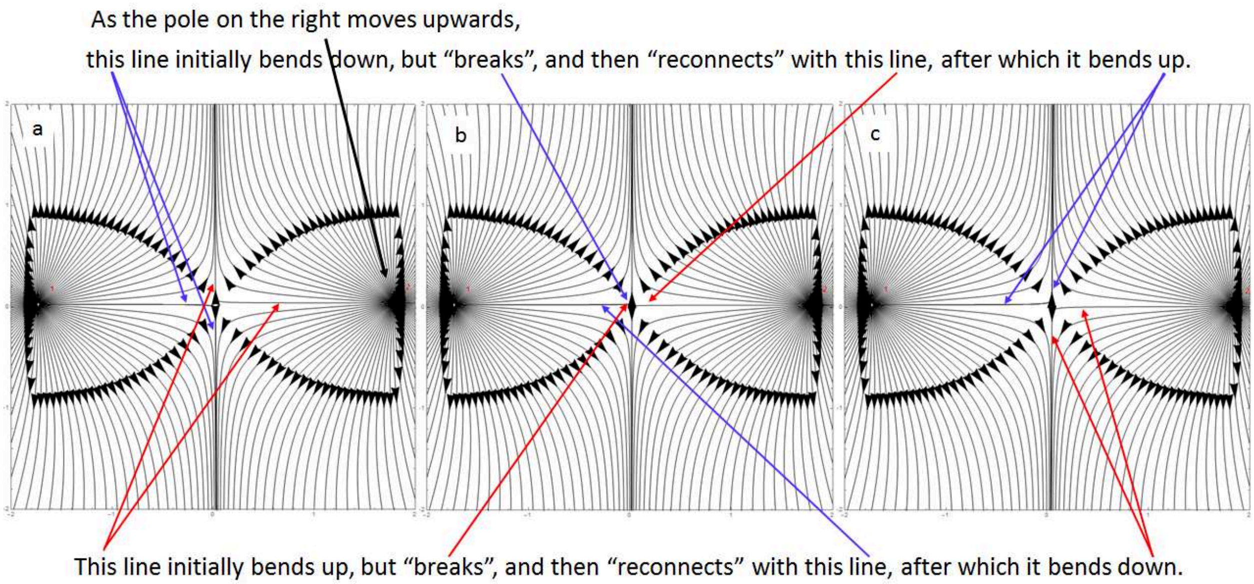


[9]Helical field lines were argued to be a result of the variation of the angular velocity of the solar surface with latitude. The differential rotation velocity results in twisting of the "frozen-in" field lines, which can lead to tubes/ropes of helical flux lines.

When the two dipoles are not collinear, the field pattern is fully 3-dimensional and the story is more complicated, as first discussed by Sweet [8]. See also, for example, [25].

2.4 Two Like Charges/Poles

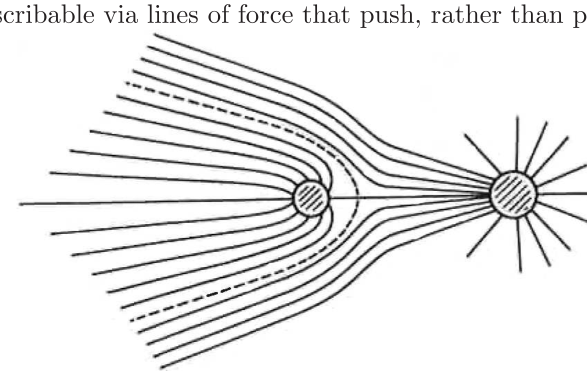
We have seen that "breaking and reconnection" of field lines occurs whenever two dipoles (electric or magnetic) move with respect to one another. It also occurs whenever two like charges/poles have a component of their motion perpendicular to their line of centers, as illustrated below.[10]



The field energy changes during such motion, but again we emphasize that one should not say the change in energy is "caused" by "breaking and reconnection", as the latter occurs at null points where the local density of field energy is negligible.

For a discussion of "breaking and reconnection" of magnetic fields lines when a magnet is tied in a knot, see [27].

[10]in a letter to Faraday of Nov. 9, 1857, Maxwell [26] discussed how the attractive force between the Sun and the Earth might be describable via lines of force that push, rather than pull.



As the Earth orbits the Sun, "breaking and reconnection" of the gravitational lines of force would occur near the null point in the figure. However, Maxwell did not comment on such details.

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