# The Rare Decay $K_L^0 \to \pi^0 \nu\bar{\nu}$

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## 1 Problem

The rare decay of the long-lived neutral $K$ meson, $K_L^0 \to \pi^0\nu\bar{\nu}$, when observed, will be considerable interest towards an understanding of the violation of the combined symmetries of charge conservation and parity ($CP$ violation).

a) Draw a Feynman diagram for this decay process. Deduce the $CP$ quantum number of the final state $\pi^0\nu\bar{\nu}$ that arises in this decay? Why don't the related decays $K_L^0 \to \pi^0 e^+e^-$ and $K_L^0 \to \pi^0\mu^+\mu^-$ lead to final states with a definite $CP$ quantum number?

b) Draw a (penguin) diagram representing the decay $K^0 \to \pi^0\nu\bar{\nu}$, and indicate the corresponding CKM matrix factors for the weak interaction, using the Wolfenstein approximation,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

Use the fact that $K_L \approx K_2 + \epsilon K_1$, where $K_{1,2} = (K_0 \pm \bar{K}_0)/\sqrt{2}$, to express the decay amplitude $A(K_L \to \pi^0\nu\bar{\nu})$ in terms of the CKM matrix parameters and the parameter $\epsilon$ (which measures 'indirect' $CP$ violation in the mixing of $K_0$ and $\bar{K}_0$).

From what is currently known about $CP$ violation, characterize the expected $CP$ violation $K_L \to \pi^0\nu\bar{\nu}$ as "direct" or "indirect".

c) **Estimate** the branching fraction for the decay $K_L \to \pi^0\nu\bar{\nu}$.

## 2 Solution

a) To determine the $CP$ quantum number of $\pi^0\nu\bar{\nu}$, we consider the $\nu\bar{\nu}$ system, combined with the $\pi^0$ with relative orbital angular momentum $L_{\pi(\nu\bar{\nu})}$. Then,

$$CP(\pi^0\nu\bar{\nu}) = C(\pi^0)P(\pi^0)C(\nu\bar{\nu})P(\nu\bar{\nu})(-1)^{L_{\pi(\nu\bar{\nu})}}.$$

We need the facts,

$$C(\pi^0) = +1, \qquad (\pi^0 \to \gamma\gamma),$$

$$P(\pi^0) = -1, \qquad (\text{crossed planes of polarization in } \pi^0 \to e^+e^-e^+e^-),$$

$$C(\nu\bar{\nu}) = (-1)^{L_{\nu\bar{\nu}}+S_{\nu\bar{\nu}}},$$

$$P(\nu\bar{\nu}) = (-1)^{L_{\nu\bar{\nu}}+1},$$

which last two results are "well-known" consequences of the Dirac equation for spin-1/2 particles.

First, what is $S_{\nu\bar{\nu}}$? In the rest frame of the $\nu\bar{\nu}$ pair, the neutrino momenta are back to back, but their spins are aligned! Hence, $S_{z,\nu\bar{\nu}} = 1 = S_{\nu\bar{\nu}}$.

We don't need to figure out $L_{\nu\bar{\nu}}$, since this cancels out of $CP$.

But we do need to know $J_{\nu\bar{\nu}}$ to figure out $L_{\pi(\nu\bar{\nu})}$. In the penguin diagram (part b)), we see that the $\nu\bar{\nu}$ pair comes from a virtual $Z^0$ boson, so we could have $J_{\nu\bar{\nu}} = 0$ or 1. But for the 2-body state, $\nu\bar{\nu}$, we have $L_z = 0$ for $z$ along the particle's motion, so $J_z = 1$ and so we must have $J_{\nu\bar{\nu}} = 1$.

Then from the facts that $J_K = 0$, $J_\pi = 0$ and $J_{\nu\bar{\nu}} = 1$, we deduce that $L_{\pi(\nu\bar{\nu})} = 1$.

Altogether,

$$CP(\pi^0\nu\bar{\nu}) = (+1)(-1)(-1)^{L_{\nu\bar{\nu}}}(-1)(-1)^{L_{\nu\bar{\nu}}}(-1)(-1) = +1.$$

In the case of charged leptons in the final states, we cannot conclude that $S_{l^+l^-} = 1$ only, since massive leptons can have both helicities. Hence the $CP$ quantum number is a mixture of $+1$ and $-1$.

b) From the penguin diagram,

we deduce that,

$$A(K^0 \to \pi^0\nu\bar{\nu}) \propto V_{ts}^* V_{td} \propto (-A\lambda^2)(A\lambda^3)(1-\rho-i\eta),$$

and therefore,

$$A(\bar{K}^0 \to \pi^0\nu\bar{\nu}) \propto V_{ts} V_{td}^* \propto (-A\lambda^2)(A\lambda^3)(1-\rho+i\eta).$$

In the diagram, the top quark could also have been a $u$ or $c$ quark. But, the unitarity of the CKM matrix implies that the first and second columns are orthogonal, *i.e.*, $V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$. Since the masses of the $u$ and $c$ quarks are "nearly" equal, the sum of the terms in the diagram involving $u$ and $c$ quarks nearly cancel, leaving terms of order $V_{ts}^* V_{td}$.

If so,

$$A(K_1 \to \pi^0\nu\bar{\nu}) \propto -A^2\lambda^5(1-\rho),$$

while,

$$A(K_2 \to \pi^0\nu\bar{\nu}) \propto iA^2\lambda^5\eta.$$

Then, since $K_L \approx K_2 + \epsilon K_1$,

$$A(K_L \to \pi^0\nu\bar{\nu}) \propto A^2\lambda^5(i\eta - \epsilon(1-\rho)).$$

Present knowledge: $\epsilon \approx 10^{-3}$, while $|\eta| \approx |1-\rho|$. Hence, $A(K_L \to \pi^0\nu\bar{\nu})$ is dominated by $A(K_2 \to \pi^0\nu\bar{\nu})$, which is called 'direct' $CP$ violation.

c) The diagram involves a loop, so a detailed calculation is messy.

For a quick estimate, note that the diagram is second order in the weak interaction $\Rightarrow$ rate $\propto G_F^4$. In addition, the square of the CKM factors yield $A^4\lambda^{10}\eta^2 \approx \lambda^{10}$.

Altogether, $\Gamma(K_L \to \pi^0\nu\bar{\nu}) \propto G_F^4\lambda^{10}$.

For comparison, the decay rate of the short-lived $K$ meson is $\Gamma(K_S) \propto G_F^2$ in the same approximation.

The $K_L$ lives about 1000 times as long as the $K_L$ (fact).

Hence $\Gamma(K_L) \approx 10^{-3}G_F^2$, and the branching fraction for $K_L \to \pi^0\nu\bar{\nu}$ is about $10^3 G_F^2\lambda^{10} \approx 10^3(10^{-5})^2(0.2)^{10} \approx 10^{-14}$.

Supposedly, more detailed calculation yields a branching fraction of $10^{-12}$.