The Poynting Vector Should be $\mathbf{E} \times \mathbf{B}/\mu_0$ Not $\mathbf{E} \times \mathbf{H}$

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When Poynting deduced his eponymous vector [1], he argued that the rate at which the electromagnetic field transfers energy to a system of electric charges and currents is given by $\mathbf{J}_{\rm free} \cdot \mathbf{E}$, where $\mathbf{J}_{\rm free}$ is the "free" (conduction) electric current and $\mathbf{E}$ is the electric field. Both of these fields would now be called macroscopic; Poynting (and Maxwell) did not consider what is now called microscopic electrodynamics. Then, using the Maxwell equation $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J}_{\rm free} + \partial \mathbf{D}/\partial t$ (in SI units) for linear media where $\mathbf{B}$ is the magnetic field, $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$ with constants ϵ and μ, Poynting deduced that

$$\begin{aligned}\int d\mathrm{Vol}\, \mathbf{J}_{\rm free} \cdot \mathbf{E} &= \int d\mathrm{Vol} \left(\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{H}) - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \right) \\ &= -\int d\mathrm{Vol} \left(\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{H}) + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \\ &= -\oint d\mathbf{Area} \cdot (\mathbf{E} \times \mathbf{H}) - \frac{\partial}{\partial t} \int d\mathrm{Vol} \left(\frac{\mathbf{D} \cdot \mathbf{E}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2} \right). \end{aligned} \tag{1}$$

From this, Poynting identified the vector flow $\mathbf{S}$ of field energy, and the volume density u of field energy as

$$\mathbf{S}_{\rm Poynting} = \mathbf{E} \times \mathbf{H}, \qquad \text{and} \qquad u_{\rm Poynting} = \frac{\mathbf{D} \cdot \mathbf{E}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2}\,. \tag{2}$$

However, the electromagnetic field also transfers energy to the "bound" electric currents, so it is more correct to consider $\mathbf{J}_{\rm total} \cdot \mathbf{E} = (\mathbf{J}_{\rm free} + \mathbf{J}_{\rm bound}) \cdot \mathbf{E}$ and use the Maxwell equation $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0\, \mathbf{J}_{\rm total} + \epsilon_0 \mu_0\, \partial \mathbf{E}/\partial t$, which leads to[1]

$$\begin{aligned}\int d\mathrm{Vol}\, \mathbf{J}_{\rm total} \cdot \mathbf{E} &= \int d\mathrm{Vol} \left(\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{B}/\boldsymbol{\mu_0}) - \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} \right) \\ &= -\int d\mathrm{Vol} \left(\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{B}/\mu_0) + \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{B}}{\mu_0} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \\ &= -\oint d\mathbf{Area} \cdot \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} - \frac{\partial}{\partial t} \int d\mathrm{Vol} \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right), \end{aligned} \tag{3}$$

and the identifications

$$\mathbf{S}_{\rm EM} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_0}\,, \qquad \text{and} \qquad u_{\rm EM} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\,. \tag{4}$$

While most textbooks on electromagnetism give Poynting's argument (1)-(2) without comment as to its misunderstanding about $\mathbf{J}_{\rm total}$, the forms of eq. (4) are advocated in [2], Sec. 54 of [3], Chap. 27 of [4], Sec. 11.9 of [5], Chap. 8 of [6], Sec. 9.6 of [7] and Chap. 5 of [8].

[1] It is likely that in 1883, Poynting considered that he should use $\mathbf{J}_{\rm total} \cdot \mathbf{E}$, but it seemed to him that the total current of electric charges was just the conduction current $\mathbf{J}_{\rm free}$.

It is noteworthy that eq. (3) can be obtained from Poynting's derivation (1) if we are aware that $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ and $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$, where $\mathbf{P}$ and $\mathbf{M}$ are the (macroscopic) volume densities of electric and magnetic dipole moments, respectively, and that $\mathbf{J}_{\rm bound} = \partial \mathbf{P}/\partial t + \boldsymbol{\nabla} \times \mathbf{M}$, where $\partial \mathbf{P}/\partial t$ is the (electric) polarization current and $\boldsymbol{\nabla} \times \mathbf{M}$ is the magnetization current.[2] The polarization densities $\mathbf{P}$ and $\mathbf{M}$ are associated with mechanical mass densities, and so could be called "electromechanical" quantities. Then, we can consider $\mathbf{D}$ and $\mathbf{H}$ to be "electromechanical" fields, while $\mathbf{E}$ and $\mathbf{B}$ are the "pure" electromagnetic fields.

We can now write

$$\begin{aligned} \int d\mathrm{Vol}\, \mathbf{J}_{\rm free} \cdot \mathbf{E} &= \int d\mathrm{Vol} \left(\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{B}/\mu_0) - \mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{M}) - \epsilon_0\, \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} - \mathbf{E} \cdot \partial \mathbf{P}/\partial t \right) \\ &= - \int d\mathrm{Vol} \left(\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{B}/\mu_0) + \frac{\mathbf{B}}{\mu_0} \cdot \frac{\partial \mathbf{B}}{\partial t} + \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} \right) - \int d\mathrm{Vol}\, \mathbf{E} \cdot \mathbf{J}_{\rm bound}, \end{aligned} \tag{5}$$

which is equivalent to the second line of eq. (3), recalling that $\mathbf{J}_{\rm free} + \mathbf{J}_{\rm bound} = \mathbf{J}_{\rm total}$.

Already in 1900, Poincaré [2] argued that the flux of energy in the electromagnetic field, described by the Poynting vector $\mathbf{S}_{\rm EM}$, is associated with a density of momentum $\mathbf{S}/c^2$ in the electromagnetic field. This argument was seconded by Abraham in 1903 [19]. This relation is more general, as discussed by Planck (1908, p. 829 of [20]), who argued that a flow $\mathbf{q}$ (with dimensions of energy per unit area per unit time) of any type of energy is associated with a momentum density $\mathbf{p} = \mathbf{q}/c^2$, where c is the speed of light in vacuum.[3] The factor $1/c^2$ suggests that this very small momentum density could be called "relativistic", as well as "hidden".

We are thus led to associate a momentum density $\mathbf{g}$ with the energy-flux vector (2), which we call[4]

$$\mathbf{g}_{\rm Abraham} = \frac{\mathbf{S}_{\rm Poynting}}{c^2} = \frac{\mathbf{E} \times \mathbf{H}}{c^2} = \epsilon_0\, \mathbf{E} \times \mathbf{B} - \frac{\mathbf{E} \times \mathbf{M}}{c^2}\,. \tag{6}$$

[2]Neither Maxwell nor Poynting enunciated a concept of the polarization density $\mathbf{P}$ of electric dipoles, and only regarded the relation between $\mathbf{D}$ and $\mathbf{E}$ as $\mathbf{D} = \epsilon_r \mathbf{E}$, where ϵ_r is now called the (relative) dielectric constant and/or the (relative) permittivity in Gaussian units, which are used in this footnote. See Art. 111 of [9] for Maxwell's use of the term polarization.

In 1885, Heaviside introduced the concept of an electret as the electrical analog of a permanent magnet [10], and proposed that the electrical analog of magnetization (density) be called electrization. He did not propose a symbol for this, nor did he write an equation such as $\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}$.

The density of electric dipoles was called the polarization by Lorentz (1892) in sec. 102, p. 465 of [12], and assigned the symbol $\mathbf{M}$.

Larmor (1895), p. 738 of [13], introduced the vector (f', g', h') for what is now written as the polarization density $\mathbf{P}$, and related it to the electric field $\mathbf{E} = (P, Q, R)$ as $(f', g', h') = (K - 1)(P, Q, R)/4\pi$, *i.e.*, $\mathbf{P} = (\epsilon_r - 1)\mathbf{E}/4\pi = (\mathbf{D} - \mathbf{E})/4\pi$. Larmor's notation was mentioned briefly on p. 91 of [14] (1898).

The symbol $\mathbf{M}$ for dielectric polarization was changed to $\mathbf{P}$ by Lorentz on p. 263 of [16] (1902), and a relation equivalent to $\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}$ was given in eq. (22), p. 265. See also p. 224, and eq. (147), p. 240 of [17] (1903), which latter subsequently appeared as eq. (142), p. 155 of the textbook [18] (1904) by Abraham. The quantity $\partial \mathbf{P}/\partial t$ is called the "polarization current" on p. 193 of [18], as part of the "displacement current" $(1/4\pi)\partial \mathbf{D}/\partial t$ (in Gaussian units).

[3]This relation has been discussed, for example, by Eckart on p. 923 of [22], endorsed by Feynman in Sec. 27.6 of [4], and attributed to Planck by Møller in eq. (13) of [23].

[4]To distinguish the field-momentum-density vector from the electric-dipole-moment density $\mathbf{P}$ we use the symbol $\mathbf{g}$ for the former, following [19].

as first introduced by Abraham [19]. Since $\mathbf{S}_{\rm Poynting}$ is an "electromechanical" quantity, $\mathbf{g}_{\rm Abraham}$ is also.

The "pure" electromagnetic-field-momentum density $\mathbf{g}_{\rm EM}$ can be obtained from the first of eq. (4),

$$\mathbf{g}_{\rm EM} = \frac{\mathbf{S}_{\rm EM}}{c^2} = \epsilon_0 \mathbf{E} \times \mathbf{B}, \tag{7}$$

recalling that $1/c^2 = \epsilon_0 \mu_0$.[5]

The electromagnetic-field-momentum density $\mathbf{g}_{\rm EM}$ can also be deduced via an argument that starts with the Lorentz-force density $\mathbf{f}_{\rm total} = \rho_{\rm total}\mathbf{E} + \mathbf{J}_{\rm total} \times \mathbf{B}$ on the total electric charge and current densities, as discussed, for example, in Sec. 11.9 of [5], Chap. 8 of [6] and Chap. 5 of [8], all of which endorse eqs. (3)-(4) and (7). See also Sec. 2.1 of [24].

We have noted that the field-momentum density is of order $1/c^2$, and so is a somewhat "hidden" quantity. Likewise, in many systems with electric currents there is "hidden" mechanical momentum of order $1/c^2$ in the currents, a small relativistic correction. See, for example, [25, 26]. In particular, in quasistatic systems with magnetization density $\mathbf{M}$ in an electric field $\mathbf{E}$ there is "hidden" mechanical momentum $\int d{\rm Vol}\, \mathbf{M} \times \mathbf{E}/c^2$, and in quasistatic systems with polarization density $\mathbf{P}$ in a magnetic field $\mathbf{B}$ there is "hidden" mechanical momentum which includes $\int d{\rm Vol}\, \mathbf{P} \times \mathbf{B}$. We infer that the Abraham momentum (6) includes the contribution from the "hidden" mechanical momentum of the magnetic dipoles but not of the electric dipoles. Similarly, the field-momentum density according to Minkowski [27], $\mathbf{g}_{\rm Minkowski} = \mathbf{D} \times \mathbf{B}$, includes (some of) the "hidden" mechanical momentum due to electric dipoles but not due to magnetic dipoles. It seems that the "perpetual" Abraham-Minkowski controversy (see, for example, [28]) is not about the "pure" electromagnetic-field momentum of eq. (7), but about the circumstances in which various expressions for "electromechanical" field momenta are relevant.

This short note was inspired by the lengthy eprint [29], that also reviewed how the macroscopic and microscopic forms of Maxwell's equations and the Lorentz force are the same in terms of the electromagnetic fields $\mathbf{E}$ *and* $\mathbf{B}$ *and the total densities of electric charge and current,* $\rho_{\rm total}$ *and* $\mathbf{J}_{\rm total}$.

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The equations on p. 6 before eqs. (4) state that the density $\mathbf{g}$ of momentum in the electromagnetic field is $\mathbf{S}/c^2$, where $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$.

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