The Poynting Vector Should be $\mathbf{E} \times \mathbf{B}/\mu_0$ Not $\mathbf{E} \times \mathbf{H}$

Kirk T. McDonald

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

(July 26, 2025)

When Poynting deduced his eponymous vector [1], he argued that the rate at which the electromagnetic field transfers energy to a system of electric charges and currents is given by $\mathbf{J}_{\rm free} \cdot \mathbf{E}$, where $\mathbf{J}_{\rm free}$ is the "free" (conduction) electric current and $\mathbf{E}$ is the electric field. Both of these fields would now be called macroscopic; Poynting (and Maxwell) did not consider what is now called microscopic electrodynamics. Then, using the Maxwell equation $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J}_{\rm free} + \partial \mathbf{D}/\partial t$ (in SI units) for linear media where $\mathbf{B}$ is the magnetic field, $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$ with constants ϵ and μ, Poynting deduced that

$$\begin{aligned}
\int d{\rm Vol}\, \mathbf{J}_{\rm free} \cdot \mathbf{E} &= \int d{\rm Vol} \left(\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{H}) - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \right) \\
&= -\int d{\rm Vol} \left(\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{H}) + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \\
&= -\oint d\mathbf{Area} \cdot (\mathbf{E} \times \mathbf{H}) - \frac{\partial}{\partial t} \int d{\rm Vol} \left(\frac{\mathbf{D} \cdot \mathbf{E}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2} \right). \qquad (1)
\end{aligned}$$

From this, Poynting identified the vector flow $\mathbf{S}$ of field energy, and the volume density u of field energy as

$$\mathbf{S}_{\rm Poynting} = \mathbf{E} \times \mathbf{H}, \qquad \text{and} \qquad u_{\rm Poynting} = \frac{\mathbf{D} \cdot \mathbf{E}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2}. \qquad (2)$$

However, the electromagnetic field also transfers energy to the "bound" electric currents, so it is more correct to consider $\mathbf{J}_{\rm total} \cdot \mathbf{E} = (\mathbf{J}_{\rm free} + \mathbf{J}_{\rm bound}) \cdot \mathbf{E}$ and use the Maxwell equation $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0\, \mathbf{J}_{\rm total} + \epsilon_0 \mu_0\, \partial \mathbf{E}/\partial t$, which leads to[1]

$$\begin{aligned}
\int d{\rm Vol}\, \mathbf{J}_{\rm total} \cdot \mathbf{E} &= \int d{\rm Vol} \left(\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{B}/\boldsymbol{\mu_0}) - \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} \right) \\
&= -\int d{\rm Vol} \left(\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{B}/\mu_0) + \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{B}}{\mu_0} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \\
&= -\oint d\mathbf{Area} \cdot \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} - \frac{\partial}{\partial t} \int d{\rm Vol} \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right), \qquad (3)
\end{aligned}$$

and the identifications

$$\mathbf{S}_{\rm EM} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_0}, \qquad \text{and} \qquad u_{\rm EM} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}. \qquad (4)$$

While most textbooks on electromagnetism give Poynting's argument (1)-(2) without comment as to its misunderstanding about $\mathbf{J}_{\rm total}$, the forms of eq. (4) are advocated in [2], Sec. 54 of [3], Chap. 27 of [4], Sec. 11.9 of [5], Chap. 8 of [6], Sec. 9.6 of [7] and Chap. 5 of [8].

[1] It is likely that in 1883, Poynting considered that he should use $\mathbf{J}_{\rm total} \cdot \mathbf{E}$, but it seemed to him that the total current of electric charges was just the conduction current $\mathbf{J}_{\rm free}$.

It is noteworthy that eq. (3) can be obtained from Poynting's derivation (1) if we are aware that $\mathbf{D} = \epsilon_0\,\mathbf{E} + \mathbf{P}$ and $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$, where $\mathbf{P}$ and $\mathbf{M}$ are the (macroscopic) volume densities of electric and magnetic dipole moments, respectively, and that $\mathbf{J}_{\rm bound} = \partial\mathbf{P}/\partial t + \boldsymbol{\nabla}\times\mathbf{M}$, where $\partial\mathbf{P}/\partial t$ is the (electric) polarization current and $\boldsymbol{\nabla}\times\mathbf{M}$ is the magnetization current.[2] The polarization densities $\mathbf{P}$ and $\mathbf{M}$ are associated with mechanical mass densities, and so could be called "electromechanical" quantities. Then, we can consider $\mathbf{D}$ and $\mathbf{H}$ to be "electromechanical" fields, while $\mathbf{E}$ and $\mathbf{B}$ are the "pure" electromagnetic fields.

We can now write

$$\int d{\rm Vol}\,\mathbf{J}_{\rm free}\cdot\mathbf{E} = \int d{\rm Vol}\left(\mathbf{E}\cdot(\boldsymbol{\nabla}\times\mathbf{B}/\mu_0) - \mathbf{E}\cdot(\boldsymbol{\nabla}\times\mathbf{M}) - \epsilon_0\,\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t} - \mathbf{E}\cdot\partial\mathbf{P}/\partial t\right)$$
$$= -\int d{\rm Vol}\left(\boldsymbol{\nabla}\cdot(\mathbf{E}\times\mathbf{B}/\mu_0) + \frac{\mathbf{B}}{\mu_0}\cdot\frac{\partial\mathbf{B}}{\partial t} + \epsilon_0\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t}\right) - \int d{\rm Vol}\,\mathbf{E}\cdot\mathbf{J}_{\rm bound}, \qquad (5)$$

which is equivalent to the second line of eq. (3), recalling that $\mathbf{J}_{\rm free} + \mathbf{J}_{\rm bound} = \mathbf{J}_{\rm total}$.

Already in 1900, Poincaré [2] argued that the flux of energy in the electromagnetic field, described by the Poynting vector $\mathbf{S}_{\rm EM}$, is associated with a density of momentum $\mathbf{S}/c^2$ in the electromagnetic field. This argument was seconded by Abraham in 1903 [19]. This relation is more general, as discussed by Planck (1908, p. 829 of [20]), who argued that a flow $\mathbf{q}$ (with dimensions of energy per unit area per unit time) of any type of energy is associated with a momentum density $\mathbf{p} = \mathbf{q}/c^2$, where c is the speed of light in vacuum.[3] The factor $1/c^2$ suggests that this very small momentum density could be called "relativistic", as well as "hidden".

We are thus led to associate a momentum density $\mathbf{g}$ with the energy-flux vector (2), which we call[4]

$$\mathbf{g}_{\rm Abraham} = \frac{\mathbf{S}_{\rm Poynting}}{c^2} = \frac{\mathbf{E}\times\mathbf{H}}{c^2} = \epsilon_0\,\mathbf{E}\times\mathbf{B} - \frac{\mathbf{E}\times\mathbf{M}}{c^2}\,. \qquad (6)$$

---

[2]Neither Maxwell nor Poynting enunciated a concept of the polarization density $\mathbf{P}$ of electric dipoles, and only regarded the relation between $\mathbf{D}$ and $\mathbf{E}$ as $\mathbf{D} = \epsilon_r\mathbf{E}$, where ϵ_r is now called the (relative) dielectric constant and/or the (relative) permittivity in Gaussian units, which are used in this footnote. See Art. 111 of [9] for Maxwell's use of the term polarization.

In 1885, Heaviside introduced the concept of an electret as the electrical analog of a permanent magnet [10], and proposed that the electrical analog of magnetization (density) be called electrization. He did not propose a symbol for this, nor did he write an equation such as $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$.

The density of electric dipoles was called the polarization by Lorentz (1892) in sec. 102, p. 465 of [12], and assigned the symbol $\mathbf{M}$.

Larmor (1895), p. 738 of [13], introduced the vector (f', g', h') for what is now written as the polarization density $\mathbf{P}$, and related it to the electric field $\mathbf{E} = (P, Q, R)$ as $(f', g', h') = (K-1)(P, Q, R)/4\pi$, *i.e.*, $\mathbf{P} = (\epsilon_r - 1)\mathbf{E}/4\pi = (\mathbf{D} - \mathbf{E})/4\pi$. Larmor's notation was mentioned briefly on p. 91 of [14] (1898).

The symbol $\mathbf{M}$ for dielectric polarization was changed to $\mathbf{P}$ by Lorentz on p. 263 of [16] (1902), and a relation equivalent to $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ was given in eq. (22), p. 265. See also p. 224, and eq. (147), p. 240 of [17] (1903), which latter subsequently appeared as eq. (142), p. 155 of the textbook [18] (1904) by Abraham. The quantity $\partial\mathbf{P}/\partial t$ is called the "polarization current" on p. 193 of [18], as part of the "displacement current" $(1/4\pi)\partial\mathbf{D}/\partial t$ (in Gaussian units).

[3]This relation has been discussed, for example, by Eckart on p. 923 of [22], endorsed by Feynman in Sec. 27.6 of [4], and attributed to Planck by Møller in eq. (13) of [23].

[4]To distinguish the field-momentum-density vector from the electric-dipole-moment density $\mathbf{P}$ we use the symbol $\mathbf{g}$ for the former, following [19].

as first introduced by Abraham [19]. Since $\mathbf{S}_{\rm Poynting}$ is an "electromechanical" quantity, $\mathbf{g}_{\rm Abraham}$ is also.

The "pure" electromagnetic-field-momentum density $\mathbf{g}_{\rm EM}$ can be obtained from the first of eq. (4),

$$\mathbf{g}_{\rm EM} = \frac{\mathbf{S}_{\rm EM}}{c^2} = \epsilon_0 \mathbf{E} \times \mathbf{B}, \tag{7}$$

recalling that $1/c^2 = \epsilon_0 \mu_0$.[5]

The electromagnetic-field-momentum density $\mathbf{g}_{\rm EM}$ can also be deduced via an argument that starts with the Lorentz-force density $\mathbf{f}_{\rm total} = \rho_{\rm total} \mathbf{E} + \mathbf{J}_{\rm total} \times \mathbf{B}$ on the total electric charge and current densities, as discussed, for example, in Sec. 11.9 of [5], Chap. 8 of [6] and Chap. 5 of [8], all of which endorse eqs. (3)-(4) and (7). See also Sec. 2.1 of [24].

We have noted that the field-momentum density is of order $1/c^2$, and so is a somewhat "hidden" quantity. Likewise, in many systems with electric currents there is "hidden" mechanical momentum of order $1/c^2$ in the currents, a small relativistic correction. See, for example, [25, 26]. In particular, in quasistatic systems with magnetization density $\mathbf{M}$ in an electric field $\mathbf{E}$ there is "hidden" mechanical momentum $\int d{\rm Vol}\, \mathbf{M} \times \mathbf{E}/c^2$, and in quasistatic systems with polarization density $\mathbf{P}$ in a magnetic field $\mathbf{B}$ there is "hidden" mechanical momentum which includes $\int d{\rm Vol}\, \mathbf{P} \times \mathbf{B}$. We infer that the Abraham momentum (6) includes the contribution from the "hidden" mechanical momentum of the magnetic dipoles but not of the electric dipoles. Similarly, the field-momentum density according to Minkowski [27], $\mathbf{g}_{\rm Minkowski} = \mathbf{D} \times \mathbf{B}$, includes (some of) the "hidden" mechanical momentum due to electric dipoles but not due to magnetic dipoles. It seems that the "perpetual" Abraham-Minkowski controversy (see, for example, [28]) is not about the "pure" electromagnetic-field momentum of eq. (7), but about the circumstances in which various expressions for "electromechanical" field momenta are relevant.

This short note was inspired by the lengthy eprint [29], that also reviewed how the macroscopic and microscopic forms of Maxwell's equations and the Lorentz force are the same in terms of the electromagnetic fields $\mathbf{E}$ *and* $\mathbf{B}$ *and the total densities of electric charge and current,* $\rho_{\rm total}$ *and* $\mathbf{J}_{\rm total}$.

References

[1] J.H. Poynting, *On the Transfer of Energy in the Electromagnetic Field*, Phil. Trans. Roy. Soc. London **175**, 343 (1884),
http://kirkmcd.princeton.edu/examples/EM/poynting_ptrsl_175_343_84.pdf

[2] H. Poincaré, *La Théorie de Lorentz et la Principe de Réaction*, Arch. Neer. **5**, 252 (1900), http://kirkmcd.princeton.edu/examples/EM/poincare_an_5_252_00.pdf
Translation: *The Theory of Lorentz and the Principle of Reaction*,
http://kirkmcd.princeton.edu/examples/EM/poincare_an_5_252_00_english.pdf
The equations on p. 6 before eqs. (4) state that the density **g** of momentum in the electromagnetic field is $\mathbf{S}/c^2$, where $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$.

[5] This argument also is given in Chap. 27 of [4].

[3] M. Mason and W. Weaver, *The Electromagnetic Field* (U. Chicago Press, 1929). kirkmcd.princeton.edu/examples/EM/mason_emf_29.pdf

[4] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. 2, Sec. 27.6. (Addison-Wesley, 1964), https://www.feynmanlectures.caltech.edu/II_27.html

[5] L. Eyges, *The Classical Electromagnetic Field* (Addison-Wesley, 1972). http://kirkmcd.princeton.edu/examples/EM/eyges_72.pdf

[6] D.J. Griffiths, *Introduction to Electrodynamics*, 4th ed. (Addison-Wesley, 2012), http://kirkmcd.princeton.edu/examples/EM/griffiths_em4.pdf

[7] E.M. Purcell and D.J. Morin, *Electricity and Magnetism*, 3rd ed. (Cambridge U. Press, 2013). http://kirkmcd.princeton.edu/examples/EM/purcell_em_13.pdf

[8] R.M. Wald, *Advanced Classical Electromagnetism* (Princeton U. Press, 2022), http://kirkmcd.princeton.edu/examples/EM/wald_22_ch5.pdf

[9] J.C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1 (Clarendon Press, 1873), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v1_73.pdf
Vol. 1, 3rd ed. (Clarendon Press, 1904), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v1_04.pdf

[10] O. Heaviside, *Electromagnetic Induction and Its Propagation*, part 12, Electrician **15**, 230 (1885), http://kirkmcd.princeton.edu/examples/EM/heaviside_eip12_electrician_15_230_85.pdf
Also on p. 488 of [11].

[11] O. Heaviside, *Electrical Papers*, Vol. 1 (Macmillan, 1894), http://kirkmcd.princeton.edu/examples/EM/heaviside_electrical_papers_1.pdf

[12] H.A. Lorentz, *La Théorie Électromagnétique de Maxwell et son Application aux Corps Mouvants*, Arch. Neérl. **25**, 363 (1892), http://kirkmcd.princeton.edu/examples/EM/lorentz_ansen_25_363_92.pdf

[13] J. Larmor, *A Dynamical Theory of the Electric and Luminiferous Medium—Part II. Theory of Electrons*, Phil. Trans. Roy. Soc. London A **186**, 695 (1895), http://kirkmcd.princeton.edu/examples/EM/larmor_ptrsla_186_695_95.pdf

[14] J.G. Leathem, *On the theory of the Magneto-Optic phenomena of Iron, Nickel, and Cobalt*, Phil. Trans. Roy. Soc. London A **190**, 89 (1897), http://kirkmcd.princeton.edu/examples/EM/leathem_ptrsla_190_89_97.pdf

[15] J. Larmor, *Æther and Matter* (Cambridge U. Press, 1900), http://kirkmcd.princeton.edu/examples/EM/larmor_aether_matter_00.pdf

[16] H.A. Lorentz, *The fundamental equations for electromagnetic phenomena in ponderable bodies, deduced from the theory of electrons*. Proc. Roy. Acad. Amsterdam **5**, 254 (1902), http://kirkmcd.princeton.edu/examples/EM/lorentz_pknaw_5_254_02.pdf

[17] H.A. Lorentz, *Weiterbildung der Maxwellschen Theorie. Elektronentheorie*, Enzykl. Math. Wiss. **5**, part II, 145 (1904),
http://kirkmcd.princeton.edu/examples/EM/lorentz_emw_5_2_145_04.pdf

[18] M. Abraham, *Theorie der Elektrizität*, Vol. 1 (Teubner, 1904),
http://kirkmcd.princeton.edu/examples/EM/abraham_foppl_elektrizitat_v1_04.pdf

[19] M. Abraham, *Prinzipien der Dynamik des Elektrons*, Ann. d. Phys. **10**, 105 (1903),
http://physics.princeton.edu/~mcdonald/examples/EM/abraham_ap_10_105_03.pdf
http://kirkmcd.princeton.edu/examples/EM/abraham_ap_10_105_03_english.pdf

[20] M. Planck, *Bemerkungen zum Prinzip der Aktion und Reaktion in der allgemeinen Dynamik*, Phys. Z. **9**, 828 (1908). See p. 829 and pp. 342-343 of [21].
http://kirkmcd.princeton.edu/examples/GR/planck_pz_9_828_08.pdf
http://kirkmcd.princeton.edu/examples/GR/planck_pz_9_828_08_english.pdf

[21] M. Giovanelli, *The practice of principles: Planck's vision of a relativistic general dynamics*, Arch. Hist Exact Sci. **78**, 305 (2024).
http://kirkmcd.princeton.edu/examples/GR/giovanelli_ahes_78_305_24.pdf

[22] C. Eckart, *The Thermodynamics of Irreversible Processes III. Relativistic Theory of the Simple Fluid*, Phys. Rev. **58**, 919 (1940). See p. 923.
http://kirkmcd.princeton.edu/examples/GR/eckart_pr_58_919_40.pdf

[23] C. Møller, *Relativistic Thermodynamics. A Strange Incident in the History of Physics*, Mat.-fys. Med. **36**-1, (1967).
http://kirkmcd.princeton.edu/examples/GR/moller_mfm_36-1_67.pdf

[24] K.T. McDonald, *Four Expressions for Electromagnetic Field Momentum* (April 10, 2006). http://kirkmcd.princeton.edu/examples/pem_forms.pdf

[25] K.T. McDonald, *On the Definition of "Hidden" Momentum* (July 9, 2012),
http://kirkmcd.princeton.edu/examples/hiddendef.pdf

[26] D.J. Griffiths, *A catalogue of hidden momenta*, Phil. Trans. Roy. Soc. London A **376**, 20180043 (2018). See the Table at the end of Sec. 3.
http://kirkmcd.princeton.edu/examples/EM/griffiths_ptrsla_376_20180043_18.pdf

[27] H. Minkowski, *Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körper*, Nachr. Ges. Wiss. Göttingen **1**, 53 (1908),
http://kirkmcd.princeton.edu/examples/EM/minkowski_ngwg_53_08.pdf
http://kirkmcd.princeton.edu/examples/EM/minkowski_ngwg_53_08_english.pdf

[28] K.T. McDonald, *Bibliography on the Abraham-Minkowski Debate* (Feb. 17, 2015),
http://kirkmcd.princeton.edu/examples/ambib.pdf

[29] B.S. Westhoff, *Electrodynamics in Matter from First Principles: A Unified and Consistent Formulation* (July 16, 2025). https://arxiv.org/abs/2504.10532