Noncontact Measurement of the Tension of a Wire

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1 The Problem

A conducting wire of mass m and length L is stretched between two fixed points with an unknown tension T. One could determine the tension by plucking the wire and observing the frequency of the vibration. Analyze the following scheme for noncontact plucking: A magnetic field of strength B is applied transversely to the wire over a length $l \ll L$ around the midpoint of the wire. A very short pulse of total charge Q is passes down the wire, which therefore starts vibrating. The voltage induced between the two ends of the vibrating wire is measured as a function of time and a Fourier analysis yields the various frequencies present.

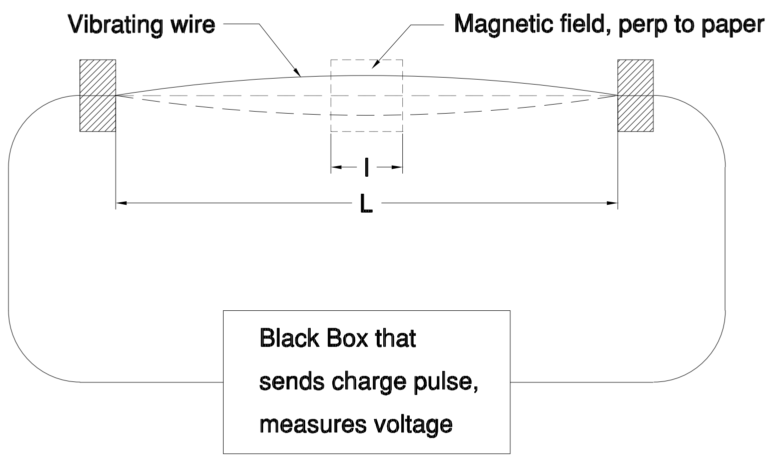

Figure 1: Scheme from noncontact measurement of wire tension.

Calculate the voltage induced at the various frequencies, and relate those frequencies to the tension in the wire. You may ignore damping effects.

A device constructed on the principles of this problem is described in [1].

2 Solution

According to Faraday, the induced voltage is $V = -\dot{\Phi}/c$ in Gaussian units, where Φ is the magnetic flux and c is the speed of light.

We write the amplitude of the transverse oscillation of the wire as $a(x, t)$. The magnetic flux through the circuit containing the wire is $\Phi \approx \Phi_0 + Bla(0, t)$, where Φ_0 is the flux when the wire is at rest, taking the origin at the midpoint of the wire. The boundary conditions on the wire are $a(L/2, t) = 0 = a(-L/2, t)$, and one of the initial conditions is $a(x, 0) = 0$ at the moment the current pulse is applied. These are sufficient to determine the amplitude as having the form,

$$a(x, t) = \sum_{\text{odd } n} a_n \cos \frac{n\pi x}{2L} \sin \omega_n t. \tag{1}$$

The motion satisfies the wave equation $a'' = \ddot{a}/v^2$, where the wave velocity is related to the tension by $v = \sqrt{TL/m}$. Hence,

$$\omega_n = \frac{n\pi v}{2L} = \frac{n\pi}{2}\sqrt{\frac{T}{mL}}. \tag{2}$$

To complete the solution we need another initial condition, corresponding to the transverse impulse due the current pulse interacting with the magnetic field. The Lorentz force on the length l of wire in the magnetic field due to current I is $F = IBl/c$. If this lasts for time Δt, the resulting impulse is $\Delta P = I\Delta t Bl/c = QBl/c$. Only a length l of the wire experiences this impulse, so the mass of this section is ml/L, and the initial velocity is QBL/mc. In sum, the second initial condition is,

$$\dot{a}(x, 0) = \begin{cases} QBL/mc, & |x| < l/2; \\ 0, & |x| > l/2. \end{cases} \tag{3}$$

From the form (1) of $a(x, t)$ we deduce that

$$\dot{a}(x, 0) = \sum_{\text{odd } n} a_n \omega_n \cos \frac{n\pi x}{2L}. \tag{4}$$

Hence, in the usual manner we evaluate the Fourier coefficients as,

$$a_n \omega_n = \frac{2}{L}\int_{-L/2}^{L/2} \dot{a}(x, 0) \cos \frac{n\pi x}{2L} dx \approx \frac{2QBl}{mc}, \tag{5}$$

for $l \ll L$.

The induced voltage is then,

$$V(t) = -\frac{Bl}{c}\dot{a}(0, t) = -\frac{2Q(Bl)^2}{mc^2} \sum_{\text{odd } n} \cos \omega_n t. \tag{6}$$

The amplitude of the voltage induced at frequency ω_n is $2Q(Bl)^2/mc^2$, independent of n. In practice, the finite values of l and Δt reduce the amplitudes of the higher harmonics.

References

[1] M.R. Convery, *A Device for Quick and Reliable Measurement of Wire Tension*, (Apr. 29, 1996), http://kirkmcd.princeton.edu/tndc/tension.pdf