INELASTIC ELECTRON SCATTERING

This topic is introduced in Perkins, and Coene & O'Neill. It is instructive to read the original paper of Bjorken and Paschos, P.R. 185, 1975 (1969). See also Feynman, 'Photon-Hadron Physics' & Close, 'Intro to Quarks & Partons'.

1. What can we learn from inelastic scattering?

The study of elastic electron-proton scattering has shown that the proton has structure – but we haven't learned too much about this structure. If the proton is made of pieces, why not break it up and look at those pieces. The break-up reaction is certainly inelastic, but why should it be initiated by electrons? A proton-proton collision is much more violent (in the sense that higher c.m. energies are available with present beams) and would seem more likely to produce direct evidence of the constituents of the proton. However this approach has failed experimentally – we now say this is due to 'quark confinement'. It has been a hard lesson to learn that it does little good to look at the pieces of the break-up of the proton in detail; the constituents are never found there. Instead it proved more fruitful to try to scatter electrons elastically off the constituents. Such a scattering yields:

a). A final state electron of different energy and angle than for elastic scattering off the whole proton.

b). A smashed proton. Try to forget this part.

The useful experiment observes only the scattered electron.

The elastic relation $E_f = \dfrac{E_i}{1 + \frac{2E_i}{M_2}\sin^2\theta/2}$ is no longer true.

The cross section must be expressed as a function of E_f as well as θ_f. We will talk about $\dfrac{d\sigma}{d\Omega\, dE_f}$

IN THE NON-RELATIVISTIC LIMIT, THE QUANTITY

$$\nu = E_i - E_f \qquad (= q_0 \text{ OF MOMENTUM TRANSFER } q_\mu)$$

IS THE ENERGY OF EXCITATION OF THE TARGET.

SO, FOR EXAMPLE, IN LOW ENERGY SCATTERING OF ELECTRONS OFF NUCLEI, WE MIGHT EXPECT PEAKS IN THE CROSS SECTION AT VALUES OF ν CORRESPONDING TO NUCLEAR LEVELS.

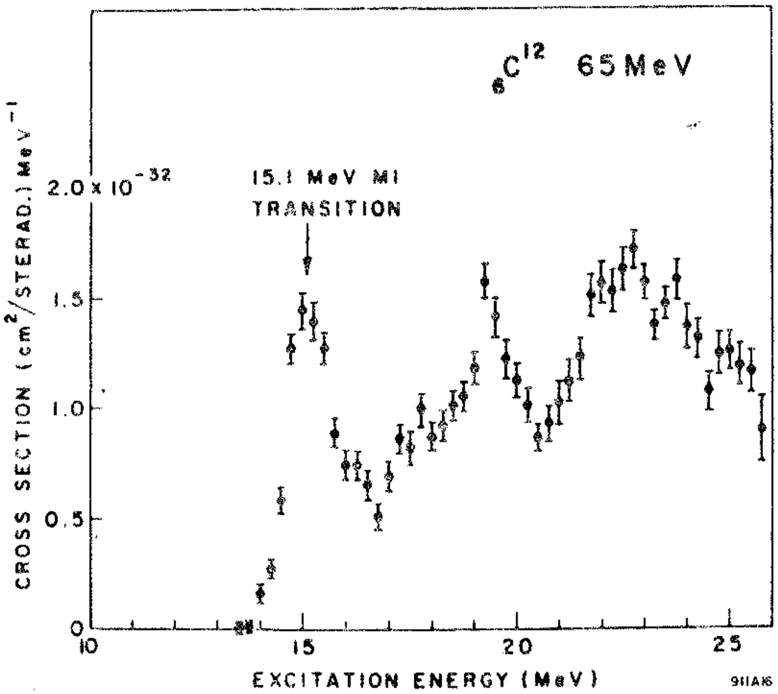
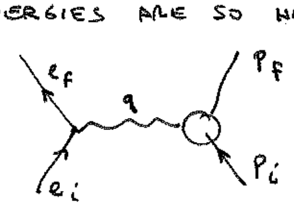


FIG. 16 --Cross section for inelastic scattering of 65 MeV electrons at 180° from C^{12}. 28

A SIMILAR THING HAPPENS IN INELASTIC ELECTRON-PROTON SCATTERING. HOWEVER, IT MAKES MORE SENSE TO DISCUSS THAT DATA IN TERMS OF THE <u>MASS</u> OF THE EXCITED PROTON WHEN THE ENERGIES ARE SO HIGH THAT RELATIVITY IS IMPORTANT. THE PICTURE IS



THE 4 VECTOR OF THE 'EXCITED PROTON' IS $P_f = P_i + \ell_i - \ell_f = P_i + q$

SO $M_f^2 = P_i^2 + q^2 + 2 P_i q$ OF COURSE $P_i^2 = M^2 = (\text{MASS OF PROTON})^2$

IN THE <u>LAB FRAME</u>, $P_i = (M, 0, 0, 0)$

So $\underline{M_f^2 = M^2 + q^2 + 2M\nu}$ (recall $\nu = q_0 = E_{1i} - E_{1f}$)

For elastic scattering $M_f^2 = M^2$ and $\underline{\nu = \dfrac{-q^2}{2M}}$ $\left(q^2 < 0\right)$

In general if we describe the scattering by q^2 and ν, we can easily calculate the mass of the 'excited proton' using the above result. $q^2 = -4 E_{1i} E_{1f} \sin^2 \theta/2$ as before.

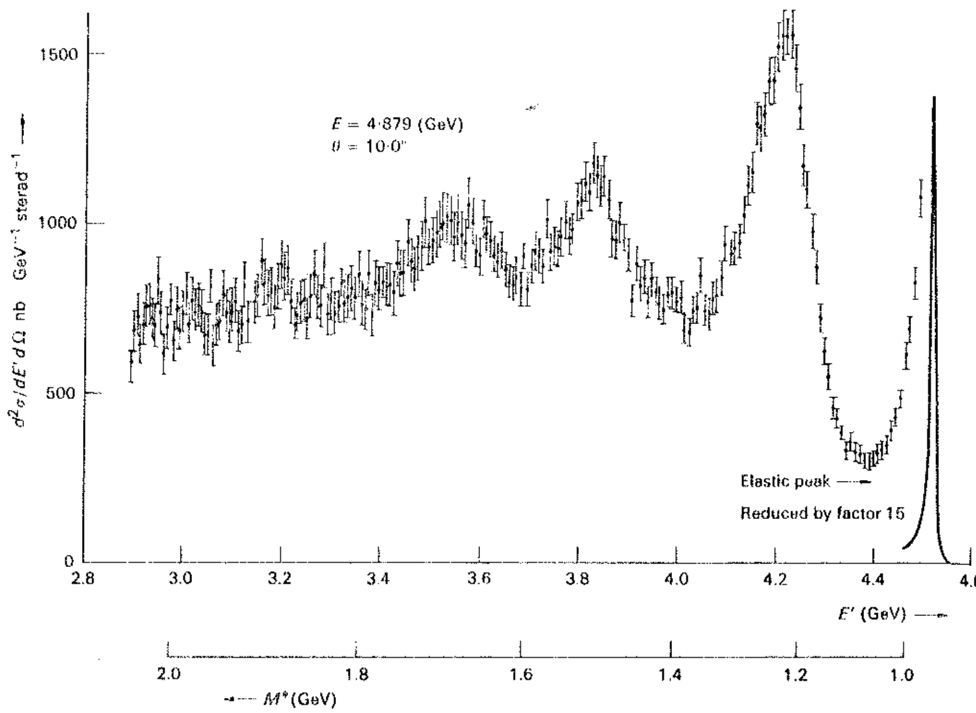
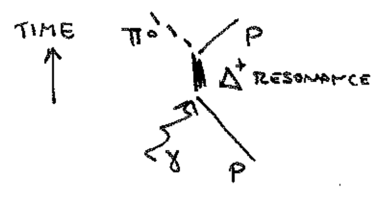


Fig. 5.25 Excitation curve of inelastic *e-p* scattering, obtained at the DESY electron accelerator (Bartel *et al.*, 1968). *E* and *E′* are the energies of incident and scattered electron, and *M** is the mass of the recoiling hadronic state. The peaks due to the pion-nucleon resonances of masses 1.24, 1.51, and 1.69 GeV are clearly visible. After corrections are made for other radiative processes in the target (bremsstrahlung), the strong elastic peak would disappear.

So inelastic electron-proton scattering certainly tells us something about the proton, but not obviously about the <u>structure</u> of the proton. In fact the data shown above could be described as demonstrating the 'virtual photoproduction of nucleon resonances', in analogy with experiments like



performed with real photons.

A CLUE AS TO HOW WE MIGHT LEARN ABOUT THE STRUCTURE OF THE PROTON IS THE PHENOMENON OF 'QUASI-ELASTIC' SCATTERING OF ELECTRONS OFF NUCLEI. SUPPOSE THE ELECTRON HITS ONLY ONE OF THE CONSTITUENTS OF THE NUCLEUS (A PROTON OR NEUTRON) AND EJECTS IT, LEAVING BEHIND A MORE OR LESS CALM NUCLEUS OF THE ELEMENT WITH Z-1 OR N-1. FROM OUR RELATION ON TOP OF p 124, THIS SHOULD OCCUR FOR $\nu \sim -\frac{q^2}{2M}$, M = PROTON MASS

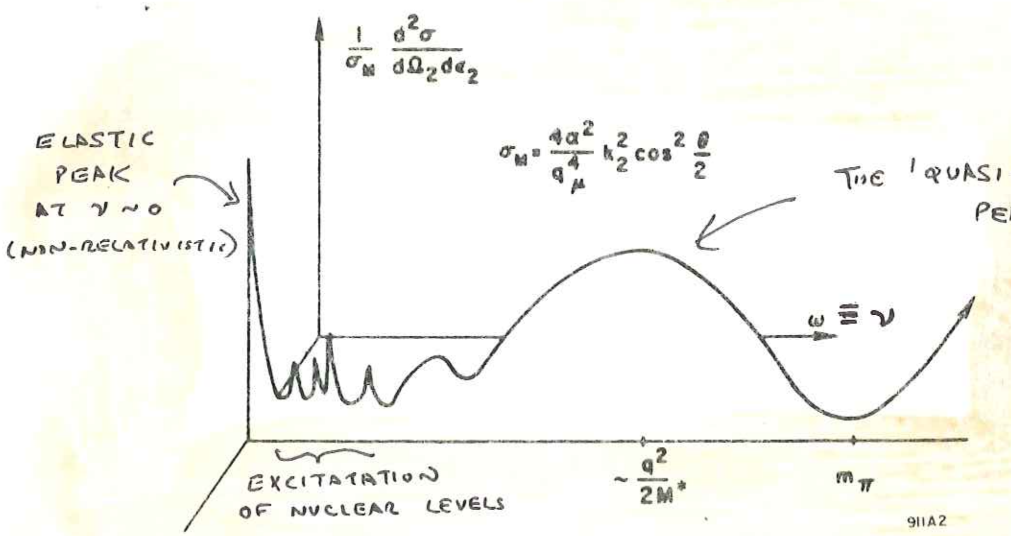


FIG. 2 --A typical nuclear double differential electron scattering cross section.

THIS IS INDEED OBSERVED, AND CAN BE REGARDED AS EVIDENCE THAT THE NUCLEUS REALLY DOES CONTAIN NUCLEONS!

IN 1964 THE QUARK MODEL WAS DEVELOPED. NAIVELY ONE EXPECTS THAT THE u AND d QUARKS HAVE MASS M/3 SINCE p = uud, n = ddu. SO PEOPLE LOOKED FOR A 'QUASI-ELASTIC' PEAK IN INELASTIC e-p SCATTERING AT $\nu = -\frac{3q^2}{2M_p}$.

THE FIGURES ON p 126 ARE FROM HAND'S TALK, pp 134, 135 OF THE 1967 SLAC ELECTRON-PHOTON CONFERENCE.

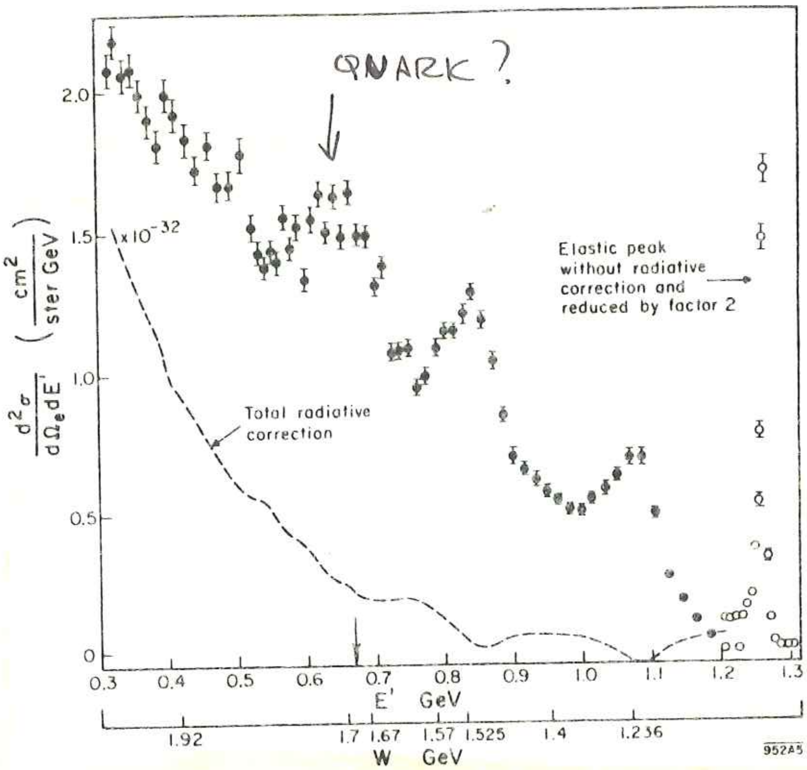


FIG. 5--Recent inelastic electron-proton scattering data from DESY (Ref. 1) $\theta_e = 47^o$, $q^2 = 40\ f^{-2}$ (q^2 varies with E', only the value at the 1238 resonance is quoted here). (On Figs. 5 - 8, the arrow shows the predicted position of the quasi-elastic peak for ground quarks of effective mass $M_p/3$, see preceding talk by J. D. Bjorken.)

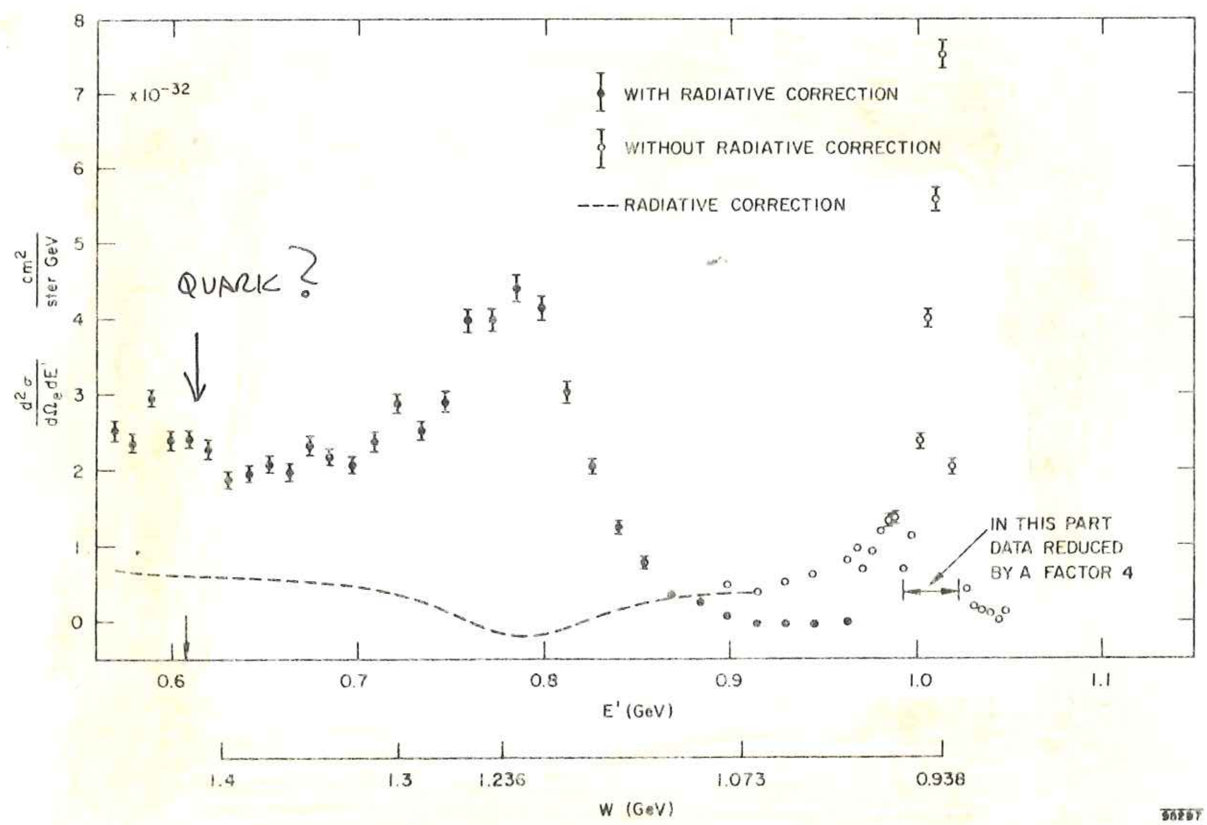


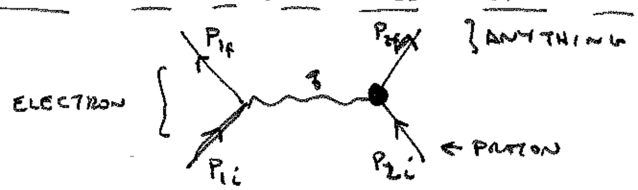
FIG. 7--DESY data, $\theta_e = 47^o$, $q^2_{1238} = 20\ f^{-2}$.

THE QUASI-ELASTIC PEAK AT $\nu = -\frac{3q^2}{2M}$ SHOULD MOVE AROUND RELATIVE TO THE NUCLEON RESONANCES OF FIXED $M_f^2 = M^2 + q^2 + 2M\nu$ IF q^2 IS VARIED.

THE EXPERIMENTAL EVIDENCE IS NOT OVERWHELMINGLY IN FAVOR OF THE PICTURE OF A PROTON AS A KIND OF ATOM CONSISTING OF 3 QUARKS. THIS WAS A BLOW TO THE QUARK MODEL, INSOFAR AS ONE HOPED THAT QUARKS WERE DYNAMICALLY MEANINGFUL OBJECTS, RATHER THAN MERELY CONVENIENT MATHEMATICAL INDICES.

AFTER DISCUSSING SOME DETAILS ABOUT THE FORM OF THE INELASTIC e-p CROSS SECTION, WE RETURN TO THE STORY OF HOW BJORKEN FOUND A NEW WAY TO USE INELASTIC e-p SCATTERING TO VALIDATE THE QUARK MODEL.

2. GENERAL FORM OF THE CROSS SECTION FOR e-p INELASTIC SCATTERING



WE FIRST COMMENT ABOUT THE CALCULATION OF CROSS-SECTIONS WHEN ONE OF THE FINAL STATE 'PARTICLES' IS NOT DEFINED – IT COULD BE 'ANYTHING'. THAT IS, THE MASS M_{2f} OF THE 4 VECTOR P_{2f} IS NOT SPECIFIED.

IF M_{2f} IS KNOWN, OUR GOLDEN RULE (p 79) IS

$$d\sigma = \frac{1}{|\bar{v}_{1i} - \bar{v}_{2i}|}\,\frac{1}{2E_{1i}}\,\frac{1}{2E_{2i}}\,|M|^2 (2\pi)^4\,\delta^4(P_{1i}+P_{2i}-P_{1f}-P_{2f})\,\frac{d^3\bar{P}_{1f}}{(2\pi)^3\,2E_{1f}}\,\frac{d^3\bar{P}_{2f}}{(2\pi)^3\,2E_{2f}}$$

WE CAN SEE WHAT TO DO IF WE TAKE NOTE OF THE USEFUL RELATION $\int d^4p\,\delta(p^2-M^2) = \int d^3\bar{p}\,dE\,\delta(E^2-\bar{p}^2-M^2) = \int \frac{d^3\bar{p}}{2E}$.

[THIS IS ONE WAY OF SHOWING THAT $\frac{d^3\bar{p}}{2E}$ IS A RELATIVISTIC INVARIANT.]

IF M_{2f} IS NOT KNOWN, THIS SUGGESTS THAT THE DENSITY OF STATES FACTOR $\frac{d^3\bar{p}_{2f}}{(2\pi)^3 2E_{2f}}$ SHOULD BE REPLACED BY $\frac{d^4 p_{2f}}{(2\pi)^3}$

THEN WE INTEGRATE OVER $d^4 p_{2f}$ TO ABSORB ALL OF THE $\delta^4(\ldots\ldots)$

AND
$$d\sigma = \frac{1}{32\pi^2}\frac{|\mathcal{M}|^2\, p_{1f}^2\, dp_{1f}\, d\Omega_{1f}}{|\bar{v}_{1i}-\bar{v}_{2i}|\, E_{1i}\, E_{2i}\, E_{1f}}$$

WE WILL CONSIDER THE <u>LAB</u> <u>FRAME</u> $E_{2i} \to M$

AND THE HIGH ENERGY LIMIT $v_{1i} \to 1$ $p_{1f} \to E_{1f}$

THEN
$$\underline{\frac{d\sigma}{dE_{1f}\, d\Omega_{1f}} = \frac{E_{1f}}{32\pi^2 E_{1i} M}|\mathcal{M}|^2}$$

WE NOW TURN TO THE MATRIX ELEMENT $\mathcal{M}$.

IN THE ONE PHOTON EXCHANGE APPROXIMATION,
$$\mathcal{M} = \frac{e^2}{q^2}\, j_\mu J^\mu$$

WHERE j_μ = ELECTRON CURRENT

J_μ = HADRON CURRENT

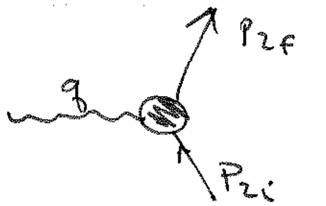
$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\cdot\frac{1}{4}\sum_{\text{SPINS}}(j_\mu J^\mu)(j_\nu^+ J^{\nu +}) = \frac{1}{4}\frac{e^4}{q^4}\sum_{\text{SPINS}} j_\mu j_\nu^+ J^\mu J^{\nu +}$$

AVERAGE OVER INITIAL ELECTRON AND PROTON SPINS

$$\equiv \frac{1}{4}\frac{e^4}{q^4}\, l_{\mu\nu} L^{\mu\nu}$$ (FOR UNPOLARIZED BEAM AND TARGET)

$$l_{\mu\nu} = \sum_{\text{SPINS}} j_\mu j_\nu^+ = 4\left(\frac{q^2}{2}\delta_{\mu\nu} + p_{1i_\mu} p_{1f_\nu} + p_{1f_\mu} p_{1i_\nu}\right)$$ (p95)

$L_{\mu\nu} = \sum_{\text{SPINS}} J_\mu J_\nu^+ \equiv$ HADRON TENSOR, CONTAINING ALL POSSIBLE INFORMATION ABOUT THE COUPLING OF A PHOTON TO 'p + ANYTHING'



$$q = P_{2f} - P_{2i}$$

$L_{\mu\nu}$ IS A TENSOR. IT MUST BE CONSTRUCTED OUT OF QUANTITIES PERTAINING TO THE ABOVE PICTURE. NO TENSORS ARE DIRECTLY INVOLED (EXCEPT, OF COURSE, $\delta_{\mu\nu}$). BUT WE CAN MAKE A TENSOR OUT OF THE PRODUCT OF TWO 4-VECTORS. SINCE WE SUM OVER SPINS, THE ONLY 4-VECTORS AVAILABLE ARE q, P_{2i} AND P_{2f}. ONLY 2 ARE INDEPENDENT. IT IS NATURAL (OR AT LEAST CONVENTIONAL) TO CHOOSE

$$q \text{ AND } P \equiv P_{2i}$$

THEN POSSIBLE CANDIDATES ARE:

$$\delta_{\mu\nu}, \quad q_\mu q_\nu, \quad P_\mu P_\nu, \quad q_\mu P_\nu + P_\mu q_\nu, \quad q_\mu P_\nu - P_\mu q_\nu$$

NOW $L_{\mu\nu}$ WILL BE <u>SYMMETRIC</u> SINCE CURRENTS ARE HERMITIAN

$$L_{\mu\nu} = L_{\mu\nu}^{+} = L_{\nu\mu}. \qquad \therefore\ q_\mu P_\nu - P_\mu q_\nu \text{ IS RULED OUT.}$$

FOUR CANDIDATES REMAIN. EACH COULD BE MULTIPLIED BY A LORENTZ SCALAR FUNCTION. I.E.

$$L_{\mu\nu} = A\,\delta_{\mu\nu} + \frac{B}{q^2} q_\mu q_\nu + \frac{C}{M^2} P_\mu P_\nu + \frac{D}{qP}\left(q_\mu P_\nu + P_\mu q_\nu\right)$$

THE FUNCTIONS A, B, C, D CAN DEPEND ONLY ON LORENTZ SCALARS. CANDIDATES ARE q^2, P^2, P_{2f}^2, qP, qP_{2f}, PP_{2f} ETC.

ONLY TWO ARE INDEPENDENT, SINCE $P^2 = M^2 =$ CONSTANT

AND $P_{2f} = q + P$ SO $P_{2f}^2 = M_{2f}^2 = M^2 + q^2 + 2(qP)$ ETC.

THE COMMON CHOICE IS $\underline{q^2}$ AND $\underline{qP}$

IN THE LAB FRAME $qP = q_0 M = \nu M$ $\qquad \underline{\nu = \frac{qP}{M}}$

SO $\underline{q^2 \text{ AND } \nu}$ ARE ALSO COMMONLY USED.

WE CAN USE CURRENT CONSERVATION TO SHOW THAT ONLY 2 OF THE 4 FUNCTIONS A, B, C, D, ARE INDEPENDENT.

FROM THE DEFINITION $L_{\mu\nu} = \sum' J_\mu J_\nu^+$ AND THE RELATION $q^\mu J_\mu = 0$ WE SEE THAT $q^\mu L_{\mu\nu} = 0$.

$$\text{or} \quad A q_\mu + B q_\mu + \frac{C(qP)}{M^2} P_\mu + \frac{D}{qP}\left(q^2 P_\mu + (qP) q_\mu\right) = 0$$

$$\text{or} \quad A + B + D = 0 \quad \text{AND} \quad C\frac{(qP)}{M^2} + D\frac{q^2}{qP} = 0$$

SINCE q_μ AND P_μ ARE ARBITRARY.

ELIMINATING A AND D,

$$L_{\mu\nu} = B\left(\frac{q_\mu q_\nu}{q^2} - \delta_{\mu\nu}\right) + \frac{C}{M^2}\left[P_\mu P_\nu - \frac{(qP)}{q^2}\left(q_\mu P_\nu + P_\mu q_\nu\right) + \frac{(qP)^2}{q^2}\delta_{\mu\nu}\right]$$

THE SECOND TERM IS ALMOST FACTORIZABLE, SO WE REARRANGE

$$L_{\mu\nu} = \left(B - \frac{C(qP)^2}{M^2 q^2}\right)\left(\frac{q_\mu q_\nu}{q^2} - \delta_{\mu\nu}\right) + \frac{C}{M^2}\left(P_\mu - \frac{(qP)}{q^2} q_\mu\right)\left(P_\nu - \frac{(qP)}{q^2} q_\nu\right)$$

IT HAS BECOME CUSTOMARY TO INTRODUCE NEW FUNCTIONS W_1 AND W_2 SO THAT

$$\boxed{L_{\mu\nu} = 4M\left\{W_1\left(\frac{q_\mu q_\nu}{q^2} - \delta_{\mu\nu}\right) + \frac{W_2}{M^2}\left(P_\mu - \frac{(qP)}{q^2} q_\mu\right)\left(P_\nu - \frac{(qP)}{q^2} q_\nu\right)\right\}}$$

$$\underline{W_{1,2} = W_{1,2}(q^2, \nu)}$$

THE FACTOR M WILL BEAUTIFY THE CROSS SECTION.

THE FACTOR 4 IS TO MAKE THE $L_{\mu\nu}$ LOOK LIKE THE RESULT OF A FEYNMAN TRACE CALCULATION.

THEN $l_{\mu\nu} L^{\mu\nu} = 16 M W_1 \left\{ -\frac{3}{2} q^2 + 2 \frac{(P_{1i} q)(P_{1f} q)}{q^2} - 2 (P_{1i} P_{1f}) \right\}$

$$+ 16 \frac{W_2}{M} \left\{ \frac{q^2}{2} \left(M^2 - \frac{(qP)^2}{q^2} \right) + 2 \left[(P_{1i} P) - \frac{(qP)(P_{1i} q)}{q^2} \right] \left[(P_{1f} P) - \frac{(qP)(P_{1f} q)}{q^2} \right] \right\}$$

IN THE <u>LAB FRAME</u> $\quad q^2 = -4 E_{1i} E_{1f} \sin^2 \theta/2$

$$qP = M\nu \qquad \nu = E_{1i} - E_{1f}$$

$$P_{1i} q = - P_{1f} q = - P_{1i} P_{1f} = q^2/2$$

$$P_{1i} P = M E_{1i} \qquad P_{1f} P = M E_{1f}$$

AND $\quad l_{\mu\nu} L^{\mu\nu} = 32 M E_{1i} E_{1f} \left[W_2 \cos^2 \theta/2 + 2 W_1 \sin^2 \theta/2 \right]$

NOTE THE APPEARANCE OF THE FAMOUS ANGULAR FACTORS FROM ANOTHER GENERAL ARGUMENT!

FROM p 128, $\quad \frac{d\sigma}{d\Omega \, dE_{1f}} = \frac{1}{32\pi^2} \frac{E_{1f}}{E_{1i} M} \cdot \frac{1}{4} \frac{e^4}{q^4} \cdot l_{\mu\nu} L^{\mu\nu}$

$$\boxed{\frac{d\sigma}{d\Omega \, dE_{1f}} = \frac{\alpha^2}{4 E_{1i}^2 \sin^4 \theta/2} \left[W_2 \cos^2 \theta/2 + 2 W_1 \sin^2 \theta/2 \right]} \qquad \left(\alpha = \frac{e^2}{4\pi} \right)$$

WHILE WE DERIVED THIS RESULT FOR INELASTIC e-p SCATTERING, WE SEE THAT OUR ARGUMENT ACTUALLY HOLDS FOR INELASTIC SCATTERING OF ELECTRONS OFF <u>ANYTHING</u>! (THE SPIN WEIGHTING FACTORS COULD ALWAYS BE ABSORBED INTO THE DEFINITIONS OF W_1 AND W_2.)

3. <u>ELASTIC SCATTERING OFF SPIN-0 AND SPIN ½ PARTICLES.</u>

ELASTIC SCATTERING IS JUST A SPECIAL CASE OF INELASTIC SCATTERING, FOR WHICH $\nu = -\frac{q^2}{2m}$ (<u>LAB FRAME</u>)

(USING $P_{2f}^2 = m^2 = (q+P)^2 = M^2 + q^2 + 2M\nu$)

SPIN-0 : RECALL THAT $\dfrac{d\sigma}{d\Omega} = \dfrac{\alpha^2}{4E_i^2 \sin^4\theta/2} \dfrac{\cos^2\theta/2}{1 + \frac{2E_i}{M}\sin^2\theta/2}$ (p 96)

THIS SUGGESTS THAT $\left.\boxed{\begin{aligned} W_1 &= 0 \\ W_2 &= \delta\left(\nu + \frac{q^2}{2M}\right)\end{aligned}}\right\}$ POINTLIKE SPIN-0

THIS IS EASILY VERIFIED, RECALLING THAT $\nu = E_i - E_f$

AND $q^2 = -4E_i E_f \sin^2\theta/2 = -4E_i(E_i - \nu)\sin^2\theta/2$

SO $$\int \delta\left(\nu + \frac{q^2}{2M}\right) dE_f = \frac{1}{1 + \frac{2E_i}{m}\sin^2\theta/2}$$

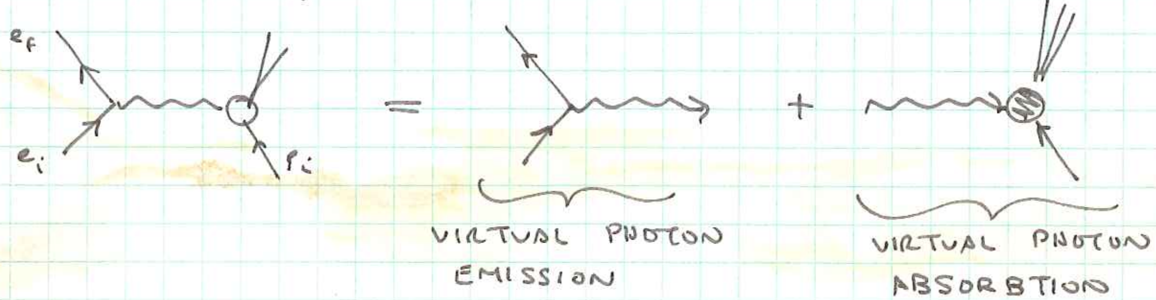
SPIN ½ : $\dfrac{d\sigma}{d\Omega} = \dfrac{\alpha^2}{4E_i^2\sin^4\theta/2} \dfrac{\left[\cos^2\theta/2 - \frac{q^2}{2M^2}\sin^2\theta/2\right]}{1 + \frac{2E_i}{m}\sin^2\theta/2}$ (p 99)

HENCE $\left.\boxed{\begin{aligned} W_1 &= -\frac{q^2}{4m^2}\delta\left(\nu + \frac{q^2}{2m}\right) \\ W_2 &= \delta\left(\nu + q^2/2m\right)\end{aligned}}\right\}$ POINTLIKE SPIN ½

NOTE THAT $W_1 = -q^2 W_2/4m^2$, OR $\underline{\nu W_2 = 2mW_1}$ AS $q^2 = -2M\nu$ CALLAN-GROSS RELATION

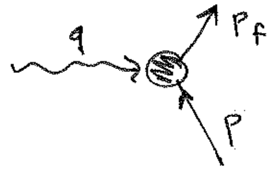
4. σ_L, σ_T AND R

THERE IS ANOTHER GENERAL VIEW OF INELASTIC e-p SCATTERING WHICH IS STILL USED IN THE LITERATURE. THIS WAS HINTED AT ON p124.



THE RIGHT HAND PICTURE IS CONCEPTUALLY RELATED TO THE ABSORPTION OF REAL PHOTONS BY A PROTON, LEADING TO SOME GENERAL FINAL STATE.

WE SHALL CALCULATE THE CROSS SECTION FOR THE VIRTUAL PHOTON ABSORPTION PROCESS.



$(q^2 < 0)$

FROM OUR DISCUSSION OF THE GOLDEN RULE FOR ARBITRARY FINAL STATES (pp 127-128) WE SEE THAT

$$\sigma = \frac{1}{|\vec{v}_\gamma - \vec{v}_p|} \frac{1}{2E_\gamma} \frac{1}{2E_p} 2\pi |\mathcal{M}|^2$$

WE WILL CALCULATE IN THE LAB FRAME, $P = (M, 0, 0, 0)$ WITH THE z AXIS TAKEN ALONG THE PHOTON'S MOMENTUM

$$q \equiv (\nu, 0, 0, k) \qquad q^2 = \nu^2 - k^2 < 0$$

SO $$\sigma = \frac{\pi}{2kM} |\mathcal{M}|^2$$

FROM OUR DISCUSSION IN SECTION 14, LECTURE 7, WE EXPECT THAT THE MATRIX ELEMENT CAN BE WRITTEN

$$\mathcal{M} = e \, \epsilon_\mu J^\mu$$

WHERE ϵ_μ = PHOTON POLARIZATION

J_μ = HADRON CURRENT

AND $$|\mathcal{M}|^2 = e^2 \epsilon_\mu \epsilon_\nu^+ \cdot \frac{1}{2} \sum_{\text{SPIN}} J^\mu J^{\nu +} = \frac{1}{2} e^2 \epsilon_\mu \epsilon_\nu^+ L^{\mu\nu}$$

AVERAGE OVER INITIAL PROTON SPINS

WHERE $L_{\mu\nu}$ IS THE HADRON TENSOR FOUND IN SECTION 2 !

IT IS CUSTOMARY TO CONSIDER THE CASE OF TRANSVERSE AND LONGITUDINAL POLARIZATION SEPARATELY.

TRANSVERSE POLARIZATION $\epsilon_\mu = (0, \epsilon_x, \epsilon_y, 0)$ WITH $\epsilon_x^2 + \epsilon_y^2 = 1$

THEN $\epsilon_\mu \epsilon^\mu = -1$ AND $\epsilon_\mu q^\mu = 0 = \epsilon_\mu P^\mu$

THEN $\epsilon_\mu \epsilon_\nu^+ L^{\mu\nu} = -4MW_1 \epsilon_\mu \epsilon^\mu = 4MW_1$

AND $$\boxed{\sigma_T = \frac{\pi e^2 W_1}{K}}$$

LONGITUDINAL POLARIZATION. WE DEFINE $Q = \sqrt{-q^2}$, $Q^2 = K^2 - \nu^2 \geq 0$

THEN $\epsilon_\mu = \left(\frac{K}{Q}, 0, 0, \frac{\nu}{Q}\right)$ SO THAT $\epsilon_\mu q^\mu = 0$ AS REQUIRED (P. 110)

AND $\epsilon_\mu \epsilon^\mu = +1$ IS OUR NORMALIZATION

THIS TIME $\epsilon_\mu P^\mu = \frac{KM}{Q}$

$$\epsilon_\mu \epsilon_\nu^+ L^{\mu\nu} = 4M\left\{-\epsilon_\mu \epsilon^\mu W_1 + \frac{W_2}{M^2}(\epsilon_\mu P^\mu)(\epsilon_\nu P^\nu)\right\} = 4M\left\{-W_1 + \frac{K^2}{Q^2} W_2\right\}$$

$$= 4M\left\{-W_1 + \left(1 - \frac{\nu^2}{q^2}\right) W_2\right\}$$

$$\boxed{\sigma_L = \frac{\pi e^2}{K}\left(-W_1 + \left(1 - \frac{\nu^2}{q^2}\right) W_2\right)}$$

HENCE σ_L AND σ_T PROVIDE AN ALTERNATE TO THE TWO STRUCTURE FUNCTIONS W_1 AND W_2

$$\boxed{\begin{aligned} W_1 &= \frac{K}{\pi e^2} \sigma_T \\ W_2 &= \frac{1}{\left(1 - \frac{\nu^2}{q^2}\right)} \frac{K}{\pi e^2} (\sigma_L + \sigma_T) \end{aligned}}$$

THE RATIO IS SOMETIMES QUOTED

$$\boxed{\frac{W_1}{W_2} = \left(1 - \frac{\nu^2}{q^2}\right) \frac{\sigma_T}{\sigma_L + \sigma_T}}$$

SOME PEOPLE ALSO DEFINE $\boxed{R \equiv \frac{\sigma_L}{\sigma_T}}$

$$\frac{W_1}{W_2} = \left(1 - \nu^2/q^2\right) \frac{1}{1+R}$$

HENCE WE COULD WRITE THE CROSS SECTION FOR INELASTIC

e-p SCATTERING (p 80) AS

$$\frac{d\sigma}{d\Omega dE_{1f}} = \frac{\alpha^2 \cos^2\theta/2}{4 E_{1i}^2 \sin^4\theta/2} \cdot W_2 \left[1 + \frac{2}{1+R}\left(1 - \nu^2/q^2\right) \tan^2\theta/2\right]$$

WE CAN ALSO WRITE THE CROSS SECTION IN TERMS OF σ_L AND σ_T,

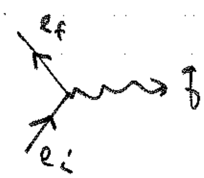
AFTER SOME ALGEBRA:

$$\frac{d\sigma}{d\Omega dE_{1f}} = \frac{\alpha}{2\pi^2} \frac{1}{Q^2} \frac{E_{1f}}{E_{1i}} \frac{k}{1-\epsilon} \left(\sigma_T + \epsilon \sigma_L\right)$$

WHERE $\epsilon \equiv \dfrac{1}{1 + 2\left(1 - \frac{\nu^2}{q^2}\right)\tan^2\theta/2}$ $\equiv$ POLARIZATION PARAMETER

THE FACTOR $\Gamma \equiv \dfrac{\alpha}{2\pi^2} \dfrac{1}{Q^2} \dfrac{E_{1f}}{E_{1i}} \dfrac{k}{1-\epsilon}$ IS SOMETIMES

CALLED THE <u>VIRTUAL</u> <u>PHOTON</u> <u>EMISSION</u> <u>SPECTRUM</u>



IT GOES LIKE $1/q^2$

THE PARAMETER ϵ MEASURES THE RELATIVE AMOUNT OF LONGITUDINAL AND TRANSVERSE PHOTON POLARIZATION. AS DISCUSSED IN SECTION 17, LECTURE 7, $\epsilon = 0$ IN THE SPECIAL BREIT FRAME.

SUPPOSEDLY, THE LAB FRAME EXPRESSION FOR ϵ CAN BE DERIVED BY TRANSFORMING THE TRANSVERSE POLARIZATION OBSERVED IN THE BREIT FRAME TO TRANSVERSE + LONGITUDINAL AS SEEN IN THE LAB.

WE CLOSE OUR DISCUSSION OF 'KINEMATICS' BY NOTING THE VALUES OF $R = \sigma_L/\sigma_T$ FOR ELASTIC SCATTERING. FROM SECTION 3

SPIN 0 $\Rightarrow$ $R \to \infty$ SINCE $W_1 = 0 \Rightarrow \sigma_T = 0$

FOR SPIN ½, $W_1 = -\frac{q^2}{4M^2} W_2$ (p 81)

$$\text{so} \quad \left(1 - \frac{\nu^2}{q^2}\right)\sigma_T = -\frac{q^2}{4M^2}(\sigma_L + \sigma_T) \qquad (\text{p } 134)$$

BUT FOR ELASTIC SCATTERING $q^2 = -2M\nu$

$$\text{so} \quad R = \frac{\sigma_L}{\sigma_T} = -\frac{4M^2}{q^2} \qquad \text{SPIN } 1/2$$

IN THE LIMIT $M^2 \ll q^2$, $R \to 0$ FOR SPIN ½.

Ph 529 LECTURE 8

5. THE PARTON MODEL AND SCALING

WE SAW IN SECTION I THAT INELASTIC e-p SCATTERING FAILED TO SHOW A CONVINCING QUASI-ELASTIC PEAK CORRESPONDING TO SCATTERING OFF A QUARK OF MASS $M/3$. BJORKEN SUGGESTED (P.R. 185, 1975 (1969)) THAT THIS IS BECAUSE THE STRONG INTERACTION IS SO STRONG THAT A QUARK STRUCK BY AN ELECTRON INTERACTS WITH ANOTHER QUARK BEFORE THE ELECTRON INTERACTION IS OVER. WE NEED A SITUATION WHERE THE QUARKS ACT INDEPENDENTLY, AT LEAST FOR THE DURATION OF THE PASSAGE OF THE ELECTRON.

THE LIMIT OF VERY HIGH ENERGY MAY PROVIDE THE REQUIRED BEHAVIOR. TO AN OBSERVER AT REST, THE INTERNAL MOTION OF A RAPIDLY MOVING OBJECT APPEARS SLOWED DOWN BY THE TIME DILATION EFFECT. FOR VERY FAST PROTONS, THE QUARKS BEHAVE ESSENTIALLY INDEPENDENTLY, WHEN VIEWED BY OUR ELECTRON PROBE. WE MAY STILL HOPE FOR EVIDENCE OF ELASTIC SCATTERING OF ELECTRON OFF INTERNAL CONSTITUENTS OF THE PROTON IF WE CONSIDER ONLY EXPERIMENTS AT VERY HIGH ENERGY.

WE DROP THE ASSUMPTION THAT THE CONSTITUENTS ARE QUARKS, AND USE FEYNMAN'S TERM PARTON TO DESCRIBE ANY SUB-PARTICLE OF WHATEVER CHARGE OR SPIN.

WE CAN ANTICIPATE THAT OUR DESCRIPTION OF THE STRUCTURE OF A FAST-MOVING OBJECT WILL NOT BE TOO SIMILAR TO THE CONCEPT OF AN ATOM. ANY INTERNAL MOTION TRANSVERSE TO THE PROTON'S DIRECTION IS NEGLIBLE IN THE HIGH ENERGY LIMIT.

WE SHALL LEARN PRIMARILY ABOUT THE LONGITUDINAL MOTION OF THE PARTONS. IF THE PROTON HAS MOMENTUM P (LARGE), THE PARTON COULD HAVE ANY LONGITUDINAL MOMENTUM BETWEEN 0 AND P. WE DEFINE (SO LONG AS $P_{PARTON} \gg M_{PARTON}$, THE VELOCITY IS $\approx c$ AND ALL PARTONS TRAVEL TOGETHER INSIDE THE PARENT PARTICLE)

$$x = \frac{P_{\parallel}(\text{PARTON})}{P(\text{PROTON})} = \text{FRACTION OF LONGITUDINAL MOMENTUM CARRIED BY THE PARTON.}$$

OUR DESCRIPTION OF THE STRUCTURE OF A FAST MOVING PROTON WE FOCUS ON THE DISTRIBUTION:

$f(x)\,dx \equiv$ PROBABILITY OF FINDING A PARTON WITH MOMENTUM FRACTION BETWEEN x AND $x+dx$.

THIS SEEMS A LESS COMPLETE DESCRIPTION THAN MIGHT BE HOPED FOR. BUT IF WE SAY AN OBJECT CONTAINS INTERNAL PARTS, THOSE PARTS NEED TO BE INDEPENDENT FOR THE STATEMENT TO HAVE MEANING. WHEN THE STRONG INTERACTION IS INVOLVED, THIS APPEARS POSSIBLE ONLY IN THE HIGH ENERGY LIMIT, FOR WHICH THE LONGITUDINAL MOMENTUM DISTRIBUTION EMERGES AS THE RELEVANT CONCEPT. DESPITE THIS APPARENT RESTRICTION, THE HIGH ENERGY PARTON IDEA HAS PROVED REMARKABLY FRUITFUL.

WE RETURN TO THE CASE OF INELASTIC e-p SCATTERING. SUPPOSE THE KINEMATIC VARIABLES ν AND q^2 ARE INDEED LARGE ENOUGH SO THAT ELASTIC SCATTERING OFF A SINGLE PARTON IS A GOOD DESCRIPTION (FOLLOWED BY SUBSEQUENT PARTON-PARTON INTERACTIONS WHICH PRODUCE THE OBSERVED DEBRIS OF THE PROTON).

Ph 529 LECTURE 8

THEN ACCORDING TO SECTION 3, WE EXPECT (FOR SPIN 0 OR SPIN ½ PARTONS) THAT THE STRUCTURE FUNCTION W_2 BECOMES

$$W_2(\nu, q^2) = Q^2\,\delta\left(\nu + \frac{q^2}{2m}\right) \qquad \text{[CALCULATED IN LAB FRAME]}$$

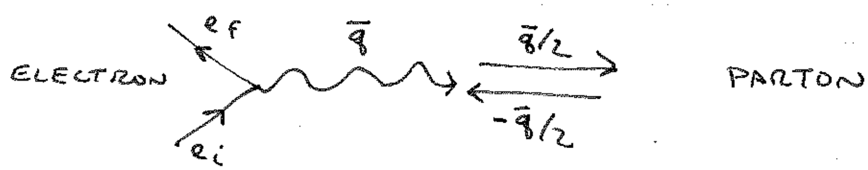
WHERE Q = PARTON CHARGE IN UNITS OF ELECTRON CHARGE, I.E. $Q_e = 1$

AND m = PARTON MASS

WE WISH TO SHOW THAT THE RELATION $\nu = -\frac{q^2}{2m}$ FOR ELASTIC SCATTERING CAN BE REWRITTEN $\nu = \frac{-q^2}{2Mx}$

M = PROTON MASS, x = PARTON MOMENTUM FRACTION DEFINED ON p 138

THE USUAL ARGUMENT IS TO CONSIDER THE SCATTERING IN A BREIT FRAME SUCH THAT THE PARTON IS SCATTERED BY 180°.



THE PHOTON HAS 4-MOMENTUM $q = (0, \bar{q})$ IN THIS FRAME, SO THE PARTON MUST HAVE MOMENTUM $\bar{q}/2$ IF IT SCATTERED ELASTICALLY. $|\bar{q}| = \sqrt{-q^2}$

WHAT IS THE INITIAL PROTON MOMENTUM IN THIS FRAME?

IN THE LAB, THE PHOTON 4 MOMENTUM IS $q = (\nu, \bar{q}_{LAB})$ WITH $|\bar{q}_{LAB}| = \sqrt{\nu^2 - q^2}$.

CHOOSING THE z AXIS ALONG $\bar{q}_{LAB}$, $q_\mu = (\nu, 0, 0, \sqrt{\nu^2 - q^2})$

TO TRANSFORM TO THE BREIT FRAME WE BOOST ALONG THE z AXIS SO THAT $\nu^* = 0 = \gamma(\nu - \beta\sqrt{\nu^2 - q^2}) \Rightarrow \beta = \frac{\nu}{\sqrt{\nu^2 - q^2}}$ AND $\gamma = \sqrt{\frac{\nu^2 - q^2}{-q^2}}$

So in the Breit frame the incident proton momentum

is $\gamma\beta M = \dfrac{\nu M}{\sqrt{-q^2}}$

Hence $x = \dfrac{\sqrt{-q^2}/2}{\nu M/\sqrt{-q^2}}$ or $\boxed{x = \dfrac{-q^2}{2M\nu}}$

and $\delta\left(\nu + \dfrac{q^2}{2m}\right) \longrightarrow \delta\left(\nu + \dfrac{q^2}{2Mx}\right)$

and $W_2 = Q^2\,\delta\left(\nu + \dfrac{q^2}{2Mx}\right)$ for scattering off a parton

with momentum (fraction) x.

If the partons have probability distribution $f(x)\,dx$

Then $W_2 = Q^2 \displaystyle\int_0^1 f(x)\,dx\,\delta\left(\nu + \frac{q^2}{2Mx}\right) = \frac{Q^2 f(x)}{-q^2/2Mx^2} = \frac{Q^2 x f(x)}{\nu}$

or $\boxed{\nu W_2 = Q^2 x\, f(x) \equiv F_2(x)}$

If there are several different types of partons of

charges Q_i and distributions $f_i(x)$

Then $\nu W_2 = x \sum_i Q_i^2 f_i(x)$

A prominent feature of this model is that $W_2(\nu, q^2)$

becomes a function of a single variable $x = -q^2/2M\nu$

X is dimensionless. This is called Bjorken scaling

and should hold when ν and q^2 are large compared

to the intrinsic energy scale — the proton mass.

Exercise: Make a similar argument for function W_1 (p.196) to show

$2MW_1 = Q^2 f(x) \equiv 2F_1(x)$. Then $2xF_1(x) = F_2(x)$, which is another form of the Callan-Gross relation.

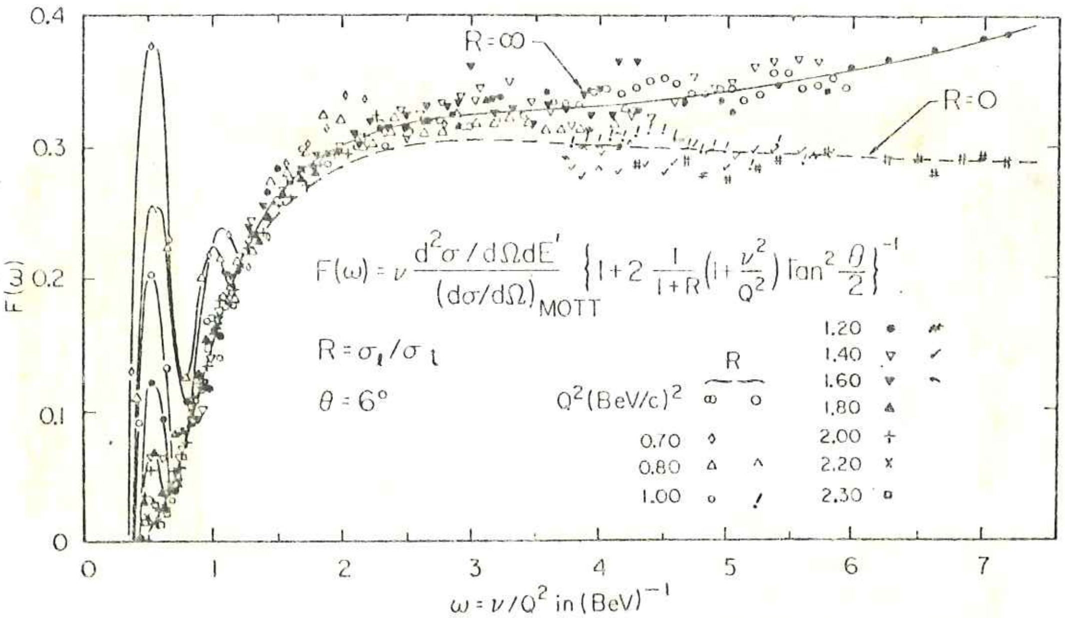


FIG. 2. Plot of the data as a function of ν/Q^2.

THIS IS THE DATA QUOTED IN BJORKEN'S PAPER SHOWING THE ONSET OF SCALING ONCE THE ENERGIES ARE ABOVE THE RESONANCE REGION.

6. PARTONS AS QUARKS

CAN WE INTERPRET THE BJORKEN SCALING AS EVIDENCE THAT PARTONS ARE QUARKS?

A FIRST TEST IS THE RATIO $R = \sigma_L/\sigma_T \lesssim 0.2$ FROM SLAC DATA. RECALL THAT SPIN-0 $\Rightarrow R \to \infty$, WHILE SPIN 1/2 $\Rightarrow R \to 0$

SO THERE IS REASONABLE CONSISTENCY THAT THE PARTONS HAVE SPIN 1/2.

ANOTHER WAY OF STATING THIS IS THAT FOR SPIN 1/2 PARTONS, THE CALLAN-GROSS RELATION $\nu W_2 = 2M_p x W_1$ SHOULD HOLD. AGAIN, THE SLAC DATA (p142) SHOW REASONABLE CONSISTENCY.

[DATA FROM THE 1975 SLAC LEPTON-PHOTON CONF.]

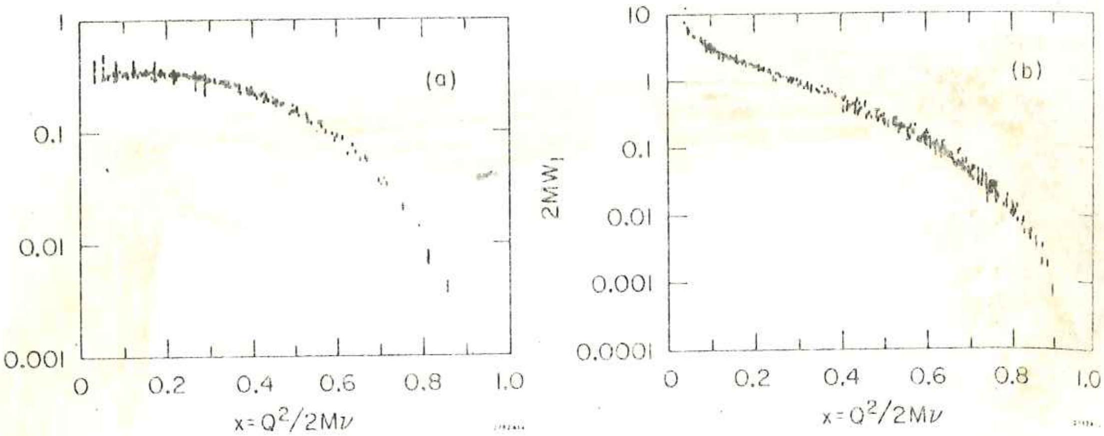


Fig. 1--Values of νW_2 and $2MW_1$ for the proton with $Q^2 > 1\ \mathrm{GeV}^2$ and $W^2 > 4\ \mathrm{GeV}^2$. $R = \sigma_L/\sigma_T$ is assumed to be zero, and values of νW_2 are extracted from cross section measurements with $\epsilon \geq 1/2$, while values of $2MW_1$ are extracted for $\epsilon \leq 1/2$. The large vertical bars below $x = 0.3$ in the graph of νW_2 are obtained from muon data taken at Fermilab.

FROM THE RELATION $\nu W_2 = \sum_i Q_i^2 \, x \, f_i(x)$

WE SHOULD BE ABLE TO CONCLUDE SOMETHING ABOUT THE PARTON CHARGES. NOTE THAT SINCE $f(x)\,dx$ IS A PROBABILITY DISTRIBUTION, $x\,f(x)\,dx$ IS THE MOMENTUM DISTRIBUTION.

i.e. $\int_0^1 x\, f_i(x)\, dx$ = TOTAL MOMENTUM CARRIED BY PARTONS OF TYPE i.

THUS $\int_0^1 \nu W_2\, dx = Q^2_{\text{AVERAGE}} \cdot$ MOMENTUM CARRIED BY CHARGED PARTONS.

EXPERIMENTALLY, $\int_0^1 \nu W_2 \sim 0.16$, SO IF ALL PARTONS ARE CHARGED, THE AVERAGE CHARGE IS CERTAINLY LESS THAN 1.

IF THE PROTON CONSISTS OF uud QUARKS, THEN

$$Q^2_{\text{AVE}} = \frac{1}{3}\left(\frac{4}{9} + \frac{4}{9} + \frac{1}{9}\right) = \frac{1}{3}$$

TO SAVE THE QUARK MODEL WE MAKE A BOLD HYPOTHESIS: HALF THE MOMENTUM OF THE PROTON IS CARRIED BY NEUTRAL PARTONS!

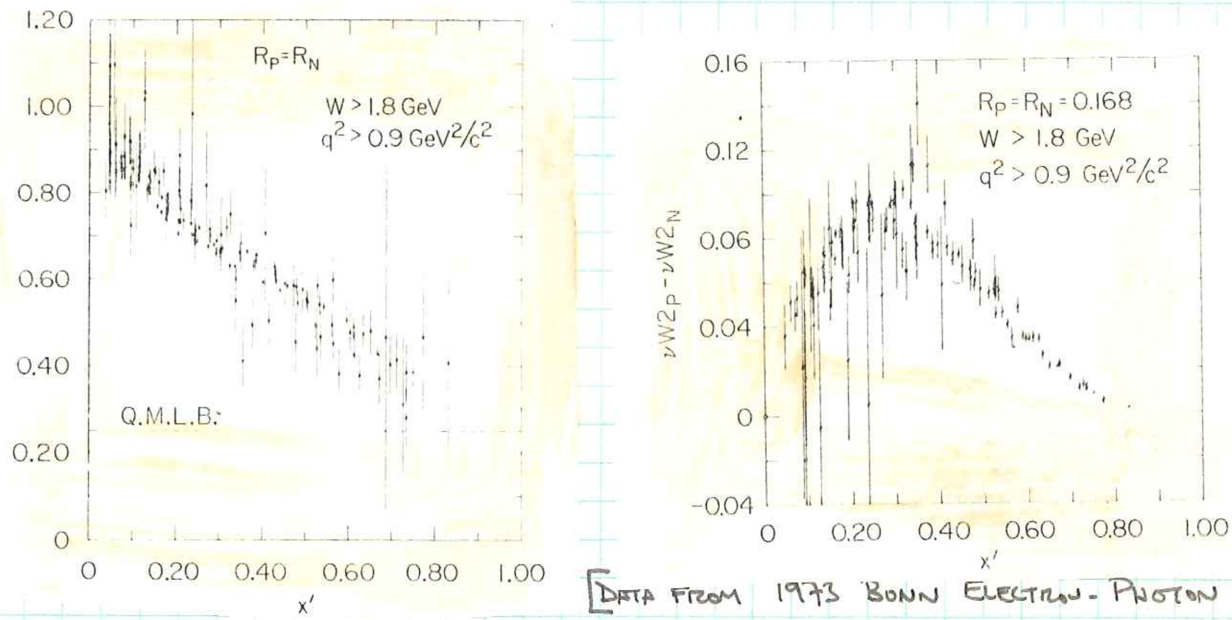
THESE COULD BE THE GLUONS WHICH BIND THE QUARKS TOGETHER.

Ph 529 Lecture 8

7. Neutron Structure Functions

Additional tests are obtained using the neutron structure function, which is extracted from inelastic e-deuteron scattering

$$\nu W_2^{n} = \nu W_2^{d} - \nu W_2^{P}$$



[Data from 1973 Bonn Electron-Photon Conf.]

In the model that $p = uud$ and $n = ddu$, we have

$$\nu W_2^{P} = x\left\{\frac{4}{9} f_u^{P} + \frac{1}{9} f_d^{P}\right\} \qquad \nu W_2^{n} = x\left\{\frac{1}{9} f_d^{n} + \frac{4}{9} f_u^{n}\right\}$$

By isospin symmetry, we expect $f_d^{n} = f_u^{P}$ and $f_u^{n} = f_d^{P}$

So $$\frac{\nu W_2^{n}}{\nu W_2^{P}} = \frac{4 f_d^{P} + f_u^{P}}{4 f_u^{P} + f_d^{P}} \Rightarrow \frac{1}{4} \le \frac{\nu W_2^{n}}{\nu W_2^{P}} \le 4$$

The data are certainly within these limits.

Note that $\left.\dfrac{\nu W_2^{n}}{\nu W_2^{P}}\right|_{x\sim 0} \to 1$ and $\left.\dfrac{\nu W_2^{n}}{\nu W_2^{P}}\right|_{x\sim 1} \to \frac{1}{4}$

A model to explain this is that in both the proton and neutron a u and d have the same distributions which are peaked at low X. They might be in a ${}^1S_0$ diquark state.

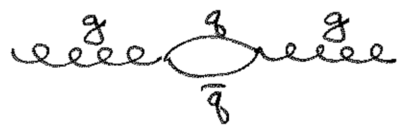
THE REMAINING u QUARK IN THE PROTON (OR d QUARK IN THE NEUTRON) HAS A DIFFERENT DISTRIBUTION WHICH IS MORE PROMINENT AS $X \to 1$

IN THIS PICTURE THE NET SPIN OF THE PROTON IS CARRIED BY THE 'LEADING' u QUARK.

8. VALENCE QUARKS AND SEA QUARKS

IT WAS ALREADY NOTED BY BJORKEN IN 1969 THAT THE SIMPLE QUARK MODEL THAT $p = uud$ SHOULD BE AUGMENTED BY A "SEA" OF QUARK ANTIQUARK PAIRS

$$p = uud + u\bar{u} + d\bar{d} + s\bar{s} + \cdots$$

THE SEA QUARK COULD COME FROM VIRTUAL PAIR PRODUCTION BY THE GLUONS: 

g = GLUON
q = QUARK

THE QUARKS COMPRISING THE SIMPLE QUARK MODEL OF A PARTICLE ARE CALLED THE VALENCE QUARKS.

BECAUSE OF THE SEA QUARKS, THE INTEGRAL $\int_0^1 f_i(x)\,dx$ = TOTAL PROBABILITY OF FINDING A QUARK, NEED NOT BE FINITE.

INDEED THE DATA ON p 142 SUGGESTS $\nu W_2(0) \approx$ CONSTANT

$\Rightarrow f(x) \sim \frac{1}{x}$ AS $X \to 0$.

HOWEVER, THE VALENCE QUARK DISTRIBUTIONS, f^v, MUST BE WELL BEHAVED. FOR THE PROTON

$$\int_0^1 f_u^{p,v}(x)\,dx = 2$$

$$\int_0^1 f_d^{p,v}\,dx = 1 \qquad \text{since } p = uud$$

A PLAUSIBLE POSSIBILITY IS THAT THE SEA QUARK DISTRIBUTION IS THE SAME FOR THE PROTON AND THE NEUTRON. THEN BY EXAMINING

$\nu W_2^{P} - \nu W_2^{n}$, ONLY VALENCE QUARK DISTRIBUTIONS REMAIN.

IN PARTICULAR, $\int_0^1 \frac{\nu W_2^{P} - \nu W_2^{n}}{x}\,dx = \int_0^1 \frac{4}{9} f_u^{p,v} + \frac{1}{9} f_d^{p,v} - \frac{1}{9} f_u^{p,v} - \frac{4}{9} f_d^{p,v}$

$$= \frac{1}{3}\int_0^1 f_u^{p,v} - f_d^{p,v} = 1/3$$

THE DATA SHOWN ON P 143 GIVE 0.28 FOR THE INTEGRAL.

ALTOGETHER THE INELASTIC SCATTERING RESULTS CONSTITUTE FAIRLY IMPRESSIVE EVIDENCE THAT THE PROTON AND NEUTRON ACTUALLY CONTAIN QUARKS.

9. The Structure Functions as $x \to 0$

WE HAVE ALREADY SEEN HOW $\nu W_2(0) \to$ CONSTANT IMPLIES

$$f^{SEA}(x) \sim \frac{1}{x} \quad \text{AS } x \to 0.$$

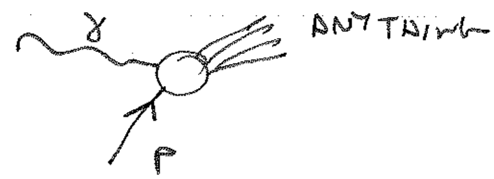
THIS CAN BE ROUGHLY EXPLAINED BY A MODEL OF GLUON BREMSSTRAHLUNG. QUARKS OF MODERATE x RADIATE GLUONS WITH A MOMENTUM SPECTRUM $\sim 1/x$ DUE TO THE STRONG QUARK-QUARK INTERACTIONS. THE GLUONS IN TURN YIELD QUARK-ANTIQUARK PAIRS BY VIRTUAL PAIR PRODUCTION, MAINTAINING THE $1/x$ SPECTRUM TO 1ST APPROXIMATION.

WE SKETCH AN ARGUMENT THAT THE VALENCE QUARK DISTRIBUTION GOES LIKE $x f^{VALENCE} \sim \sqrt{x}$ AS $x \to 0$

THE ARGUMENT IS BASED ON THE REGGE POLE PHENOMENOLOGY, WHICH IS OUTSIDE THE SCOPE OF THESE NOTES.

EMPIRICALLY, THE REGGE MODEL WORKS VERY WELL FOR STRONG INTERACTIONS AT HIGH ENERGY AND LOW MOMENTUM TRANSFER.

This is a region in which the partons don't appear to act independently $\Rightarrow$ complicated. In our case we apply the model to the virtual photo absorption

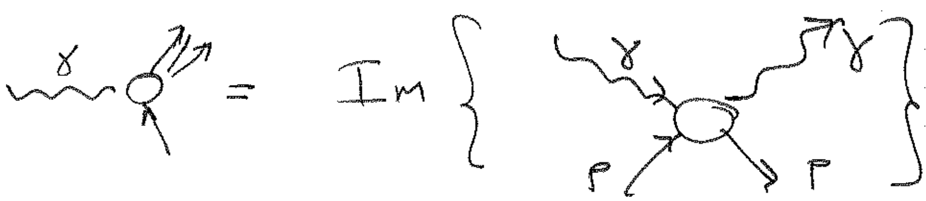


The energy variable is ν and q^2 = momentum transfer, of of course. Large ν, small $q^2 \Rightarrow x = \frac{-q^2}{2M\nu} \to 0$

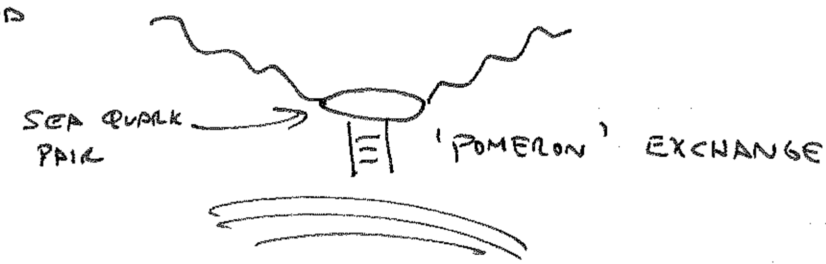
so Regge may help us in this limit. He tells us that the cross section will vary like $\nu^{\alpha_0 - 1} \sim x^{1-\alpha_0}$ where α_0 = intercept of the relevant Regge trajectory.

A picture argument taken from Altarelli et al Nuc Phys B69, 531 (1974).

The amplitude for the <u>inelastic</u> photo absorption is related to the imaginary part of the <u>elastic</u> scattering amplitude:

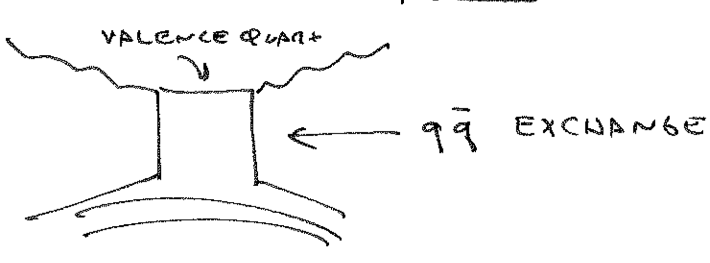


If the γ hits a sea quark pair, no quantum numbers need be exchanged



The so-called Pomeron exchanges $\Rightarrow \alpha_0 = 1 \Rightarrow \nu W_2 \to$ constant as $x \to 0$

But if the γ hits a valence quark, <u>meson</u> exchange results!



THE LEADING MESON TRAJECTORY IS THE ρ WITH $\alpha_0 = \frac{1}{2}$

SO $\nu W_2 \sim x^{1-\frac{1}{2}} = \sqrt{x}$ AS $x \to 0$

IN REASONABLE AGREEMENT WITH THE DATA. [$\nu W_2^p - \nu W_2^n$, p 142]

10. The Structure Functions as $x \to 1$

THE SO CALLED Drell-Yan-West RELATION STATES

THAT $\nu W_2 \longrightarrow (1-x)^{p-1}$ AS $x \to 1$ WHERE

$$F_{\text{elastic}}(q^2) \to \frac{1}{q^p} \quad \text{as } q \to \infty$$

THAT IS, WE HAVE A RELATION BETWEEN THE ELASTIC AND INELASTIC STRUCTURE FUNCTIONS IN LIMITING CASES.

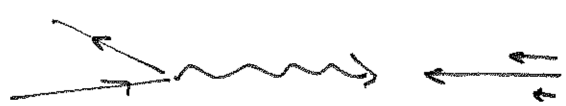
WE SAW THAT $F_p(q^2) \sim \frac{1}{q^4}$ (p 100)

SO THE SUGGESTION IS $\nu W_2^p \sim (1-x)^3$ AS $x \to 1$

THE DATA SHOWN ON p 142 ARE IN REASONABLE AGREEMENT WITH THIS CLAIM.

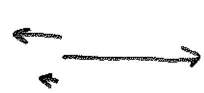
WE GIVE AN INCOMPLETE SKETCH ON AN ARGUMENT TAKEN FROM FEYNMAN (PHOTON-HADRON PHYSICS p 140)

CONSIDER ELECTRON PROTON SCATTERING IN THE BREIT FRAME



} INCIDENT PROTON CONSISTING OF SEVERAL PARTONS

THE PHOTON STRIKES A SINGLE PARTON, REVERSING ITS MOMENTUM:



} FINAL HADRON STATE.

PH 529 LECTURE 8

IF THE SCATTERING IS TO BE ELASTIC OVER THE WHOLE PROTON THE FINAL STATE CONFIGURATION MUST BE ESSENTIALLY THE SAME AS THE INITIAL, BUT REVERSED.

THIS IS UNLIKELY UNLESS ALL PARTON MOMENTA (EXCEPT THE STRUCK PARTON) ARE SMALL. ONLY SMALL X PARTONS ARE ESSENTIALLY THE SAME NO MATTER WHICH WAY THEIR MOMENTA POINT. GIVEN THE PHOTON MOMENTUM q, 'SMALL' IS OF THE ORDER $1/q$.

THE PARTON MODEL INTERPRETATION OF THE ELASTIC FORM FACTOR IS $F(q^2) \sim$ PROB THAT NO PARTON HAS $X \gtrsim \frac{1}{q}$

(EXCEPT THE STRUCK PARTON)

IF THE SEA DISTRIBUTION VARIES LIKE $\frac{C}{X}$ AS $X \to 0$ $(C = \text{CONSTANT})$

THEN THE AVERAGE NUMBER OF PARTONS WITH $X > 1/q$

IS $\bar{n} \sim \int_{1/q}^{1} \frac{c}{x}\,dx = -C \ln(1/q)$

THE PROBABILITY THAT NO SUCH PARTONS ARE OBSERVED GIVEN AVERAGE $\bar{n}$ IS $P(0) = \frac{\bar{n}^{\,0} e^{-\bar{n}}}{0!} = e^{-\bar{n}} = \frac{1}{q^C}$

IE $F_{\text{ELASTIC}}(q^2) \sim \frac{1}{q^C}$, AT LEAST FOR q LARGE.

TURNING TO INELASTIC SCATTERING, WE NOTE THAT AS $X \to 1$ $\nu W_2 \sim$ PROB THAT STRUCK PARTON HAS MOMENTUM X, AND THAT ALL OTHER PARTON MOMENTA SUM TO $1-X$.

FEYNMAN CLAIMS HE CAN SHOW THAT THE PROBABILITY THAT ALL PARTONS BUT ONE HAVE TOTAL MOMENTUM <u>LESS THAN</u> $1-X$ VARIES LIKE $(1-X)^C$ AS $X \to 1$, WHERE C = SAME CONSTANT AS BEFORE.

THEN $\int_x^1 f(x)\,dx \sim (1-x)^C \Rightarrow f(x) \sim (1-x)^{C-1}$

CAN YOU FILL IN THE DETAILS?

11. <u>FITS TO THE QUARK DISTRIBUTIONS</u>

AS THE INELASTIC $e-p$ DATA ACCUMULATED IT BECAME POPULAR TO TRY TO EXTRACT THE INDIVIDUAL QUARK DISTRIBUTIONS. THE FIRST ATTEMPT WAS KUTI & WEISSKOPF, P.R. <u>D4</u>, 3418 (1971). WE REFER TO ONE OF THE LAST FITS PRIOR TO THE INCLUSION OF QCD CORRECTIONS: FIELD AND FEYNMAN, P.R. <u>D15</u>, 2590 (1977).

THEY FIT
$$u(x) = f_u^P = \text{VALENCE} + \text{SEA} \sim \sqrt{x}\,(1-x)^3 + u_{SEA}$$
$$d(x) = f_d^P = \text{VALENCE} + \text{SEA} \sim \sqrt{x}\,(1-x)^4 + d_{SEA}$$
$$\bar{u}(x) = u_{SEA} \sim (1-x)^{10}$$
$$\bar{d}(x) = d_{SEA} \sim (1-x)^7$$
$$s(x) = \bar{s}(x) \sim (1-x)^8$$

SEE FIGURES ON THE NEXT PAGE.

IN THIS FIT, VALENCE QUARKS CARRY 27% OF THE PROTON MOMENTUM, SEA QUARKS CARRY 22% AND GLUONS CARRY 51%.

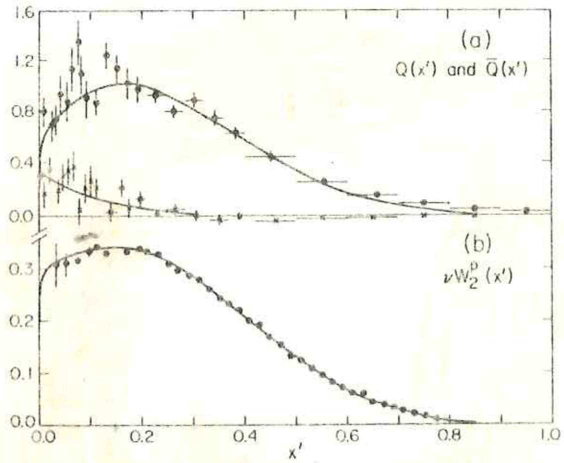


FIG. 3. Comparison of the quark distribution functions (shown in Fig. 4) with data that were used to help determine them. (a) Momentum carried by quarks $Q(x) = xu(x) + xd(x)$, and antiquarks $\bar{Q}(x) = x\bar{u}(x) + x\bar{d}(x)$. The data are from the Gargamelle neutrino collaboration (Ref. 9). (b) Fit to $\nu W_2^{ep}(x')$. The data are from Bodek *et al.* (see caption of Fig. 2).

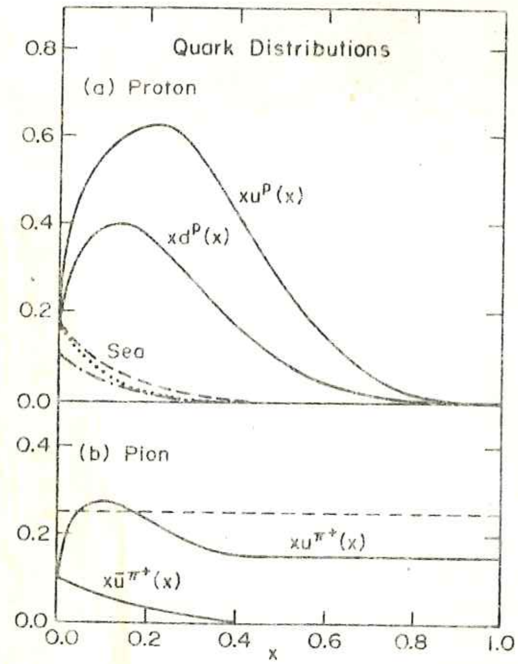


FIG. 4. (a) Quark distribution within the proton. We have assumed that as $x \to 1$ $xu(x) \sim (1-x)^3$ and $xd(x) \sim (1-x)^4$. In addition, we take an unsymmetrical sea with $x\bar{d}(x)$ (dashed curve) greater than $x\bar{u}(x)$ (dotted curve) and $xs(x) = x\bar{s}(x) = 0.1\ (1-x)^8$ (dash-dot curve). (b) Quark distributions within a pion. As $x \to 1$ we expect that $xu^{\pi^+}(x) \to$ constant. The dashed curve represents a first guess of $xu^{\pi^+}(x) = 0.25$ and $x\bar{u}^{\pi^+}(x) = 0$. This is refined by knowledge of large-$p_\perp$ hadron data (Fig. 16) yielding the solid curves.