PH 529 LECTURE 5

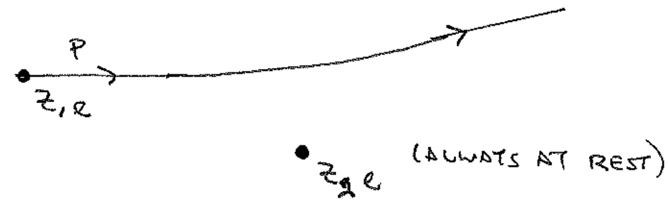
THE ELECTROMAGNETIC STRUCTURE OF MATTER

WE NOW BEGIN A SOMEWHAT MORE DETAILED LOOK INTO HIGH ENERGY PHYSICS. AS OUR FIRST TOPIC WE WILL FOLLOW THE EVOLUTION OF OUR UNDERSTANDING OF THE STRUCTURE OF MATTER AS REVEALED BY ELECTROMAGNETIC SCATTERING EXPERIMENTS

THIS PROGRAM OF STUDY WAS INITIATED BY RUTHERFORD, WHO FIRST SHOWED THAT ATOMS HAVE A VERY SMALL NUCLEUS, AND LATER (~1925) THAT THE NUCLEUS HAS SIZE $\sim A^{1/3}\cdot$ FERMIS.

HIS WORK WAS BASED ON COMPARISON OF α-ATOM AND α-NUCLEUS SCATTERING EXPERMENTS WITH THE CLASSICAL RUTHERFORD CROSS SECTION

$$\frac{d\sigma}{d\Omega} = \frac{z_1^2 z_2^2 e^4}{4 (pV)^2 \sin^4 \theta/2}$$



($\hbar \neq c \neq 1$)

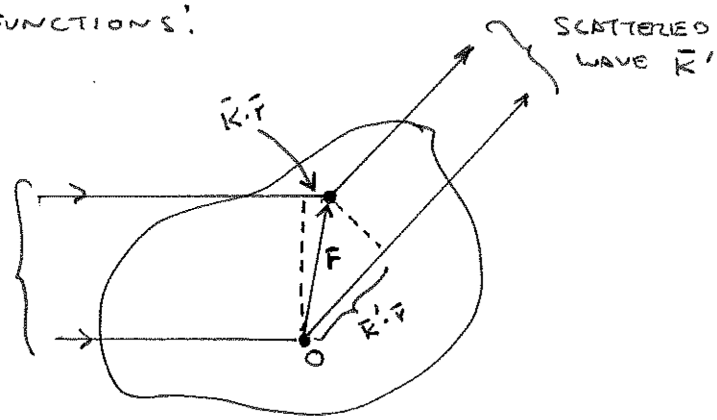
WE WILL FOLLOW SUBSEQUENT WORK TO EXPLORE THE CHARGE DISTRIBUTION OF THE NUCLEUS, THE PROTON, MESONS, AND EVENTUALLY TO INQUIRE AS TO THE MOTION OF QUARKS INSIDE THESE PARTICLES.

IN ADDITION TO THE INTRINSIC INTEREST OF THE SUBJECT, IT IS A GOOD STARTING POINT BECAUSE IT ALLOWS US TO INTRODUCE TECHNIQUES FOR ACTUAL CALCULATIONS OF CROSS SECTIONS, INCLUDING EFFECTS OF RELATIVITY AND SPIN (DIRAC EQUATION). THE LECTURE NOTES ARE SOMEWHAT DETAILED, PERHAPS MORE SO THAN IS JUSTIFIED FOR AN INTRODUCTORY COURSE. CHAPTER 7 OF PERKINS OFFERS A MORE RAPID OVERVIEW OF THE SAME MATERIAL, WHICH SHOULD ALLOW YOU TO KEEP YOUR BEARINGS.

X-RAY DIFFRACTION [LAUE (1912)]

THE STUDY OF X-RAY DIFFRACTION BY CRYSTALS MAY HAVE BEEN THE FIRST INVESTIGATION OF AN EXTENDED CHARGE DISTRIBUTION VIA ELECTROMAGNETIC RADIATION. THIS WORK INTRODUCED THE IDEA OF 'FORM FACTORS' OR 'STRUCTURE FUNCTIONS'.

THE INCIDENT PLANE WAVE HAS ELECTRIC FIELD $\bar{E} = \bar{E}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$



WE LOOK FOR A SCATTERED WAVE $\sim e^{i(\bar{k}'\cdot\bar{r} - \omega t)}$

THE SCATTERED WAVE IS THE SUM OF PIECES FROM ALL PARTS OF THE CRYSTAL

THE STRENGTH OF THE SCATTERED ELECTRIC FIELD CAUSED BY A VOLUME dV AT $\bar{r}$ IS PROPORTIONAL TO $\rho(\bar{r})\,dV$, AND HAS A PHASE CHANGE RELATIVE TO A SCATTER AT THE ORIGIN OF $\Delta\varphi = (\bar{k} - \bar{k}')\cdot\bar{r} \equiv \bar{q}\cdot\bar{r}$

$$\text{So } E_{SCAT} \sim \int \rho(r)\, e^{i\bar{q}\cdot\bar{r}}\, dVol \equiv F(q) = \text{FORM FACTOR}$$

THE OBSERVED INTENSITY GOES LIKE $I \sim |F(q)|^2$

THE CHARGE DISTRIBUTION IS RECONSTRUCTED BY A FOURIER TRANSFORM OF THE OBSERVED DATA.

AGAIN WE SEE THE NEED FOR HIGH ENERGIES. TO PROBE A STRUCTURE OF SIZE Δr WILL REQUIRE A SPREAD OF FREQUENCIES SUCH THAT $\Delta q > \frac{1}{\Delta r}$. IN PARTICULAR, $K_{MAX} > \frac{1}{\Delta r}$. ACCORDING TO EINSTEIN, $E = \hbar c K$, SO $E_{MAX} > \frac{\hbar c}{\Delta r}$ OR $E(MeV) > \frac{200}{\Delta r \text{ (FERMIS)}}$

PH 529 LECTURE 5

NON-RELATIVISTIC QUANTUM MECHANICS (WITHOUT SPIN)

THE WAVE-PARTICLE DUALITY OF QUANTUM MECHANICS COMBINES KEY FEATURES OF RUTHERFORD'S PARTICLE SCATTERING AND LAUE'S WAVE SCATTERING. (MOST OF THESE NOTES ARE TAKEN FROM LANDAU & LIFSHITZ, Q.M. §139. SEE ALSO, FLÜGGE, PRACTICAL Q.M. §108, AND MOTT & MASSEY, THEORY OF ATOMIC COLLISIONS V.1 ETC)

1. THE BORN APPROXIMATION

WE CONSIDER A FIXED (= HEAVY) SCATTERING CENTER WHICH EXERTS A FORCE DERIVABLE FROM A POTENTIAL $U(\bar{r})$. THE BEAM PARTICLE IS SPINLESS AND HAS VELOCITY v_0. WE CONSIDER ONLY <u>ELASTIC COLLISIONS</u>, SO THE FINAL VELOCITY IS v_0 ALSO.

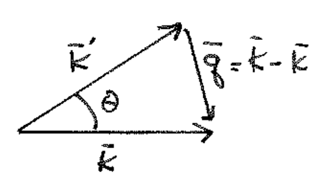
WE LOOK FOR A WAVE FUNCTION OF THE BEAM PARTICLE LIKE

$$\psi = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

INCIDENT PLANE WAVE SCATTERED SPHERICAL WAVE.

THE FIRST BORN APPROXIMATION IS THAT

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int U(\bar{r})\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r}$$



THE FOURIER TRANSFORM OF THE <u>POTENTIAL</u>

AGAIN $\bar{q} = \bar{k} - \bar{k}'$ SO $q = 2k\sin\frac{\theta}{2} = \frac{2mv_0}{\hbar}\sin\theta/2$

$\hbar q$ IS THE <u>MOMENTUM TRANSFER</u>.

WE DEFINE THE DIFFERENTIAL CROSS SECTION IN THE MANNER APPROPRIATE FOR <u>WAVES</u>:

$$d\sigma = \frac{\text{FLUX INTO SOLID ANGLE } d\Omega}{\text{INCIDENT FLUX}}$$

NOW, INCIDENT FLUX $= v \left| e^{ikz} \right|^2 = v$

$$\text{FLUX INTO } d\Omega = v\, r^2 d\Omega \left| \frac{f(\theta)\, e^{ikr}}{r} \right|^2 = v\, d\Omega \left| f(\theta) \right|^2$$

SO $$\frac{d\sigma}{d\Omega} = \left| f(\theta) \right|^2$$

(THIS IS BASED ON A NORMALIZATION OF 1 INCIDENT PARTICLE PER UNIT VOLUME.)

2. <u>COULOMB SCATTERING OFF A POINT CHARGE</u>

$$U(\bar{r}) = \frac{\alpha}{r}$$

THE FOURIER INTEGRAL IS NOT WELL BEHAVED. THE USUAL TRICK (WENTZEL, 1927) IS TO USE $U(\bar{r}) = \frac{\alpha}{r} e^{-\mu r}$ AND LET $\mu \to 0$

FIRST, WE NOTE THAT FOR SPHERICALLY SYMMETRIC POTENTIALS $U(\bar{r}) = U(r)$,

$$\int = 2\pi \int_0^\infty r^2 dr\, U(r) \int_{-1}^{1} e^{iqrx}\, dx = \frac{4\pi}{q} \int r\, U(r) \sin qr\, dr$$

SO $$f(\theta) = -\frac{2m\alpha}{\hbar^2 q} \int_0^\infty e^{-\mu r} \sin qr\, dr = -\frac{2m\alpha}{\hbar^2} \frac{1}{\mu^2 + q^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2\alpha^2}{\hbar^4} \frac{1}{(\mu^2+q^2)^2}$$

LETTING $\mu \to 0$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2\alpha^2}{(\hbar q)^4} = \frac{\alpha^2}{4m^2 v_0^4 \sin^4 \theta/2}$$

<u>RUTHERFORD !</u>

3. COULOMB SCATTERING OFF AN EXTENDED CHARGE DISTRIBUTION

THE CHARGE DISTRIBUTION $\rho(\bar{r})$ SETS UP THE ELECTRICAL POTENTIAL $\phi(\bar{r})$ WHICH OBEYS $\nabla^2\phi = \frac{-\rho}{\epsilon_0}$ (M.K.S.)

THE POTENTIAL <u>ENERGY</u> $U(\bar{r}) = eZ_1\phi(\bar{r})$ WHERE eZ_1 IS THE CHARGE OF THE INCIDENT PARTICLE.

RATHER THAN WORKING OUT $\phi(\bar{r})$ AND PLUGGING INTO THE BORN INTEGRAL, SOMEONE NOTICED A CLEVER TRANSFORMATION. ONE DERIVATION IS:

WE WANT $\int U(\bar{r})\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r} = eZ_1 \int \phi(\bar{r})\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r}$

DEFINE $\phi_q = \int \phi\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r}$. SIMILARLY $\rho_q = \int \rho\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r}$

THE INVERSE TRANSFORMS ARE $\phi = \frac{1}{(2\pi)^3}\int \phi_q\, e^{-i\bar{q}\cdot\bar{r}}\, d^3\bar{q}$; $\rho = \frac{1}{(2\pi)^3}\int \rho_q\, e^{-i\bar{q}\cdot\bar{r}}\, d^3\bar{q}$

BUT $\nabla^2\phi = \frac{-\rho}{\epsilon_0}$ SO $-q^2\phi_q = \frac{-\rho_q}{\epsilon_0}$ OR $\phi_q = \frac{\rho_q}{\epsilon_0 q^2}$

HENCE OUR DESIRED INTEGRAL IS $\frac{eZ_1}{\epsilon_0 q^2}\int \rho(\bar{r})\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r}$

AGAIN, WE DEFINE $\boxed{F(q) = F(\theta) = \int \rho(\bar{r})\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r} = \text{FORM FACTOR}}$

THE FOURIER TRANSFORM OF THE CHARGE DISTRIBUTION.

NOTE THAT $F(0) = \int \rho(\bar{r})\, d^3\bar{r} = \text{TOTAL CHARGE} = Q_2 = eZ_2$

THUS $f(\theta) = \frac{-m}{2\pi\epsilon_0\hbar^2}\,\frac{Z_1Z_2e^2}{q^2}\,\frac{F(\theta)}{F(0)}$

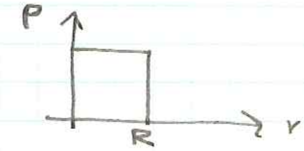
AND
$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{z_1^2 z_2^2 \alpha^2}{4m^2 v_0^2 \sin^4 \theta/2}}_{\text{RUTHERFORD}} \left|\frac{F(\theta)}{F(0)}\right|^2$$

THUS THE SQUARE OF THE FORM FACTOR MEASURES THE DEVIATION FROM RUTHERFORD SCATTERING.

$$F(\theta) = \text{CONSTANT} \iff \rho(r) = Q\delta(\bar{r})$$

4. SCATTERING OFF A UNIFORM SPHERE OF CHARGE

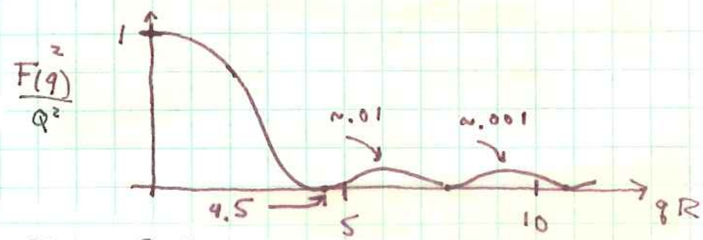
$$\rho(\bar{r}) = \rho_0 \quad \text{FOR} \quad r \le R$$



THIS PROBLEM IS NOT COMMONLY TREATED IN CLASSICAL MECHANICS. IT REPRESENTS THOMSON'S MODEL OF THE "NUCLEUS" IF WE TAKE $R \sim 1\,\text{Å}$

$$F(q) = \int \rho(\bar{r}) e^{i\bar{q}\cdot\bar{r}} d^3\bar{r} = \frac{4\pi\rho_0}{q}\int_0^R r \sin qr\, dr = \frac{4\pi\rho_0}{q^3}\left(\sin qR - qR\cos qR\right)$$

THIS OSCILLATES SO $|F(q)|^2$ GIVES SOMETHING LIKE A DIFFRACTION PATTERN



TO GET A SENSE, EXPAND F IN POWERS OF q

$$F(q) \approx \frac{4\pi\rho_0}{q^3}\left(\left(qR - \frac{(qR)^3}{6} + \frac{(qR)^5}{120} - \dots\right) - qR\left(1 - \frac{(qR)^2}{2} + \frac{(qR)^4}{24} - \dots\right)\right)$$

$$= \frac{4}{3}\pi R^3 \rho_0 \frac{3}{(qR)^3}\left(\frac{(qR)^3}{3} - \frac{(qR)^5}{30} + \dots\right)$$

$$= Q\left(1 - \frac{(qR)^2}{10} + \dots\right)$$

$F(0) = Q$ AS REQUIRED

THE 1ST ZERO OF THE DIFFRACTION PATTERN IS AT $q \sim \sqrt{10}/R$

AS CLAIMED EARLIER, TO OBSERVE THE SCATTERING OFF A STRUCTURE OF SIZE R, WE NEED $q_{MAX} \sim \frac{1}{R}$ TO RESOLVE THE CHARACTERISTIC FEATURE OF THE CROSS SECTION.

WE ALSO NOTE THAT IF $q \gg \frac{1}{R}$, WE ENTER A REGION WHERE $|F(q)|^2 \sim \frac{1}{(qR)^4}$, AND IN EFFECT $\frac{d\sigma}{d\Omega} \approx 0$. THAT IS, WE GET NO LARGE ANGLE SCATTERINGS OF HIGH ENERGY PARTICLES OFF A BROAD DISTRIBUTION.

THIS ALLOWED RUTHERFORD TO ELIMINATE THOMSON'S MODEL (THO WE USED CLASSICAL ARGUMENTS, OF COURSE). LATER WHEN RUTHERFORD DESIRED TO MEASURE THE SIZE OF THE NUCLEUS, HE WOULD LOOK FOR A FORM FACTOR WITH STRUCTURE AS DISCUSSED HERE.

5. EXPANSION OF THE FORM FACTOR

IN THE PRECEDING EXAMPLE WE FOUND IT USEFUL TO MAKE A POWER SERIES EXPANSION OF $F(q)$. IN GENERAL, IF WE ASSUME SPHERICAL SYMMETRY FOR THE CHARGE DISTRIBUTION, WE HAVE

$$F(q) = \int \rho(r)\, e^{i\bar{q}\cdot\bar{r}}\, d^3\bar{r} = 2\pi \int \rho r^2 dr \int_{-1}^{1} e^{iqr\cos\theta}\, d\cos\theta = \frac{4\pi}{q}\int \rho r \sin qr\, dr$$

$$\approx \frac{4\pi}{q}\int \rho r \left(qr - \frac{1}{6}(qr)^3 + \dots\right) dr$$

$$= \int \rho\, d^3\bar{r} - \frac{q^2}{6}\int \rho r^2 d^3\bar{r} + \dots$$

OF COURSE $Q = \int \rho\, d^3\bar{r}$ = TOTAL CHARGE

WE DEFINE $\langle r^2 \rangle = \frac{1}{Q}\int r^2 \rho(r)\, d^3\bar{r}$ ≡ SQUARE OF THE CHARGE RADIUS

THEN $F(q) = Q\left(1 - \frac{q^2\langle r^2\rangle}{6} + \dots\right)$

NOTE THE $\langle r^2 \rangle$ CAN BE NEGATIVE ! THIS OCCURS IF THE NEGATIVE CHARGE IS AT A LARGER RADIUS THAN THE POSITIVE, AS IN AN ATOM.

6. SCATTERING OFF ATOMS

$$\rho(\bar{r}) = \underbrace{Z_2 e\, \delta(\bar{r})}_{\text{NUCLEAR CHARGE}} - \underbrace{\rho_-(r)}_{\text{ELECTRON CHARGE DENSITY}} \quad \text{WHERE} \quad \int \rho_-(r)\, d^3r \equiv Z_2 e$$

WE CONSIDER BEAMS OF LOW ENOUGH ENERGY THAT THE NUCLEUS IS STILL POINT-LIKE.

$$F_{ATOM} = Z_2 e - F_-(q^2) \approx Z_2 e - (Z_2 e)\left(1 - \frac{q^2 \langle r^2 \rangle}{6}\right)$$

$$= Z_2 e \frac{q^2 \langle r^2 \rangle}{6}$$

NOTE THAT THE q^2 CANCELS IN THE EXPRESSION FOR $f(\theta)$;

$$f(\theta) = -\frac{m}{2\pi\epsilon_0 \hbar^2} \frac{Z_1 e}{q^2} F(\theta) = \frac{-m\alpha}{3\hbar^2} \langle r^2 \rangle \qquad \left[\alpha = \frac{e^2}{4\pi\epsilon_0}\right]$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{m^2 \alpha^2}{9\hbar^4} \langle r^2 \rangle^2 = \underline{\underline{\text{CONSTANT}}}$$

$$\sigma_{TOT} = \frac{4\pi m^2 \alpha^2}{9 \hbar^4} \langle r^2 \rangle^2$$

THIS IS ANALAGOUS TO CLASSICAL BILLIARD BALL SCATTERING :

$\frac{d\sigma}{d\Omega}$ ISOTROPIC ; σ_{TOT} FINITE.

IF WE WRITE $\sigma_{TOT} = \pi R_{eff}^2$

THEN $R_{eff} = \frac{2}{3} \frac{m\alpha}{\hbar^2} \langle r^2 \rangle$

IF THE INCIDENT PARTICLE IS AN ELECTRON, $\frac{m\alpha}{\hbar^2} = \frac{Z_2}{a_0}$

a_0 = FIRST BOHR RADIUS $= 4\pi\epsilon_0 \hbar^2 / me^2$

$$R_{eff} = \frac{2 Z_2}{3} \frac{\langle r^2 \rangle_{ATOM}}{a_0}$$

WHICH IS ABOUT THE SAME SIZE AS THE ATOM ITSELF.

THAT IS, IF THE BEAM MISSES THE ATOM, THERE IS NO SCATTERING — THE ATOM APPEARS NEUTRAL; BUT IF THE BEAM PENETRATES THE ATOM, SCATTERING CAN OCCUR.

IF THE BEAM ENERGY IS VERY LARGE $\left(k \gg \frac{1}{\sqrt{\langle r^2 \rangle}} \right)$

THEN THE SCATTERING OFF THE ELECTRON CLOUD LEADS TO A RAPIDLY OSCILLATING FORM FACTOR, F_-, WHICH CAN EFFECTIVELY BE IGNORED. THEN ALL THAT IS LEFT IS RUTHERFORD SCATTERING OFF THE NUCLEUS!

7. <u>SCATTERING OF ELECTRONS OFF SPIN-0 NUCLEI</u>

THE CLASSICAL RELATION OF $r_{MIN} = \frac{\alpha}{E}$ IN RUTHERFORD SCATTERING MEANS THAT BEAMS OF MANY MeV ARE NEEDED TO PROBE THE CHARGE DISTRIBUTION OF THE NUCLEUS. THESE DID NOT BECOME AVAILABLE UNTIL AFTER WORLD WAR II. MEANWHILE IT WAS LEARNED THAT BEAMS OF ν'S OR PROTONS GAVE GIANT CROSS-SECTIONS COMPARED TO THE EXPECTATIONS OF RUTHERFORD — THE <u>STRONG INTERACTION</u> WAS DISCOVERED. SO IT APPEARED THAT ONLY BEAMS

OF ELECTRONS COULD BE USED TO YIELD INFORMATION ON THE CHARGE DISTRIBUTION OF NUCLEI. THE EXPERT IN THIS FIELD IS R. HOFSTADTER. SEE ANN REV NUC SCI 7, 231 (1957).

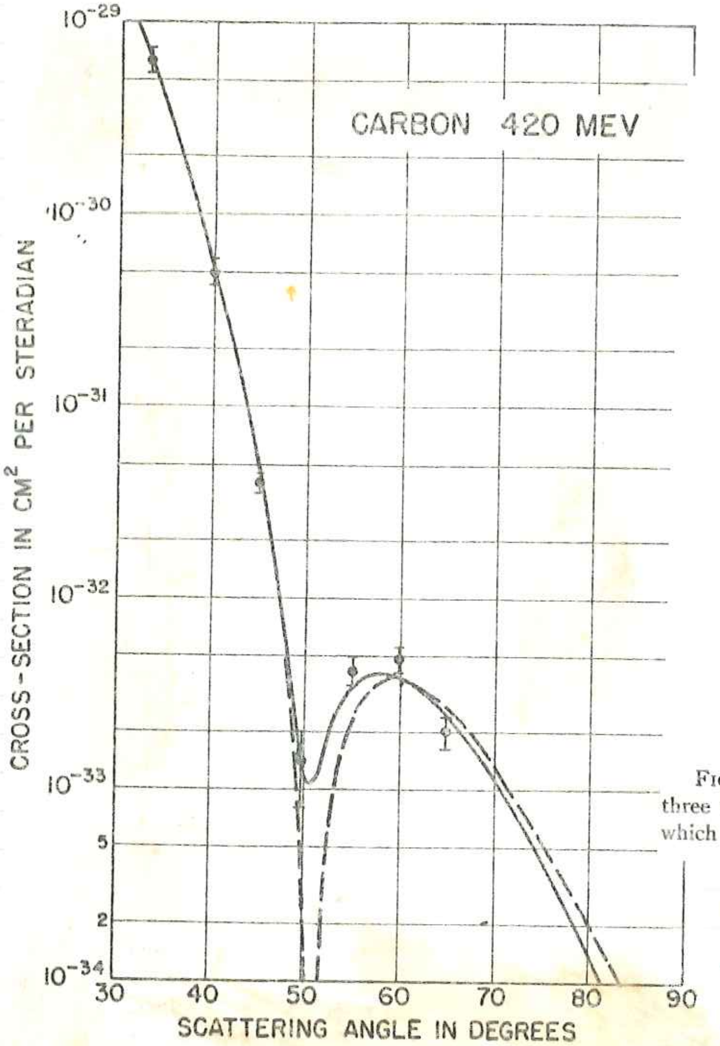


FIG. 4. Recent data in C[12] observed by Sobottka & Hofstadter (10) at an incident electron energy of 420 Mev. Two theoretical curves are presented for comparison. The dashed curve is the Born approximation for a harmonic-well charge distribution corresponding to Fig. 3 ($\alpha=4/3$). The solid line is the accurate phase-shift calculation of D. G. Ravenhall, which appears to fit the experimental points rather well.

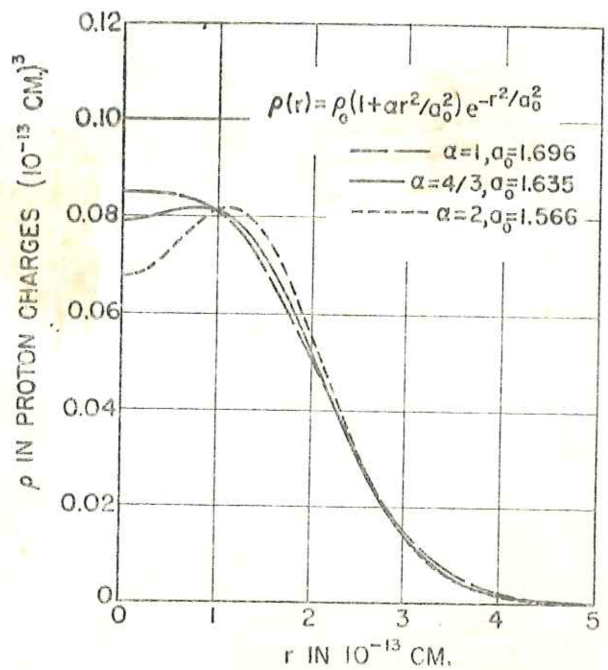


FIG. 3. The charge-density distribution for the harmonic-well nuclear model for three different values of α. The abscissa is correct only for the case of carbon, for which $\alpha=4/3$.

THE DIP IN $\rho(r)$ NEAR $r=0$ IS PRESUMABLY THE EFFECT OF THE P-WAVE PROTONS.

IN THE 1950'S YUKAWA'S MESON THEORY WAS QUITE POPULAR. THEN K-P, OR π-P SCATTERING MIGHT BE EXPECTED TO TELL US ABOUT THE MESON DISTRIBUTION OF THE PROTON. BUT THE MESON THEORY WAS NOT 'CRISP' ENOUGH TO ALLOW CLEAR-CUT INTERPRETATIONS, AND LITTLE DETAILED UNDERSTANDING CAME FROM THE PURELY STRONG-INTERACTION EXPERIMENTS. RATHER, IT WAS THE WELL-UNDERSTOOD ELECTROMAGNETIC INTERACTION WHICH PROVED TO BE THE MOST EFFECTIVE PROBE OF HADRON STRUCTURE.

8. THE FORM FACTOR OF THE NEUTRAL PION – π^0

WE CAN BRING OUR STORY OF ELECTROMAGNETIC STRUCTURE OF SPINLESS MATTER UP TO THE PRESENT BY CONSIDERING THE SPIN-ZERO MESONS.

THE CASE OF THE π^0 IS EASY. SINCE THE π^0 IS ITS OWN ANTI PARTICLE, THE CHARGE DISTRIBUTION MUST BE IDENTICALLY ZERO

HENCE $F_{\pi^0}(q^2) = 0$

THIS IS ALSO TRUE FOR THE η^0 MESON.

IN THE QUARK MODEL, $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

SO WE HAVE A PICTURE OF NON VANISHING DISTRIBUTIONS ρ^+ AND ρ^-, BUT $\rho_{TOT} = \rho^+ - \rho^- = 0$.

9. FORM FACTOR OF THE CHARGED PIONS – $\pi^\pm$

SINCE THE π^- IS THE ANTIPARTICLE OF THE π^+, NOTHING PREVENTS THEM FROM HAVING A NON-TRIVIAL CHARGE DISTRIBUTION. THE PROBLEM IS HOW TO MEASURE IT, SINCE PIONS WON'T SIT STILL TO BE SHOT AT.

ONE METHOD IS TO SHOOT THE PIONS AT THE ELECTRONS! IF WE SHOOT VERY HIGH ENERGY PIONS AT AN ATOM, THE PION IS MOST LIKELY TO HIT THE NUCLEUS, CAUSING A MESS (INELASTIC COLLISION). BUT IF A RECOIL ELECTRON IS DETECTED, THE REACTION MAY BE $\pi + e$ ELASTIC SCATTERING. THE SCATTERING ANGLES ARE QUITE SMALL, AND ONLY A LIMITED RANGE OF q^2 CAN BE EXPLORED WITH THIS TECHNIQUE. THE MAIN RESULT WILL BE THE CHARGE RADIUS OF THE PION, USING

$$F(q^2) = e\left(1 - \frac{q^2}{6}\langle r^2\rangle + \dots\right)$$

THE FIGURE IS FROM

G. T. ADYLOV ET AL,

PHYS LETT <u>51B</u>, 402 (1974)

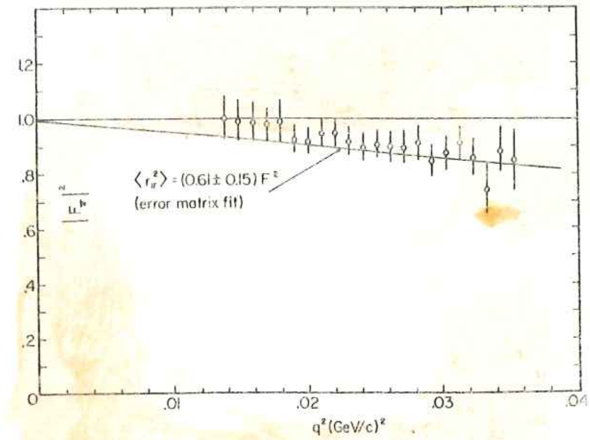
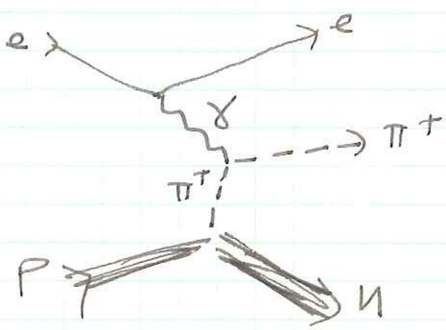


Fig. 1. $|F_\pi|^2$ versus momentum transfer squared. The errors shown are the diagonal of the error matrix. Solid curves are shown for a point pion, and for the best fit to the data.

ANOTHER METHOD INVOLVES THE INELASTIC SCATTERING OF ELECTRONS OFF PROTONS IN THE REACTION $ep \to en\pi^+$

ONE CAN IMAGINE THAT INSIDE THIS REACTION IS THE <u>ELASTIC</u> SUBREACTION $e\pi^+ \to e\pi^+$



NEEDLESS TO SAY, THE ANALYSIS OF THIS EXPERIMENT IS SOMEWHAT COMPLICATED. THE DATA (NEXT PAGE) SHOW THAT $F_{\pi^+}(q^2) \sim \dfrac{1}{q^2}$ AT LARGE q^2. THERE IS NO SIGN OF A DIFFRACTION DIP. BUT THEN, THE QUARK MODEL SAYS THAT THE $u\bar{d}$ IS AN S-WAVE STATE.

DATA FROM C.J. BEBEK ET AL →

P.R. D17, 1693 (1978).

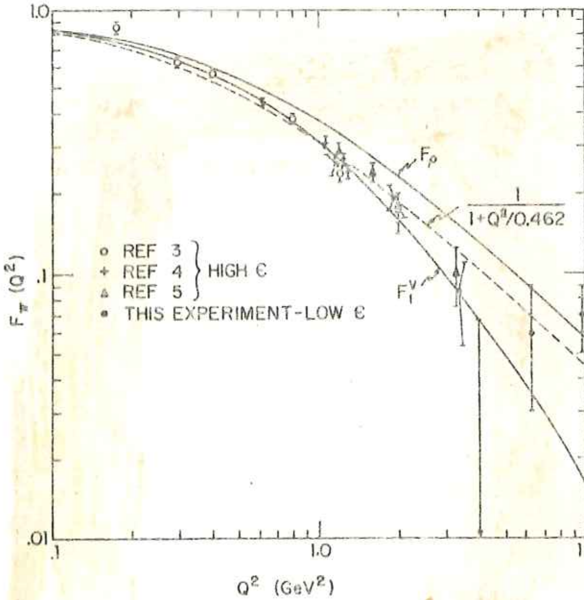


FIG. 5. The values of the pion form factor derived from the data reported in this and earlier experiments using the measured transverse cross section and the t-channel Born term. Also shown are the ρ pole form factor and the isovector nucleon form factor (solid lines), as well as the result of a simple pole fit (broken line).

THE FIT $F_\pi \approx \dfrac{1}{(.68)^2 + Q^2}$

LEADS TO A CHARGE DISTRIBUTION (see p. 62)

$$\rho(r) \sim \frac{e^{-.68r}}{r} \quad \text{OR} \quad \frac{e^{-\frac{r}{0.29}}}{r}$$

$\hbar = c = 1$

WITH r IN FERMIS

THIS DOES NOT APPEAR TO BE A TYPICAL S-WAVE CHARGE DISTRIBUTION.

→ I AM UNCERTAIN AS TO THE INTERPRETATION OF THE PION FORM FACTOR IN THE QUARK MODEL CONTEXT. I WOULD BE GLAD TO BE ENLIGHTED.

NOTE: IF

$\rho(r) = \dfrac{e^{-\frac{r}{r_0}}}{r}$

THEN $\langle r^2\rangle^{\frac{1}{2}} = \sqrt{6}\, r_0$

SO $r_0 = .29$ FERMI

$\Rightarrow \langle r^2\rangle^{\frac{1}{2}} = .71 f$

AS IN THE LAST LINE OF THE TABLE

TABLE VIII. A summary of the more recent determinations of the pion electromagnetic radius by the several methods that have been used for this purpose.

Group	Method	$\langle r_\pi^2\rangle^{1/2}$ (F)
Berezhnev *et al.* (1973) Ref. 16	Inverse electroproduction $n+p\to e^+e^-n$	0.73 ±0.13
Adylov *et al.* (1974) Ref. 17	Pion-electron scattering $E=50$ GeV	0.78 ±0.10
Dubnička and Dunbrajs (1974) Ref. 18	Alternate analysis of data from Adylov *et al.* (1974)	0.71 ±0.05
Bebek *et al.* (1976) Ref. 5	Electroproduction above resonance region up to $Q^2=4$ GeV^2	0.704 ±0.025
Bardin *et al.* (1977) Ref. 19	Electroproduction in resonance region up to $Q^2=0.12$ GeV^2	$0.74^{+0.11}_{-0.13}$
Quenzer *et al.* (1977) Ref. 20	Colliding-beam measurement of $e^+e^-\to\pi^+\pi^-$	0.676 ±0.008
Dally *et al.* (1977) Ref. 21	Pion-electron scattering $E=100$ GeV	0.56 ±0.04
This experiment BEBEK (1978)	Electroproduction above resonance region up to $Q^2=10$ GeV^2	0.711 ±0.018

10. CHARGE RADIUS OF THE NEUTRAL KAON – K^0

THE K^0 AND ITS ANTIPARTICLE, $\bar{K}^0$, ARE DISTINCT. THUS IT IS POSSIBLE FOR THE K^0 TO HAVE A NON-ZERO CHARGE RADIUS, $\sqrt{\langle r^2 \rangle}$. THE K^0 IS $d\bar{s}$ IN THE QUARK MODEL. CRUDELY SPEAKING, THE d QUARK WEIGHS 300 MeV, WHILE THE s QUARK WEIGHS 500 MeV. HENCE THE C.M. IS CLOSER TO THE $\bar{s}$ THAN THE d QUARK, BY $1/8$ OF THE SEPARATION BETWEEN THE d & THE $\bar{s}$.

WE CAN ESTIMATE THIS SEPARATION AS TWICE THE CHARGE RADIUS OF THE π^+ – 1.4 FERMIS. HENCE A SIMPLE ESTIMATE IS $\langle r^2 \rangle_{K^0} \approx (.18 \text{ FERMIS})^2$

THE SIGN SHOULD BE NEGATIVE SINCE THE d QUARK $(Q = -\frac{1}{3}e)$ IS FARTHER OUT.

AMAZINGLY, $\langle r^2 \rangle_{K^0}$ HAS BEEN MEASURED IN AN EXCEEDINGLY INTRICATE EXPERIMENT BASED ON THE REACTION

$$K_L^0 + e \to K_S^0 + e$$

SEE W.R. MOLZON ET AL PHYS. REV LETT 41, 1213 (1978).

THEY FIND $\langle r^2 \rangle = -.054 \pm .026$

SO $\sqrt{\langle r^2 \rangle} = .23 \begin{smallmatrix} +.05 \\ -.06 \end{smallmatrix}$ IN REASONABLE AGREEMENT WITH OUR PHENOMENOLOGICAL ESTIMATE.

11. CHARGE RADIUS OF THE K^-

A RECENT EXPERIMENT ON THE REACTION $K^- + e \to K^- + e$, I.E. SCATTERING OF K^- OFF ATOMIC ELECTRONS, CLAIMS

$\sqrt{\langle r^2 \rangle} = .53 \pm .05$. E.B. DALLY ET AL. P.R.L. 45, 232 (1980)

THE SAME GROUP GOT A PION CHARGE RADIUS OF $.56 \pm .04$ IN A SIMILAR EXPERIMENT.

ELASTIC SCATTERING OF 2 SPIN-LESS PARTICLES, INCLUDING RELATIVITY

$$a + b \rightarrow a' + b'$$

THIS TOPIC IS SOMEWHAT ACADEMIC, SINCE THE INTERESTING SPIN-0 PARTICLES (π, K, η MESONS) ALL INTERACT STRONGLY AND DECAY QUICKLY. SO ELECTROMAGNETIC SCATTERING IS ONLY A SMALL CORRECTION. BUT WE CAN USE THIS CASE TO INTRODUCE THE EFFECTS OF RELATIVITY WITHOUT THE COMPLICATION OF SPIN.

TYPICAL 'THEORY' REFERENCES: BJORKEN & DRELL, LANDAU & LIFSHITZ, JEREMY BERNSTEIN

TYPICAL 'PHENOMENOLOGY' BOOKS: W.S.C. WILLIAMS, CHENG & O'NEILL, PERKINS

1. INTRODUCING RELATIVITY

INSTEAD OF SCALARS, WE WANT TO DEAL WITH LORENTZ INVARIANTS. INSTEAD OF VECTORS, WE NOW USE 4-VECTORS

NOTATION $\quad A = A_\mu = (A_0, \bar{A}) \qquad A^2 = A_0^2 - \bar{A}^2$; METRIC $= \begin{pmatrix} 1 & & & \\ & -1 & & 0 \\ & 0 & -1 & \\ & & & -1 \end{pmatrix}$

SUMMATION CONVENTION: $ab = a_\mu b^\mu = a_0 b_0 - \bar{a}\cdot\bar{b}$

WE SET $\hbar = c = 1$, AND NOTE THAT $\hbar c = 197$ MeV FERMI TO CONVERT BACK TO PRACTICAL UNITS.

PREVIOUSLY WE CONSIDERED SCATTERING OF PLANE WAVES, $\psi = e^{i\bar{k}\cdot\bar{r}}$

THE APPROPRIATE GENERALIZATION IS $\psi = e^{-ipx} = e^{-iEt}\, e^{i\bar{p}\cdot\bar{r}}$

(SINCE $\hbar = 1$, $\bar{p} = \bar{k}$) $\qquad (px = p_\mu x^\mu)$

RECALL THAT IN CALCULATING CROSS SECTIONS, WE USED A NORMALIZATION OF ONE PARTICLE PER UNIT VOLUME. WE ANTICIPATE A PROBLEM SINCE VOLUME IS NOT A LORENTZ INVARIANT, VOLUME SUFFERS A LORENTZ CONTRACTION BY $\frac{1}{\gamma}$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ IN SWITCHING FRAMES.

BUT ENERGY IS BOOSTED BY THE SAME FACTOR γ.

HENCE WE EXPECT WE MUST NORMALIZE $\psi \sim \frac{1}{\sqrt{E}} e^{-iPX}$

SO $\psi^*\psi$ BEHAVES LIKE A RELATIVISTIC SCALAR DENSITY.

WE CAN PURSUE THIS THOUGHT FURTHER BY EXAMINING THE RELATIVISTIC WAVE EQUATION (KLEIN, GORDON 1926)

$$(\Box^2 + m^2)\psi = 0 \qquad \text{IN FREE SPACE}$$

$$\left[\Box^2 = \partial_\mu \partial^\mu \quad , \partial_\mu = \left(\frac{\partial}{\partial t}, -\bar{\nabla}\right)\right]$$

IE $\quad E^2 - \bar{p}^2 + m^2 = 0 \quad$ FOR A PARTICLE OF MASS M.

WE LOOK FOR A RELATIVISTIC PROBABILITY 4-VECTOR $j_\mu = (\rho, \bar{j})$ WHICH OBEYS THE CONSERVATION LAW $\partial_\mu j^\mu = 0 = \frac{\partial \rho}{\partial t} + \bar{\nabla}\cdot\bar{j}$

SIMILAR TO THE NON-RELATIVISTIC ARGUMENT:

$$\psi^*(\Box^2 + m)\psi - \psi(\Box^2 + m^2)\psi^* = 0$$

$$\psi^*\partial_\mu\partial^\mu\psi - \psi\partial_\mu\partial^\mu\psi^* = 0$$

$$\partial^\mu(\psi^*\partial_\mu\psi - \psi\partial_\mu\psi^*) = 0$$

HENCE WE CAN IDENTIFY $\rho = i\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$ AS THE PARTICLE DENSITY

(THE FACTOR i IS INSERTED TO MAKE ρ REAL)

THEN FOR $\psi = e^{-iPX}$, $\rho = 2E$ PARTICLES PER UNIT VOLUME

THUS TO NORMALISE TO <u>ONE</u> PARTICLE PER UNIT VOLUME,

$$\boxed{\psi = \frac{1}{\sqrt{2E}} e^{-iPX}}$$

WE UNDERSTAND THIS TO BE RELATIVISTICALLY INVARIANT FROM OUR PRECEDING ARGUMENT.

A WORRY: THE PARAMETER $E = p_0$ IN THE WAVE FUNCTION COULD WELL BE NEGATIVE. THIS LED KLEIN & GORDON TO DOUBT THE SIGNIFICANCE OF THEIR EQUATION. DIRAC'S INSIGHT WAS THAT THESE 'NEGATIVE ENERGY' WAVES REPRESENT ANTIPARTICLES. THE MATING OF RELATIVITY AND QUANTUM MECHANICS PRODUCED A DRAMATIC AND UNEXPECTED OFFSPRING! THE FIRST SPINLESS ANTIPARTICLES ($\pi^{\pm}$ MESONS) WERE DISCOVERED IN 1947, SO FOR 20 YEARS THE FORMALISM SKETCHED ABOVE WAS CONSIDERED 'USELESS.'

2. RELATIVISTIC ELECTROMAGNETIC SCATTERING

THE NON-RELATIVISTIC BORN APPROXIMATION WAS TRANSFORMED INTO A VERY USEFUL RELATIVISTIC FORM BY FEYNMAN (~1948).

BORN TOLD US TO CONSIDER THE FOURIER TRANSFORM OF THE INTERACTION ENERGY $e\Phi$.

FEYNMAN TELLS US TO CONSIDER THE 4-DIMENSIONAL FOURIER TRANSFORM OF THE RELATIVISTIC INTERACTION $j_\mu A^\mu$

j_μ IS THE 'CURRENT' OF THE SCATTERED PARTICLE

A_μ IS THE 4 VECTOR POTENTIAL SET UP BY THE SCATTERING CENTER.

THE RELATIVISTIC VIEWPOINT NO LONGER CONSIDERS FIXED SCATTERING CENTERS; INDEED WE MUST HAVE A KIND OF SYMMETRY BETWEEN SCATTERER AND SCATTEREE.

THE SYMMETRIC INTERACTION ENERGY IS SOMETHING LIKE

$$\iint \frac{\rho_1(\bar{r}_1)\,\rho_2(\bar{r}_2)}{r_{12}|_{RET}}\,d\bar{r}_1\,d\bar{r}_2 + \iint \frac{\bar{j}_1(\bar{r}_1)\cdot\bar{j}_2(\bar{r}_2)}{r_{12}\ _{RET}}\,d\bar{r}_1\,d\bar{r}_2$$

WE MAKE A LEAP THAT THAT APPROPRIATE FOURIER TRANSFORM CAN BE EXPRESSED AS

$$\frac{\mathcal{J}_{1\mu}\,\mathcal{J}_2^{\mu}}{q^2}$$

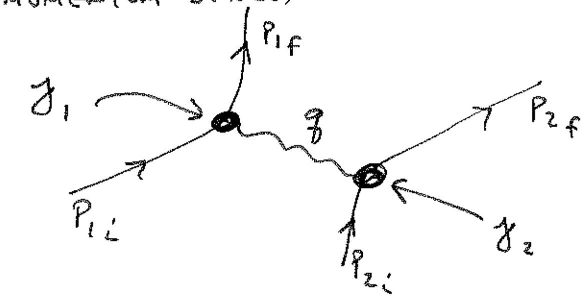
WHERE $\mathcal{J}_{1,2\,\mu}$ ARE THE FOURIER TRANSFORMED CURRENTS. (SORRY ABOUT CONFUSING NOTATION)

AND $q^2 = q_\mu q^\mu$ WHERE $q_\mu = P_{1i} - P_{1f} = P_{2f} - P_{2i}$ = 4-MOMENTUM TRANSFER.

RECALL THAT $\frac{1}{|\vec{r}|}$ TRANSFORMED TO $\frac{1}{|\vec{q}|^2}$

THE CURRENTS $\mathcal{J}_{1,2}$ CONTAIN THE INFORMATION AS TO THE ELECTROMAGNETIC STRUCTURE OF THE PARTICLES.

FEYNMAN EXPRESSED THE SCATTERING MATRIX ELEMENT IN PICTURES (IN MOMENTUM SPACE)



IN THE 1ST APPROXIMATION, THE MOMENTUM TRANSFER IS CARRIED BY A SINGLE PHOTON OF MOMENTUM 4-VECTOR q_μ. THIS EXCHANGE LEADS TO A FACTOR $1/q^2$ IN THE MATRIX ELEMENT, CALLED THE PHOTON PROPAGATOR. THE REST OF THE MATRIX ELEMENT CONSISTS OF TWO VERTEX FACTORS — THE 'CURRENTS' $\mathcal{J}_1$ & $\mathcal{J}_2$.

REMARK ON ELECTRICAL UNITS. IF WE WISH TO USE C.G.S. UNITS FOR WHICH $\alpha = e^2/\hbar c$ THEN WE MUST ACTUALLY WRITE

PHOTON PROPAGATOR $= \frac{4\pi}{q^2}$. IF WE USE $1/q^2$, LATER WE SET $\alpha = \frac{e^2}{4\pi\hbar c}$

3. THE FORM OF THE CURRENT FOR SPINLESS PARTICLES

THE CURRENT $\mathcal{J}_\mu$ IS NOT COMPLETELY ARBITRARY – IT MUST BE A 4-VECTOR. IN MOMENTUM SPACE, THE ONLY 4-VECTORS ASSOCIATED WITH A SPINLESS PARTICLE ARE THE INITIAL AND FINAL MOMENTA P_i AND P_f, OR EQUIVALENTLY, $q = P_i - P_f$, AND $\mathcal{P} = P_i + P_f$.

HENCE $\quad \mathcal{J}_\mu = F\,\mathcal{P}_\mu + G\,q_\mu$

WHERE F AND G ARE SCALAR FUNCTIONS, WHICH MUST SURELY BE PROPORTIONAL TO THE CHARGE. ADDITIONALLY, THE ARGUMENTS OF F AND G MUST BE LORENTZ SCALARS. THE ONLY POSSIBILITIES ARE q^2, $\underline{\mathcal{P}}^2$ AND m^2, WHERE m = MASS. (ALSO CHARGE, e)

NOW $q^2 + \mathcal{P}^2 = 4m^2$, SO IN EFFECT q^2 IS THE ONLY <u>VARIABLE</u> LORENTZ SCALAR AVAILABLE.

$$F = F(q^2)\,, \quad G = G(q^2).$$

ELECTROMAGNETISM SATISFIES AN ADDITIONAL RELATION: <u>CURRENT CONSERVATION</u>. IN COORDINATE SPACE, $\partial_\mu \mathcal{J}^\mu = 0$.

THE FOURIER TRANSFORM OF THIS IS $\quad \underline{q_\mu \mathcal{J}^\mu = 0}$

HENCE WE MUST HAVE $G = 0$ (AS $q^2 \neq 0$ WHILE $q\mathcal{P} = 0$)

$$\mathcal{J}_\mu = F(q^2)\,\mathcal{P}_\mu = F(q^2)\,(P_i + P_f)_\mu \qquad \text{SPIN - 0}$$

$F(q^2)$ IS, OF COURSE, THE <u>FORM FACTOR</u>.

FOR A POINT CHARGE, WE EXPECT $F(q^2) = e$

THE MATRIX ELEMENT OF 1ST ORDER ELECTROMAGNETIC SCATTERING OF SPINLESS PARTICLES IS THEN

$$M = \frac{F_1(q^2)\, F_2(q^2)}{q^2} \quad P_{1\mu} P_2^{\mu}$$

4. THE BREIT FRAME

WE CLAIM THAT THE FUNCTION $F(q^2)$ IS THE RELATIVISTIC FORM FACTOR, PRESUMABLY RELATED TO THE CHARGE DISTRIBUTION. BUT BECAUSE OF THE LORENTZ CONTRACTION THE CHARGE DISTRIBUTION APPEARS TO DEPEND ON THE VELOCITY OF THE PARTICLE. A POSSIBILITY IS TO REFER TO THE CHARGE DISTRIBUTION IN THE PARTICLE'S REST FRAME. HOWEVER, IN RELATIVISTIC SCATTERING, THE PARTICLE CANNOT BE AT REST BOTH BEFORE AND AFTER THE COLLISION. HENCE A PRICE OF RELATIVITY IS SOME LOSS OF THE CLASSICAL INTERPRETATION OF THE FORM FACTOR.

$F(q^2)$ HAS A WELL DEFINED MEANING IN TERMS OF A SCATTERING EXPERIMENT. WE CAN FORMALLY DEFINE

$$\rho(x) = \frac{1}{(2\pi)^4} \int e^{iqx} F(q^2)\, d^4q \qquad [x = (t, \bar{x}) \quad \text{4 VECTOR}]$$

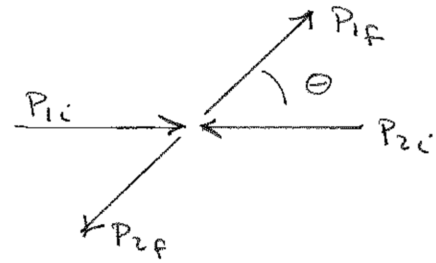
IN GENERAL ρ IS A FUNCTION OF TIME, WHICH SEEMS UNSATISFACTORY. THE BEST WE CAN DO IS CHOOSE A FRAME IN WHICH $q_0 = 0$. THEN $\rho = \rho(\bar{x})$ ONLY

SUCH A FRAME IS CALLED A BREIT FRAME, OR BRICK WALL FRAME

SINCE $q = P_i - P_f$, SUCH A FRAME IMPLIES $E_i = E_f$.

FOR ELASTIC SCATTERING THE C.M. FRAME IS ALSO A BREIT FRAME. THE MOMENTA $\bar{p}_i$, $\bar{p}_f$ DO NOT VANISH, HOWEVER.

C.M. FRAME:



$q = (0, \bar{p}_{1i} - \bar{p}_{1f})$

SO WHILE $\rho = \rho(\vec{r})$, THIS DOES NOT EXACTLY DESCRIBE A 'REST-FRAME' CHARGE DISTRIBUTION. IN THE LIMIT OF LOW MOMENTA (THE NON-RELATIVISTIC LIMIT), WE DO RECOVER OUR PREVIOUS MEANING OF ρ - ESPECIALLY FOR THE HEAVIER OF THE TWO PARTICLES.

5. CALCULATING THE CROSS SECTION

TO GO FROM THE MATRIX ELEMENT, M, TO A CROSS SECTION, WE NEED THE RELATIVISTIC VERSION OF FERMI'S GOLDEN RULE:

$$d\sigma = \underbrace{\frac{1}{|\bar{v}_1 - \bar{v}_2|}}_{\substack{\text{FLUX} \\ \text{FACTOR}}} \underbrace{\frac{1}{2E_{1i}} \frac{1}{2E_{2i}}}_{\substack{\text{NORMALIZATION} \\ \text{OF INITIAL} \\ \text{STATES}}} |M|^2 (2\pi)^4 \underbrace{\delta^4(p_{1i} + p_{2i} - p_{1f} - p_{2f})}_{\substack{\text{CONSERVATION} \\ \text{OF ENERGY-} \\ \text{MOMENTUM}}} \underbrace{\frac{d^3 p_{1f}}{2E_{1f}(2\pi)^3} \frac{d^3 p_{2f}}{2E_{2f}(2\pi)^3}}_{\substack{\text{DENSITY OF FINAL} \\ \text{STATES, INCLUDING} \\ \text{NORMALIZATION}}}$$

SINCE M IN EFFECT ONLY INCORPORATES THE e^{-ipx} FACTORS OF THE WAVE FUNCTIONS, THE NORMALIZATIONS, $\frac{1}{\sqrt{2E}}$, MUST BE ADDED NOW. OTHERWISE THIS IS VERY SIMILAR TO THE CORRESPONDING NON-RELATIVISTIC VERSION. (THIS IS USUALLY DISCUSSED IN BOOKS UNDER THE HEADING "S-MATRIX").

SINCE THE CROSS SECTION HAS THE MEANING OF AN AREA PERPENDICULAR TO THE LINE OF FLIGHT, IT SHOULD BE AN INVARIANT. THE FACTORS $\frac{d^3P}{E}$ ARE INDEED LORENTZ INVARIANT, AS DISCUSSED ON P. 73.

IT TURNS OUT THAT $|\bar{v}_1 - \bar{v}_2| E_1 E_2 = \sqrt{(P_{1\mu} P_2^\mu)^2 - m_1^2 m_2^2}$

SO ALL FACTORS OF $d\sigma$ ARE INVARIANTS, AS DESIRED.

WE WILL HAVE OCCASION TO CONSIDER CROSS SECTIONS IN BOTH THE C.M. FRAMES, AND THE LAB FRAME FOR WHICH PARTICLE 2 IS INITIALLY AT REST. IN EITHER CASE, ONE INTEGRATES OVER 4 FINAL STATE VARIABLES TO USE UP THE DELTA FUNCTIONS.

WE RECALL THAT $\int \delta(f(x))\,dx = \int \frac{\delta(f)\,df}{f'}$

SO $\int \delta^4(p_{1i}+p_{2i}-p_{1f}-p_{2f}) \frac{d^3p_{1f}\, d^3p_{2f}}{E_{1f} E_{2f}} \rightarrow \frac{p_{1f}^3\, d\Omega_{1f}}{E\, p_{1f}^2 - E_{1f}\, \bar{p}_{1f}\cdot\bar{P}}$

WHERE $p_{1f} = |\bar{p}_{1f}|$, $E = E_{TOTAL} = E_{1i} + E_{2i}$; $\bar{P} = \bar{P}_{TOTAL} = \bar{p}_{1i} + \bar{p}_{2i}$ ETC.

$$\text{SO} \quad \frac{d\sigma}{d\Omega_1} = \frac{1}{64\pi^2} \frac{p_{1f}^3 |M|^2}{|\bar{v}_{1i}-\bar{v}_{2i}| E_{1i} E_{2i} \left(E\, p_{1f}^2 - E_{1f}\, \bar{p}_{1f}\cdot\bar{P}\right)}$$

6. C.M. FRAME $\quad \bar{P} = 0$, $\bar{p}_{2i} = -\bar{p}_{1i}$, $\bar{v}_{1i} = \frac{\bar{p}_{1i}}{E_{1i}}$ $\quad \bar{v}_{2i} = -\frac{\bar{p}_{1i}}{E_{2i}}$

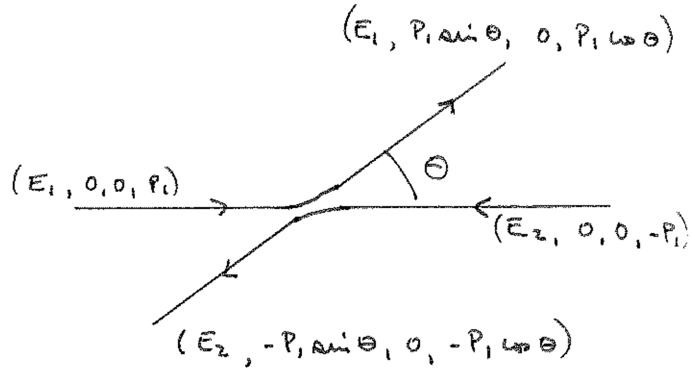
$$\Rightarrow |\bar{v}_{1i} - \bar{v}_{2i}| E_{1i} E_{2i} = p_{1i} E \quad ; \quad p_{1i} = |\bar{p}_{1i}|$$

AND $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E^2} \frac{p_{1f}}{p_{1i}} |M|^2$ (TRUE FOR ANY 2-BODY REACTION IF ALL SPIN FACTORS ARE PUT IN $|M|^2$)

FOR ELASTIC SCATTERING, $p_{1f} = p_{1i}$ AND $\left.\frac{d\sigma}{d\Omega}\right|_{EL} = \frac{1}{64\pi^2 E^2} |M|^2$

FOR OUR SPECIAL CASE OF SPIN-0 SCATTERING

$$|M|^2 = \frac{F_1^2(q^2)\, F_2^2(q^2)\, \left(P_{1\mu} P_2^{\mu}\right)^2}{q^4}$$



$$q^2 = (p_{1i} - p_{1f})^2 = -p_1^2\left(\sin^2\theta + (1-\cos\theta)^2\right) = -4\, p_1^2 \sin^2\theta/2$$

$$P_1 = p_{1i} + p_{1f} = \left(2E_1,\ p_1\sin\theta,\ 0,\ p_1(1+\cos\theta)\right)$$

$$P_2 = p_{2i} + p_{2f} = \left(2E_2,\ -p_1\sin\theta,\ 0,\ -p_1(1+\cos\theta)\right)$$

$$P_{1\mu} P_2^{\mu} = 4E_1E_2 + p_1^2\left(\sin^2\theta + (1+\cos\theta)^2\right) = 4\left(E_1E_2 + p_1^2\cos^2\theta/2\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E^2}\,\frac{\left(E_1E_2 + p_1^2\cos^2\theta/2\right)^2}{p_1^4\sin^4\theta/2}\, F_1^2(q^2)\, F_2^2(q^2)$$

IN THE <u>NON-RELATIVISTIC LIMIT</u>, $p_1 \ll E_1$, $E_1 \sim M_1$, $E_2 \sim M_2$ AND $q^2 \to 0$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 (M_1+M_2)^2}\,(M_1M_2)^2\,\frac{F_1^2(0)\, F_2^2(0)}{p_1^4\sin^4\theta/2}$$

TO COMPARE TO RUTHERFORD, WE CONSIDER $M_1 \ll M_2$, SO M_2 REMAINS ESSENTIALLY AT REST. NOTING $(KE_1) = \frac{p_1^2}{2M_1}$)

$$\frac{d\sigma}{d\Omega} \to \frac{1}{256\pi^2}\,\frac{F_1^2(0)\, F_2^2(0)}{(KE_1)^2\sin^4\theta/2}$$

SO IF INDEED $F_1(0) = e$ AND $F_2(0) = e$

THEN $\displaystyle\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16(KE_1)^2}\,\frac{1}{\sin^4\theta/2}$ WITH $\displaystyle\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

RUTHERFORD!

THIS SLIGHTLY ARTIFICIAL EXAMPLE ILLUSTRATES THE TYPICAL METHOD OF ATTACK IN RELATIVISTIC QUANTUM MECHANICS: THE MOST GENERALLY ALLOWED BEHAVIOR IS ESTABLISHED FORMALLY, MAINTAINING CONSISTENCY WITH RELATIVITY & QUANTUM MECHANICS. THEN AN INTERPRETATION IS SOUGHT BY CONSIDERING LIMITING CASES IN REALMS WHERE OUR 'CLASSICAL INTUITION' APPLIES.

7. <u>LAB FRAME</u> AS A FINAL EXERCISE TO THIS SECTION, WE PUT $\bar{P}_{2i} = 0 \Rightarrow E_{2i} = M_2$. THE TARGET IS AT REST.

$$\bar{P}_{TOT} = \bar{P}_{1i}, \quad \bar{v}_{1i} = \frac{\bar{P}_{1i}}{E_{1i}}, \quad |\bar{v}_{1i} - \bar{v}_{2i}|\, E_{1i} E_{2i} = P_{1i} M_2$$

$$E P_{1f}^2 - E_{1f}\, \bar{P}_{1f} \cdot \bar{P} = P_{1f} \left(E P_{1f} - E_{1f} P_{1i} \cos\theta \right) \qquad \theta \equiv \text{SCATTERING ANGLE}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{P_{1f}^2\, |M|^2}{P_{1i} M_2 \left(E P_{1f} - E_{1f} P_{1i} \cos\theta \right)} \qquad \text{LAB FRAME}$$

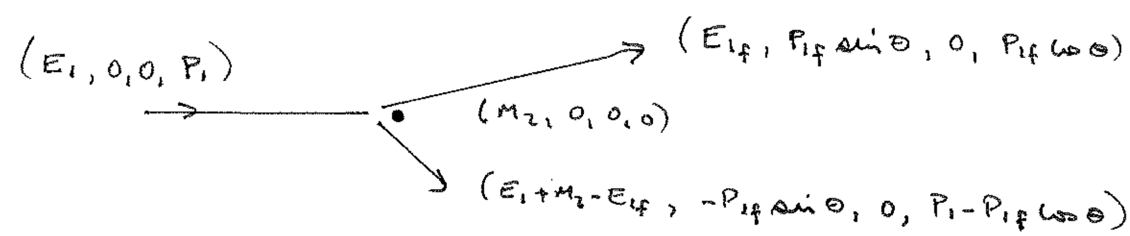
THIS TIME WE CONSIDER THE <u>EXTREME</u> <u>RELATIVISTIC</u> <u>LIMIT</u>, $E_1 \gg M_1$ AND ALSO SUPPOSE $M_1 \ll M_2$ BUT $M_2 \ll E_1$

THEN $P_{1i} \to E_{1i}$, $P_{1f} \to E_{1f}$ AND $E \to E_{1i} + M_2$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{E_{1f}}{E_{1i} M_2^2} \frac{|M|^2}{1 + \frac{2E_{1i}}{M_2} \sin^2\theta/2}$$

THE EXTRA ANGULAR FACTOR IS THE EFFECT OF THE TARGET RECOIL.

AGAIN, $|M|^2 = F_1^2 F_2^2 \left(P_{1\mu} P_2^{\mu} \right)^2 / q^4$



$$q^2 = (P_{1i} - P_{1f})^2 = (E_1 - E_{1f})^2 - P_{1f}^2 \sin^2\theta - (P_1 - P_{1f}\cos\theta)^2$$
$$= 2(m_1^2 - E_1 E_{1f} - P_1 P_{1f} \cos\theta)$$

$$P_{1\mu} = (E_1 + E_{1f},\ P_{1f}\sin\theta,\ 0,\ P_1 + P_{1f}\cos\theta)$$
$$P_{2\mu} = (2M_2 + E_1 - E_{1f},\ -P_{1f}\sin\theta,\ 0,\ P_1 - P_{1f}\cos\theta)$$

$$P_{1\mu} P_2^{\mu} = E_1^2 - E_{1f}^2 + 2M_2(E_1 + E_{1f}) + P_{1f}^2 \sin^2\theta - (P_1^2 - P_{1f}^2\cos^2\theta) = 2M_2(E_1 + E_{1f})$$

AGAIN CONSIDER THE LIMIT $m_1 \ll m_2 \ll E_1$

$$q^2 \rightarrow -4E_1 E_{1f} \sin^2\theta/2$$

TO ELIMINATE E_{1f}, NOTE THAT $M_2^2 = P_{2f}^2 = (E_1 + M_2 - E_{1f})^2 - E_{1f}\sin^2\theta - (E_1 - E_{1f}\cos\theta)^2$

SO $E_{1f} = \dfrac{E_1}{1 + \frac{2E_1}{M_2}\sin^2\theta/2}$ AND $E_1 + E_{1f} = 2E_1 \left(\dfrac{1 + \frac{E_1}{M_2}\sin^2\theta/2}{1 + \frac{2E_1}{M_2}\sin^2\theta/2} \right)$

$$\frac{d\sigma}{d\Omega} \rightarrow \frac{\alpha^2}{4E_1^2} \left(\frac{1 + \frac{E_1}{M_2}\sin^2\theta/2}{1 + \frac{2E_1}{M_2}\sin^2\theta/2} \right)^2 \frac{F_1^2(q^2) F_2^2(q^2) / F_1^2(0) F_2^2(0)}{\sin^4\theta/2}$$

IF $E_1 \gg M_2$

$$\frac{d\sigma}{d\Omega} \rightarrow \frac{\alpha^2}{16E_1^2} \frac{F_1^2(q^2) F_2^2(q^2) / F_1^2(0) F_2^2(0)}{\sin^4\theta/2}$$

FOR POINT LIKE PARTICLES, RUTHERFORD'S FORMULA HOLDS IN THE EXTREME RELATIVISTIC LIMIT IN THE LAB FRAME!