ELECTROMAGNETIC INTERACTIONS

THIS INTERACTION IS CONSIDERED TO BE WELL UNDERSTOOD. AS SUCH IT IS SELDOM STUDIED FOR ITS OWN SAKE ANYMORE, EXCEPT FOR OCCASIONAL TESTS OF QED (≡ QUANTUM ELECTRODYNAMICS) AT EVER SMALLER DISTANCES. RATHER THIS INTERACTION CAN PROVIDE PROBES OF LESS UNDERSTOOD PROCESSES.

WE WILL USE E & M TO SKETCH SOME FEATURES OF THE FEYNMAN DIAGRAM APPROACH. THIS IS A GRAPHIC WAY OF REPRESENTING TERMS IN A PERTURBATION EXPANSION OF THE INTERACTION.

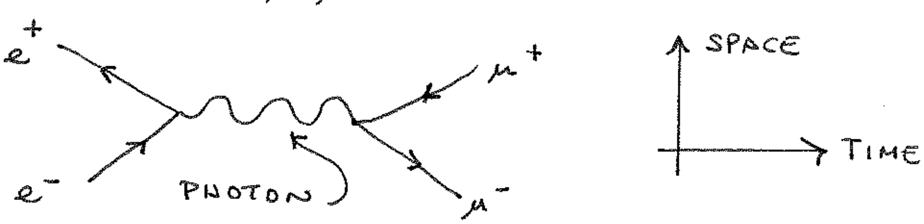
ELECTROMAGNETIC INTERACTIONS INVOLE THE EMISSION, ABSORPTION AND EXCHANGE OF PHOTONS. THE FEWER THE NUMBER OF PHOTONS, THE SIMPLER THE INTERACTION (≡ LOWER ORDER IN PERTURBATION).

ONE OF THE CONCEPTUALLY SIMPLEST HIGH ENERGY REACTIONS IS THAT MENTIONED IN LECTURE 1:

$$e^+e^- \to \mu^+\mu^-$$

THE ELECTRON-POSITRON PAIR CAN ANNIHILATE PRODUCING A SINGLE PHOTON, WHICH IN TURN PRODUCES A $\mu^+\mu^-$ PAIR

THE FEYNMAN DIAGRAM:



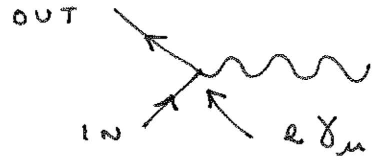
WE SELDOM EXPLICITLY MENTION THE AXES IN A FEYNMAN DIAGRAM, BUT THEY MAY BE THOUGHT OF AS SPACE AND TIME AS SHOWN.

THE DIAGRAM IS TO HELP US EVALUATE THE MATRIX ELEMENT

M_{fi} = (FINAL | $H_{\text{INTERACTION}}$ | INITIAL). AS DISCUSSED IN LECTURE 1, THE CALCULATION IS DONE IN MOMENTUM SPACE, CONSISTENT WITH THE EXPERIMENTAL SITUATION OF SCATTERING EXPERIMENTS IN WHICH DIRECTIONS AND ENERGIES ARE OBSERVED WELL, AND POSITIONS KNOWN POORLY.

THE 'FEYNMAN RULES' ALLOW US TO READ OFF THE MATRIX ELEMENT FROM THE DIAGRAM. THIS TASK CONSISTS OF IDENTIFYING THE VERTEX FACTORS AND THE INTERNAL PROPAGATORS.

A VERTEX IN QED IS THE EMISSION OR ABSORPTION OF A PHOTON



THE VERTEX FACTOR IS THE PRODUCT OF e = ELECTRIC COUPLING = CHARGE AND THE SPIN FACTOR γ_μ – A 4-VECTOR COMPRISED OF DIRAC MATRICES

THE SPIN FACTOR γ_μ IS TO BE SANDWICHED BETWEEN THE IN AND OUT STATE VECTORS – DIRAC SPINORS, TO MAKE UP A PIECE OF THE MATRIX ELEMENT:

$$\langle \text{out} | e\gamma_\mu | \text{in} \rangle$$

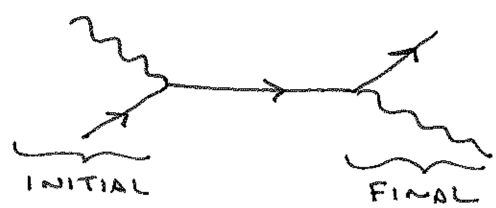
THIS PART OF MATRIX ELEMENT MAY BE THOUGHT OF AS DESCRIBING THE POLARIZATION OF THE PHOTON – A 4-VECTOR IN RELATIVISTIC QUANTUM MECHANICS.

THE PROPAGATOR IS A MEASURE OF THE ENERGY OF THE INTERACTION, OR RATHER ITS FOURIER TRANSFORM INTO MOMENTUM SPACE.

IN OUR EXAMPLE A PHOTON IS EXCHANGED, OR PROPAGATED BETWEEN THE INITIAL AND FINAL STATES.

IN COMPTON SCATTERING WE SAY THAT THE ELECTRON IS PROPAGATED.

$\gamma + e \rightarrow \gamma + e$:



RECALL YUKAWA'S IDEA THAT THE FIELD DUE TO THE EXCHANGE (PROPAGATION) OF A QUANTUM OF MASS M IS DESCRIBED BY THE POTENTIAL

$$\frac{e^{-rm}}{r} \qquad (\hbar = c = 1)$$

THE FOURIER TRANSFORM OF THIS IS $\int e^{i\bar{q}\cdot\bar{r}} \frac{e^{-rm}}{r} d^3\bar{r} \sim \frac{1}{\bar{q}^2 + m^2}$

WHERE $\bar{q}$ = 3-MOMENTUM OF THE PROPAGATED OBJECT.

THIS NON-RELATIVISTIC ARGUMENT GIVES A SENSE OF THE RELATIVISTIC RESULT:

PHOTON PROPAGATOR $\sim \dfrac{1}{q^2}$ WHERE $q^2 = q_\mu q^\mu = q_0^2 - \vec{q}^{\,2}$

q_μ = 4-MOMENTUM

SPIN-ZERO PROPAGATOR $\sim \dfrac{1}{q^2 - m^2} = \dfrac{1}{q_0^2 - (\vec{q}^{\,2} + m^2)}$ EXAMPLE: π-MESON

SPIN ½ PROPAGATOR $\sim \dfrac{q_\mu \gamma_\mu + m}{q^2 - m^2}$ EXAMPLE: ELECTRON

COMPLETE CALCULATIONS OF THE SQUARE OF THE MATRIX ELEMENT ARE "STRAIGHTFORWARD BUT LENGTHY." HERE WE INDICATE HOW THE BASIC NOTIONS OF THE FEYNMAN METHOD CAN BE USE TO MAKE QUICK ESTIMATES OF CROSS-SECTIONS.

AN IMPORTANT OBSERVATION IS THAT FOR EACH PHOTON VERTEX WE GET ONE FACTOR OF THE CHARGE, e, IN THE MATRIX ELEMENT.

THUS $\sigma_{e^+e^- \to \mu^+\mu^-} \sim |M|^2 \sim e^4 \sim \alpha^2$ WHERE $\alpha = e^2/\hbar c$

(FOR THE 1-PHOTON EXCHANGE DIAGRAM ON p. 27)

RECALL ALSO THAT σ HAS DIMENSIONS OF $(\text{LENGTH})^2 \sim \dfrac{1}{(\text{ENERGY})^2}$

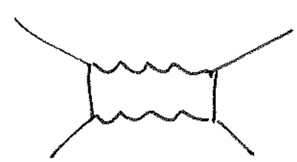
SO IF $E_{cm} \gg m_e$ AND m_μ, WE EXPECT

$$\sigma_{e^+e^- \to \mu^+\mu^-} \sim \frac{\alpha^2}{E_{cm}^2}$$ WITHOUT ANY DETAILED CALCULATION

WITH A LOT OF WORK WE FIND $\sigma = \dfrac{4\pi}{3} \dfrac{\alpha^2}{E_{cm}^2}$ 'EXACTLY' IN THE 1-PHOTON APPROXIMATION

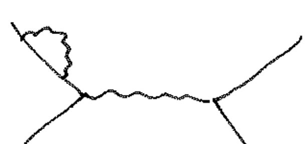
HIGHER ORDER TERMS INVOLVE MORE PHOTONS. 2ND ORDER DIAGRAMS ARE LIKE

BREMSSTRAHLUNG

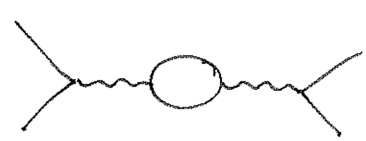


2 PHOTON EXCHANGE

VERTEX CORRECTION



ELECTRON PROPAGATOR CORRECTION



PHOTON PROPAGATOR CORRECTION

OF THESE DIAGRAMS, THE VERTEX AND PROPAGATOR CORRECTIONS LEAD TO THE FAMOUS INFINITIES OF QED. COLLECTING THE INFINITE NUMBER OF SUCH HIGHER ORDER DIAGRAMS TOGETHER, THEIR NET EFFECT IS TO CHANGE THE APPARENT CHARGE AND MASS OF THE VARIOUS PARTICLES. BUT WE CAN ONLY MEASURE THE 'APPARENT' QUANTITIES, AND CANNOT DIRECTLY ACCESS THE 'BARE' CHARGE OR MASS. AS THESE CORRECTIONS ARE ALWAYS PRESENT, AND ALWAYS THE SAME SIZE, WE CAN IN EFFECT IGNORE THEM, AND DEAL ONLY WITH THE APPARENT OR CORRECTED QUANTITIES.

THIS IDEA IS CALLED RENORMALISATION. IT SEEMS QUITE REASONABLE, EXCEPT FOR THE FACT THAT THE SIZE OF THE CORRECTION IS INFINITE!

ENERGY AND MOMENTUM CONSERVATION

IN ALL FUNDAMENTAL PROCESS OBSERVED SO FAR, ENERGY AND MOMENTUM ARE CONSERVED. AS SUCH THE ENERGY-MOMENTUM 4-VECTOR OF THE INTERNALLY PROPAGATED PARTICLE MAY APPEAR ODD AT FIRST.

EXAMPLE: $e^+e^- \to \mu^+\mu^-$ AT A COLLIDING BEAM FACILITY

AS THE ELECTRON AND POSITRON COLLIDE HEAD ON, THEIR TOTAL MOMENTUM IS ZERO. THE THE 4 VECTOR OF THE PHOTON IS

$$q_\mu = e^+_\mu + e^-_\mu = (E_{e^+} + E_{e^-}, \bar{0}) = (E_{cm}, \bar{0})$$

THE "MASS" OF THE PHOTON IS $m^2 = q_\mu q^\mu = q^2 = E_{cm}^2 \neq 0$

EVEN IN THE COMPTON SCATTERING DIAGRAM 

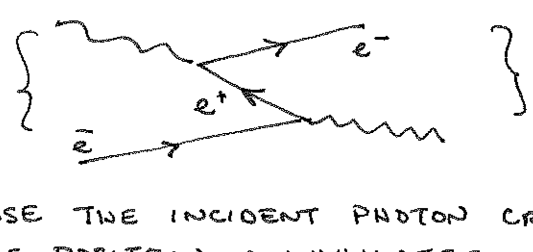
THE INTERNAL ELECTRON HAS A 4-VECTOR WITH MASS $\neq m_e$!

THIS STATE OF AFFAIRS CANNOT LAST, THO HEISENBERG ALLOWS IT FOR A SHORT WHILE: $\Delta t \sim \hbar/\Delta E$

WE CALL THESE INTERNAL STATES 'VIRTUAL PARTICLES', AND SOMETIMES SAY THAT THEY ARE 'OFF THE MASS SHELL'.

FEYNMAN'S VIEW OF ANTIPARTICLE'S

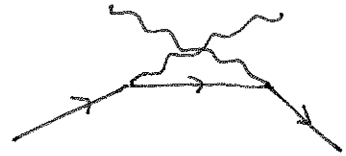
ANOTHER POSSIBLE DIAGRAM FOR COMPTON SCATTERING IS

INITIAL 

FINAL

IN THIS CASE THE INCIDENT PHOTON CREATES AN ELECTRON-POSITRON PAIR. THEN THE POSITRON ANNIHILATES THE INCIDENT ELECTRON TO CREATE THE FINAL PHOTON.

THIS LOOKS VERY DIFFERENT, BUT IS TOPOLOGICALLY EQUIVALENT TO THE DIAGRAM



IN WHICH THE ELECTRON RADIATES THE FINAL PHOTON SLIGHTLY BEFORE IT ABSORBS THE INITIAL ONE.

THIS LED TO THE INTERPRETATION THAT A POSITRON IS JUST AN ELECTRON MOVING BACKWARDS IN TIME! IN THE ABOVE DIAGRAMS, ANTIPARTICLES HAVE BEEN INDICATED WITH ARROWS POINTED OPPOSITE TO THEIR DIRECTION OF MOTION. THEN THE ARROWHEADS FLOW NICELY AROUND THE VERTICES.

COMPTON SCATTERING CROSS SECTION

AS FOR $e^+e^- \to \mu^+\mu^-$, THE COMPTON REACTION $\gamma + e \to \gamma + e$ HAS 2 PHOTON VERTICES. SO AGAIN WE ARE QUICKLY LED TO THE ESTIMATE

$$\sigma \sim \frac{e^4}{E_{cm}^2} \sim \frac{\alpha^2}{E_{cm}^2}$$

FOR LOW ENERGY PHOTONS INCIDENT ON AN ELECTRON AT REST $(E_\gamma \ll M_e)$ WE HAVE $E_{cm} \approx M_e$, AND $\sigma \sim \alpha^2 / M_e^2$

THIS IS JUST THE THOMSON CROSS SECTION NOTED ON P. 24

$$\sigma_{THOMSON} = \frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2 = \frac{8\pi}{3}\alpha^2\left(\frac{\hbar}{Mc}\right)^2 \to \frac{8\pi}{3}\frac{\alpha^2}{M^2} \quad \text{IF } \hbar = c = 1$$

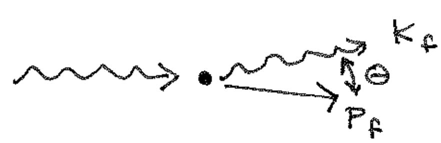
AGAIN $\hbar/Mc \equiv$ COMPTON WAVELENGTH OF THE ELECTRON.

ANOTHER LIMIT IS VERY HIGH ENERGY PHOTONS $(E_\gamma \equiv K \gg m_e)$ INCIDENT ON ELECTRONS AT REST.

THEN $E_{cm}^2 = \left[(K,0,0,K) + (M,0,0,0)\right]^2 \approx 2MK$

$$\text{SO} \quad \sigma \to \frac{\alpha^2}{2MK}$$

WE MAY ALSO ESTIMATE A CORRECTION DUE TO BAD BEHAVIOR OF THE ELECTRON PROPAGATOR FOR SMALL ANGLE SCATTERS



THE MATRIX ELEMENT $\sim \dfrac{1}{q^2 - M^2}$

NOW WE CAN WRITE $q = P_f + k_f$ (4-VECTORS), NOTING THAT P_f DESCRIBES A RELATIVISTIC ELECTRON, EVEN FOR THE INITIAL ELECTRON AT REST, So $q^2 - M^2 = \underset{0}{\underset{\shortparallel}{k_f^2}} + \underset{M^2}{\underset{\shortparallel}{p_f^2}} + 2k_f\cdot p_f - M^2 = 2\left(k_{0_f}p_{0_f} - \bar{k}_f\cdot\bar{p}_f\right) \to 0$ AS $\begin{cases}\theta \to 0 \\ p_{0f} \to \infty\end{cases}$

TO AVOID ESTIMATING AN INFINITE CROSS-SECTION WE MUST KEEP TRACK OF THE SMALL DEPARTURE OF $q^2 - M^2$ FROM ZERO.

WE RELATE p_0 TO $\bar{p}$ BY $p_0 = \sqrt{\bar{p}^2 + M^2} \approx |\bar{p}| + \dfrac{M^2}{2|\bar{p}|}$

SO $q^2 - M^2 \sim 2k_0\left(|\bar{p}|(1-\cos\theta) + \dfrac{M^2}{2|\bar{p}|}\right) \approx \dfrac{k_0}{|\bar{p}|}\left(|\bar{p}|^2\theta^2 + M^2\right)$

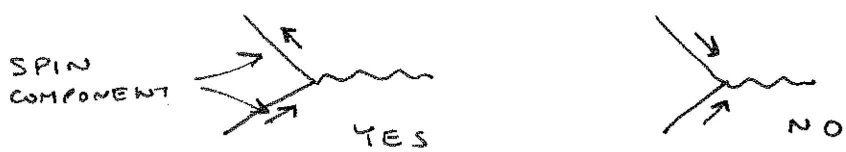
HENCE WE MIGHT EXPECT $\sigma \sim \dfrac{\alpha^2}{2MK}\displaystyle\int \frac{d\cos\theta}{\left(|\bar{p}|^2\theta^2 + M^2\right)^2}$ · OTHER FACTOR
$\nwarrow$ SQUARE OF PROPAGATOR

THE OTHER FACTOR IS REQUIRED TO MAKE OUR CORRECTION DIMENSIONLESS, AND EASY TO CALCULATE. ∴ IT GOES LIKE $|\bar{p}|^4$

BUT WE MUST TAKE NOTE OF ANOTHER FEATURE OF THE ELECTROMAGNETIC INTERACTION NOT YET EVIDENT : <u>HELICITY</u> <u>CONSERVATION</u>

RECALL HELICITY ≡ COMPONENT OF SPIN ALONG A PARTICLE'S DIRECTION OF MOTION.

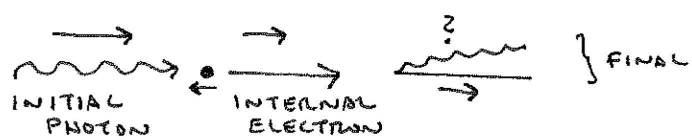
IN THE EXTREME RELATIVISTIC LIMIT THE ELECTROMAGNETIC INTERACTION CONSERVES THE HELICITY OF THE SPIN ½ PARTICLES AT EACH PHOTON VERTEX. WE SHALL TRY TO DEMONSTRATE THIS LATER.



AT <u>HIGH</u> ENERGIES

IN THE PRESENT CASE, HELICITY CONSERVATION TELLS US THAT THE COMPTON CROSS SECTION MUST VANISH AS $\theta \to 0$ (AT HIGH ENERGIES)

THE CONFIGURATION  DOES NOT COUPLE AS THIS WOULD INPUT $S_z = 3/2$ FOR THE INTERNAL ELECTRON!



[RECALL THAT AT $\theta = 0$, ORBITAL ANGULAR MOMENTUM COMPONENT L_z MUST VANISH]

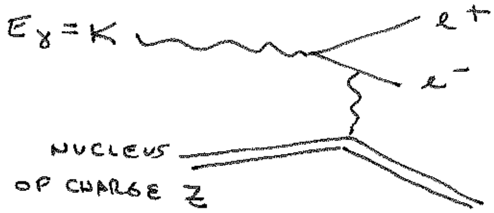
TO CONSERVE ELECTRON HELICITY AND TOTAL ANGULAR MOMENTUM AT $\theta = 0$ WOULD REQUIRE A FINAL PHOTON OF HELICITY ZERO – WHICH CAN'T HAPPEN.

WE INFER THAT THE MATRIX ELEMENT VANISHES LIKE θ TO INSURE COMPATIBILITY WITH HELICITY CONS. & CONS. OF ANGULAR MOMENTUM.

THEN $\sigma \sim \dfrac{\alpha^2}{MK}\displaystyle\int \frac{\theta^2\, d\cos\theta\, |\bar{p}|^4}{\left(|\bar{p}|^2\theta^2 + M^2\right)^2} \sim \frac{\alpha^2}{MK}\int_0^\infty \frac{\theta^2\, d\theta^2}{\left(\theta^2 + M^2/|\bar{p}|^2\right)^2} \sim \frac{\alpha^2}{MK}\ln\frac{|p|^2}{M^2}$

OR $\underline{\sigma \sim \dfrac{\alpha^2}{MK}\ln\dfrac{K}{M}}$ NOTING $|\bar{p}_f| \sim K$ [KLEIN-NISHINA (1928)]

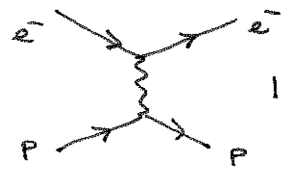
WITH THIS TYPE OF ARGUMENT ONE MAY ALSO ESTIMATE THE CROSS SECTION FOR PAIR PRODUCTION BY A HIGH ENERGY PHOTON IN THE VICINITY OF A HEAVY NUCLEUS (BETHE-HEITLER (1934)).



$$\sigma_{B\text{-}H} \approx \frac{Z^2 \alpha^3}{M^2} \ln \frac{K}{M}$$

[SEE BOOK OF T. D. LEE, CHAP. 8]

RUTHERFORD SCATTERING



1 PHOTON EXCHANGE

WE KNOW THAT THE TOTAL CROSS SECTION IS INFINITE - THE ELECTRIC FIELD EXTENDS TO INFINITY. WE WILL TRY TO ESTIMATE A DIFFERENTIAL CROSS SECTION.

RUTHERFORD CONSIDERED $\frac{d\sigma}{d\Theta}$, FOR SCATTERING OF THE LIGHTER PARTICLE BY ANGLE Θ.

FOR A QUICK ESTIMATE USING OUR DIMENSIONAL ARGUMENTS IT IS SIMPLER TO CALCULATE

$\frac{d\sigma}{dq^2}$ WHERE q^2 = SQUARE OF THE EXCHANGED PHOTON'S 4-MOMENTUM - A RELATIVISTIC INVARIANT OF THIS PROCESS. OF COURSE, q^2 HAS DIMENSIONS $(\text{ENERGY})^2$

$$\text{so} \quad \frac{d\sigma}{dq^2} \sim \frac{1}{(\text{ENERGY})^4}$$

ANY 2 BODY PROCESS $a+b \to c+d$ HAS ONLY 2 INDEPENDENCE RELATIVISTIC INVARIANTS OF DIMENSIONS $(\text{ENERGY})^2$. THESE ARE OBTAINED BY ADDING THE KINEMATIC 4-VECTORS IN VARIOUS WAYS:

MANDELSTAM VARIABLES

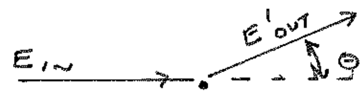
$$\begin{cases} s \equiv (a+b)^2 = (c+d)^2 = E_{cm}^2 \\ t \equiv (a-c)^2 = (b-d)^2 = [\text{4 MOMENTUM TRANSFER}]^2 \ (= q^2 \text{ ABOVE}) \\ u \equiv (a-d)^2 = (b-c)^2 \end{cases}$$

NOTE THAT $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$

THE DIAGRAM APPROACH SUGGESTS THAT WHILE s WAS THE RELEVANT VARIABLE FOR $e^+e^- \to \mu^+\mu^-$ (AN "S-CHANNEL" PROCESS), t IS MORE IMPORTANT FOR $ep \to ep$ (A "t-CHANNEL" PROCESS)

$$\text{SO} \quad \frac{d\sigma}{dq^2} = \frac{d\sigma}{dt} \sim \frac{\alpha^2}{t^2} = \frac{\alpha^2}{q^4}$$

IN THE LABORATORY FRAME, THE ELECTRON 4-VECTORS RELATE TO q:



$$q = (E, 0, 0, E) - (E', E'\sin\theta, 0, E'\cos\theta) \qquad (E, E' \gg m_e)$$

$$q^2 = 2EE'(1-\cos\theta) = 4EE'\sin^2\theta/2$$

$$dq^2 = 2EE'\,d\cos\theta$$

$$\text{SO} \quad \frac{d\sigma}{d\cos\theta} \sim \frac{\alpha^2}{8EE'\sin^4\theta/2}$$

WHICH IS VERY CLOSE TO RUTHERFORD'S RESULT $(E' \to E \to \frac{1}{2}mv_0^2)$

THE WEAK INTERACTION

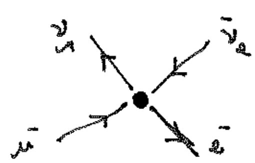
FERMI (1934) GAVE A VIEW OF THE WEAK INTERACTION WHICH IS A GOOD FIRST APPROXIMATION.

THIS VIEW IS EVEN MORE PRECISE IF WE ADOPT THE NOTION THAT THE SPIN-1/2 QUARKS AND LEPTONS ARE THE INTERACTING PARTICLES.

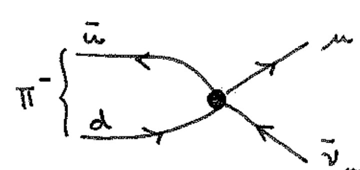
THE WEAK FORCE WAS OBSERVED INITIALLY ONLY IN NUCLEAR β-DECAYS, AND LATER IN THE DECAYS OF VARIOUS MESONS AND HADRONS. FROM THIS IT WAS ESTABLISHED THAT THE RANGE OF THE WEAK FORCE IS VERY SHORT, CERTAINLY LESS THAN 10^{-13} CM.

FERMI SUPPOSED THAT THE FORCE WAS ZERO RANGE - A <u>CONTACT</u> <u>FORCE</u>. BUT OTHERWISE IT CONNECTED SPIN-1/2 PARTICLES, AS DOES ELECTROMAGNETISM.

EXAMPLES: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$



$\pi^- \to \mu^-\bar{\nu}_\mu$



FERMI WROTE THE INTERACTION AMPLITUDE AS THE PRODUCT OF TWO VERTEX FACTORS:

$$\text{AMPLI} \sim G\,(\nu_\mu|\gamma^\alpha|\mu^-)(e^-|\gamma_\alpha|\bar{\nu}_e) \qquad \text{ETC}$$

COMPARING WITH ELECTROMAGNETISM, WE SEE THAT THE WEAK COUPLING CONSTANT, G, TAKES THE PLACE OF BOTH THE $(\text{CHARGE})^2$ AND THE PROPAGATOR. HENCE G HAS DIMENSIONS $\frac{1}{(\text{ENERGY})^2} \sim \frac{1}{(\text{MASS})^2}$.

AN IMPORTANT ADVANCE CAME IN 1956 WHEN LEE AND YANG NOTED THAT THE WEAK INTERACTION VIOLATES PARITY CONSERVATION.

RECALL THAT THE PARITY TRANSFORMATION CONSISTS OF REPLACING ALL POSITION VECTORS $\bar{r}$ BY $-\bar{r}$, LEAVING TIME UNCHANGED. (THIS IS EQUIVALENT TO REFLECTION IN A MIRROR, FOLLOWED BY A ROTATION OF 180° ABOUT AN AXIS ⊥ TO THE MIRROR) THUS

VELOCITY $v = \frac{d\bar{r}}{dt} \to -\bar{v}$; MOMENTUM $\bar{p} \to -\bar{p}$

BUT ANGULAR MOMENTUM $\bar{L} = \bar{r} \times \bar{p} \to (-\bar{r}) \times (-\bar{p}) = \bar{L}$

A PRACTICAL CONSEQUENCE IS THAT IN A PARITY-CONSERVING INTERACTION THERE CAN BE NO NET ANGULAR-MOMENTUM CORRELATION WITH DIRECTION OF MOTION: TERMS LIKE $\langle \bar{L} \cdot \bar{p} \rangle$ MUST VANISH.

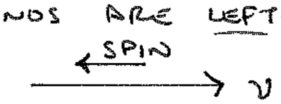
BUT SUCH TERMS TURN OUT TO BE LARGE AFTER A WEAK INTERACTION: WU ET AL (1957). THEY ARE, HOWEVER, COMPLICATED TO MEASURE.

IN 1957 FEYNMAN AND GELL-MANN, AMONG OTHERS, MODIFIED FERMI'S THEORY TO ACCOMODATE PARITY VIOLATION. THEY WROTE, FOR μ DECAY,

$$\text{AMPL} \sim G \, (\nu_\mu | \gamma_\mu (1-\gamma_5) | \mu^-)(e^- | \gamma_\mu (1-\gamma_5) | \bar{\nu}_e)$$

THIS IS THE SO-CALLED V - A INTERACTION WHICH VIOLATES PARITY 'MAXIMALLY'.

THE MOST PROMINENT CONSEQUENCE OF THIS CONCEPT IS THAT NEUTRINOS HAVE ONLY ONE SPIN COMPONENT RATHER THAN 2 AS IS NORMAL FOR SPIN-½ PARTICLES. WE SAY THAT NEUTRINOS ARE LEFT-HANDED. (ANTI-NEUTRINOS ARE RIGHT-HANDED.)



THUS IN $\pi^- \to \mu^- \bar{\nu}_\mu$ FOR π'S AT REST

IS THE ONLY POSSIBLE SPIN ORIENTATION, AS THE π HAS SPIN ZERO.

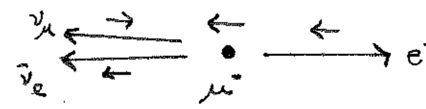
IN THE V-A THEORY, ANY HIGH ENERGY $(E \gg m)$ SPIN-½ PARTICLE CAN HAVE A SIGNIFICANT WEAK INTERACTION ONLY IF ITS SPIN ORIENTATION IS LEFT HANDED.

NOTE THAT IN OUR PICTURE OF $\pi^- \to \mu^- \bar{\nu}$, THE μ^- IS RIGHT HANDED DUE TO ANGULAR-MOMENTUM CONSERVATION. WE INFER THAT THIS DECAY IS ACTUALLY SOMEWHAT SUPPRESSED, AND WOULD BE ALMOST FORBIDDEN IF $M_\pi \gg M_\mu$ (SINCE $E_\mu \sim M_\pi$)

COMPARE THE DECAY $\pi^- \to e^- \bar{\nu}_e$. IN THIS CASE $M_e \ll M_\pi$ BUT $E_e \sim \frac{M_\pi}{2}$ AND INDEED THIS DECAY IS VERY RARE.

IN CONTRAST CONSIDER THE DECAY $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

THE DECAY CONFIGURATION SHOWN AT RIGHT HAS ALL PARTICLES LEFT HANDED, ALL ANTI PARTICLES RIGHT HANDED, AND SO IS NOT SUPPRESSED.



FROM THESE PICTURES WE CAN EXTRACT SOME DIMENSIONAL ESTIMATES FOR THE DECAY RATE :

$$\Gamma = \frac{1}{\tau} \sim |\text{AMPLI}|^2 \qquad \text{so} \qquad \Gamma = G^2 \cdot \text{FACTOR}$$

NOW G HAS DIMENSIONS $\frac{1}{(\text{MASS})^2}$, WHILE Γ HAS DIMENSIONS MASS

THUS THE FACTOR MUST HAVE DIMENSIONS OF $(\text{MASS})^5$

$$\left.\begin{array}{lll} \text{For} & \pi \to \mu \bar{\nu} & \Gamma \sim G^2 M_\pi^{\alpha} M_\mu^{5-\alpha} \\ & \pi \to e \bar{\nu} & \Gamma \sim G^2 M_\pi^{\alpha} M_e^{5-\alpha} \end{array}\right\} \text{ WITH THE SAME } \alpha \text{ IF UNIVERSALITY HOLDS}$$

$$\mu \to e \bar{\nu} \nu \qquad \Gamma \sim G^2 M_\mu^{\beta} M_e^{5-\beta} \qquad (\alpha \neq \beta)$$

WE SAW THAT $\pi \to e \bar{\nu}$ IS SUPPRESSED TO THE EXTENT THAT M_e IS SMALL. AN EDUCATED GUESS IS THAT AMPLI $\sim M_e$ TO PROVIDE THIS SUPPRESSION.

$$\Gamma_{\pi \to e \bar{\nu}} \sim G^2 M_\pi^3 M_e^2$$
$$\Gamma_{\pi \to \mu \bar{\nu}} \sim G^2 M_\pi^3 M_\mu^2$$

[SEE TABLE III, APPENDIX G OF PERKINS FOR THE EXPERIMENTAL FACTS]

THERE IS NO SUPPRESSION IN $\mu \to e \bar{\nu} \nu$, WHICH THEN SUGGESTS

$$\Gamma_{\mu \to e \bar{\nu} \nu} \sim G^2 M_\mu^5$$

A COMPLETE CALCULATION YIELDS $\Gamma_{\mu \to e \bar{\nu} \nu} = \dfrac{G^2 M_\mu^5}{192\,\pi^3}$

COMPARISON OF THIS RESULT WITH EXPERIMENTAL DATA YIELDS THE VALUE

$$\underline{G \sim \frac{10^{-5}}{M_{\text{PROTON}}^2}} \qquad \text{A MEMORABLE FACT}$$

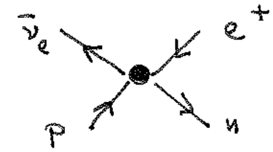
EXERCISE: ESTIMATE THE LIFETIME OF THE NEUTRON: $n \to p e^- \bar{\nu}_e$

EXPERIMENT: $\tau = 925 \pm 11$ SEC

HIGH ENERGY NEUTRINO SCATTERING

IF WE CAN PRODUCE A BEAM OF ANTI-NEUTRINOS, THEN WE CAN INDUCE THE 'INVERSE β-DECAY' REACTION

$$\bar{\nu}_e + p \to n + e^+$$



THE CROSS SECTION FOR THIS GOES LIKE $\sigma \sim G^2 \cdot$ FACTOR

NOW $\sigma \sim \frac{1}{(\text{ENERGY})^2}$; $G^2 \sim \frac{1}{(\text{ENERGY})^4}$ $\Rightarrow$ FACTOR $\sim (\text{ENERGY})^2$

AT HIGH ENERGIES THE ONLY RELEVANT ENERGY IS E_{CM}, AND AS BEFORE $E_{CM}^2 \sim 2 M_p E_\nu$ FOR A PARTON AT REST IN THE LAB.

$$\text{SO} \quad \sigma \sim G^2 M_p E_\nu$$

NUMERICALLY, FOR $E_\nu \sim 1 \text{ GeV} \sim M_p$ $\quad (M_p = 938 \text{ MeV}/c^2)$

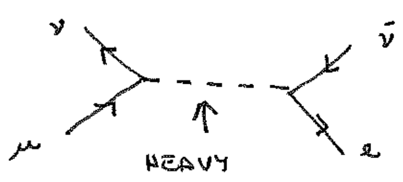
$$\sigma \sim \frac{10^{-10}}{M_p^{\,4}} \cdot M_p^2 \sim \frac{10^{-10}}{M_p^2} \sim 10^{-10}\left(\frac{1}{5}\text{ FERMI}\right)^2 \sim 10^{-38}\text{ cm}^2$$

THIS ESTIMATE IS NOT TOO FAR FROM EXPERIMENT FACT.

NOTE HOWEVER THAT $\sigma \sim E_\nu$, SO AS E_ν RISES THE CROSS SECTION GETS INFINITELY BIG. THIS SEEMS CONTRARY TO THE IDEA OF A CONTACT INTERACTION, AND INDICATES A FUNDAMENTAL "WEAKNESS" OF THE FERMI-FEYNMAN-GELL-MANN THEORY, FIRST POINTED OUT BY HEISENBERG IN 1936.

YUKAWA ALREADY SAW A WAY OUT. WE COULD MAKE THE WEAK INTERACTION APPEAR MORE LIKE THE ELECTROMAGNETIC INTERACTION, WHILE REMAINING SHORT RANGE, IF A HEAVY QUANTUM WERE EXCHANGED

$\mu \to e \bar{\nu} \nu \quad \longrightarrow$



THE HEAVY PROPAGATOR IS $\frac{1}{q^2 - M_{HEAVY}^2} \sim \frac{1}{M_{HEAVY}^2}$ FOR 'LOW ENERGY'

REACTIONS SUCH AS MUON DECAY. THEN WE EXPECT THERE IS ALSO A DIMENSIONLESS WEAK COUPLING CONSTANT $g^2/\hbar c$ SUCH THAT

$$\text{AMPLI} \sim G \sim \frac{g^2}{\hbar c}\,\frac{1}{M_{HEAVY}^2} \qquad (\text{LOW ENERGY LIMIT})$$

UNIFICATION OF THE WEAK AND ELECTROMAGNETIC INTERACTIONS

THE IDEA OF WEINBERG AND SALAM (1967) IS THAT $g = e$, THAT THE BASIC COUPLING STRENGTH OF THE ELECTROMAGNETIC & WEAK FORCE IS THE SAME, ALTHOUGH THE PROPAGATORS AND VERTEX SPIN FACTORS HAVE SOME DIFFERENCES. (ACTUALLY, SINCE THE ν HAS NO ELECTRIC CHARGE, BUT DOES INTERACT WEAKLY, THE INTRODUCED A NEW CHARGE, g', FOR NEUTRINOS. NUMERICALLY $g' \sim e$.)

THEN $$G = \frac{10^{-5}}{M_p^2} = \frac{e^2}{\hbar c}\frac{1}{M_{HEAVY}^2} = \frac{1}{137}\frac{1}{M_{HEAVY}^2}$$ SO $M_{HEAVY} \sim 30\, M_p$

ACTUALLY WEINBERG & SALAM PREDICTED A COMPLICATION: THERE SHOULD BE BOTH CHARGED AND NEUTRAL HEAVY QUANTA, RELATED BY AN ADDITIONAL PARAMETER, THE 'WEINBERG ANGLE Θ_W' (WHICH RELATES THE TWO CHARGES g AND g')

$$M_{W^\pm} = \frac{37\,\text{GeV}}{\sin\Theta_W} \qquad M_{Z^0} = \frac{M_{W^\pm}}{\cos\Theta_W}$$

THE $W^\pm$ ARE THE CHARGED QUANTA WHICH CARRY THE WEAK FORCE OF FERMI'S THEORY. THE Z^0 IS A HEAVY PHOTON, WHICH CARRIES A NEW FORM OF THE WEAK FORCE, THE 'NEUTRAL CURRENT.' THIS LAST EFFECT WAS FIRST OBSERVED IN 1973.

DURING THE '70'S THE WEINBERG ANGLE WAS MEASURED IN VARIOUS WAYS TO BE $$\Theta_W \sim .45 - .5 \quad \Rightarrow \quad M_W \sim 80\,\text{GeV}, \quad M_Z \sim 90\,\text{GeV}$$

THIS IS ALMOST EXACTLY THE MASSES FOUND BY RUBBIA ET AL IN 1982-83 WHEN THE $W^\pm$ AND Z^0 WERE FIRST OBSERVED DIRECTLY. THEY TOOK ADVANTAGE OF THE DIVERGENT NATURE OF THE PROPAGATOR: $\dfrac{1}{q^2 - M^2}$. WHEN THE C.M. ENERGY OF A PROCESS LIKE $u + \bar{d} \to \mu^+ \nu$ REACHES M_W THE CROSS-SECTION GETS VERY LARGE (- THO NOT ACTUALLY INFINITE AS THE W HAS A SHORT LIFETIME $\Rightarrow$ FINITE SPREAD OF MASSES $\Rightarrow$ 'RESONANCE' BEHAVIOR OF THE PROPAGATOR AS INDICATED IN LECTURE 1)

NOW THAT THE W MASS IS KNOWN WE MAY CALCULATE THE CHARACTERISTIC RANGE OF THE WEAK INTERACTION: $$r \sim \frac{1}{M_W} \sim \frac{1}{100\,M_P} \sim 2 \times 10^{-16}\ \text{cm}$$

HENCE FERMI'S IDEA OF A CONTACT INTERACTION IS QUITE GOOD FOR NUCLEAR PROCESSES WITH SCALE 10^{-13} cm.

THE UNIFICATION OF THE WEAK AND ELECTROMAGNETIC INTERACTIONS IS A FIRST LARGE STEP TOWARDS FULFILLING EINSTEIN'S VISION OF THE UNITY OF ALL FORCES IN NATURE.

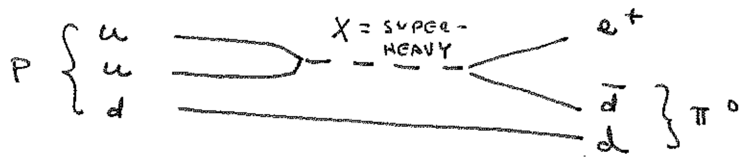
Ph 529 Lecture 3

Grand Unification

Following the success of the Weinberg-Salam model, Georgi & Glashow (1974) suggested a unification of the strong, weak and electromagnetic interactions. This might occur at extremely high energies when the exchange of a super-heavy object could provide direct coupling of leptons to quarks.

This theory has 2 accessible predictions:

- $\Theta_{\text{Weinberg}} \sim 1/2$, which is nearly true
- Protons can decay, with a mean lifetime of 10^{31-33} years



$$P \to e^+ \pi^0$$

Thus far there is no clear evidence for proton decay, with a limit of about a 10^{32} year lifetime. But watch the newspapers for updates.

We can give a very approximate sense of the grand unification argument.

We mentioned in lecture 2 that the strong force carried by the gluons obeys 'asymptotic freedom.' Roughly, if one includes higher order gluon exchange (multiple-gluon effects), the effect is to weaken the apparent strength of the 1 gluon process! The higher the energy, the weaker the interaction.

If q = characteristic energy of the process, the strength then depends on q:

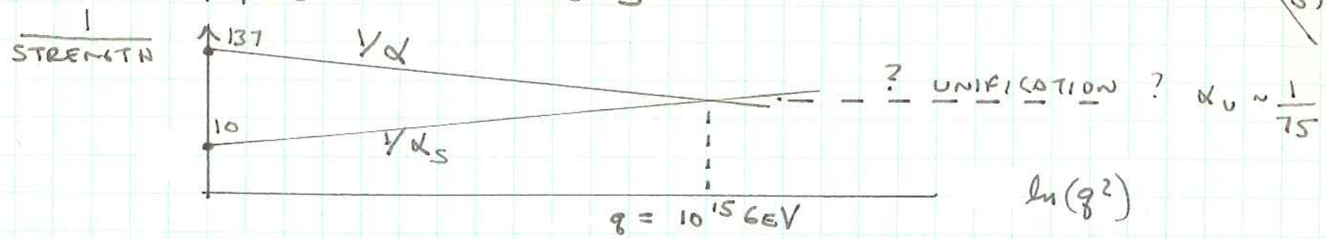
$$\frac{1}{\alpha_s} \approx 10 + \ln\left(q^2/(10\,\text{GeV})^2\right) \qquad \text{A } \underline{\text{very}} \text{ slow change}$$

Likewise, even the electromagnetic interaction is subject to an apparent increase of strength at very high energies:

$$\frac{1}{\alpha} \approx 137 - \ln\left(q^2/(10\,\text{GeV})^2\right)$$

We say that at high energies we probe deeper into the charge distribution and see more of the 'bare' charge, which is larger than the apparent charge at low energies.

If we plot the strong and electromagnetic coupling constants as a function of energy, they eventually cross



TO HAVE $1/\alpha_s(q^2) = 1/\alpha(q^2)$ EACH MUST CHANGE BY $\sim$ 65 UNITS FROM THEIR VALUES AT $q \sim 10$ GeV. THIS SUGGESTS THAT AT $q \sim 10^{15}$ GeV THE INTERACTIONS HAVE EQUAL STRENGTH: $\alpha = \alpha_s \sim 1/75 \equiv \alpha_U$

PERHAPS THE EVENTUAL EQUALITY OF THE INTERACTION STRENGTHS IS ACTUALLY A MANIFESTATION OF THEIR BASIC UNITY. THIS IN TURN SUGGESTS THE EXISTENCE OF THE X PARTICLE, SKETCHED ON p 38, WHICH MIGHT HAVE MASS $\sim 10^{15}$ GeV.

THEN THE PROTON DECAY RATE $\Gamma \sim \text{MASS} \sim \alpha_u^2 \dfrac{M_p^5}{M_X^4} \sim 10^{-64}$ GeV BY OUR DIMENSIONAL ARGUMENTS.

SO $\tau = \frac{1}{\Gamma}$ = LIFETIME = 10^{64} GeV^{-1} $\sim 10^{40}$ SEC $\sim 10^{33}$ YEAR USING 1 SEC $\approx \dfrac{1}{10^{-24}\text{ GeV}}$ AND 1 YEAR $\sim \pi \times 10^7$ SEC.

GRAVITY

THE GRAVITATIONAL FORCE $F = \dfrac{GMM'}{r^2}$ HAS A COUPLING CONSTANT WITH DIMENSIONS $G \sim \dfrac{10^{-38}}{M_p^2}$ (i.e. NEWTON'S CONSTANT CONVERTED TO OUR UNITS!) IN A UNIFIED VIEW, FOLLOWING THE ABOVE PATTERN, WE MIGHT EXPECT

$$G \sim \left(\frac{g^2}{\hbar c}\right) \frac{\hbar c}{M^2}$$

WHERE $g^2/\hbar c \approx 1$ IS THE TRUE DIMENSIONLESS COUPLING CONSTANT.

THEN $M \sim 10^{19} M_p$ = PLANCK MASS

(AFTER PLANCK WHO FIRST NOTED THE SPECIAL INTEREST OF $M \equiv \sqrt{\frac{\hbar c}{G}}$ IN 1905!)

IT IS FERVENTLY BELIEVED BY ALL THAT WHEN WE REACH ENERGIES OF $10^{19} M_p$ IN LABORATORY REACTIONS OF FUNDAMENTAL PARTICLES THAT WE WILL AT LAST UNDERSTAND HOW TO UNIFY ALL 4 FUNDAMENTAL INTERACTIONS. THE RECENT 10-DIMENSIONAL STRING THEORIES MAY GIVE INSIGHT AS TO HOW THIS WILL HAPPEN...