Neutral Kaon Decays and CP Violation

Along with the discovery of parity non-conservation in 1956, another milestone in weak-interaction physics is the discovery in 1964 of CP violation in K^0 decays. At the outset we remark that CP violation has a poorly understood role in the nature of things. It is only a small correction to the weak interaction which remains CP conserving in most instances. This contrasts with parity violation which was found to be 'maximal' - i.e. no right-handed couplings at all. Even today there is no known cause for CP violation. It is imagined that it gives us a view of some very-high-energy phenomenon which is otherwise invisible in all other presently accessible phenomena. Theories of the very-high-energy regime such as grand unification still have trouble explaining CP violation. In a 'practical' sense, CP violation gives us a clue as to the apparent T non-invariance of the universe. Also it allows us to tell the difference between matter and antimatter in a way that is independent of terms like + and - , or left and right (as discussed in Sec. 6 below).

Well before CP violation was discovered it was realized that the K^0-$\bar{K}^0$ system has many peculiar and interesting features. Before discussing CP violation we briefly describe strangeness oscillations, and regeneration.

References: Lee & Wu, 'Decays of Neutral K Mesons' Ann. Rev. Nuc. Sci. 16, 511 (1966)
Kleinknecht, 'CP Violation and K^0 Decays', Ann. Rev. Nuc. Sci. 26, 1 (1976)
Cronin, Rev. Mod. Phys. 53, 373 (1981).

1. Strangeness Oscillations

In 1952 Pais [P.R. 86, 663 (1952)] invented the idea of strangeness to explain the fact that K's & Λ's always seemed to be produced together in reactions like $pp \to p\Lambda K^+$, $\pi^- p \to \Lambda K^0$. In Pais' view, the $\bar{K}^0$ has opposite strangeness from the K^0, and so must be a distinct particle. It was clear from the start that strangeness is not conserved in the weak interaction, so that as $K^0 \to \pi^+\pi^-$ we would also expect $\bar{K}^0 \to \pi^+\pi^-$. It is rather odd (not to say strange) that $K^0 \neq \bar{K}^0$ but they both decay the same way. Indeed Fermi objected to the whole strangeness scheme because of this. The skepticism of Fermi led Gell-Mann & Pais [P.R. 97, 1387 (1955)] to a remarkable deduction.

If $K^0 \to 2\pi$ and also $\bar{K}^0 \to 2\pi$ via the weak interaction, then one can imagine the 2nd order process $K^0 \leftrightarrow 2\pi \leftrightarrow \bar{K}^0$.

Hence we have a 2 state system with a non-zero transition amplitude between the states. Then we know from quantum mechanics that the K^0 and $\bar{K}^0$ are not the eigen states of the system.

RATHER THERE WILL BE A PAIR OF LINEAR COMBINATIONS OF THE
K^0 AND $\bar{K}^0$ WHICH ACTUALLY HAVE DEFINITE LIFETIMES. THESE
ARE CALLED $K^0_{SHORT} = K^0_S$ AND $K^0_{LONG} = K^0_L$

THESE STATE WILL HAVE DIFFERENT MASSES WHERE

$$\Delta M \sim \text{AMPLITUDE} \left(K^0 \to 2\pi \to \bar{K}^0\right) \sim G^2 M_K^5 \sin^2 \theta_C$$

(WE PURSUE THIS IN MORE DETAIL IN SECS 5 AND 7 BELOW)

WE NOTE THAT $\dfrac{1}{\tau_{SHORT}} \approx \Gamma_{K^0 \to 2\pi} \sim \langle 2\pi | \text{WEAK} | K^0 \rangle^2 \sim \langle \bar{K}^0 | \overset{2ND}{ORDER} | K^0 \rangle \sim \Delta M$

SO THAT WE CAN ANTICIPATE $\Delta M \tau_{SHORT} \sim 1$

BY INVOKING CP INVARIANCE WE CAN MAKE A SHORT CUT IN THE
DERIVATION OF THE EXPRESSIONS FOR K^0_S AND K^0_L IN TERMS OF K^0 AND $\bar{K}^0$.
THIS WAS ALREADY SKETCHED ON P. 309

WITH $P|K^0\rangle = -|\bar{K}^0\rangle$ AND $C|K^0\rangle = -|\bar{K}^0\rangle$ WE HAVE $CP|K^0\rangle = +|\bar{K}^0\rangle$
THUS IF WE DEFINE $|K^0_1\rangle = \dfrac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}$ AND $|K^0_2\rangle = \dfrac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$

THEN $CP|K^0_1\rangle = +|K^0_1\rangle$ AND $CP|K^0_2\rangle = -|K^0_2\rangle$

WE IDENTIFY $K^0_S = K^0_1$ SO THAT IT CAN DECAY INTO 2π WHICH
HAS CP $= +1$ WHEN $\ell_\pi = 0$. ALL THIS ASSUMES CP CONSERVATION
IN THE WEAK INTERACTION — WHICH HOLDS FOR THE V-A THEORY.
THEN WITH $K^0_L = K^0_2$, IT CAN DECAY TO 3π BUT NOT 2π, AND WILL
CLEARLY HAVE THE LONGER LIFETIME DUE TO THE LACK OF PHASE SPACE
FOR THE 3π DECAY.

NOW THINGS BEGIN TO GET INTERESTING. SUPPOSE WE PRODUCE A
K^0 IN A STRONG INTERACTION SUCH AS $\pi^- p \to K^0 \Lambda$. BEFORE DISCUSSING
ITS DECAY WE MUST CONVERT THE K^0 TO THE APPROPRIATE STATE OF
K^0_S AND K^0_L, AS THE LATTER ARE THE EIGENSTATES OF THE WEAK
INTERACTION WHICH GOVERNS THE DECAY. NOW

$$|K^0\rangle = \frac{|K^0_S\rangle + |K^0_L\rangle}{\sqrt{2}}$$

AS THE $|K^0_S\rangle$ STATE DECAYS AWAY WITH TIME WE CAN DESCRIBE
ITS WAVE FUNCTION AS $\psi_S(t) = \psi_S(0)\, e^{-iM_S t}\, e^{-\frac{\Gamma_S}{2} t}$ $\Gamma_S = \dfrac{1}{\tau_S}$

SO THAT THE PROBABILITY OF STILL HAVING A K^0_S AT TIME t
IS $P(t) = P_S(0)\, e^{-\Gamma_S t}$

LIKEWISE $\psi_L(t) = \psi_L(0)\, e^{-iM_L t}\, e^{-\frac{\Gamma_L}{2} t}$

THUS $\psi_{K^0}(t) = \frac{1}{\sqrt{2}} \left(e^{-im_S t} e^{-\frac{\Gamma_S}{2} t} + e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} \right)$

SO THAT THE PROBABILITY THAT A K^0 REMAINS AT TIME t IS

$$P_{K^0}(t) = \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2 e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \cos \Delta m t \right]$$

THIS SHOWS HOW THE K^0 DOES NOT HAVE A DEFINITE LIFETIME!

EVEN MORE BIZARRE IS THAT WHILE THE K^0 IS DYING AWAY, THERE IS SOME PROBABILITY THAT IT HAS TURNED INTO A $\bar{K}^0$. NAMELY

$|\bar{K}^0\rangle = \frac{|K_S^0\rangle - |K_L^0\rangle}{\sqrt{2}}$ SO $\psi_{\bar{K}^0}(t) = \frac{1}{\sqrt{2}} \left(e^{-im_S t} e^{-\frac{\Gamma_S}{2} t} - e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} \right)$

AND SO $P_{\bar{K}^0}(t) = \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \cos \Delta m t \right]$

WE CAN NOW ANSWER FERMI'S QUESTION AND SHOW THAT THERE IS EXPERIMENTAL MEANING TO THIS ANALYSIS. A K^0 PRODUCED IN THEN REACTION $\pi^- P \to K^0 \Lambda$ WILL BE MOVING WITH VELOCITY $v \sim c$ (IF $E_K \gtrsim 1 \text{GeV}$). SO WE CAN PLACE MATTER IN THE PATH OF THE KAON AT VARIOUS DISTANCES FROM THE PRODUCTION POINT, THEREBY SAMPLING ITS HISTORY IN TIME. IF A K^0 IS STILL PRESENT WE MIGHT OBSERVE REACTIONS LIKE

$K^0 P \to K^+ n$

Figure 7.3. Strangeness oscillations and regeneration for K^0 and $\bar{K}^0$.

BUT IF A $\bar{K}^0$ IS PRESENT WE CAN HAVE $\bar{K}^0 P \to \Lambda^0 \pi^+$, WHICH IS COMPLETELY IMPOSSIBLE IF ONLY K^0'S REMAIN. THE LATTER REACTION WAS INDEED OBSERVED IN A BUBBLE CHAMBER, WITH INCIDENT π^- BEAM, IN 1956.

FURTHERMORE, IF ONE MEASURES THE NUMBER OF $\bar{K}^0$ AS A FUNCTION OF TIME, ONE CAN DETERMINE $|\Delta m|$, BY DIRECT OBSERVATION OF THE 'STRANGENESS OSCILLATION'. THE FIRST EXPERIMENT OF THIS TYPE TO PERFORM A SIGNIFICANT MEASUREMENT OF $|\Delta m|$ WAS BY FITCH ET AL [NUOVO CIMENTO $\underline{22}$, 1160 (1961)].

2. REGENERATION

SHORTLY AFTER THE STRANGENESS OSCILLATION ANALYSIS, PAIS & PICCIONI [P.R. $\underline{100}$, 1487 (1955)] NOTED ANOTHER UNUSUAL POSSIBILITY. SUPPOSE WE PRODUCE A BEAM OF K^0 PARTICLES AND THEN WAIT UNTIL THE K_S^0 COMPONENT HAS DIED AWAY, SAY AFTER $\sim 10 \tau_S$. WE ARE THEN LEFT WITH A K_L^0 BEAM OF $\frac{1}{4}$ THE ORIGINAL INTENSITY ($\tau_L \sim 600 \tau_S$; SEE ABOVE FIGURE).
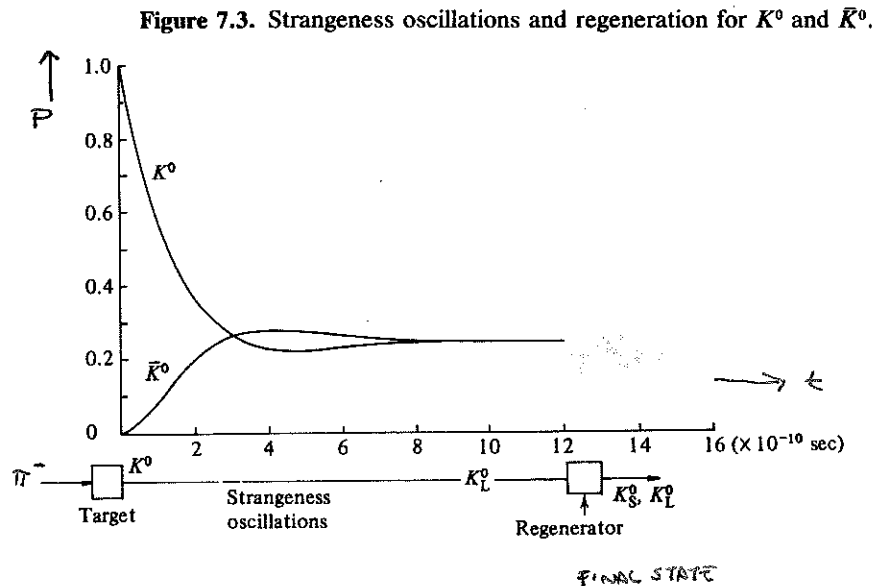
If this beam strikes a target, strong interactions can occur. For this it is appropriate to again use a description in terms of K^0 and $\bar{K}^0$ rather than K^0_L and K^0_S, as strangeness is conserved in the strong interaction. Because the $\bar{K}^0$ has the same strangeness as the $\bar{\Lambda}^0$, it interacts with matter more readily than the K^0. So the $\bar{K}^0$ component of the beam is attenuated more than the K^0 component as the beam passes thru matter. The wave function of the unscattered beam particles is no longer

$$|K^0_L\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} \quad \text{but rather} \quad \frac{f|K^0\rangle - \bar{f}|\bar{K}^0\rangle}{\sqrt{2}}$$

where f and $\bar{f}$ are the amplitudes for a K^0 and $\bar{K}^0$ to pass thru the matter without scattering. The unscattered beam now consists of

$$\frac{f+\bar{f}}{2}\frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} + \frac{f-\bar{f}}{2}\frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} = \frac{f+\bar{f}}{2}|K^0_L\rangle + \frac{f-\bar{f}}{2}|K^0_S\rangle$$

Once the beam has emerged from the absorbing matter, we can again observe $K^0_S \to \pi\pi$ decays. We say that a K^0_S component of the beam has been __REGENERATED__
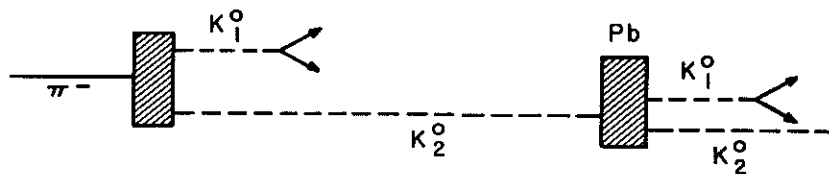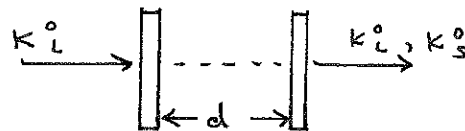

Figure 10.3. Regeneration of K^0_1

Good [P.R. 110, 550 (1958)] was the first to realize that the regeneration phenomenon can be used to measure $\Delta m = m_L - m_S$. We discuss a variation due to Fitch.

A K^0_L beam impinges on 2 thin regenerator slabs separated by a distance d. The measurement consists of counting the number of $K \to \pi^+\pi^-$ decays observed after the second slab, as a function of gap size d.

It is simpler in the following to think of d as the proper time needed for a kaon to cross the gap. Then the amplitude for a K^0_S to emerge from the 2nd slab is

$$\rho_1 e^{-im_S d} e^{-\frac{\Gamma_S}{2}d} + \rho_2 e^{-im_L d} e^{-\frac{\Gamma_L}{2}d} \qquad \text{if } \rho_i \ll 1$$

where $\rho_i = $ (complex) amplitude to regenerate a K^0_S in slab i

The exponential factors indicate that the K^0_S component produced in the 1st slab can decay before emerging from the 2nd slab; and

THAT THE K_L^0 COMPONENT CAN DECAY WHILE CROSSING THE GAP. THEN

$$P_{K_s^0}(d) = |\rho_1|^2 e^{-\Gamma_s d} + |\rho_2|^2 e^{-\Gamma_L d} + 2|\rho_1||\rho_2| e^{-\frac{\Gamma_s+\Gamma_L}{2}d} \cos(\Delta m d + \phi_1 - \phi_2)$$

ϕ_i = PHASE OF REGENERATION AMPLITUDE ρ_i

OVER THE YEARS VERY PRECISE MEASUREMENTS OF Δm HAVE BEEN MEASURED WITH THIS TECHNIQUE, WHICH IS A CONCEPTUAL VARIATION ON THE DOUBLE SLIT INTERFERENCE OF LIGHT.
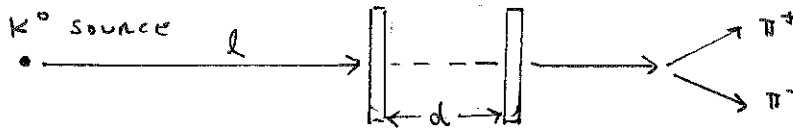
RECENT RESULTS:
$$|\Delta m| \tau_s = 0.477 \pm .003$$
$$|\Delta m|' = 3.521 \pm .003 \times 10^{-6} eV$$
$$\frac{|\Delta m|}{M_K} \approx 0.7 \times 10^{-14}$$

THE LAST RATIO IS CERTAINLY THE SMALLEST DIMENSIONLESS QUANTITY MEASURED IN HIGH ENERGY PHYSICS.

THE SIGN OF Δm HAS BEEN DETERMINED BY ANOTHER VARIATION ON THE REGENERATION TECHNIQUE, BY USING A K^0 BEAM.

TWO BLOCKS OF MATTER ARE PLACED RELATIVELY CLOSE TO A K^0 SOURCE, SO THAT BOTH K^0_S AND K_L^0 STRIKE THE BLOCK. THE EFFECT IS BASED ON THE INTERFERENCE BETWEEN THE K_S^0 COMPONENT OF THE BEAM, AND THE K_S^0 COMPONENT REGENERATED FROM K_L^0 IN THE TWO REGENERATORS.

TO DETERMINE THE SIGN OF Δm WE MUST KNOW THE PHASE OF THE STRONG INTERACTION AMPLITUDE $f - \bar{f}$ WHICH IS RESPONSIBLE FOR THE REGENERATION OF THE K_S^0. EVEN THE ALGEBRA NEEDED TO KEEP TRACK OF ALL THIS IS SOMEWHAT COMPLICATED, BUT THE EXPERIMENTS SHOW THAT

$$M_L > M_S$$

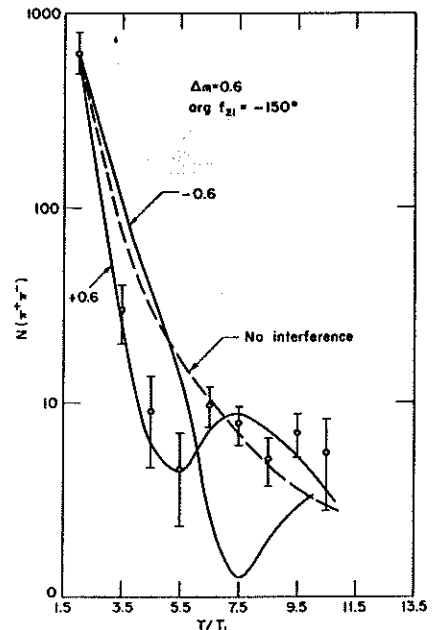[JOVANOVICH ET AL P.R.L. 17, 1075 (1966)]

[MEHLHOP ET AL, P.R. 172, 1613 (1968)]

FIG. 17. Distribution of $N(\pi^+\pi^-)$ versus T/T_1 or $D/A+1.9$ where D is the distance from the target to the regenerator and A is the mean decay length of K_1^0. [Piccioni (58) and (84).]

EXERCISE: SHOW RATE $\to \pi^+\pi^- \sim e^{-\Gamma_s(d+\ell)}$
$$+|\rho|^2\left[e^{-(\Gamma_L \ell + \Gamma_s d)} + e^{-\Gamma_L(\ell+d)}\right] + 2|\rho|^2 e^{-\Gamma_L \ell - \frac{\Gamma_L+\Gamma_s}{2}} \cos(\Delta m d + \phi)$$
$$+ 2|\rho| e^{-\Gamma_s d - \frac{\Gamma_s+\Gamma_L}{2}\ell} \cos(\Delta m \ell + \phi) + 2|\rho| e^{-\frac{\Gamma_s+\Gamma_L}{2}(d+\ell)} \cos(\Delta m(\ell+d) + \phi)$$

TAKING $\rho_1 = \rho_2 = \rho$. THE SIGN OF Δm CAN BE FIXED COMPARING THE 3 INTERFERENCE TERMS.

3. Discovery of CP Violation

In 1964 Christenson et al [P.R.L. 13, 138 (1964)] and Abashian et al [P.R.L. 13, 243 (1964)] observed the decay $K^0_L \to \pi^+ \pi^-$, which is forbidden by CP invariance. Christenson et al measured

$$\frac{\Gamma_{K^0_L \to \pi^+ \pi^-}}{\Gamma_{K^0_L \to ALL}} = 2.0 \pm 0.4 \times 10^{-3}$$

This result agrees well with the best value now measured of $2.03 \pm .05 \times 10^{-3}$, which shows the quality of the initial measurement. Of course one must be very careful that the observed $\pi^+ \pi^-$ are not due to the CP conserving process of regeneration of K^0_s via K^0_L interactions with matter. (Paradox: is $K^0_L \to K^0_s$ regeneration really CP conserving?)

The decay $K^0_L \to \pi^+ \pi^-$ is sometimes termed 'vacuum regeneration' under the supposition that the K^0_L turned into a K^0_s even in the absence of any absorber. (For this to be possible the K^0_L and K^0_s states cannot be orthogonal, as discussed in sec. 5 below). The vacuum regeneration phenomenon allows many further variations on the already intricate subject of regeneration, which has led to many beautiful experiments. We sketch briefly the formalism used in some of these endeavors.

We start at $t=0$ with a general neutral kaon state

$$a_S |K^0_s\rangle + a_L |K^0_L\rangle$$

and observe the rate of decay to $\pi^+ \pi^-$ as a function of time. The amplitude to decay at time t is

$$a_S e^{-i M_s t} e^{-\frac{\Gamma_s}{2} t} \langle \pi^+ \pi^- | weak | K^0_s\rangle + a_L e^{-i M_L t} e^{-\frac{\Gamma_L}{2} t} \langle \pi^+ \pi^- | weak | K^0_L\rangle$$

We introduce the CP violation parameter $\eta_{+-} \equiv \dfrac{\langle \pi^+ \pi^- | weak | K^0_L\rangle}{\langle \pi^+ \pi^- | weak | K^0_s\rangle}$

Then decay amplitude $\sim a_S e^{-i M_s t} e^{-\frac{\Gamma_s}{2} t} + \eta_{+-} a_L e^{-i M_L t} e^{-i \frac{\Gamma_L}{2} t}$

And decay rate $\approx |a_S|^2 e^{-\Gamma_s t} + |\eta_{+-} a_L|^2 e^{-\Gamma_L t} + 2 |a_S| |a_L| |\eta_{+-}| e^{-\frac{\Gamma_s + \Gamma_L}{2} t} \cos(\Delta m t + \phi)$

where ϕ = phase of $(a_S \eta^*_{+-} a^*_L) = \phi_{a_S} - \phi_{\eta_{+-}} - \phi_{a_L}$

For example, in the original experiment of Christenson et al they had a K^0_L beam $\Rightarrow a_S = 0$, $a_L = 1$

Then $|\eta_{+-}|^2 = \dfrac{\Gamma_{K^0_L \to \pi^+ \pi^-}}{\Gamma_{K^0_L \to ALL}} \cdot \dfrac{\Gamma_{K^0_L \to ALL}}{\Gamma_{K^0_s \to \pi^+ \pi^-}} = 2 \times 10^{-3} \cdot 2.5 \times 10^{-3} \sim 5 \times 10^{-6}$

↑ known from earlier studies

$\Rightarrow |\eta_{+-}| \sim 2.2 \times 10^{-3}$

IF INSTEAD ONE STARTS WITH A K^0 BEAM AND OBSERVES $\pi^+\pi^-$ DECAYS BEFORE THE K_S'S COMPONENT HAS DIED AWAY, MORE INFORMATION CAN BE OBTAINED. IN THIS CASE $a_S = 1 = a_L$ AT $t = 0$, AND

$$\text{RATE} \to \pi^+\pi^- \approx e^{-\Gamma_S t} + 2|\eta_{+-}| e^{-\frac{\Gamma_S t}{2}} \cos(\Delta m t - \phi_{+-})$$

IGNORING $|\eta_{+-}|^2$, AND SETTING $e^{-\Gamma_L t} \sim 1$ FOR SMALL t.

THE FIRST EXPERIMENT OF THIS TYPE WAS THAT OF FITCH ET AL [P.R.L. 15, 73 (1965)]. THIS DEMONSTRATED THAT THE $\pi^+\pi^-$ PRODUCED BY THE CP VIOLATING DECAYS OF K^0_L REALLY DO INTERFERE WITH $K^0_S \to \pi^+\pi^-$, A NON-TRIVIAL RESULT OF QUANTUM MECHANICS.

THE MOST ACCURATE EXPERIMENT OF THIS TYPE IS THAT OF GEWENIGER ET AL [PHYS. LETT B48, 483 (1974), SEE FIG →] WHO MEASURED

$$|\eta_{+-}| = 2.30 \pm .035 \times 10^{-3}$$

$$\phi_{+-} = 45.9 \pm 1°$$
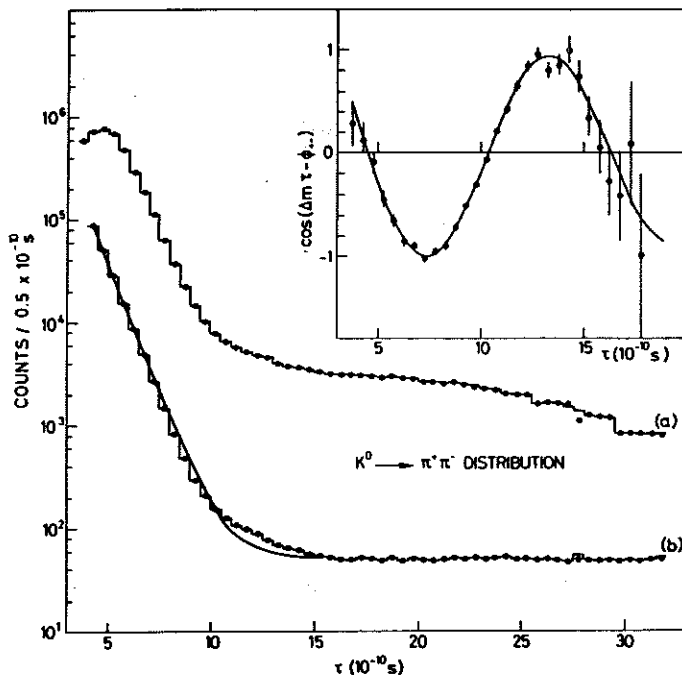
Figure 10 Time distribution of $K \to \pi^+\pi^-$ events from a coherent mixture of K_L and K_S produced in pure strangeness states (77). (a) Events (histogram) and fitted distribution (dots). (b) Events corrected for detection efficiency (histogram), fitted distribution with interference term (dots), and without interference term (curve). Insert: interference term as extracted from data (dots) and fitted term (line).
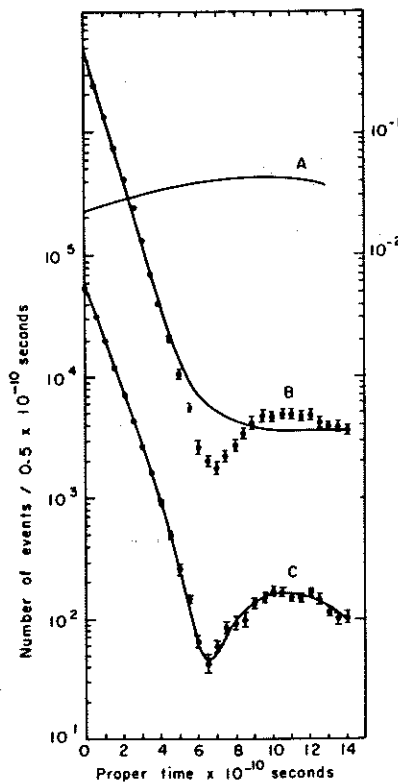
BECAUSE η_{+-} IS SO SMALL THE INTERFERENCE EFFECT IS SMALL. THE INTERPRETATION OF THE DATA IS VERY SIMPLE HOWEVER.

THE INTERFERENCE EFFECT CAN BE ENHANCED AT EXPENSE OF CRISPNESS OF INTERPRETATION. ONE STARTS WITH A K^0_L BEAM ($a_L = 1$) AND PASSES IT THRU A THIN REGENERATOR, INDUCING A K^0_S COMPONENT OF SIZE $a_S = \rho$ CHOSEN $\approx \eta_{+-}$. THEN SETTING $t = 0$ AS THE BEAM EMERGES FROM THE REGENERATOR WE FIND

$$\text{RATE} \to \pi^+\pi^- \sim |\rho|^2 e^{-\Gamma_S t} + |\eta_{+-}|^2$$

$$+ 2|\rho||\eta_{+-}| e^{-\frac{\Gamma_S t}{2}} \cos(\Delta m t - \phi_{+-} + \phi_\rho)$$

DETAILS OF THE STRONG INTERACTION PARAMETERS $|\rho|$ AND ϕ_ρ MUST BE KNOWN TO COMPLETE THE ANALYSIS. CARRITHERS ET AL [P.R.L. 34, 1240 (1975); SEE FIG →] FIND $\phi_{+-} = 45.8 \pm 2.8°$ BY THIS TECHNIQUE.

Figure 8 Time distribution of $K \to \pi^+\pi^-$ decays behind a carbon regenerator (71). (a) Detection efficiency for kaon momentum interval $5 < p_K < 6$ GeV/c. (b) Efficiency-corrected data summed over p_K; curve shows fit without interference. (c) Data for $5 < p_K < 6$ GeV/c with fit including interference.

4. CP VIOLATION IN $K_L^0 \to \pi^0 \pi^0$

Both K^0 and $\bar{K}^0$ decay to $\pi^0 \pi^0$, so we also expect evidence for CP violation in the form of the decay $K_L^0 \to \pi^0 \pi^0$. This is much harder to demonstrate experimentally as the π^0's decay to 2γ very quickly. Nonetheless several experiments have succeeded in analyzing the 4γ final state. To describe the results we define the CP violating parameter

$$\eta_{00} \equiv \frac{\langle \pi^0 \pi^0 | \text{weak} | K_L^0 \rangle}{\langle \pi^0 \pi^0 | \text{weak} | K_S^0 \rangle}$$

The best determination of η_{00} comes from 2 experiments which use the regeneration technique and a K_L^0 beam. They found

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right| \approx \begin{cases} 1.00 \pm .06 & [\text{Holder et al, Phys. Lett. } \underline{B40}, 141 \ (1972)] \\ 1.03 \pm .07 & [\text{Banner et al, P.R.L. } \underline{28}, 1597 \ (1972)] \end{cases}$$

The phase angle ϕ_{00} has been measured in a vacuum regeneration type experiment, with a K^0 beam, Christenson et al [P.R.L. $\underline{43}$, 1209, 1212 (1979)] observed both $\pi^+ \pi^-$ and $\pi^0 \pi^0$ decays in the same experiment. They found

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right| \approx 1.00 \pm .09$$

$$\phi_{+-} = 41.7 \pm 3.5°$$

$$\phi_{00} = 55.7 \pm 5.8°$$

Very recent results indicate

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right| = \begin{cases} .985 \pm .015, & \text{Bernstein et al} \\ & \text{PRL } \underline{54}, 1631 \ (1985). \\ 1.006 \pm .024, & \text{Black et al,} \\ & \text{P.R.L. } \underline{54}, 1628 \ (1985). \end{cases}$$

Figure 7.7. Intensity of $\pi^+ \pi^-$ and $\pi^0 \pi^0$ as a function of proper time τ. The exponential decay is modified by a sinusoidal term shown in the lower half of the diagrams. The curve through the lower data is a best fit to $|\eta|$ and ϕ. (From Christenson et al. 79. Reprinted with permission.)
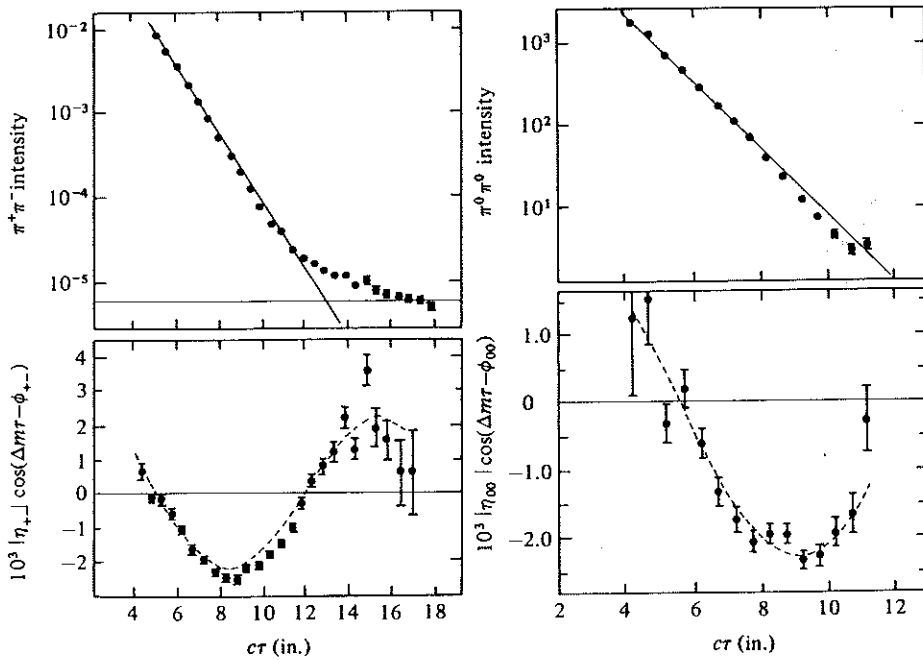
5. FURTHER PARAMETRIZATION OF CP VIOLATION

We now categorize the nature of the observed CP violation in somewhat more detail. An immediate question is whether CP violation implies CPT violation, T violation or both. We will not examine this question in these notes, but simply state that the observed CP violation is experimentally consistent with CPT invariance. We then infer that T violation does occur, although evidence for this is indirect. For details, refer to the review of Lee and Wu mentioned on p. 326.

WE NOW RETURN TO THE QUESTION OF THE RELATION BETWEEN
THE STRONG INTERACTION EIGENSTATES K^0 AND $\bar{K}^0$, AND THE WEAK
INTERACTION EIGENSTATES K^0_L AND K^0_S.   WE RELAX THE ASSUMPTION
OF CP INVARIANCE WHICH SIMPLIFIED THE PREVIOUS DISCUSSION ON P. 326.

WE CONSIDER A STATE $\psi = \alpha |K^0\rangle + \beta |\bar{K}^0\rangle$ PRODUCED IN A STRONG
INTERACTION. NOW WE FOLLOW ITS SUBSEQUENT HISTORY AS GOVERNED BY
THE WEAK INTERACTION. THE SCHRÖDINGER EQUATION TELLS US

$$i \frac{\partial \psi}{\partial t} = H \psi \qquad \text{WHERE THE HAMILTONIAN } H \text{ IS A } 2 \times 2 \text{ MATRIX}$$

$$H = \begin{pmatrix} \langle K^0|H|K^0\rangle & \langle K^0|H|\bar{K}^0\rangle \\ \langle \bar{K}^0|H|K^0\rangle & \langle \bar{K}^0|H|\bar{K}^0\rangle \end{pmatrix} \equiv \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

BECAUSE THE KAONS CAN DECAY, H HAS THE FORM $M - \frac{i\Gamma}{2}$ WHERE

M AND Γ ARE HERMITIAN MATRICES: $M_{12} = M_{21}^*$ ETC (WIGNER-WEISSKOPF, 1932)
M IS USUALLY CALLED THE MASS MATRIX; Γ IS THE DECAY MATRIX.

WE SEEK THE EIGENSTATES OF H, I.E. THE STATES K^0_L AND K^0_S WHICH
HAVE DEFINITE LIFETIMES. TO BE BRIEF WE ASSUME THAT CPT INVARIANCE
HOLDS, SO THAT K^0 AND $\bar{K}^0$ ARE INDEED ANTIPARTICLES. THEN WE HAVE
$H_{11} = H_{22}$; OTHERWISE $M_{K^0} \neq M_{\bar{K}^0}$.   WE THEN FIND THE

EIGENVALUES OF H VIA $|H - \lambda I| = 0 \Rightarrow \lambda = H_{11} \pm \sqrt{H_{12} H_{21}}$

IF WE WRITE $p^2 \equiv i H_{12}$ AND $q^2 \equiv i H_{21}$

THEN $\lambda_{LONG} = H_{11} - i p q \equiv M_L - i\frac{\Gamma_L}{2}$

$\lambda_{SHORT} = H_{11} + i p q \equiv M_S - i\frac{\Gamma_S}{2}$

AND, WITH SOME EFFORT, $|K^0_L\rangle = \dfrac{p|K^0\rangle - q|\bar{K}^0\rangle}{\sqrt{|p|^2 + |q|^2}}$ $|K^0_S\rangle = \dfrac{p|K^0\rangle + q|\bar{K}^0\rangle}{\sqrt{|p|^2 + |q|^2}}$

IT IS COMMON TO EXPRESS $|K^0_S\rangle$ AND $|K^0_L\rangle$ IN TERMS OF THE
CP EIGENSTATES $|K^0_{1,2}\rangle = \dfrac{|K^0\rangle \pm |\bar{K}^0\rangle}{\sqrt{2}} \Rightarrow |K^0\rangle = \dfrac{|K^0_1\rangle + |K^0_2\rangle}{\sqrt{2}}$; $|\bar{K}^0\rangle = \dfrac{|K^0_1\rangle - |K^0_2\rangle}{\sqrt{2}}$

AND $|K^0_L\rangle = \dfrac{(p-q)|K^0_1\rangle + (p+q)|K^0_2\rangle}{\sqrt{2}\sqrt{|p|^2 + |q|^2}}$ $|K^0_S\rangle = \dfrac{(p+q)|K^0_1\rangle + (p-q)|K^0_2\rangle}{\sqrt{2}\sqrt{|p|^2 + |q|^2}}$

WE DEFINE $\epsilon \equiv \dfrac{p-q}{p+q}$

$$|K^0_L\rangle = \frac{|K^0_2\rangle + \epsilon|K^0_1\rangle}{\sqrt{2}\sqrt{1 + |\epsilon|^2}} = \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2}\sqrt{1 + |\epsilon|^2}}$$

$$|K^0_S\rangle = \frac{|K^0_1\rangle + \epsilon|K^0_2\rangle}{\sqrt{2}\sqrt{1 + |\epsilon|^2}} = \frac{(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2}\sqrt{1 + |\epsilon|^2}}$$

CLEARLY CP INVARIANCE $\Rightarrow$ $\epsilon = 0$.   HENCE ϵ IS OFTEN REFERRED TO AS THE MEASURE OF CP VIOLATION IN THE K^0_L AND K^0_S WAVE FUNCTIONS.

IT IS USEFUL TO DERIVE ANOTHER EXPRESSION FOR THE CP VIOLATION PARAMETER ϵ:

$$\epsilon = \frac{p-q}{p+q} = \frac{p^2-q^2}{p^2+2pq+q^2} \approx \frac{p^2-q^2}{4pq} \qquad \text{NOTING } \epsilon \ll 1 \Rightarrow p \approx q$$

$$= \frac{i(H_{12}-H_{21})}{2i(\lambda_L-\lambda_S)} = \frac{M_{12}-i\frac{\Gamma_{12}}{2} - M_{12}^* + i\frac{\Gamma_{12}^*}{2}}{2\left[(M_L-M_S) - \frac{i}{2}(\Gamma_L-\Gamma_S)\right]} \qquad \left(\begin{array}{c}\text{AS } M \text{ \& } \Gamma \text{ ARE}\\ \text{HERMITIAN}\end{array}\right)$$

$$= \frac{\text{Im}\,\Gamma_{12}/2 + i\,\text{Im}\,M_{12}}{i(\Gamma_S-\Gamma_L)/2 + (M_L-M_S)}$$

FOR THE NEUTRAL KAONS, $\Gamma_S \sim 600\,\Gamma_L$, AND $(M_L-M_S)\tau_S \sim \frac{1}{2}$

$\Rightarrow$ $\Delta M \sim \Gamma_S/2$, AND WE CAN WRITE

$$\epsilon \sim \frac{1+i}{\Gamma_S}\left(\text{Im}\,M_{12} - i\,\text{Im}\,\Gamma_{12}/2\right)$$

IT MAY ALSO BE WORTH NOTING THAT $\langle K^0_L | K^0_S \rangle \approx 2\,\text{Re}\,\epsilon$

THAT IS, THE WEAK EIGENSTATES ARE NOT ORTHOGONAL, IF CP VIOLATION OCCURS, WHICH LEADS TO THE POSSIBILITY OF VACUUM REGENERATION NOTED ON p.331.

FURTHER CONSTRAINT ON ϵ CAN BE FOUND BE CONSIDERING THE $K^0_L \to \pi\pi$ DECAYS. WE WILL RELATE ϵ TO η_{+-} AND η_{00}

AS NOTED BEFORE THE $\pi\pi$ SYSTEM WITH $J=0$ CAN HAVE EITHER $I=0$ OR $I=2$. THE $\Delta I = \frac{1}{2}$ RULE SUGGESTS THAT TRANSITIONS TO THE $I=2$ $\pi\pi$ COMBO WILL BE SUPPRESSED.   WE WRITE

$$a_0 \equiv \langle \pi\pi, I=0,0 | \text{WEAK} | K^0 \rangle \qquad \xrightarrow{\text{CPT}} \qquad a_0 = \langle \pi\pi, I=0,0 | \text{WEAK} | \bar{K}^0 \rangle$$

$$a_2 \equiv \langle \pi\pi, I=2,0 | \text{WEAK} | K^0 \rangle \qquad a_2^* = \langle \pi\pi, I=2,0 | \text{WEAK} | \bar{K}^0 \rangle$$

WE HAVE CHOSEN THE PHASE OF $|K^0\rangle$ SO THAT a_0 IS REAL.
[WE IGNORE A COMPLICATION DUE TO STRONG FINAL-STATE INTERACTIONS AMONG THE 2π.   SEE THE REVIEW OF LEE & WU]

FROM C-G TABLES $|\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}}\,|\pi\pi, I=2,0\rangle - \sqrt{\frac{1}{3}}\,|\pi\pi, I=0,0\rangle$

$$|\pi^+\pi^-\rangle = \sqrt{\frac{1}{3}}\,|\pi\pi, I=2,0\rangle + \sqrt{\frac{2}{3}}\,|\pi\pi, I=0,0\rangle$$

THUS

$$\langle \pi^+\pi^- | \text{WEAK} | K^0_L \rangle \approx \left(\sqrt{\frac{1}{3}}\langle I=2,0| + \sqrt{\frac{2}{3}}\langle I=0,0|\right)|\text{WEAK}|\frac{\left((1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right)}{\sqrt{2}}$$

$$\approx \frac{2}{\sqrt{3}}\,a_0\left(\epsilon + \frac{i}{\sqrt{2}}\,\frac{\text{Im}\,a_2}{a_0}\right)$$

Similarly $\quad \langle \pi^0 \pi^0 | \text{weak} | K_L^0 \rangle = \frac{2}{\sqrt{3}} a_0 \left(-\frac{\epsilon}{\sqrt{2}} + i \frac{Im\, a_2}{a_0} \right)$

$$\langle \pi^+ \pi^- | \text{weak} | K_S^0 \rangle \approx \frac{2 a_0}{\sqrt{3}} + O\left(\frac{a_2}{a_0}\right)$$

$$\langle \pi^0 \pi^0 | \text{weak} | K_S^0 \rangle \approx \frac{-2 a_0}{\sqrt{6}} + O\left(\frac{a_2}{a_0}\right)$$

Ignore these as leading to 2nd order pieces in η_{+-} & η_{00}

Then $\quad \eta_{+-} = \dfrac{\langle \pi^+ \pi^- | \text{weak} | K_L^0 \rangle}{\langle \pi^+ \pi^- | \text{weak} | K_S^0 \rangle} = \epsilon + \dfrac{i}{\sqrt{2}} \dfrac{Im\, a_2}{a_0} \equiv \epsilon + \epsilon'$

$\eta_{00} = \dfrac{\langle \pi^0 \pi^0 | \text{weak} | K_L^0 \rangle}{\langle \pi^0 \pi^0 | \text{weak} | K_S^0 \rangle} = \epsilon - \dfrac{2i}{\sqrt{2}} \dfrac{Im\, a_2}{a_0} = \epsilon - 2\epsilon'$

$\epsilon' = \dfrac{i}{\sqrt{2}} \dfrac{Im\, a_2}{a_0} = $ CP violation in $\Delta I = \frac{3}{2}$ amplitude

From experiment $\quad \dfrac{\eta_{+-}}{\eta_{00}} \sim 1 \implies \left| \dfrac{\epsilon'}{\epsilon} \right| < 0.005 \qquad (1992)$

It is perhaps no great surprise that $\epsilon'/\epsilon < .01$ as this is typical of $\Delta I = 3/2$ transitions. It is possible, of course, that $\epsilon' = 0$.

Thus we conclude that $\quad \epsilon \sim \eta_{+-} \sim \eta_{00}$

The results discussed on p 322 for $\phi_{+-} \implies$ phase of $\epsilon \sim 45°$

From p 335, $\quad \epsilon \sim \dfrac{1+i}{\Gamma_S} \left(Im\, M_{12} - i\, Im\, \dfrac{\Gamma_{12}}{2} \right)$

To get phase of $\epsilon \sim 45°$ we must have $\quad Im\, \Gamma_{12} \ll Im\, M_{12}$

And so $\quad \epsilon \sim (1+i) \dfrac{Im\, M_{12}}{\Gamma_S} \sim 2(1+i) \dfrac{Im\, M_{12}}{\Delta M} \sim (1+i) \dfrac{Im\, M_{12}}{Re\, M_{12}}$

We will compare this result with constraints from the Kobayashi-Maskawa mixing scheme in section 7.

6. CP Violation in the decay $K^0 \to \pi l \nu$

We compare the decays $K_L^0 \to \pi^+ e^- \bar{\nu}$ and $K_L^0 \to \pi^- e^+ \nu$. If CP invariance were to hold then the decay rates for these must be equal, as $CP | \pi^+ e^- \bar{\nu} \rangle = | \pi^- e^+ \nu \rangle$

Note that $| \pi^+ e^- \bar{\nu} \rangle$ is not an eigenstate of CP; both K_L^0 and K_S^0 can decay to this.

THUS THE QUANTITY $\dfrac{\text{RATE}\,(K^0_L \to \pi^- e^+ \nu) - \text{RATE}\,(K^0_L \to \pi^+ e^- \bar{\nu})}{\text{RATE}\,(K^0_L \to \pi^- e^+ \nu) + \text{RATE}\,(K^0_L \to \pi^+ e^- \bar{\nu})}$

WILL GIVE FURTHER EVIDENCE OF CP VIOLATION IF IT IS NON-VANISHING.
WE GIVE A BRIEF ANALYSIS OF THIS, ALONG WITH ANOTHER RESULT WHICH
DOES NOT DEPEND ON CP VIOLATION.

IN THE QUARK PICTURE OF WEAK DECAYS WE NATURALLY HAVE
THE $\Delta S = \Delta Q$ RULE (P. 306) THUS $K^0 \to \pi^- e^+ \nu$ BUT $K^0 \not\to \pi^+ e^- \nu$

AND $\bar{K}^0 \to \pi^+ e^- \nu$ BUT $\bar{K}^0 \not\to \pi^- e^+ \nu$

LET $f \equiv$ AMPLITUDE $(K^0 \to \pi^- e^+ \nu)$. THEN ASSUMING CPT INVARIANCE

$f^* =$ AMPLITUDE $(\bar{K}^0 \to \pi^+ e^- \nu)$

WE AGAIN CONSIDER A NEUTRAL KAON STATE WHICH HAS COMPOSITION
$$a_S |K^0_S\rangle + a_L |K^0_L\rangle \qquad \text{AT } t = 0$$

WE CALCULATE THE AMPLITUDE $A^{\pm}$ FOR THIS STATE TO DECAY TO $\pi^{\mp} e^{\pm} \nu$
AT TIME t. NOTE THAT A^+ DEPENDS ONLY ON THE K^0 COMPONENT OF
OUR STATE AT TIME t. REFER TO p334 FOR $K^0_{L,S}$ IN TERMS OF $K^0, \bar{K}^0$.

$$A^+(t) \sim a_S\, e^{-i M_S t}\, e^{-\frac{\Gamma_S}{2} t}\, (1+\epsilon)\, f \;+\; a_L\, e^{-i M_L t}\, e^{-\frac{\Gamma_L}{2} t}\, (1+\epsilon)\, f$$

$$A^-(t) \sim a_S \qquad '' \qquad (1-\epsilon)\, f^* \;+\; a_L \qquad '' \qquad (1-\epsilon)\, f^*$$

THE DECAY RATES $N^{\pm} = |A^{\pm}|^2$ ARE THEN

$$N^{\pm}(t) \sim (1 \pm 2\,\mathrm{Re}\,\epsilon)\left[|a_S|^2 e^{-\Gamma_S t} + |a_L|^2 e^{-\Gamma_L t} \pm 2|a_S||a_L|\, e^{-\frac{\Gamma_S + \Gamma_L}{2} t}\, \cos(\Delta M t + \phi) \right]$$

$\phi =$ PHASE OF $a_S^* a_L$

$$\frac{N^+ - N^-}{N^+ + N^-} \sim \frac{2|a_S||a_L|\, e^{-\frac{\Gamma_S + \Gamma_L}{2} t}\, \cos(\Delta M t + \phi)}{|a_S|^2 e^{-\Gamma_S t} + |a_L|^2 e^{-\Gamma_L t}} + 2\,\mathrm{Re}\,\epsilon \qquad \left(\begin{array}{c}\text{TIMES A FACTOR}\\ \sim 1 - \epsilon\end{array}\right)$$

TWO CASES ARE OF INTEREST:

a. A K^0_L BEAM $\Rightarrow a_S = 0,\ a_L = 1$ AND $\delta = \dfrac{N^+ - N^-}{N^+ + N^-} = 2\,\mathrm{Re}\,\epsilon$

THIS ALLOWS EXPERIMENTAL DETERMINATION OF THE CP VIOLATING
PARAMETER ϵ INDEPENDENT OF THE $\pi\pi$ DECAY MODE. THE BEST RESULT
OF THIS TYPE IS FROM GEWENIGER ET AL [PHYS. LETT. B48, 483 (1974)]
THEY FOUND $\delta = 3.41 \pm .18 \times 10^{-3}$

$\Rightarrow \mathrm{Re}\,\epsilon = 1.7 \pm .09 \times 10^{-3}$ (IF CPT & $\Delta S = \Delta Q$ GOOD)

THIS IS QUITE CONSISTENT WITH THE PREVIOUS VALUES OF ϵ, LEADING TO
A COMBINED RESULT $|\epsilon| = 2.28 \pm .05 \times 10^{-3}$ $\phi_\epsilon \sim 45°$

6.   A K^0 Beam $\Rightarrow$ $q_s = 1 = q_L$

$$\delta(t) \sim \frac{e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta m\, t + \phi)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}} + 2\,\mathrm{Re}\,\epsilon$$

This gives a very dramatic interference effect when $t < 10\,\tau_S$, which provides one of the best ways to measure ΔM.

$\Big[$ Note that the same result holds if we start with a $\overline{K^0}$ beam. This is **not** a CPT violation, as mentioned on p 176. $\Big]$

Figure 7.11. Charge asymmetry in the decay $K^0 \to \pi^\pm e^\pm \overset{(-)}{\nu}$ as a function of proper time. The curve is a fit to (7.103). Both the K_L–K_S interference and the CP-violating offset are clearly shown. (From Gjesdal et al. 74. Reprinted with permission.)
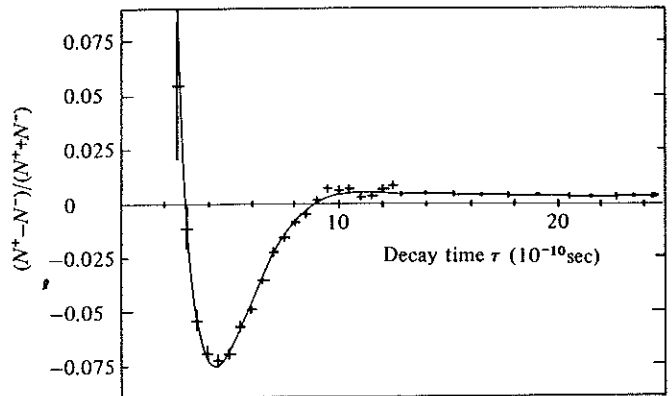
We note that the asymmetry δ measured with a K^0_L beam allows one to 'safely' distinguish the universe from the anti-universe. Namely, if a being from another universe comes to earth, do you really want to shake hands? If (s)he is made of antimatter a diplomatic incident might occur. But we can ask the beings to perform an experiment in their own universe before they come, to determine whether they are made of matter or anti-matter.

- Produce a beam of long lived neutral kaons
- Measure the charge asymmetry in $K^0_L \to \pi \ell \nu$

If the lepton which is produced more frequently is made of the same kind of matter as their atoms, we don't want them to come here!

7. CP VIOLATION AND THE KOBAYASHI - MASKAWA MATRIX

We have claimed that the K–M matrix and the 6 quark scheme was a place for CP violation. We now sketch how this works. This is not an explanation of CP violation, but it does relate CP violation to other features of the weak interaction of quarks.

We have seen that most features of CP violation can be related to the single parameter

$$\epsilon \sim \frac{\partial_m \Gamma_{12}/2 + i\, \partial_m M_{12}}{i(\Gamma_S - \Gamma_L)/2 + (M_L - M_S)} \approx (1 + i)\frac{\mathrm{Im}\, M_{12}}{\mathrm{Re}\, M_{12}} \qquad (\text{pp } 335, 336)$$

where $M_{12} = \langle \overline{K^0} | \text{weak} | K^0 \rangle$. On p 318 we indicated how this transition might be due to a second order weak interaction.
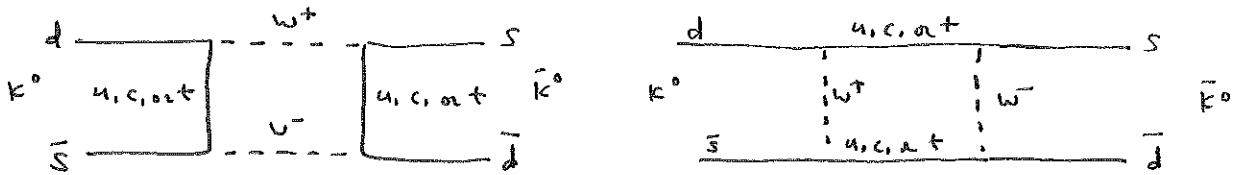
IN THE 6 QUARK SCHEME u, c OR t QUARKS CAN BE EXCHANGED INSIDE THE DIAGRAMS.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (KM) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{array}{c|ccc} & d & s & b \\ \hline u & C_1 & S_1 C_3 & S_1 S_3 \\ c & -S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{-i\delta} \\ t & S_1 S_2 & -C_1 S_2 C_3 - C_2 S_3 e^{i\delta} & -C_1 S_2 S_3 + C_2 C_3 e^{i\delta} \end{array}$$

$$S_i = \sin\theta_i, \quad C_i = \cos\theta_i, \quad \text{etc.}$$

THE AMPLITUDE $M_{12} = \langle \bar{K}^0 | \text{WEAK} | K^0 \rangle$ THEN VARIES LIKE

$$\left(\frac{V_{ud} V_{us}}{q^2 - M_u^2} + \frac{V_{cd} V_{cs}}{q^2 - M_c^2} + \frac{V_{td} V_{ts}}{q^2 - M_t^2} \right)^2 \qquad \leftarrow \begin{array}{l} \text{SQUARE SINCE THERE} \\ \text{ARE 2 QUARK EXCHANGES} \end{array}$$

$$= \left(\frac{S_1 C_1 C_3}{q^2 - M_u^2} + \frac{-S_1 C_1 C_2^2 C_3 + S_1 S_2 C_2 S_3 e^{i\delta}}{q^2 - M_c^2} + \frac{-S_1 C_1 S_2^2 C_3 - S_1 S_2 C_2 S_3 e^{i\delta}}{q^2 - M_t^2} \right)^2$$

$$\sim \left(S_1 C_1 C_3 \left(\frac{1}{q^2 - M_u^2} - \frac{C_2^2}{q^2 - M_c^2} - \frac{S_2^2}{q^2 - M_t^2} \right) - \frac{S_1 S_2 C_3 S_3 e^{i\delta} (M_t^2 - M_c^2)}{(q^2 - M_c^2)(q^2 - M_t^2)} \right)^2$$

THEN $\epsilon = (1+i) \dfrac{\text{Im } M_{12}}{\text{Re } M_{12}} \approx (1+i) \dfrac{(S_1 C_1 C_3)(S_1 S_2 C_3 S_3 \sin\delta)}{(S_1 C_1 C_3)^2} \sim S_2 S_3 \sin\delta$

PUTTING $C_i \approx 1$.   FROM THE ANALYSIS OF $b \to u$ AND $b \to c$ DECAYS (P 325) WE CONCLUDES $S_2 \sim .08$ AND $S_3 \sim .04$

HENCE $\epsilon \sim (1+i) \, 2.8 \times 10^{-3} \sin\delta$

AT FIRST SIGHT THE AGREEMENT IS RATHER IMPRESSIVE. $\delta \sim 90°$ WILL DO.

APPARENTLY HOWEVER THE DETAILED NUMERICAL AGREEMENT IS NOT SO GOOD IF THE CALCULATION IS DONE MORE PRECISELY. ELLIS ET AL [NUC. PHYS. B109, 213 (1976)], OR GINSPARG ET AL [P.R.L. 50, 1415 (1983)]

CLAIM $\epsilon \sim \dfrac{1}{10} S_2 S_3 \sin\delta \left[-1 + S_2 \dfrac{M_t^2}{M_c^2} + \ln\left(\dfrac{M_t^2}{M_c^2}\right) \right]$

THEN IT IS CLAIMED THAT THE SMALL MEASURED VALUES OF S_2 AND S_3 REQUIRE THE MASS OF THE TOP QUARK TO BE $M_t > 50 \text{ GeV}$!

The K-M analysis puts no special restriction on ϵ', the CP violating parameter for $K^0_L \to (\pi\pi, I=2)$. In CP conserving decays we noted that $\dfrac{a_{3/2}}{a_{1/2}} \sim .03 - .05$ (p 302)

Already it is known that $\dfrac{\epsilon'}{\epsilon} < .01$. Gilman & Hagelin [Phys. Lett. B133, 443 (1983)] argue that ϵ'/ϵ depends on M_t in such a way that $M_t \sim 50 GeV \Rightarrow \epsilon'/\epsilon \sim .01$

People at Princeton are presently involved in an experiment to measure ϵ'/ϵ to accuracy .002. Perhaps their eventual result can be interpreted as indicating the top quark mass....

8. CP Violation in D^0, B^0 and T^0 Meson Systems

In the 6 quark model there are several more meson pairs which might show CP violation!

$$D^0 - \bar{D}^0 \quad = \quad c\bar{u} \ \& \ \bar{c}u$$

$$B^0_d \quad \bar{B}^0_d \quad = \quad b\bar{d} \ \& \ \bar{b}d$$

$$B^0_s \quad \bar{B}^0_s \quad = \quad b\bar{s} \ \& \ \bar{b}s$$

$$T^0_u \quad \bar{T}^0_u \quad = \quad t\bar{u} \ \& \ \bar{t}u$$

$$T^0_c \quad \bar{T}^0_c \quad = \quad t\bar{c} \ \& \ \bar{t}c$$

However it will not be easy to demonstrate the CP violation in these systems even if it exists.

Reviewing the situation for K^0 decay we note several features conspired to make CP violation measurement possible

- $\tau_L \gg \tau_S$ so K^0_L & K^0_S are experimentally quite distinct

- τ_L long enough to observe rate. vs t

- All decays of K^0_S are to CP eigenstates. Hence interference between $K^0_S \to$ CP state & $K^0_L \to$ CP state will be fairly strong

- CP violating parameter ϵ not too small. $\epsilon \sim 2 \times 10^{-3}$

On the first 3 counts the heavy quark systems are not favored.

FOR EXAMPLE, A DECAY OF D^0 TO A CP EIGENSTATE IS

$$D^0 \to \pi^+ \pi^- \quad \text{OR} \quad D^0 \to K^+ K^-$$
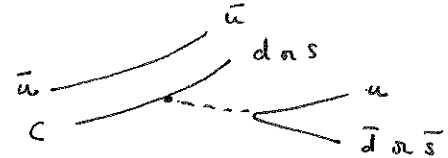
EITHER WAY   AMPLI $\sim \sin \theta_C$

SO RATE $\sim \sin^2 \theta_C$ AND THESE DECAYS ARE SUPPRESSED.

IN FACT $\dfrac{D^0 \to K^+ K^-}{D^0 \to ALL} = 2 \pm 0.8 \times 10^{-3}$ HAS BEEN MEASURED

THEN WE MUST EXPECT $\tau_{D^0_L} \approx \tau_{D^0_S}$ AS BOTH CAN GO TO ALMOST ALL POSSIBLE FINAL STATES WITH EQUAL RATES.

A LOOK AT THE K-M MATRIX (p 339) SHOWS SIMILAR DIFFICULTY FOR THE T^0 MESONS.   THUS A FAVORED DECAY WILL BE LIKE

$$t\bar{u} \to b\bar{u} + u\bar{d} \sim c\bar{s} \qquad \text{WHILE} \quad t\bar{u} \to d\bar{u} + u\bar{d} \sim (s_1 s_2)^2$$
$$\to s\bar{u} + u\bar{s}$$

THE BEST HOPE SEEMS TO BE FOR THE B^0_s AS $b\bar{s} \to c\bar{s} + \bar{c}s = F^+ F^-$ IS A DOMINANT HADRONIC DECAY MODE. THE B^0_s HAS NOT BEEN OBSERVED YET, BUT IF IT PROVES TO HAVE A LIFETIME $> 10^{-12}$ SEC, AS THE B^0_d APPARENTLY DOES, EXPERIMENTS MAY BE POSSIBLE.

WE CAN MAKE SIMPLIFIED ESTIMATES OF THE CP VIOLATING PARAMETER ϵ AS IN SECTION 7.

$$\epsilon \sim \frac{\text{Im } M_{12}}{\text{Re } M_{12}}$$

D^0 $\epsilon \sim \dfrac{\text{Im } V_{cs} V_{su}}{\text{Re } V_{cd} V_{du}} \sim \dfrac{s_1 s_2 s_3 c_3 \sin \delta}{s_1 c_1 c_2} \sim s_2 s_3 \sin \delta$

SMALL

BUT

B^0_s $\epsilon \sim \dfrac{\text{Im } V_{bc} V_{cs}}{\text{Re } V_{bc} V_{cs}} \sim \dfrac{c_1 c_2 s_2 c_3^2 \sin \delta}{(c_1^2 c_2^2 c_3 s_3 + c_1 c_2 c_3 s_2 \cos \delta)} \sim \dfrac{s_2 \sin \delta}{s_3 + s_2 \cos \delta}$

BIG ??

DIRECT RECONSTRUCTION OF $B^0_s \to F^+ F^-$ DECAYS MAY PROVE DIFFICULT.

A BETTER EXPERIMENTAL SIGNAL WILL PROBABLY BE IN THE CHARGE ASYMMETRY IN $B^0_s \to h \ell^\pm_\nu$

FOR $\bar{B}^0_s - B^0_s$ PAIRS PRODUCED IN $e^+ e^-$ COLLISIONS, ONE WOULD LOOK FOR 2 LEPTONS, ONE FROM EACH MESONS, AND MEASURE $\dfrac{N^{++} - N^{--}}{N^{++} + N^{--}}$ . . . .

[UPDATE: CP VIOLATION IN THE B MESON SYSTEM TURNED OUT TO BE LARGE ....]

A Footnote on $D^0 \bar{D}^0$ Mixing

A less dramatic possibility than CP violation in the $D^0 \bar{D}^0$ (or $B^0 \bar{B}^0$...) system is charm (or beauty) oscillations. This is the analogue of strangeness oscillations discussed on p. 326. Because the D^0 lifetime is so short, actual patterns of oscillation are very difficult to demonstrate.

But we can consider experiments which integrate over the entire lifetime of the D^0 mesons. For example, a prominent semi leptonic decay of the D^0

is $D^0 \to K^- \mu^+ \nu$

while $\bar{D}^0 \to K^+ \mu^- \bar{\nu}$
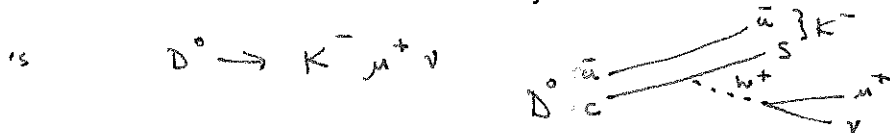
If we start with a pure D^0 state, we can ask what fraction of the decay muons are μ^-, due to $D^0 \to \bar{D}^0$ mixing?

Comparing with the expressions for K^0 and $\bar{K}^0$ on top of p 328, we infer:

$$\frac{\# \text{ of } \mu^-}{\# \text{ of } \mu^+} = \frac{\int_0^\infty P_{\bar{D}^0}(t)\, dt}{\int_0^\infty P_{D^0}(t)\, dt} = \frac{\Delta M^2 + \frac{\Delta \Gamma^2}{4}}{2\Gamma_{AVE}^2 + \Delta M^2 - \frac{\Delta\Gamma^2}{4}}$$

where $\Delta M = M_{D_L^0} - M_{D_S^0}$; $\Delta\Gamma = \Gamma_{D_S^0} - \Gamma_{D_L^0}$; $\Gamma_{AVE} = \frac{\Gamma_{D_S^0} + \Gamma_{D_L^0}}{2}$

In an actual experiment, we are likely to have produced a $D^0 \bar{D}^0$ pair rather than only a D^0 (why?). This complicates matters, but allows a measurement of

$$\frac{\#(\mu^+\mu^+) + \#(\mu^-\mu^-)}{\#(\mu^+\mu^-)}$$

Here, like sign muons occur if one of the D^0 or $\bar{D}^0$ has 'mixed' to its antiparticle. An analysis of the above ratio is given by Kingsley, Phys. Lett. **63B**, 329 (1976). He claims the ratio should be

$$\frac{\Delta M^2 + \frac{\Delta\Gamma^2}{4}}{2\Gamma_{AVE}^2 + \Delta M^2 - \Delta\Gamma^2/4} \qquad \text{Also !}$$

The best (tho none too good) limit on this ratio is < 0.006 [Louis et al. P.R.L. 56, 1027 (1986)]

Since $\Gamma_{AVE} \sim \frac{1}{4 \times 10^{-13} s}$, we infer $M_{D_L^0} - M_{D_S^0} < 10^{-4} eV$
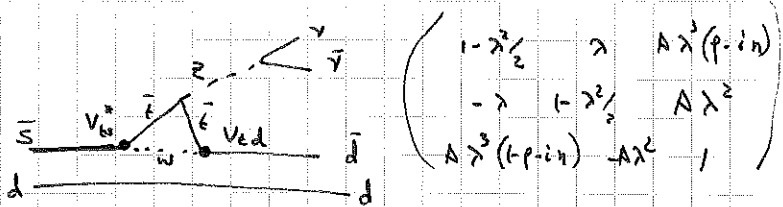
(Compare with $M_{K_L^0} - M_{K_S^0} \sim 3 \times 10^{-6} eV$). Similarly $\frac{\Delta\Gamma}{\Gamma_{AVE}} < .2$.

[UPDATE: The mixing ratio was finally measured to be about 0.003 in 2006.]

Needs better experiments !

3/12/98

$K^0 \to \pi^0 \nu\bar{\nu}$ PENGUIN DIAGRAM

$$\begin{pmatrix} 1-\tfrac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\tfrac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$A(K^0 \to \pi^0 \nu\bar{\nu}) \sim V_{ts}^* V_{td} \sim (-A\lambda^2)(A\lambda^3)(1-\rho-i\eta)$

$A(\bar{K}^0 \to \pi^0 \nu\bar{\nu}) \sim V_{ts}^* V_{td}^* \sim (-A\lambda^2)(A\lambda^3)(1-\rho+i\eta)$

IN MORE DETAIL: $A \sim \dfrac{V_{us}^* V_{ud}}{m_u} + \dfrac{V_{cs}^* V_{cd}}{m_c} + \dfrac{V_{ts}^* V_{td}}{m_t}$

BUT $V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$

$K_1 = \dfrac{K_0 + \bar{K}_0}{\sqrt{2}}$ $K_2 = \dfrac{K_0 - \bar{K}_0}{\sqrt{2}}$

$A(K_1 \to \pi^0 \nu\bar{\nu}) \sim -A^2\lambda^5(1-\rho)\sqrt{2}$ CP CONSERVING

$A(K_2 \to \pi^0 \nu\bar{\nu}) \sim +iA^2\lambda^5\eta\sqrt{2}$ CP VIOLATING, SINCE $\sim \eta$

$K_L = K_2 + \epsilon K_1$

$A(K_L \to \pi^0 \nu\bar{\nu}) \sim \sqrt{2}A^2\lambda^5\left(i\eta - \epsilon(1-\rho)\right)$

$|\eta| \sim |1-\rho|$

SO $A(K_L \to \pi^0 \nu\bar{\nu})$ IS UNDERLINED DOMINATED   BY $A(K_2 \to \pi^0 \nu\bar{\nu})$

WHICH IS CALLED   DIRECT CP VIOLATION

THE SMALLER PIECE, $\sim \epsilon(1-\rho)$ IS CALLED   INDIRECT CP VIOLATION, OR

CP VIOLATION IN THE MIXING, SINCE IT DEPENDS ON $K_L \neq K_2$ DUE TO

CP VIOLATION.

? $CP(\pi^0 \nu\bar{\nu}) = C(\pi^0) P(\pi^0) C(\nu\bar{\nu}) P(\nu\bar{\nu}) (-1)^{L_{\pi^0(\nu\bar{\nu})}}$

$C(\pi^0) = +1$ $P(\pi^0) = -1$ $C(\nu\bar{\nu}) = -1^{L_{\nu\bar{\nu}} + S_{\nu\bar{\nu}}}$ $P(\nu\bar{\nu}) = (-1)^{L_{\nu\bar{\nu}}+1}$

$S_{\nu\bar{\nu}}$!   GO TO REST FRAME OF $\nu\bar{\nu}$ $\bar{\nu} \xleftarrow{\quad+\quad} \xrightarrow{\quad+\quad} \nu$ $S_{\nu\bar{\nu}}|_z = -1 \Rightarrow S_{\nu\bar{\nu}} = 1$

$\nu\bar{\nu}$ COMES FROM $Z \Rightarrow J=0$ or 1, BUT CANNOT HAVE $J=0, S=1$

SO MUST HAVE $J_{\nu\bar{\nu}} = 1$

(DON'T NEED TO KNOW $L_{\nu\bar{\nu}}$!)

$J_{K^0} = 0$, $J_{\pi^0} = 0$, $J_{\nu\bar{\nu}} = 1 \Rightarrow L_{\pi(\nu\bar{\nu})} = 1$

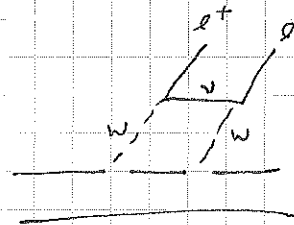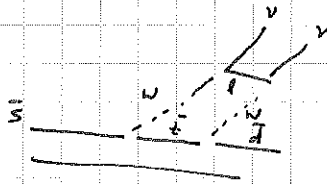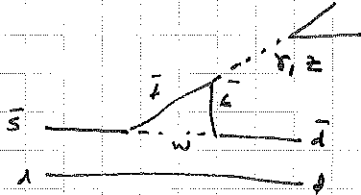SO $CP(\pi^0 \nu\bar{\nu}) = (+1)(-1)(-1)^{L_{\nu\bar{\nu}}}(-1)(-1)^{L_{\nu\bar{\nu}}}(-1)(-1) = +1$

? WHAT IS DIFFERENT IF $K_0 \to \pi^0 e^+ e^-$ or $\pi^0 \mu^+\mu^-$

CANNOT CONCLUDE THAT $S_{\ell\bar{\ell}} = 1$ ONLY, THOUGH THIS IS A GOOD APPROX

FOR $e^+ e^-$! SINCE THE e's ARE RELATIVISTIC

$CP(\pi^0 e^+ e^-)$ MOSTLY $+1$)

$CP(\pi^0 \mu^+\mu^-)$ SOME $+1$, SOME -1

WHAT ABOUT DIAGRAMS

FOR $\ell^+ \ell^-$, THE γ DIAGRAM MAY DOMINATE, BUT SHOULDN'T CHANGE $\mathscr{C}P$ ARGUMENT ABOUT V_{td}.