# The Quark Model

References: The 60's – 'The Quark Model' by Kokkedee, Benjamin (1969)
'Models of Elementary Particles' by Feld, Blaisdell (1969)
The 70's – 'Intro to Quarks & Partons' by Close, Academic Press (1979)
'Quark Models' by Rosner, in 'Techniques & Concepts of High Energy Physics' ed. by Ferbel, Plenum Press (1981)

We now pursue the classification of hadron states into broader schemes than isospin = SU(2). The historical path leading to the model of colored quarks and gluons was not as direct as our rapid introduction may indicate. In real life people seem slow to take giant steps if it leads to only one new result. The construction of a compelling edifice with many interlocking aspects takes many people many years, despite the heroic efforts of individuals whom we honor in retrospect.

We have already noted a simple relation which combines isospin with the strangeness quantum number:

$$Q = I_3 + \frac{B+S}{2} \qquad \left[\text{Gell-Mann – Nishijima (1953)}\right]$$

A further step was taken by Sakata, Prog. Theor. Phys. 16, 686 (1956), who attempted to extend the Fermi-Yang model (p.25) to include the $\Lambda^0$ particle, which has $S=-1$. He considered mesons to consist of $B\bar{B}$ pairs, where $B = p, n$, or $\Lambda$. It was realized that this scheme had something to do with the group SU(3), and Ohnuki (CERN Conf. 1960) gave a classification of an 'octet' of pseudoscalar mesons very similar to our present view. A new state was predicted: $\frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n} - 2\Lambda\bar{\Lambda})$, which was discovered as the $\eta(548)$ meson in 1961. (To some extent this state is already anticipated in the Fermi-Yang model, p.25).

However, the Sakata model had trouble accomodating the baryon states. The picture was that baryons were $B\bar{B}B$ combinations of the basic triplet. The 27 such combinations include states like $p\bar{\Lambda}p$ with charge 2 and strangeness +1 which don't seem to be found in nature. Also, experimental evidence showed that the lowest mass mesons have $J^P = 0^-$. Then one might expect the lowest mass $B\bar{B}B$ states to have $J^P = 0^- + \frac{1}{2}^+ = \frac{1}{2}^-$ rather than $\frac{1}{2}^+$.

A big step was made by Gell-Mann [Caltech Report CTSL-20 (1961); P.R. 125, 1067 (1962)] and Ne'eman [Nuc. Phys. 26, 222 (1961)] who noted that baryon states could be classified into octets and decuplets such as found in SU(3) group theory (p 21). They predicted a new state of strangeness −3, and a definite mass for it, which was indeed discovered in 1964. [Barnes et al. P.R.L. 12, 204 (1964)]. Initially they gave no explanation of the SU(3) classification in terms of a constituent model such as the Fermi-Yang-Sakata scheme.

THIS WAS PROVIDED IN 1964 BY GELL-MANN [PHYS. LETT 8, 214 (1964)] AND ZWEIG [CERN REPORT TH-401, 412 (1964)] WHO INTRODUCED QUARKS WITH FRACTIONAL BARYON NUBER AND FRACTIONAL ELECTRIC CHARGE. THE BOLDNESS OF THIS STEP IS REFLECTED IN THE FACT THAT EVEN TODAY THERE IS NO DIRECT EVIDENCE FOR THE EXISTENCE OF FREE QUARKS (FAIRBANK ASIDE). COMBINING THE SU(3) SYMMETRY GENERATED BY 3 QUARK 'FLAVORS' u, d & s WITH SPIN THEY CONSIDERED THE SU(6) SYMMETRY. ONE ADVANTAGE OF THIS IS THAT ALL KNOWN BARYONS COULD BE FIT INTO SYMMETRIC MULTIPLETS IN SU(6) SPACE; OTHER SU(6) MULTIPLETS ARE NOT POPULATED. EVEN THIS RESULT IS SOMEWHAT DISQUIETING AS THE BARYONS ARE STATES OF SPIN 1/2 QUARKS AND SO OUGHT TO HAVE ANTISYMMETRIC WAVE FUNCTIONS ACCORDING TO FERMI-DIRAC STATISTICS.

BETWEEN 1964 AND 1974 THE AWARENESS SLOWLY GREW THAT ONE SHOULD ATTACH AN ADDITIONAL TRIPLET OF LABELS (OBEYING YET ANOTHER SU(3) SYMMETRY) TO THE QUARKS. THE LABELS DESCRIBE THE 'COLOR' OF THE QUARKS. BUT AFTER INTRODUCING THE NEW SYMMETRY, ONE NEEDS AN IMMEDIATE RESTRICTION THAT ALL OBSERVEABLE PARTICLES ARE 'COLORLESS'. THIS MAKES THE BARYON COLOR WAVE FUNCTION ANTISYMMETRIC, RESTORING THE USUAL RELATION BETWEEN SPIN AND STATISTICS. LIKE THE QUARKS, COLOR APPEARS TO BE UNOBSERVABLE IN THE LABORATORY, SO WE ARE BUILDING AN EDIFICE WITH QUITE A FEW INVISIBLE FEATURES. MUCH OF OUR PRESENT ACCEPTANCE OF THESE NOTIONS IS BASED ON THE EMERGENCE OF A DYNAMICAL THEORY INCORPORATING COLOR AS THE 'CHARGE' OF THE STRONG FORCE BETWEEN QUARKS - THE SO-CALLED THEORY OF <u>QUANTUM</u> <u>CHROMODYNAMICS</u> (QCD). THE ABILITY OF A MODEL TO CLASSIFY PARTICLE STATES IS NOT SUFFICIENT EVIDENCE THAT A FUNDAMENTAL NEW LEVEL OF PHYSICAL CONCEPTS HAS BEEN ACHIEVED. [TWO OF THE 4 KEY INITIAL PAPERS ON SU(3) AND QUARKS WERE NEVER FORMALLY PUBLISHED.]

ANOTHER ASPECT OF CLASSIFICATION SCHEMES CONCERNS THE NUMBER OF QUARK FLAVORS. IF 3 FLAVORS, WHY NOT 4, 5 ....? ALREADY IN 1964 4 FLAVOR SCHEMES WERE CONSIDERED. NONETHELESS IT CAME AS A SURPRISE WHEN THE FIRST OBSERVATION OF STATES OF THE 4TH FLAVOR, CHARM $\equiv$ c, WAS MADE IN THE FORM OF LONG-LIVED $c\bar{c}$ STATES (1974). A FIFTH FLAVOR, BOTTOM $\equiv$ b, WAS DISCOVERED IN $b\bar{b}$ STATES IN (1977), AND THE SEARCH GOES ON.

A RECENT BOOK ON GROUP THEORY TECHNIQUES BY AN EXPERT PRACTITIONER IS 'LIE ALGEBRAS IN PARTICLE PHYSICS' BY H. GEORGI, BENJAMIN (1982). HE INCLUDES THE FOLLOWING PARAGRAPHS:

GROUP THEORY VERSUS PHYSICS

Notice that I have resisted the temptation to include the c quark with the u, d and s quarks in the 4 dimensional representation of an SU(4) symmetry. Let me spell out the reasons.

One of the nice things about the QCD quark model is that it explains the success of Gell-Mann's SU(3), because the u, d and s quark mass differences are small compared to the QCD scale parameter $\Lambda$. The mass difference between the c quarks and any of the light quarks, on the other hand, is large compared to $\Lambda$. SU(4) should not be a useful approximate symmetry.

There is a more general moral here. A symmetry principle should not be an end in itself. Sometimes the physics of a problem is so complicated that symmetry arguments are the only practical means of extracting information about the system. Then, by all means use them. But, do not stop looking for an explicit dynamical scheme that makes more detailed calculation possible. Symmetry is a tool that should be used to determine the underlying dynamics, which must in turn explain the success (or failure) of the symmetry arguments. Group theory is a useful technique, but it is no substitute for physics.

## 1. THE u, d & s QUARK WAVE FUNCTIONS OF THE MESONS

AS NOTED IN THE SAKATA MODEL, IT IS EASIER TO CONSTRUCT THE WAVE FUNCTIONS FOR MESONS THAN FOR BARYONS. IN THE QUARK MODEL, MESONS ARE $q\bar{q}$ STATES.

$$\psi_{MESON} = \psi_{QUARK\ FLAVOR} \cdot \psi_{QUARK\ SPIN} \cdot \psi_{QUARK\ ATOM} \cdot \psi_{QUARK\ COLOR}$$

WE CONCENTRATE ON THE FLAVOR AND SPIN FACTORS. BY $\psi_{QUARK\ ATOM}$ WE MEAN THE POSSIBILITY THAT THE $q\bar{q}$ STATE HAS ORBITAL ANGULAR MOMENTUM, AND EVEN RADIAL EXCITATIONS, AS FOUND IN THE HYDROGEN ATOM. FOR THE PRESENT WE IGNORE THESE POSSIBILITIES AND CONSIDER ONLY $n=1$ S WAVE STATES. THE FACTOR $\psi_{QUARK\ COLOR}$ IS THE SAME FOR ALL MESONS, AND IS A COLOR SINGLET STATE. WE DISCUSS THIS BRIEFLY LATER.

THE SPIN WAVE FUNCTIONS POSSIBLE FOR 2 SPIN 1/2 PARTICLES ARE IMMEDIATELY FAMILIAR:

| | $S=0$ | $S=1$ |
|---|---|---|
| $S_z = 1$ | | $\uparrow\uparrow$ |
| $0$ | $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ | $\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$ |
| $-1$ | | $\downarrow\downarrow$ |

TURNING TO THE QUARK FLAVOR WAVE FUNCTION, IT MAY BE USEFUL TO BEGIN WITH THE CASE OF ONLY 2 QUARK FLAVORS u AND d. TOGETHER THEY FORM AN ISOSPIN DOUBLET, AND THE POSSIBLE $q\bar{q}$ WAVE FUNCTIONS CAN BE SAID TO OBEY AN SU(2) CLASSIFICATION.

RECALL OUR ARGUMENT ON P. 185 AS TO NUCLEON-ANTINUCLEON COMBOS

| | $I=0$ | $I=1$ |
|---|---|---|
| $I_3 = 1$ | | $u\bar{d}$ |
| $I_3 = 0$ | $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ | $\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$ |
| $I_3 = -1$ | | $d\bar{u}$ |

WE SHOULD BE A LITTLE MORE PRECISE. THE 'FIRST' OF THE 2 PARTICLES IN A $q\bar{q}$ STATE COULD EITHER BE THE $q$ OR THE $\bar{q}$. SO FOR THE COMBO $u\bar{d}$ WE CAN ACTUALLY HAVE 2 DISTINCT SITUATIONS. THE CONCEPT OF G-PARITY HELPS ORGANIZE THESE.

$$G\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} \qquad G\begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} = -\begin{pmatrix} u \\ d \end{pmatrix} \qquad \text{so} \qquad \begin{aligned} G(u\bar{d}) &= -d\bar{u} \\ G(\bar{d}u) &= -u\bar{d} \end{aligned}$$

HENCE $G\left[\frac{1}{\sqrt{2}}(u\bar{d} \pm \bar{d}u)\right] = \mp \frac{1}{\sqrt{2}}(u\bar{d} \pm \bar{d}u)$

THUS EACH $q\bar{q}$ ISOSPIN MULTIPLET 'SPLITS' INTO 2 MULTIPLETS, ONE WITH $G=+1$ AND THE OTHER WITH $G=-1$.

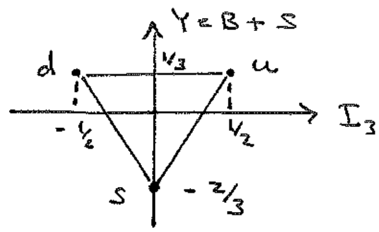
THE $G=-1$ STATES ARE SYMMETRIC UNDER INTERCHANGE OF QUARK AND ANTIQUARK. AS QUARKS ARE FERMIONS THE OVERALL $q\bar{q}$ WAVE FUNCTION SHOULD BE ANTISYMMETRIC. WE WILL SHOW LATER THAT THE COLOR PART OF THE $q\bar{q}$ WAVE FUNCTION IS SYMMETRIC. THE COMBINED SYMMETRY OF THE SPIN AND FLAVOR WAVE FUNCTIONS MUST THEN BE ANTISYMMETRIC, WHICH GIVES US THE RULES:

THE $I=1$ MESONS WITH SPIN 0 MUST HAVE $G=-1$
" " " 1 " " $G=+1$

[& THE REVERSE FOR THE $I=0$ STATES]

WE READILY IDENTIFY THE $I=1$ SPIN-0 AND SPIN-1 MULTIPLETS WITH THE $\pi$ AND $\rho$ MESONS, RESPECTIVELY.

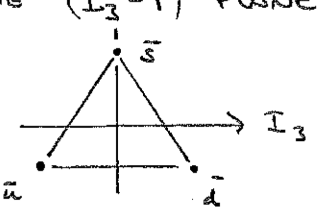
FOR THE CASE OF 3 QUARK FLAVORS $u, d$ & $s$, A USEFUL PICTURE SHOWS THE QUARK AND ANTIQUARK TRIPLETS ON THE $(I_3 - Y)$ PLANE.




THE "3" MULTIPLET

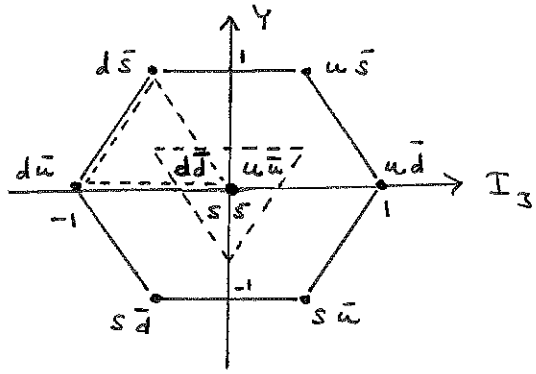
$\left[Q = I_3 + \frac{Y}{2}\right]$

| | $Q$ | $I_3$ | $B$ | $S$ |
|---|---|---|---|---|
| $u$ | $2/3$ | $1/2$ | $1/3$ | $0$ |
| $d$ | $-1/3$ | $-1/2$ | $1/3$ | $0$ |
| $s$ | $-1/3$ | $0$ | $1/3$ | $-1$ |




THE "$\bar{3}$" MULTIPLET

TO OBTAIN THE $q\bar{q}$ STATES WE SCHEMATICALLY MULTIPLY THE 3 MULTIPLET BY THE $\bar{3}$, PLACING THE CENTER OF THE $\bar{3}$ MULTIPLET IN TURN ON EACH OF THE 3 STATES OF THE 3 MULTIPLET. THE 9 STATES SO PRODUCED CAN BE PRESENTED AS A HEXAGON WITH 3 STATES AT THE CENTER




THE ONLY PROBLEM CONCERNS THE 3 STATES IN THE CENTER. ONE PARTICULAR COMBINATION IS NOTABLE

$$\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

THIS IS THE GENERALIZATION OF THE ISOSINGLET STATE, AND IS CALLED AN SU(3) SINGLET.

ONE OF THE OTHER 3 CENTRAL STATES IS TAKEN AS $\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$ SO THE ISOTRIPLET DISCUSSED ABOVE CAN BE FOUND INSIDE THE SU(3) CLASSIFICATION. THE 3RD STATE MUST BE ORTHOGONAL TO THE OTHER 2, YIELDING

$$\frac{1}{\sqrt{6}}\left(2s\bar{s} - u\bar{u} - d\bar{d}\right) \qquad \text{AN ISOSINGLET ALSO.}$$

THE LAST 2 CENTRAL STATES COMBINE WITH THE OUTER 6 TO FORM THE SU(3) <u>OCTET</u>. THE RESULT OF THE MULTIPLICATION IS SYMBOLIZED

$$3 \times \bar{3} = 1 + 8$$

OF COURSE ALL 9 OF THESE STATES MUST BE GROUPED INTO $G = \pm 1$ COMBOS BEFORE IDENTIFYING THEM WITH MESONS. HOWEVER, TO SAVE INK, PEOPLE USUALLY DO NOT WRITE OUT THE FULL G PARITY WAVE FUNCTIONS, BUT ASSUME THE READER IS FAMILIAR WITH THE NECESSARY PROCEDURE

i.e. $\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$ STANDS FOR $\frac{1}{\sqrt{6}}\left[\left(u\bar{u} + d\bar{d} + s\bar{s}\right) \pm \left(\bar{u}u + \bar{d}d + \bar{s}s\right)\right]$

WITH + OR − DEPENDING ON THE CONTEXT. (GOOD LUCK!)

GROUP THEORY ACCOUNTANTS SAY THAT THE SU(2) SYMMETRY OF SPIN IS COMBINED WITH THE SU(3) FLAVOR SYMMETRY TO GIVE THE TOTAL SYMMETRY SU(6). [PEOPLE SELDOM SEEM TO GO SO FAR AS TO SAY THAT WHEN COLOR IS ADDED WE ARE DEALING WITH SU(18)]. IN SU(6) THE BASIC QUARK AND ANTI QUARK MULTIPLETS ARE CALLED 6 AND $\bar{6}$. THE MULTIPLICATION RULE IS $6 \times \bar{6} = 1 + 35$.

BUT SINCE THE 35-PLET INCLUDES BOTH SPIN 0 AND SPIN 1 STATES IT SEEMS PREFERABLE TO STICK WITH THE SPIN-FLAVOR VIEW OF THE WAVE FUNCTIONS. PEOPLE SOMETIMES WRITE

$$35 = (1,8) + (3,8) + (3,1) \qquad \text{THEN } 1 = (1,1)$$

SU(6) MULTIPLET

SU(3) FLAVOR MULTIPLET

SU(2) SPIN MULTIPLET

WE CAN INSERT A BRIEF REMARK ABOUT COLOR AS AN ACCOUNTING DEVICE. IN LECTURE 14 WE CONSIDER ADDITIONAL CONSEQUENCES OF THE COLOR SCHEME. THE HYPOTHESIS IS THAT EACH QUARK FLAVOR ACTUALLY COMES IN 3 COLORS, SAY $r$, $g$ AND $b$. THESE 3 COLORS FORM AN SU(3) TRIPLET, AND IF QUARKS ARE COMBINED, HIGHER SU(3) MULTIPLETS OF COLOR ARE FORMED ALSO. NATURE SEEMS TO ALLOW ONLY 'COLORLESS' STATES TO EXIST (AT PRESENTLY ACCESSIBLE ENERGIES), WHICH IS ASSOCIATED IN SOME WAY WITH THE FACT OF QUARK CONFINEMENT. A SINGLE QUARK IS NOT COLORLESS - BEING A COLOR TRIPLET. BUT A $q\bar{q}$ STATE CAN BE EITHER A COLOR OCTET OR SINGLET, ACCORDING TO THE SU(3) MULTIPLICATION RULES JUST CONSIDERED. THE COLOR SINGLET IS THE FAVORED 'COLORLESS' STATE, HAVING EQUAL AMOUNTS OF $r$, $g$ & $b$.

$$q\bar{q} \text{ COLOR SINGLET} = \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$$

WE COMBINE THE COLOR WAVE FUNCTION WITH THE FLAVOR WAVE FUNCTION IN AN EXAMPLE:

$$\frac{u\bar{s} + \bar{s}u}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{6}}\left\{u_r\bar{s}_{\bar{r}} + \bar{s}_{\bar{r}}u_r + u_g\bar{s}_{\bar{g}} + \bar{s}_{\bar{g}}u_g + u_b\bar{s}_{\bar{b}} + \bar{s}_{\bar{b}}u_b\right)$$

THIS DOES NOT ALTER THE EXCHANGE SYMMETRY OF THE FLAVOR STATE, SO OUR PREVIOUS ARGUMENT ABOUT THE OVERALL SYMMETRY OF THE $q\bar{q}$ WAVE FUNCTION IS UNCHANGED.

THE TABLE SUMMARIZES THE IDENTIFICATION OF ACTUAL MESONS WITH THE QUARK WAVE FUNCTIONS. THERE IS A COMPLICATION WITH THE $\omega$ AND $\phi$ MESONS WHICH WE DISCUSS IN SEC. 4.

| | MASS | SU(3) | SPIN | ISPIN | $T_3$ | Y | C | quark wave function |
|---|---|---|---|---|---|---|---|---|
| $\pi^{\pm}$ | 139.6 | 8 | 0 | 1 | $\pm 1$ | 0 | 0 | $u\bar{d}(d\bar{u})$ |
| $\pi^0$ | 135.0 | 8 | 0 | 1 | 0 | 0 | 0 | $(u\bar{u}-d\bar{d})/\sqrt{2}$ |
| $K^{\pm}$ | 493.7 | 8 | 0 | $\frac{1}{2}$ | $\pm\frac{1}{2}$ | $\pm 1$ | 0 | $u\bar{s}(s\bar{u})$ |
| $K^0(\overline{K^0})$ | 497.7 | 8 | 0 | $\frac{1}{2}$ | $\mp\frac{1}{2}$ | $\pm 1$ | 0 | $d\bar{s}(s\bar{d})$ |
| $\eta$ | 548 | 8 | 0 | 0 | 0 | 0 | 0 | $(2s\bar{s}-u\bar{u}-d\bar{d})/\sqrt{6}$ |
| $\eta'$ | 958 | 1 | 0 | 0 | 0 | 0 | 0 | $(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$ |
| $\rho^{\pm}$ | 776 | 1 | 1 | 1 | $\pm 1$ | 0 | 0 | $u\bar{d}(\bar{d}u)$ |
| $\rho^0$ | | 1 | 1 | 1 | 0 | 0 | 0 | $u\bar{d}(\bar{d}u)$ |
| $\omega$ | 782 | $1\oplus 8$ | 1 | 0 | 0 | 0 | 0 | $(u\bar{u}+d\bar{d})/\sqrt{2}$ |
| $K^{*\pm}$ | 892 | 1 | 0 | $\frac{1}{2}$ | $\pm\frac{1}{2}$ | $\pm 1$ | 0 | $u\bar{s}(s\bar{u})$ |
| $K^{*0}(\overline{K^{*0}})$ | 899 | 1 | 1 | $\frac{1}{2}$ | $\mp\frac{1}{2}$ | $\pm 1$ | 0 | $d\bar{s}(s\bar{d})$ |
| $\phi$ | 1020 | $1\oplus 8$ | 1 | 0 | 0 | 0 | 0 | $s\bar{s}$ |
| $D^{\pm}$ | 1868 | $\bar{3}(3)$ | 0 | $\frac{1}{2}$ | $\pm\frac{1}{2}$ | $\frac{1}{3}$ | $\pm 1$ | $c\bar{d}(d\bar{c})$ |
| $D^0(\overline{D^0})$ | 1863 | $\bar{3}(3)$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\mp\frac{1}{3}$ | $\pm 1$ | $c\bar{u}(u\bar{c})$ |
| $F^{\pm}$ | – | $\bar{3}(3)$ | 0 | 0 | 0 | $\pm\frac{2}{3}$ | $\pm 1$ | $c\bar{s}(s\bar{c})$ |
| $D^{*\pm}$ | 2009 | $\bar{3}(3)$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\mp\frac{1}{3}$ | $\pm 1$ | $c\bar{d}(d\bar{c})$ |
| $D^{*0}(\overline{D^{*0}})$ | 2006 | $\bar{3}(3)$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\mp\frac{1}{3}$ | $\pm 1$ | $c\bar{u}(u\bar{c})$ |
| $\eta_c$ | 2980 | 1 | 0 | 0 | 0 | 0 | 0 | $c\bar{c}$ |
| $J/\psi$ | 3097 | 1 | 1 | 0 | 0 | 0 | 0 | $c\bar{c}$ |
| $\Upsilon$ | 9460 | 1 | 1 | 0 | 0 | 0 | 0 | $b\bar{b}$ |

2. Quark Wave Functions of the Baryons

These are more difficult to construct. We can see a problem if we know the SU(3) multiplication rule for building a $qqq$ state:

$$\nabla \times \nabla = 3 \times 3 = 3^* + 6 = \Delta + \nabla$$

$3 + 3^* = 1 + 8$ as before $\qquad 3 \times 6 = 8 + 10 =$  + 

$$\text{So} \quad 3 \times 3 \times 3 = 1 + 8 + 8' + 10$$

antisymmetric (1) mixed symmetry (8 + 8') symmetric (10)

In nature we find a candidate baryon octet and decuplet. But the mixed symmetry of the SU(3) octets makes identification with particles confusing. The situation is improved in SU(6):

$$6 \times 6 \times 6 = 20 + 70 + 70 + 56$$

symmetric (56)

And the SU(6) 56-plet can be decomposed as $(2, 8) + (4, 10)$, where the first number is the spin and the second the SU(3) multiplet.

All of the lowest mass spin ½ and spin 3/2 baryons can be accomodated in the symmetric 56-plet of SU(6). The eventual introduction of color gave a mechanism for making the overall wave function antisymmetric, as suitable for a state composed of fermions. The 3 particle color singlet state is antisymmetric, according to the multiplication rules just given. Explicitly

$$qqq_{\text{color}} = \frac{1}{\sqrt{6}}\left(rgb - rbg + brg - bgr + gbr - grb\right)$$

Consider now the flavor decuplet. Both the spin 3/2 and SU(3) decuplet wave functions are symmetric – as they are the highest multiplets constructible out of 3 quarks.

$ddd \quad udd \quad uud \quad uuu = \Delta$
$dds \quad uds \quad uus = \Sigma^*$
$dss \quad uss = \Xi^*$
$sss = \Omega^-$

Thus the $\Delta^{++}$ with $S_z = 3/2$ is just $u\uparrow u\uparrow u\uparrow$

while the $\Delta^+$ is $\frac{1}{\sqrt{3}}(uud + udu + duu) \cdot \text{spin}$ etc.

The only slightly complicated case is the $uds$ state at the center of the decuplet. But it is still symmetric:

$$\Sigma^{*0} = \frac{1}{\sqrt{6}}\left(uds + usd + dus + dsu + sud + sdu\right) \cdot \text{spin}$$

FOR THE BARYON OCTET, BOTH THE SPIN AND FLAVOR WAVE FUNCTIONS HAVE MIXED SYMMETRY. WE MUST CONSTRUCT A COMBINED WAVE FUNCTION WHICH IS SYMMETRIC, AND ORTHOGONAL TO THE $S_z = \pm 1/2$ STATES OF THE DECUPLET. THE OCTET WAVE FUNCTIONS WILL NOT SIMPLY FACTOR INTO A SPIN PART AND A FLAVOR PART.

FOR EXAMPLE, THE PROTON IS A $uud$ COMBO. CONSIDERING THIS AS A 2+1 GROUPING, THE $uu$ PIECE HAS $I = 1$. HENCE IT MUST ALSO HAVE $S = 1$ SO AS TO GIVE A SYMMETRIC $uu$ WAVE FUNCTION IN TERMS OF FLAVOR AND SPIN. (THE COLOR FACTOR WILL MAKE THE FINAL WAVE FUNCTION ANTISYMMETRIC...). WE CAN NOW USE THE C-G TABLES TO MAKE THE PROPER COMBINATION OF THE $uu$ AND $d$ SPIN STATES

$$P_{SPIN} = |\tfrac{1}{2}, \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\ \underset{uu_{SPIN}}{|1,1\rangle}\ \underset{d_{SPIN}}{|\tfrac{1}{2}, -\tfrac{1}{2}\rangle} - \sqrt{\tfrac{1}{3}}\ |1,0\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$\text{OR} \qquad P\uparrow = \sqrt{\tfrac{2}{3}}\ u\uparrow u\uparrow d\downarrow - \sqrt{\tfrac{1}{3}}\left(\frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}}\right) d\uparrow \quad + \text{PERMUTATIONS}$$

$$= \frac{1}{\sqrt{18}}\begin{pmatrix} 2u\uparrow u\uparrow d\downarrow & - u\uparrow u\downarrow d\uparrow & - u\downarrow u\uparrow d\uparrow \\ + 2u\uparrow d\downarrow u\uparrow & - u\uparrow d\uparrow u\downarrow & - u\downarrow d\uparrow u\uparrow \\ + 2d\downarrow u\uparrow u\uparrow & - d\uparrow u\uparrow u\downarrow & - d\uparrow u\downarrow u\uparrow \end{pmatrix}$$

ALL 6 OUTER STATES OF THE OCTET ARE SIMILAR

| | Mass (MeV) | SU(3) | Spin | Ispin | $T_3$ | Y | c | | quark wave function (+ cyclic permutations) |
|---|---|---|---|---|---|---|---|---|---|
| P | 938.3 | 8 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 0 | P | $\vert uud\rangle(2\vert{+}{+}{-}\rangle - \vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| N | 939.6 | 8 | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | 0 | N | $-\vert ddu\rangle(2\vert{+}{+}{-}\rangle - \vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| Λ | 1115.6 | 8 | $\frac{1}{2}$ | 0 | 0 | 0 | 0 | Λ | $(\vert uds\rangle - \vert dus\rangle)(\vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/2\sqrt{3}$ |
| $\Sigma^+$ | 1189.4 | 8 | $\frac{1}{2}$ | 1 | 1 | 0 | 0 | $\Sigma^+$ | $\vert uus\rangle(2\vert{+}{+}{-}\rangle - \vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| $\Sigma^0$ | 1192.5 | 8 | $\frac{1}{2}$ | 1 | 0 | 0 | 0 | $\Sigma^0$ | $(\vert uds\rangle + \vert dus\rangle)(2\vert{+}{+}{-}\rangle - \vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/6$ |
| $\Sigma^-$ | 1197.3 | 8 | $\frac{1}{2}$ | 1 | $-1$ | 0 | 0 | $\Sigma^-$ | $\vert dds\rangle(2\vert{+}{+}{-}\rangle - \vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| $\Xi^0$ | 1314 | 8 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-1$ | 0 | $\Xi^0$ | $\vert uss\rangle(\vert{+}{+}{-}\rangle + \vert{+}{-}{+}\rangle - 2\vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| $\Xi^-$ | 1321 | 8 | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-1$ | 0 | $\Xi^-$ | $\vert dss\rangle(\vert{+}{+}{-}\rangle + \vert{+}{-}{+}\rangle - 2\vert{-}{+}{+}\rangle)/3\sqrt{2}$ |
| $\Delta^{++}$ | | 10 | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | 1 | 0 | $\Delta^{++}$ | $\vert uuu\rangle\vert{+}{+}{+}\rangle$ (no permutations) |
| $\Delta^+$ | 1230 | 10 | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{1}{2}$ | 1 | 0 | $\Delta^+$ | $\vert uud\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Delta^0$ | 1234 | 10 | $\frac{3}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | 1 | 0 | $\Delta^0$ | $\vert udd\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Delta^-$ | | 10 | $\frac{3}{2}$ | $\frac{3}{2}$ | $-\frac{3}{2}$ | 1 | 0 | $\Delta^-$ | $\vert ddd\rangle\vert{+}{+}{+}\rangle$ (no permutations) |
| $\Sigma^{*+}$ | 1382 | 10 | $\frac{3}{2}$ | 1 | 1 | 0 | 0 | $\Sigma^{++}$ | $\vert uus\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Sigma^{*0}$ | 1382 | 10 | $\frac{3}{2}$ | 1 | 0 | 0 | 0 | $\Sigma^{*0}$ | $(\vert uds\rangle + dus\rangle)\vert{+}{+}{+}\rangle/\sqrt{6}$ |
| $\Sigma^{*-}$ | 1387 | 10 | $\frac{3}{2}$ | 1 | $-1$ | 0 | 0 | $\Sigma^{*-}$ | $\vert dds\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Xi^{*0}$ | 1532 | 10 | $\frac{3}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-1$ | 0 | $\Xi^{*0}$ | $\vert uss\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Xi^{*-}$ | 1535 | 10 | $\frac{3}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-1$ | 0 | $\Xi^{*-}$ | $\vert dss\rangle\vert{+}{+}{+}\rangle/\sqrt{3}$ |
| $\Omega^-$ | 1672 | 10 | $\frac{3}{2}$ | 0 | 0 | $-2$ | 0 | $\Omega^-$ | $\vert sss\rangle\vert{+}{+}{+}\rangle$ (no permutations) |
| $\Lambda_c^+$ | 2273 | $\bar{3}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{2}{3}$ | 1 | $\Lambda_c^+$ | $(\vert udc\rangle - \vert duc\rangle)(\vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/2\sqrt{3}$ |
| – | ? | $\bar{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{3}$ | 1 | – | $(\vert usc\rangle - \vert suc\rangle)(\vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/2\sqrt{3}$ |
| – | ? | $\bar{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | 1 | – | $(\vert dsc\rangle - \vert sdc\rangle)(\vert{+}{-}{+}\rangle - \vert{-}{+}{+}\rangle)/2\sqrt{3}$ |

THE WAVE FUNCTIONS OF THE CENTRAL uds COMBOS CAN BE FOUND BY FIRST NOTING THAT THE $\Sigma^0$ IS AN ISOSPIN PARTNER OF THE $\Sigma^+$, AND SO SHOULD HAVE A SIMILAR 2+1 QUARK GROUPING – THE ud PAIR SHOULD HAVE S=1

$$\Sigma^0\uparrow = \sqrt{\tfrac{2}{3}}\left(\frac{u\uparrow d\uparrow + d\uparrow u\uparrow}{\sqrt{2}}\right)s\downarrow - \sqrt{\tfrac{1}{3}}\left(\frac{u\uparrow d\downarrow + u\downarrow d\uparrow + d\downarrow u\uparrow + d\uparrow u\downarrow}{2}\right)s\uparrow$$

+ PERMUTATIONS

THE $\Lambda^0$ IS ORTHOGONAL TO THE $\Sigma^0$, SO THE ud PAIR HAS S=0 RATHER THAN 1:

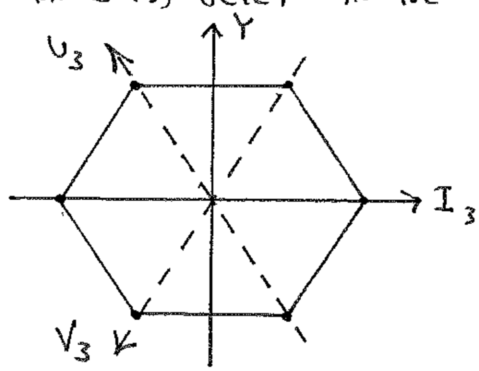
$$\Lambda^0\uparrow = \tfrac{1}{2}(u\uparrow d\downarrow - u\downarrow d\uparrow + d\downarrow u\uparrow - d\uparrow u\downarrow)s\uparrow \quad + \text{PERMUTATIONS.}$$

THE TABLE ON p 238 SUMMARIZES THESE RESULTS COMPACTLY.

## 3. GELL-MANN–OKUBO MASS FORMULA

IN OUR DISCUSSION OF ISOSPIN WE HAVE REMARKED HOW THE NEUTRON-PROTON MASS DIFFERENCE, $\pi^+ - \pi^0$ DIFFERENCE, ETC., SHOW THAT ISOSPIN IS NOT A COMPLETELY GOOD SYMMETRY. THE SITUATION WITH SU(3) IS EVEN WORSE! THE MASS DIFFERENCE BETWEEN THE PROTON AND $\Xi$, OR $\pi$ AND K IS QUITE SUBSTANTIAL. THE ORIGIN OF THESE MASS DIFFERENCES REMAIN UNCLEAR TODAY, ALTHOUGH IN LECTURE 14 WE WILL GIVE A COLORED-QUARK MODEL 'EXPLANATION'. GELL-MANN HAD THE IDEA THAT THE SU(3) SYMMETRY 'BREAKING' OBEYS PATTERNS WHICH CAN BE RELATED TO OTHER FEATURES OF SU(3) ITSELF.

CONSIDER AGAIN AN SU(3) OCTET IN THE $(I_3 - Y)$ PLANE.




NOTE THAT

$$I_3 + U_3 + V_3 = 0$$

IF SU(3) WERE TRULY A GOOD SYMMETRY THEN WE OUGHT TO BE ABLE TO USE AS AXES EITHER OF THE 2 DASHED LINES LABELLED $U_3$ AND $V_3$ (ALONG WITH CORRESPONDINGLY ROTATED Y AXES). FOR EACH CHOICE OF AXES WE WOULD DERIVE NEW SU(2) SYMMETRIES FORMALLY IDENTICAL TO ISOSPIN, WHICH HAVE BEEN CALLED U-SPIN AND V-SPIN. BUT FROM THE OBSERVED MASSES OF THE BARYONS IT IS CLEAR THAT U-SPIN AND V-SPIN SYMMETRY IS NOT WELL RESPECTED BY NATURE. IT TAKES CONSIDERABLE COURAGE TO PERSEVERE IN THE FACE OF THESE DIFFICULTIES!

Guided by the apparently large dependence of mass on hypercharge Y, Gell-Mann and Okubo (1962) suggested that the mass of an SU(3) state can be derived from an interaction with simple dependence on $U_3$, or equivalently $V_3$, rather than Y itself

i.e. $M = \langle \psi | H | \psi \rangle$ with $H = M_0 + A U_3$ as the Hamiltonian

$M_0$ is an SU(3) scalar, while $U_3$ is a 'vector' operator in the $I_3 - Y$ plane of SU(3) space. That is, the coefficient A can depend on the U-spin multiplet, but not on the $U_3$ component within the multiplet. The $U_3$ operator certainly 'breaks' the SU(3) symmetry, but in an organised way.

A 'simple' application of this idea is to the U=1 triplet within the baryon octet, all members of which have electric charge zero. Of course, without the U-spin idea the following result seems rather unlikely.

The U=1 multiplet consists of $|U=1,1\rangle = n$, $|1,0\rangle =$ mixture of $\Sigma^0$ & $\Lambda^0$, and $|1,-1\rangle = \Xi^0$. We can evaluate the appropriate mixture of $\Sigma^0$ & $\Lambda^0$ by noting that the U-axis is 120° from the I-axis. A rotation by 120° in the SU(3) plane takes states of definite I & $I_3$ into states of definite U and $U_3$

$$\begin{pmatrix} |U=1,0\rangle \\ |U=0,0\rangle \end{pmatrix} = \begin{pmatrix} \cos 120^\circ & \sin 120^\circ \\ -\sin 120^\circ & \cos 120^\circ \end{pmatrix} \begin{pmatrix} \Sigma^0 \\ \Lambda^0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda^0 \\ -\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda^0 \end{pmatrix}$$

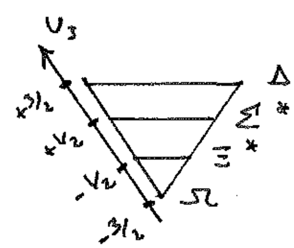
Now we can evaluate the mass $= \langle \psi | H | \psi \rangle$:

$$M_n = M_0 + A$$

$$M_{|U=1,0\rangle} = M_0 = \tfrac{1}{4} M_{\Sigma^0} + \tfrac{3}{4} M_{\Lambda^0} \qquad \Longrightarrow A = \frac{M_\Xi - M_n}{2} \sim 190\,\text{MeV}$$

$$M_{\Xi^0} = M_0 - A$$

Hence $$\underbrace{2(M_N + M_{\Xi^0})}_{4514\text{ MeV}} = \underbrace{M_{\Sigma^0} + 3M_{\Lambda^0}}_{4541\text{ MeV}} \qquad \text{Good to } 1\%$$




Given this success, we consider the baryon decuplet. The constants $M_0$ and A need not be the same for the U-spin quartet $(\Delta^-, \Sigma^{*-}, \Xi^{*-}, \Omega^-)$ as for the triplet just considered. But we can write

$$M_{\Delta^-} = M_0' + \tfrac{3}{2}A'$$
$$M_{\Sigma^{*-}} = M_0' + \tfrac{1}{2}A'$$
$$M_{\Xi^{*-}} = M_0' - \tfrac{1}{2}A'$$
$$M_{\Omega^-} = M_0' - \tfrac{3}{2}A'$$

$$\Longrightarrow \underbrace{M_{\Sigma^{*-}} - M_{\Delta^-}}_{152\text{ MeV}} = \underbrace{M_{\Xi^{*-}} - M_{\Sigma^{*-}}}_{149\text{ MeV}} = \underbrace{M_{\Omega^-} - M_{\Xi^{*-}}}_{139\text{ MeV}}$$

$$A' \sim 145\text{ MeV}$$

GELL-MANN USED THIS RELATION TO PREDICT THE MASS, AS WELL AS THE QUANTUM NUMBERS, OF THE $\Omega^-$, WHICH WAS DISCOVERED IN 1964.

BY FURTHER SU(3) CONSIDERATIONS, THE MASS FORMULA CAN BE EXTENDED TO RELATE MEMBERS OF DIFFERENT U-SPIN MULTIPLETS. THE RESULT IS USUALLY GIVEN IN TERMS OF I AND Y RATHER THAN I AND U:

$$M = M_0 + M_1 Y + M_2 \left[ I(I+1) - Y^2/4 \right]$$

## 4. Quark Model of the Mass Formula

THE GELL-MANN - OKUBO MASS FORMULA WAS DERIVED ENTIRELY FROM CONSIDERATIONS OF SU(3) PRIOR TO THE EMERGENCE OF THE QUARK MODEL. THE LATTER GIVES A SOMEWHAT DIFFERENT INSIGHT INTO THE MASS FORMULA, WHICH IS MORE READILY ILLUSTRATED FOR MESONS RATHER THAN BARYONS.

IN THE QUARK MODEL WE SIMPLY ASSIGN MOST OF THE MASS DIFFERENCE OF THE OBSERVED PARTICLES TO POSSIBLE MASS DIFFERENCES AMONG THE QUARKS. A USEFUL PARAMETRIZATION TURNS OUT TO BE

$$M_u \sim M_d \equiv M \qquad \text{WHILE} \qquad M_s \equiv M + \Delta$$

IN ADDITION THERE MIGHT BE A PIECE OF THE MASS DUE TO THE QUARK-QUARK INTERACTION ENERGY. THIS PIECE WILL DEPEND ON THE SU(3) MULTIPLET UNDER CONSIDERATION, BUT WILL BE THE SAME FOR ALL MEMBERS WITHIN A MULTIPLET IN THE FIRST APPROXIMATION THAT $M_s = M_u = M_d$. WE INDICATE A POSSIBLY BETTER APPROXIMATION IN LECTURE 14. SMALL ADDITIONAL CORRECTIONS DUE TO ELECTROMAGNETIC EFFECTS WILL BE EXPLORED IN THE NEXT SECTION.

WE SUPPOSE $$M = \sum M_{\text{QUARK}} - U(\text{SPIN, SU(3) MULTIPLET})$$
↑ ENERGY, NOT U-SPIN

WE ANTICIPATE A DIFFICULTY IN APPLYING THIS TO THE PSEUDOSCALAR MESONS. $M_\pi$ IS SO SMALL THAT THE BINDING ENERGY U IS LARGE COMPARED TO $M_\pi$, AND THE WHOLE IDEA OF A NON-RELATIVISTIC ANALYSIS IS SOMEWHAT DUBIOUS. EMPIRICALLY WE FIND THAT $(\text{MASS})^2$ FOR THE PSEUDOSCALAR MESONS OBEY PATTERNS CONSISTENT WITH THE ABOVE HYPOTHESIS. WE ARE CONTINUALLY IMPRESSED BY THE APPROXIMATE NATURE OF THE QUARK MODEL, AS WELL AS BY ITS SPECTACULAR SUCCESSES!

WE BEGIN WITH THE VECTOR MESONS, ALL OF WHOSE MASSES ARE ABOVE 750 MeV.

THUS $\rho = u\bar{d} \Rightarrow M_\rho = M_u + M_{\bar{d}} - U(3,8) = 2M - U(3,8) \equiv M_8 = 770\,MeV$

$K^* = u\bar{s} \Rightarrow M_{K^*} = 2M + \Delta - U(3,8) = M_8 + \Delta = 892\,MeV$

WE INFER $\Delta = M_s - M_u \sim 122\,MeV$

THE SU(3) STATE AT THE CENTER OF THE VECTOR MESON OCTET IS

$$\phi_8 = \frac{1}{\sqrt{6}}(2s\bar{s} - u\bar{u} - d\bar{d})$$

THE SU(3) SINGLET STATE IS

$$\phi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

THE NOTATION ANTICIPATES THAT THE OBSERVED $\omega$ AND $\phi$ MESONS HAVE DIFFERENT QUARK CONTENT THAN THE $\phi_1$ AND $\phi_8$ STATES, AS WE NOW SHOW.

$$M_{\phi_8} = \tfrac{4}{3} M_s + \tfrac{1}{3} M_u + \tfrac{1}{3} M_d - U(3,8) = M_8 + \tfrac{4}{3}\Delta$$

$$M_{\phi_1} = \tfrac{2}{3}(M_u + M_d + M_s) - U(3,1) = M_1 + \tfrac{2}{3}\Delta$$

DEFINING $M_1 \equiv 2M - U(3,1)$

BUT NOTE ALSO THAT THE MASS OPERATOR HAS NON-ZERO TRANSITION MATRIX ELEMENT BETWEEN STATES $\phi_1$ AND $\phi_8$

$$\langle \phi_8 | M | \phi_1 \rangle \sim \frac{2\sqrt{2}}{3}\Delta \cdot a$$

WHERE $a$ IS THE APPROPRIATE OVERLAP INTEGRAL OF THE SPATIAL WAVE FUNTIONS OF $\phi_1$ AND $\phi_8$. THE MAIN IDEA IS THAT THE SU(3) SYMMETRY BREAKING $M_s \neq M_u$ INTRODUCES A MIXING BETWEEN THE $q\bar{q}$ SINGLET AND OCTETS.

THE OBSERVED PARTICLE STATES $\omega$ AND $\phi$ MUST BE EIGENSTATES OF THE MASS OPERATOR, AND SO WILL BE SUITABLE LINEAR COMBOS OF THE STATES $\phi_1$ AND $\phi_8$. THE APPROPRIATE TRANSFORMATION IS JUST A ROTATION:

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_1 \end{pmatrix}$$

THIS WILL DIAGONALIZE THE MASS MATRIX, YIELDING EIGENVALUES $M_\phi$ & $M_\omega$

NAMELY
$$M = \begin{pmatrix} M_8 + \tfrac{4}{3}\Delta & \frac{2\sqrt{2}}{3}\Delta a \\ \frac{2\sqrt{2}\Delta a}{3} & M_1 + \frac{2\Delta}{3} \end{pmatrix} = R^{-1} M' R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} M_\phi & \\ & M_\omega \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

DIAGONALIZED MASS MATRIX

THEN $\det M = \det M' \Rightarrow M_\phi M_\omega = M_1 M_8 + \tfrac{2}{3}\Delta(2M_1 + M_8) + \tfrac{8}{9}\Delta^2(1 - a^2)$

VARIOUS COMPONENTS OF THE MATRIX EQUATION $M = R^{-1} M' M$ YIELD

$$\begin{cases} M_\phi + M_\omega = M_1 + M_8 + 2\Delta \\ \tan 2\theta = \dfrac{4\sqrt{2}/3\,\Delta a}{M_8 - M_1 + \frac{2\Delta}{3}} \end{cases}$$

WITH THE OBSERVED VALUES $M_\phi = 1020\,MeV$, $M_\omega = 780\,MeV$

AND $M_8 = 770\,MeV$, $\Delta = 122\,MeV$ AS ABOVE, WE FIND

$$M_1 \approx 790\,MeV \qquad a \sim 1 \qquad \text{AND} \qquad \Theta \sim 40^\circ$$

THIS SOLUTION IS VERY CLOSE TO THE 'IDEAL' MIXING CASE $M_1 \approx M_8$, $a \approx 1$ FOR WHICH $\Theta \sim 35^\circ$ WITH $\cos\Theta = \sqrt{2/3}$, $\sin\Theta = \sqrt{1/3}$

IN THE 'IDEAL' CASE
$$\phi = \sqrt{2/3}\,\phi_8 + \sqrt{1/3}\,\phi_1 = s\bar{s}$$
$$\omega = -\sqrt{1/3}\,\phi_8 + \sqrt{2/3}\,\phi_1 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

THE SU(3) SYMMETRY BREAKING ACTUALLY MAKES THE OBSERVED $\phi$ AND $\omega$ WAVE FUNCTIONS SIMPLER THAN THE PURE SU(3) STATES $\phi_1$ & $\phi_8$!

WE MAY ALSO NOTE THE MASS FORMULA

$$M_\omega + M_\phi = M_1 + M_8 + 2\Delta \sim 2(M_8 + \Delta) \approx 2 M_{K^*}$$

THIS IS THE EQUIVALENT OF THE GELL-MANN-OKUBO OCTET FORMULA DESCRIBED ON p 240.

WHEN WE CONSIDER THE PSEUDOSCALAR MESONS, THE CLAIM IS THAT THE ABOVE FORMALISM SHOULD BE ALTERED BY THE REPLACEMENT $M \to M^2$ EVERYWHERE. IF YOU BUY THIS LINE, THEN

$$M_\pi^2 = M_8^2 \qquad \text{(WITH A NEW } M_8\text{)}$$
$$M_K^2 = M_8^2 + \delta$$

THE $I = 0$ OCTET AND SINGLET STATES $\eta$ AND $\eta'$ ARE MIXED:

$$M_\eta^2 + M_{\eta'}^2 = M_1^2 + M_8^2 + 2\delta$$
$$M_\eta^2 M_{\eta'}^2 = M_1^2 M_8^2 + \tfrac{2}{3}\delta\left(2M_1^2 + M_8^2\right) + \tfrac{8}{9}\delta^2\left(1 - a^2\right)$$
$$\tan 2\Theta = \frac{4\sqrt{2}/3\;\delta a}{M_8^2 - M_1^2 + \tfrac{2}{3}\delta}$$

THE DATA $M_\pi = 135\,MeV$, $M_\eta = 548$, $M_{\eta'} = 958$ $M_K = 495$

YIELD $M_1 = 863$, $M_8 = M_\pi$ AND $\Theta = -11^\circ$ WITH $a = 1/2$

AS $\Theta \sim 0$, PEOPLE OFTEN WRITE $\eta = \frac{1}{\sqrt{6}}\left(2s\bar{s} - u\bar{u} - d\bar{d}\right)$; $\eta' = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$

BUT OCCASIONALLY ONE MUST REMEMBER THE PRESENCE OF A SMALL MIXING.

In the case of the baryons we would expect to use our formalism as for the vector mesons. For the baryon decuplet the situation is simple

$$M_\Delta = 3M - U(4,10)$$
$$M_{\Sigma^*} = M_\Delta + \Delta \leftarrow = M_s - M_u$$
$$M_{\Xi^*} = M_\Delta + 2\Delta$$
$$M_{\Omega^-} = M_\Delta + 3\Delta$$

This is well observed experimentally, with $\Delta \sim 145$ MeV, compared to $\Delta \sim 122$ MeV inferred from the vector mesons.

For the baryon octet the mass formula $M = \sum M_q - U(2,8)$ leads to

$$M_\Lambda = M_\Sigma = M_p + \Delta$$
$$M_\Xi = M_p + 2\Delta$$

But $M_\Sigma - M_\Lambda \sim 80$ MeV, while from $M_{\Lambda^0}$ and $M_\Xi$ we infer $\Delta \sim 180$ MeV)

Extra assumptions are required to patch things up, and to reproduce the Gell-Mann - Okubo mass formula. It is impressive that the early work on SU(3) gave more direct systematization of the baryon octet mass splittings than the later quark model. This may have slowed the acceptance of the quark model as a dynamical theory, as compared to a mere bookkeeping device.

## 5. Electromagnetic Mass Corrections

Small mass corrections can be attributed to the differences in electric and magnetic fields among the various particles. (The total contribution to the mass from electromagnetic effects is notoriously difficult to calculate!) A simple assumption is that particles of the same charge and in the same SU(3) multiplet have the same sized correction due to electromagnetic effects. This assumption seems unlikely to be completely correct since $M_p \neq M_{\Sigma^+}$ indicates strong differences in mass between like charged particles, but it might be accurate to $\sim 15\%$ for the electromagnetic correction. If we proceed anyway, we estimate for the baryon octet

$$\delta M_p = \delta M_{\Sigma^+}$$
$$\delta M_N = \delta M_{\Xi^0}$$
$$\delta M_{\Sigma^-} = \delta M_{\Xi^-}$$

If we also suppose that the mass difference between members of the same isospin multiplet is entirely due to electromagnetic effects, then

$$M_p - M_N = \delta M_p - \delta M_N = \delta M_{\Sigma^+} - \delta M_{\Xi^0} \quad (-\delta M_{\Sigma^-} + \delta M_{\Xi^-})$$
$$= M_{\Sigma^+} - M_{\Sigma^-} + M_{\Xi^-} - M_{\Xi^0}$$

DATA: $-1.3 \pm .005 \stackrel{?}{=} -8.00 \pm .08 + 6.42 \pm .7 = -1.68 \pm .7$

FROM THE QUARK MODEL POINT OF VIEW, THIS AGREEMENT MAY BE SOMEWHAT FORTUITOUS. IN THIS MODEL WE CAN IDENTIFY 3 SOURCES OF MASS CORRECTIONS WITHIN AN ISOSPIN MULTIPLET.

1. THE NUMBER OF STRANGE QUARKS IS CONSTANT WITHIN AN ISOSPIN MULTIPLET, BUT THE $u$ & $d$ COMPOSITION VARIES. LET

$$\delta_1 = M_u - M_d$$

THEN WE EXPECT A MASS CORRECTION $\delta M_1 = \delta_1 \cdot \#$ OF $u$ QUARKS

CLASSICALLY, SINCE $Q_u = -2Q_d$ WE WOULD EXPECT

$$\delta_1 \sim \frac{Q_u^2 - Q_d^2}{r} > 0$$

BUT THE FITS DESCRIBED SHORTLY SUGGEST $\delta_1 < 0$ !

2. THE ELECTROSTATIC INTERACTION BETWEEN QUARKS CONTRIBUTES MASS SHIFT

$$\delta M_2 = \frac{1}{2} \sum_{\text{PAIRS}}' \frac{Q_i Q_j}{r} \equiv \delta_2 \sum_{ij}' Q_i Q_j \qquad \left(\begin{array}{l}\text{MEASURING } Q\\ \text{IN UNITS OF } e\end{array}\right)$$

WE ESTIMATE $\delta_2 \sim e^2/r \approx \frac{\alpha}{r} \sim \frac{\alpha}{1\,\text{FERMI}} \sim \frac{197\,\text{MeV}}{137} \sim 1.5\,\text{MeV}$

3. THE MAGNETOSTATIC INTERACTION BETWEEN QUARK MAGNETIC MOMENTS ALSO CONTRIBUTES A MASS CORRECTION. SUCH INTERACTIONS WERE ANALYZED BY FERMI (1930, FOR ATOMS NOT QUARKS!). A SEMI-CLASSICAL DISCUSSION IS GIVEN BY JACKSON, E&M, PP 186-187. THE RESULT IS

$$\delta M_3 = -\frac{8\pi}{3} \sum_{\text{PAIRS}}' |\psi_{\text{PAIR}}(0)|^2 \, \bar{\mu}_i \cdot \bar{\mu}_j$$

FOR AN S WAVE STATE THE DIPOLE-DIPOLE INTERACTION AVERAGES TO ZERO, UNLESS THE 2 MOMENTS ARE RIGHT ON TOP OF ONE ANOTHER.
ASSUMING THE QUARKS ARE IDEAL DIRAC SPIN ½ PARTICLES, $\bar{\mu} = \frac{Q}{2M}\bar{\sigma}$

SO
$$\delta M_3 = -\frac{2\pi}{3} \sum_{\text{PAIRS}}' \frac{|\psi(0)|^2}{M_i M_j} (\bar{\sigma}_i \cdot \bar{\sigma}_j)\, Q_i Q_j$$
$$\equiv -\delta_3 \sum_{\text{PAIRS}}' Q_i Q_j (\bar{\sigma}_i \cdot \bar{\sigma}_j)$$

AS IN OUR DISCUSSION OF POSITRONIUM, P174, $|\psi(0)|^2 \sim M^3$

SO WE ESTIMATE $\delta M_3 \sim e^2 M \sim \alpha M \sim \frac{300\,\text{MeV}}{137} \sim 2\,\text{MeV}$

ALSO NOTE THAT $\bar{\sigma}_i \cdot \bar{\sigma}_j = \sigma_{xi}\sigma_{xj} + \sigma_{yi}\sigma_{yj} + \sigma_{zi}\sigma_{zj}$

GIVES $\begin{cases} +1 & \text{WHEN EVALUATED ON A TRIPLET SPIN STATE (NO MATTER WHAT } S_z) \\ -3 & \text{" SINGLET "} \end{cases}$

When considering the baryon decuplet which was spin $3/2$, all quark pairs must be in $S=1$ triplet states. For the baryon octet, both triplet and singlet $qq$ pairs occur. We can make the needed evaluations of $\vec{\sigma}_i \cdot \vec{\sigma}_j$ using our table of quark wave functions, p 238, but this is quite a chore. The table summarizes this effort.

*Table 15.6 Quark Model hfs Mass Splitting Among the Baryons*

| Particle | Mass (MeV/$c^2$) | $\delta M_1/\delta_1$ | $\delta M_2/\delta_2$ | $\delta M_3/\delta_3$ | Differences | Measured (MeV/$c^2$) |
|---|---|---|---|---|---|---|
| $P$ | 938.26 | 2 | 0 | $-\frac{16}{9}$ | $\delta_1 + \frac{1}{3}(\delta_2 - \delta_3)$ | $(-1.2933 \pm 0.0001)$ |
| $N$ | 939.55 | 1 | $-\frac{1}{3}$ | $-\frac{13}{9}$ | | |
| $\Sigma^+$ | 1189.43 | 2 | 0 | $-\frac{16}{9}$ | | |
| $\Sigma^0$ | 1192.55 | 1 | $-\frac{1}{3}$ | $-\frac{1}{9}$ | $2\delta_1 - \frac{1}{3}(\delta_2 + 7\delta_3)$ | $(-8.00 \pm 0.18)$ |
| $\Sigma^-$ | 1197.42 | 0 | $\frac{1}{3}$ | $\frac{5}{9}$ | $\delta_1 - \frac{2}{3}(\delta_2 + \delta_3)$ | $(-4.87 \pm 0.07)$ |
| $\Lambda^0$ | 1115.57 | 1 | $-\frac{1}{3}$ | $-\frac{5}{9}$ | | |
| $\Xi^0$ | 1314.7 | 1 | $-\frac{1}{3}$ | $-\frac{13}{9}$ | $\delta_1 - \frac{2}{3}(\delta_2 + 3\delta_3)$ | $(-6.6 \pm 0.7)$ |
| $\Xi^-$ | 1321.3 | 0 | $\frac{1}{3}$ | $\frac{5}{9}$ | | |
| $\Delta^{++}$ | 1236.0 | 3 | $\frac{4}{3}$ | $-\frac{4}{3}$ | | |
| $\Delta^+$ | — | 2 | 0 | 0 | $2\delta_1 + \frac{5}{3}(\delta_2 - \delta_3)$ | $(-0.45 \pm 0.85)$ |
| $\Delta^0$ | — | 1 | $-\frac{1}{3}$ | $\frac{1}{3}$ | $3\delta_1 + \delta_2 - \delta_3$ | $(-7.9 \pm 6.8)$ |
| $\Delta^-$ | — | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | | |
| ${}_{\Delta}Y^+$ | 1382.2 | 2 | 0 | 0 | | |
| ${}_{\Delta}Y^0$ | — | 1 | $-\frac{1}{3}$ | $\frac{1}{3}$ | $2\delta_1 - \frac{1}{3}(\delta_2 - \delta_3)$ | $(-5.8 \pm 3.1)$ |
| ${}_{\Delta}Y^-$ | 1388.0 | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | | |
| ${}_{\Delta}\Xi^0$ | 1528.9 | 1 | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\delta_1 - \frac{2}{3}(\delta_2 - \delta_3)$ | $(-4.9 \pm 2.2)$ |
| ${}_{\Delta}\Xi^-$ | 1533.8 | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | | |
| $\Omega^-$ | 1672.4 | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | | |

Fitting the data in the table, one finds

$$\delta_1 = -1.9 \text{ MeV}$$
$$\delta_2 = 3.2 \text{ MeV}$$
$$\delta_3 = 1.3 \text{ MeV}$$

Certainly $\delta_2$ and $\delta_3$ are of the expected magnitude, using our crude estimates. But $\delta_1 < 0$ remains surprising.

Application of this technique to the mesons is somewhat less successful. In particular one would infer that $\delta_1 = -7$ MeV.

## 6. Baryon Magnetic Moments

An interesting hypothesis of the quark model is that the "anomalous" magnetic moments of the baryons can be explained as due to the Dirac magnetic moments of the quarks:

$$\mu_q = \frac{Q_q}{2M_q} (\hbar)$$

So far we have no precise predictions for the various $M_q$, but we have the relations $M_u \sim M_d \Rightarrow \mu_d = -\frac{1}{2}\mu_u$

and $M_s = M_u + \Delta$ with $\Delta = M_\Lambda - M_p = 175$ MeV as one estimate among several

WE ESTIMATE THE BARYON MAGNETIC MOMENTS BY SIMPLY ADDING UP THE VARIOUS QUARK MOMENTS WEIGHTING THEM BY THE PROBABILITY THAT VARIOUS SPIN UP OR SPIN DOWN QUARKS APPEAR INSIDE A BARYON. OUR METHOD OF CONSTRUCTING THE BARYON WAVE FUNCTIONS ON PP 238-239 IS USEFUL FOR THIS. IN PARTICULAR, WE NOTE THAT IF 2 QUARKS ARE IN A SPIN SINGLET STATE, WE GET NO NET MOMENT FROM THAT PAIR.

THUS $\mu_p = \frac{2}{3}(2\mu_u - \mu_d) + \frac{1}{3}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$ FROM p 238

$$\left.\begin{aligned} \mu_{\Sigma^0} &= \tfrac{2}{3}(\mu_u + \mu_d - \mu_s) + \tfrac{1}{3}\mu_s = \tfrac{2}{3}(\mu_u + \mu_d) - \tfrac{1}{3}\mu_s \\ \mu_{\Lambda^0} &= \mu_s \end{aligned}\right\} \text{p } 239$$

THE OTHER STATES ON THE OUTSIDE OF THE OCTET ALL HAVE FORM $q_a q_a q_b$

$\Rightarrow \quad \mu = \frac{4}{3}\mu_a - \frac{1}{3}\mu_b$

IN PARTICULAR $\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$

USING $M_u = M_d$ WE HAVE $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$ NO MATTER WHAT $M_u$ IS

THIS IS THE MOST MODEL INDEPENDENT PREDICTION WE CAN MAKE, AND IT IS RATHER WELL SATISFIED: $\left.\frac{\mu_n}{\mu_p}\right|_{EXPT} = -.685$

FROM THE OBSERVED VALUE $\mu_p = 2.79$ NUCLEAR MAGNETONS $\left(\frac{e}{2M_p}\right)$ WE INFER $M_u = 336$ MeV.

FROM THE OBSERVED VALUE $\mu_\Lambda = -.614$ (N.M) WE INFER $M_s = 510$ MeV

THEN $M_s - M_u = 174$ MeV, REMARKABLY CLOSE TO $M_\Lambda - M_p = 175$ MeV

THEN WE HAVE $\mu_u = 1.863$ (N.M), $\mu_d = -.932$ (N.M); $\mu_\Lambda = -.614$ (N.M)

| | PREDICTION (N.M.) | DATA (N.M.) | |
|---|---|---|---|
| $\mu_{\Sigma^+} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s$ | 2.69 | $2.38 \pm .02$ | ANKENBRANDT, PRL 51, 863 (1983) |
| $\mu_{\Sigma^-} = \frac{4}{3}\mu_d - \frac{1}{3}\mu_s$ | -1.04 | $-1.14 \pm .05$ | HERTZOG, PRL 51, 1131 (1983) |
| $\mu_{\Xi^0} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_u$ | -1.44 | $-1.25 \pm .015$ | COX, PRL 46, 877 (1981) |
| $\mu_{\Xi^-} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_d$ | -.51 | $-.69 \pm .04$ | RAMIEKA, PRL 52, 581 (1984) |
| $\mu_{\Sigma^0 \to \Lambda} = \sqrt{\frac{1}{3}}(\mu_d - \mu_u)$ | -1.61 | $-1.59 \pm .07$ | PETERSON, PRL 57, 949 (1986) |

EX: DERIVE THIS PREDICTION. $\mu_{\Sigma^0 \to \Lambda}$ ENTERS IN THE RATE OF THE DECAY $\Sigma^0 \to \Lambda\gamma$

THE RECENT RESULT OF COX ET AL P.R.L. 46, 877 (1981) THAT $\mu_{\Xi^0} = -1.25 \pm .015$ N.M. IS IN DISAGREEMENT WITH OUR MODEL CALCULATION BY ~12 STANDARD DEVIATIONS!

AN INTERESTING TECHNICAL QUESTION IS: HOW ARE THE HYPERON MOMENTS MEASURED? THESE ARE SHORT-LIVED PARTICLES SO THEY DON'T WAIT FOR US TO MEASURE THEM SLOWLY.

THERE ARE 3 STEPS

- PRODUCE POLARIZED HYPERONS. IN A REACTION LIKE $\pi^- p \to \Lambda^0 K^0$ IT IS FOUND THAT THE $\Lambda^0$ POLARIZATION VARIES LINEARLY WITH TRANSVERSE MOMENTUM OF THE $\Lambda^0$.
- PRECESS THE HYPERON MOMENT BY APPLYING A MAGNETIC FIELD
  $$\vec{\mu} \times \vec{B} = \text{TORQUE} = \frac{d\vec{L}}{dt} = \frac{\hbar}{2|\vec{\mu}|}\frac{d\vec{\mu}}{dt}$$
  THE DIRECTION OF $\vec{\mu}$ VS. TIME NOW DEPENDS ON $|\vec{\mu}|$
- OBSERVE THE DIRECTION OF $\vec{\mu}$ AT THE MOMENT THE HYPERON DECAYS VIA A PARITY VIOLATING DECAY SUCH AS $\Lambda \to p\pi^-$. SUCH A DECAY CAN HAVE A $\vec{\mu}\cdot\vec{P}$ CORRELATION.

ONLY PARTICLES WHICH HAVE PARITY VIOLATING DECAYS CAN BE MEASURED BY THIS TECHIQUE. AS SUCH THE ONLY OTHER PARTICLE WHOSE MOMENT MIGHT BE MEASURED IS THE $\Omega^-$. THE VECTOR MESONS HAVE MAGNETIC MOMENTS ($\mu_{\rho^+} = \mu_u - \mu_d$ IF $S_z = 1$) BUT THESE WILL PROBABLY NEVER BE MEASURED... CONCEIVABLY THE MOMENT OF THE CHARMED PARTICLE STATE $udc \equiv \Lambda_c(2282)$ COULD BE MEASURED.

EX: WHY IS THERE NO MEASUREMENT OF $\mu_{\Sigma^0}$?

IN THE LITERATURE YOU WILL OFTEN FIND A SLIGHTLY SIMPLER ANALYSIS OF THE MAGNETIC MOMENTS. PEOPLE ASSUMED $\mu_s = \mu_d = -\frac{1}{2}\mu_u$, WHICH IS MORE SU(3) SYMMETRIC THAN OUR HYPOTHESES.

[THERE IS ANOTHER APPROACH TO ESTIMATING QUARK MASSES BASED ON THE THEORY OF 'CURRENT ALGEBRAS.' THE RESULTS ARE $M_u \sim 1\text{-}2$ MeV, $M_d \sim 5$ MeV, $M_s \sim 150\text{-}200$ MeV, I.E. THE ΔM ARE AS FOUND ABOVE, BUT ALL MASSES ARE LESS BY ~330 MeV. SEE THE BOOK OF T.D. LEE, p 582 ff. $M_u \neq 0$ IS MORE BELOVED BY THEORISTS THAN EXPERIMENTALISTS!]

[PERKINS CLAIMS THE 'PREDICTIONS' IN HIS TABLE 5.5 COME FROM THE PARAMETERS IN HIS EQ. (5-39). I BELIEVE THIS IS FALSE.]