Ph 406 Problem Set 1

Due: Feb. 10, 1993

(1) In high-energy physics a system of units in which $\hbar = c = 1$ is sometimes used. In this system show that length $\propto 1/m$, time $\propto 1/m$, energy $\propto m$ and momentum $\propto m$. If m is taken as the mass of a proton, what are the magnitudes of the units of length and of time?

(2) In the elastic scattering of 200 MeV electrons through 11° by a gold foil, it is found that the scattered intensity is 70% of that expected for point nuclei. Calculate the r.m.s. radius of the gold nucleus.

(3) An electron with energy E scatters off a stationary target of mass M, transferring momentum p and energy $\nu = E - E'$, where E' is the electron's final energy. The 4-momentum transfer q is given by $q^2 = p^2 - \nu^2/c^2$. Find an expression for W, the mass of the recoiling hadronic system in an *inelastic* collision in terms of M, ν and q^2. Show that for an *elastic* $(W \equiv M)$ collision $M = q^2/2\nu$. If the electron scattering angle is θ show that, neglecting the electron mass, $q^2c^2 = 4EE'\sin^2(\theta/2)$. In electron scattering off carbon at $E = 194$ MeV and $\theta = 135°$ a peak at $\nu = 5.58$ MeV and a broad peak near $\nu = 51$ MeV are observed. Account for their origin and explain why the peak near 51 MeV is broad.

(4) (a) Show that a negative muon captured in an S-state by a nucleus of charge Ze and mass A will spend a fraction $f \simeq 0.25A(Z/137)^3$ of its time in nuclear matter, and that in time t it will travel a total distance $fct(Z/137)$ in nuclear matter. (b) The law of radioactive decay of free muons is $dN/dt = -\lambda_d N$, where $\lambda_d = 1/\tau$ is the decay constant and the lifetime $\tau = 2.16\ \mu s$. For a negative muon captured in an atom Z, the decay constant is $\lambda = \lambda_d + \lambda_c$, where λ_c is the probability of nuclear capture per unit time. For aluminum ($Z = 13$, $A = 27$) the mean lifetime of negative muons is 0.88 μs. Calculate λ_c, and using the expression for f in (a), compute the interaction mean free path Λ for a muon in nuclear matter. (c) From the magnitude of Λ in (b) estimate the magnitude of the coupling constant in the reaction $\mu^- + p \to n + \nu$, assuming that a coupling constant of unity corresponds to a mean free path equal to the range of nuclear forces.

(5) The cross-section for the reaction $\pi^- + p \to \Lambda + K^0$ at 1-GeV/c incident momentum is approximately 1 mb (10^{-27} cm^2). Both Λ- and K^0-particles decay with a mean lifetime of about 10^{-10} s. From this information, estimate the relative magnitude of the couplings responsible for the production and decay, respectively, of the Λ- and K^0-particles.

(A SIMPLE ESTIMATE MIGHT BE BASED ON THE FACT THAT THE SIZES OF THE π, p, Λ, & K PARTICLES ARE ALL ABOUT 1 FERMI.)

OR, USE DIMENSIONAL ANALYSIS + $M_\pi \sim M_p/6$; $M_\Lambda \sim M_p$; $M_K \sim M_p/2$.

"COUPLING STRENGTH" = DIMENSIONLESS FACTOR IN σ OR τ.