Princeton University
Physics 104
Spring Term 2004

Learning Guide No. 2 Electric Potential, Capacitance

Reading Assignment: Tipler and Mosca -fifth edition, Chapters 23 and 24.

Problem I

The following questions test your knowledge of electric potentials. If you have any difficulty, reread Tipler and Mosca -fifth edition, Chapter 23.

1. Do electrons tend to go to regions of *low potential* or *high potential*? **Key 53**
2. Suppose that the earth has a net charge that is not zero. Can one still adopt the earth as a standard reference point of potential and assign the potential $V = 0$ to it? **Key 7**
3. What scheme can you devise to ensure that the electric potential in a given region of space will have a constant value? **Key 16**

Problem II

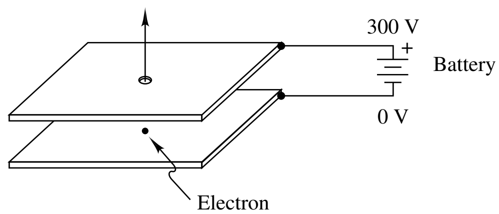
Consider a point charge with $q = 3.0 \times 10^{-8}$ C.

1. What is the radius of an equipotential surface having a potential of 60 V? **Key 28**
2. Are surfaces whose potentials differ by a constant amount (say, 1.0 V) evenly spaced in radius? **Key 57**

If your answer is incorrect, try Helping Question 1.

3. What work must be done against electric forces to move an electron from a rest position at some point on the 60-V equipotential surface to a rest position infinitely far from the stationary 3.0×10^{-8} C charge? Express your answer in electron-volts (eV) and in joules (J). See Helping Question 2 if needed. **Key 51**

Problem III

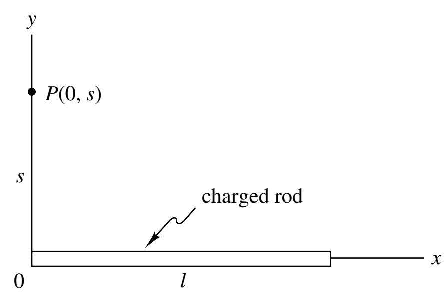


An idealized device for accelerating electrons can be made as shown in the diagram. Two metal plates are placed in a vacuum and connected to a voltage supply that puts negative charges on the lower plate and positive charges on the upper plate until there is a potential difference of 300 V between the plates. An electron, which may be "boiled off" a piece of hot metal not shown, starts from (nearly) rest at the bottom plate. It is accelerated upward until it passes through a hole in the top plate.

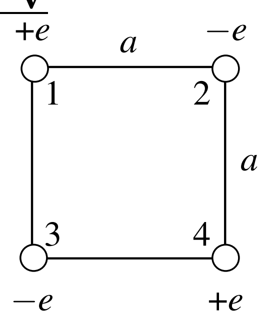
How fast is the electron going when it passes through the hole? (Recall that $e = -1.6 \times 10^{-19}$ C and $m_e = 9.11 \times 10^{-31}$ kg.) If you have trouble, see Helping Questions 2, 3, and 4. **Key 60**

Problem IV

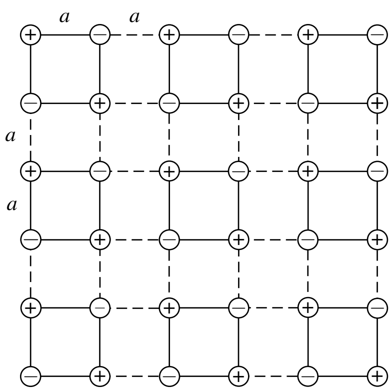
A thin glass rod of length $l = 1.0$ m is rubbed with a furry object until it has a uniformly distributed charge, $Q = 10^{-9}$ C. What is the electric potential at a perpendicular distance $s = 0.5$ m from its end? Helping Questions 5 through 9 may be needed. **Key 36**



Problem V

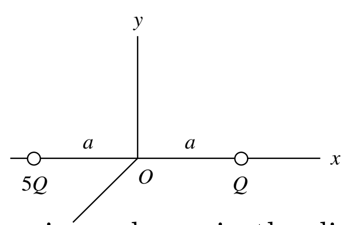


A quartet of point charges $+e, -e, +e, -e$ is arranged at the corners of a square of side $a = 1$ Å (1 Å$= 10^{-10}$ m). Find the potential energy of this charge configuration, relative to the case in which $a = \infty$. See Helping Questions 10 through 14 if you have trouble. **Key 56**



Now, suppose we have nine such arrangements of point charges, themselves arrayed in a square as shown, such that the point charges are all the same distance a away from their nearest neighbors. Note that in this arrangement, each point charge is attracted to its (four) nearest neighbors, so that the whole array tends to hold itself together. This sort of arrangement, extended into three dimensions, is the basis of crystals of ionic compounds such as NaCl and LiF. Such crystals are held together primarily by the electrostatic attractions among their constituent ions.

Problem VI-optional



A charge Q and a charge $5Q$ are located on the x-axis as shown in the diagram. The fields produced by these charges are observed at a point with coordinates (X, Y, Z).

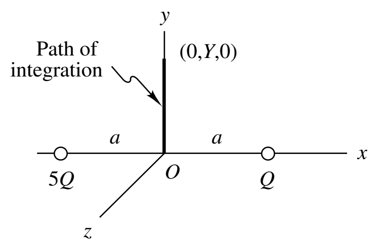
1. Find the electric field at the point (X, Y, Z) by using Coulomb's law. ($E_x = ?$ $E_y = ?$ $E_z = ?$) See Helping Questions 15 and 16 if you have trouble, and Helping Question 17 if you are still having trouble. **Key 1**

2. Find the potential V at the point (X, Y, Z) by using the $1/r$ law. If you have trouble, see Helping Questions 15 and 18. **Key 55**

3. Find the electric field at the point (X, Y, Z) by differentiating the potential you found in part (2). If you don't remember which equation to use, see Tipler and Mosca -fifth edition, Eqs. 23-17(a-c) and/or 23-18. **Key 8**

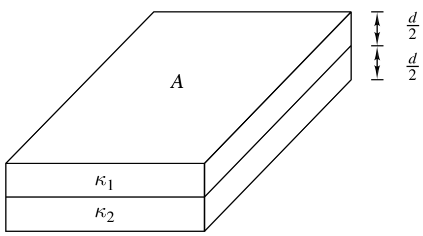
4. Find the potential difference between a point $(0, Y, 0)$ on the y-axis and the origin of the coordinate system $(0, 0, 0)$ by integrating $-\mathbf{E}\cdot d\boldsymbol{\ell}$ over a straight-line path between these two points. Help in setting up the integral can be found in Helping Questions 19 to 23. **Key 52**

5. Compute $V(0, Y, 0) - V(0, 0, 0)$ using the results of part (2) and compare your result in part (4). Do they agree? **Key 14**



6. If you explicitly evaluated $V(0, Y, 0) - V(0, 0, 0)$ by integrating $-\mathbf{E} \cdot d\boldsymbol{\ell}$ along a different path, would you get the same answer? **Key 47**

Problem VII



A parallel plate capacitor has capacitance C_0 and plate separation d. Two dielectric slabs, of constants κ_1 and κ_2, each of thickness $d/2$, are inserted between the plates. Charges Q and $-Q$ are put on the upper and lower plates.

1. Without the dielectrics, what is the field $\mathbf{E}_0$ between the plates (in terms of Q, C_0, and d)? **Key 4**
2. *With* the dielectrics, what are the fields in dielectric 1 and dielectric 2? Stuck? See Helping Question 24. **Key 48**
3. What is the potential difference between the two plates? If you need to, see Helping Question 25. **Key 10**
4. What is the capacitance? **Key 23**

Problem VIII

A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected, and the plates are pulled apart until their separation is $3d$. Derive expressions in terms of A, d, and V for

1. The new potential difference. **Key 39**
2. The initial and the final stored energy. **Key 50**
3. The work required to separate the plates. **Key 17**

If you have trouble, see Helping Questions 26 and 27.

Problem IX

A potential difference of 500 V is applied to a 2.0-μF capacitor and an 8.0-μF capacitor connected in series.

1. What are the charge and the potential difference for each capacitor? **Key 34**
2. The charged capacitors are reconnected with their positive plates together and their negative plates together, no external voltage being applied. What are the charge and the potential difference for each? **Key 11**
3. The charged capacitors in part (1) are reconnected with plates of opposite sign together. What are the charge and the potential difference for each? **Key 25**

If you have trouble, Helping Question 28 should help.

Problem X

A sphere of radius R is filled with a *uniform* volume charge density and has total charge Q.

1. Find the electrostatic energy density at distance r from the center of the sphere for $r < R$ and $r > R$. If help is needed, see Helping Question 29. **Key 3**
2. Compute the total electrostatic energy. **Key 19**
3. Explain why the value obtained in part (2) is greater than that for a spherical conductor of radius R carrying a total charge Q. **Key 44**

Problem XI

When uranium ^{235}U captures a neutron, it splits into two nuclei (and emits several neutrons that can cause other uranium nuclei to split). Assume that the fission products are two equally charged nuclei with charge $+46e$ and that these nuclei are essentially at rest just after fission and separated by twice their radius $2R = 1.3 \times 10^{-14}$ m.

1. Calculate the electrostatic potential energy of the fission products. This is approximately the energy released per fission. If troubled, try Helping Question 30. **Key 30**
2. How many fissions per second are needed to produce 1 MW of power in a reactor? **Key 5**
3. How much ^{235}U will be consumed in a year in such a reactor? **Key 9**

HELPING QUESTIONS

1. What is the potential at a distance r from a charge q? **Key 46**
2. How is the change in the electron's potential energy related to the change in potential? **Key 12**
3. What principle can be used to provide the answer immediately? **Key 41**
4. What equation expresses this principle for the case at hand? **Key 45**
5. What is the expression for the potential due to a continuous charge distribution? If you are wrong, review Tipler and Mosca -fifth edition, Section 23-4. **Key 18**
6. Let λ be the charge per meter on the rod. What is the charge dq on an element dx of the rod? If you don't get the right answer, see Tipler and Mosca -fifth edition, Equation 22-3. **Key 54**
7. In terms of s and the coordinate x of an element of charge, what is the distance of the element of the charge from P? **Key 2**
8. Express the answer to the problem as an integral over x, using the ends of the rod as limits. **Key 38**
9. Using a table of integrals, obtain the result.
10. With only charge 1 in place, what is the electric potential at corner 2? If you are wrong, review Tipler and Mosca -fifth edition, Section 23-2. **Key 22**
11. As charge 2 is brought from ∞ to corner 2, what is the change in its potential energy? If you don't get the correct answer, reread Tipler and Mosca -fifth edition, Section 23-2. **Key 27**
12. With charges 1 and 2 in place, what is the electric potential at corner 3? If your answer is wrong, review Tipler Section 23-2. **Key 15**
13. Continue the procedure begun in Helping Questions 10 through 12. What is the next step? **Key 58**

14. What is the sum of the changes in potential energy required to assemble all four charges?

Thought question: Call the value of the ith charge Q_i $(i = 1, 2, 3, 4)$ and let $\mathbf{r}_i$ be its final position. By imagining the charges to be assembled in the order 1, 2, 3, 4, you obtained

$$V_{\text{total}} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1Q_2}{|\mathbf{r}_1 - \mathbf{r}_2|} + \left(\frac{Q_1Q_3}{|\mathbf{r}_1 - \mathbf{r}_3|} + \frac{Q_2Q_3}{|\mathbf{r}_2 - \mathbf{r}_3|}\right) + \left(\frac{Q_1Q_4}{|\mathbf{r}_1 - \mathbf{r}_4|} + \frac{Q_2Q_4}{|\mathbf{r}_2 - \mathbf{r}_4|} + \frac{Q_3Q_4}{|\mathbf{r}_3 - \mathbf{r}_4|}\right)\right].$$

Can you write this expression in a way that makes it apparent that the order in which the charges were assembled is irrelevant? **Key 6**

15. What is the distance between the observation point (X, Y, Z) and the position $(-a, 0, 0)$ of the charge $5Q$? **Key 26**

16. What are the components (n_x, n_y, n_z) of the vector $\hat{\mathbf{n}}$ of unit length that points from the charge $5Q$ to the observation point? This vector gives the field $\mathbf{E}$ produced at the observation point by the charge $5Q$:

$$\mathbf{E}_{5Q} = \frac{5Q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{\mathbf{n}}.$$

Key 59

17. The electric field produced by several charges is equal to the ________________ of the fields produced by each charge separately. **Key 40**

18. The potential produced by several charges is equal to the ______________ of the potentials produced by each charge separately, **Key 37**

19. Let R denote the distance from the origin to a point $(0, Y, 0)$ on the path. Then we can write the displacement vector $\mathrm{d}\boldsymbol{\ell}$ from one point to another along the path in the form $\mathrm{d}\boldsymbol{\ell} = \mathbf{n}\mathrm{d}R$. What is $\mathbf{n}$? **Key 49**

20. Write the integral in the form

$$\Delta V = \int_0^Y (\text{something})\,\mathrm{d}R,$$

where "something" is written in terms of $\mathbf{E}$ and $\mathbf{n}$. **Key 21**

21. Using your explicit expression for $\mathbf{E}$, write out the integral for ΔV. **Key 33**

22. What substitution will enable you to do the integral easily? **Key 24**

23. What is the resulting integral? **Key 20**

24. What is the effect of introducing a dielectric of constant κ in a region of field $\mathbf{E}_0$? **Key 31**

25. What is the potential difference across the dielectric κ_1? **Key 42**

26. Does the charge change when the plates are moved? **Key 32**
27. What is the new value of the capacitance? **Key 43**
28. What do you know about the charges on capacitors connected in series, and in parallel? **Key 13**
29. What is the energy density u in an electric field? **Key 29**
30. What is the electrostatic potential energy of two charges, q_1 and q_2, separated by a distance $2R$? **Key 35**

ANSWER KEY

1.

$$E_x = \frac{5Q}{4\pi\epsilon_0}\frac{X+a}{[(X+a)^2+Y^2+Z^2]^{3/2}} + \frac{Q}{4\pi\epsilon_0}\frac{X-a}{[(X-a)^2+Y^2+Z^2]^{3/2}};$$
$$E_y = \frac{5Q}{4\pi\epsilon_0}\frac{Y}{[(X+a)^2+Y^2+Z^2]^{3/2}} + \frac{Q}{4\pi\epsilon_0}\frac{Y}{[(X-a)^2+Y^2+Z^2]^{3/2}};$$
$$E_z = \frac{5Q}{4\pi\epsilon_0}\frac{Z}{[(X+a)^2+Y^2+Z^2]^{3/2}} + \frac{Q}{4\pi\epsilon_0}\frac{Z}{[(X-a)^2+Y^2+Z^2]^{3/2}}$$

2. $r = (s^2+x^2)^{1/2}$

3.

$$u = \frac{Q^2}{32\pi^2\epsilon_0 r^4}, \text{ for } r > R;$$
$$u = \frac{Q^2 r^2}{32\pi^2\epsilon_0 R^6}, \text{ for } r < R.$$

4. $|\mathbf{E_0}| = Q/C_0 d$

5. 2.67×10^{16} fissions per second.

6.

$$V_{\text{total}} = \frac{1}{2}\sum_{i\neq j}^{i,j}\frac{1}{4\pi\epsilon_0}\frac{Q_iQ_j}{|\mathbf{r}_i-\mathbf{r}_j|}$$

In doing the sum one must count each pair $(i,\,j)$ only once- that's why there's a factor of 1/2 in front.

7. Yes, as long as the potential difference between two points on the surface is negligible.

8. Should be the same as key 1.

9. 328 g

10.

$$V = \frac{Q}{2C_0}\left(\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right)$$

11. $Q_2 = 3.2\times 10^{-4}$ C;
$Q_8 = 12.8\times 10^{-4}$ C
$\Delta V_2 = \Delta V_8 = 160$ V.

12. $\Delta U = -e\,\Delta V$, where e is the electron charge.

13. Capacitors connected in series have equal charges. When connected in parallel the potential difference across the capacitors is the same for each capacitor.

14. Yes, hopefully

15.

$$V_3 = \frac{e}{4\pi\epsilon_0 a}\left(\frac{1}{\sqrt{2}}-1\right)$$

16. Put a conducting surface around the (empty) region.

17. $W = \epsilon_0 AV^2/d$

18.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{\mathrm{d}q}{r}$$

19.

$$U = \frac{3Q^2}{20\pi\epsilon_0}\frac{1}{R}$$

20.

$$\Delta V = -\frac{6Q}{4\pi\epsilon_0}\int_{|a|}^{\sqrt{a^2+y^2}}\frac{\mathrm{d}r}{r^2}$$

21.

$$\begin{aligned}\Delta V = &-\int_0^Y \mathrm{d}R\,[E_x(0,R,0)n_x] \\ &-\int_0^Y \mathrm{d}R\,[E_y(0,R,0)n_y] \\ &-\int_0^Y \mathrm{d}R\,[E_z(0,R,0)n_z] \\ = &-\frac{6Q}{4\pi\epsilon_0}\int_0^Y\frac{R\,\mathrm{d}R}{(a^2+R^2)^{3/2}}\end{aligned}$$

22. $V_2 = e/4\pi\epsilon_0 a$

23.
$$C = C_0 \frac{2\kappa_1\kappa_2}{(\kappa_1+\kappa_2)}$$

24. $r = \sqrt{a^2+R^2}$; $\mathrm{d}r = R\,\mathrm{d}R/r$

25. $Q_2 = Q_8 = 0$; $\Delta V_2 = \Delta V_8 = 0$.

26. $r = \sqrt{(X+a)^2+Y^2+Z^2}$

27. $\Delta U_2 = -e^2/4\pi\epsilon_0 a$

28. 4.5 m

29. $u = \frac{1}{2}\epsilon_0 E^2$

30.
$$U = k\frac{(46)^2 e^2}{2R} = 3.75\times 10^{-11}\ \mathrm{J}$$

31. $E_0 \to E_0/\kappa$

32. No

33.
$$\Delta V = -\frac{6Q}{4\pi\epsilon_0}\int_0^Y \frac{R\,\mathrm{d}R}{(a^2+R^2)^{3/2}}$$

34. $Q_2 = Q_8 = 8.0\times 10^{-4}$ C;
$\Delta V_2 = 400$ V; $\Delta V_8 = 100$ V.

35. $U = kq_1q_2/2R$

36.
$$V = \frac{Q}{4\pi\epsilon_0 l}\left[\ln\left(\frac{x}{s}+\sqrt{1+\frac{x^2}{s^2}}\right)\right]\Bigg|_0^l$$
$$= 13\ \mathrm{V}$$

37. Sum

38.
$$V = \frac{\lambda}{4\pi\epsilon_0}\int \mathrm{d}x\,(x^2+s^2)^{-1/2}$$

39. $V_{\mathrm{new}} = 3V$

40. Vector sum

41. Conservation of energy

42.
$$\Delta V = \text{field}\times\text{distance} = \frac{E_0}{\kappa_1}\frac{d}{2}$$

43.
$$C_{\mathrm{new}} = \frac{\epsilon_0 A}{3d};\ \frac{1}{3}\text{ of } C_{\mathrm{old}}$$

44. For a spherical conducting shell of radius R, all of the charge Q resides on the conductor—i.e., at $r = R$. The electric field inside such a shell is zero, but for $r > R$ the electric field is the same as that for the uniform volume distribution. Hence the total electrostatic energy is less for the conducting shell.

45. $\frac{1}{2}mv^2 - e\,\Delta V = 0$

46. $V = q/4\pi\epsilon_0 r$

47. You'd better believe it!

48. $E_1 = Q/C_0 d\kappa_1$, $E_2 = Q/C_0 d\kappa_2$

49. $\mathbf{n}$ = unit vector in the direction of the path. $n_x = 0$; $n_y = 1$; $n_z = 0$.

50.
$$\text{Initial energy} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2$$
$$\text{Final energy} = \frac{3}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2$$

51. $W = 60\ \mathrm{eV} = 9.6\times 10^{-18}$ J

52.
$$\Delta V = V(0,Y,0) - V(0,0,0)$$
$$= \frac{6Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{a^2+Y^2}} - \frac{1}{|a|}\right)$$

53. High potential because their charge is negative.

54. $\mathrm{d}q = \lambda\,\mathrm{d}x = (10^{-9}\ \mathrm{C/m})\,\mathrm{d}x$

55.

$$
\begin{aligned}
V = & \frac{5Q}{4\pi\epsilon_0} \frac{1}{\sqrt{(X+a)^2 + Y^2 + Z^2}} \\
& + \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{(X-a)^2 + Y^2 + Z^2}}
\end{aligned}
$$

56.

$$
\begin{aligned}
U &= \frac{e^2}{4\pi\epsilon_0 a} \left[\sqrt{2} - 4\right] \\
&= -5.9 \times 10^{-18} \text{ J}
\end{aligned}
$$

57. No

58. Multiply the answer to Helping Question 12 by $+e$ to get the change in potential energy required to bring charge 3 into position.

59. $n_x = (X + a)/r;\ n_y = Y/r;\ n_z = Z/r$

60. $v = 1.0 \times 10^7$ m/s, which is 1/30 the speed of light.