

Princeton University

Physics 104

Spring Term 2004

Learning Guide No. 1 Electric Charge and Field, Gauss's Law

Reading Assignment: Tipler and Mosca -fifth edition, Chapters 21, 22.

**Note:** There is nothing unfamiliar about Coulomb's law if you replace

$$\mathbf{F}_{\text{grav}} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}$$

by

$$\mathbf{F}_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

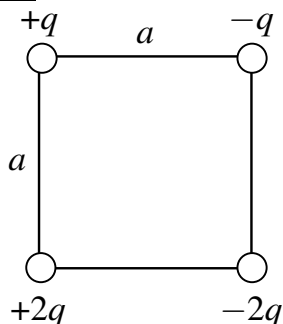
where

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|}.$$

**Problem I**

1. One coulomb is a very large charge. What fraction of electrons must be removed from a penny in order to have a charge of +1 C? See Helping Questions 1 and 2. **Key 1**
2. What force would be exerted by two such pennies separated by 0.1 m? (Express your answer not only in newtons, but in terms of some "imaginative" units such as the weight of a number of pyramids of Cheops, or the force on the moon due to the earth, for example.) **Key 6**

**Problem II**



1. Charges are located on the corners of a square as shown. If  $q = 10^{-7}$  C and  $a = 5.0$  cm, what is the resultant force (magnitude and direction) on the charge at the lower left? See Helping Question 3 for help with the procedure.

**Key 10**

2. What is the electric field (magnitude and direction) at that point?

**Key 18**

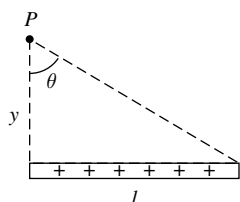
**Problem III**

1. One of the greatest obstacles to the millennium of energy production by nuclear fusion is Coulomb's law. In order to fuse two deuterons ( $^2\text{H}$ ) of radius  $\simeq 2 \times 10^{-13}\text{cm}$  into one helium atom ( $^4\text{He}$ ), one must overcome this force. How much work must be done to bring a pair of deuterons from infinite separation to a separation of  $2 \times 10^{-13}\text{cm}$ ? **Key 3**
2. At approximately what temperature do the deuterium nuclei have just enough kinetic energy to overcome this "potential barrier"? Helping Questions 4 and 5 may be needed. **Key 19**

**Problem IV**

Point charge  $A$  with a charge of  $2 \times 10^{-7}\text{ C}$  and point charge  $B$  with a charge of  $5 \times 10^{-8}\text{ C}$  are 15 cm apart.

1. What is the electric field at  $A$  due to charge  $B$ ? **Key 23**
- If your answer is incorrect, review Tipler Section 22-3.
2. What is the electric field at  $B$  due to charge  $A$ ? **Key 2**
  3. What is the magnitude of the force on charge  $A$ ? **Key 7**
  4. What is the magnitude of the force on charge  $B$ ? **Key 15**
  5. Look at your answers above. Why are the forces equal? **Key 26**

**Problem V**

1. A total charge  $Q$  is spread uniformly along a rod of length  $\ell$ . Find the electric field  $\mathbf{E}$  at the point  $P$  a distance  $y$  away from one end of the rod. If you need help, see Helping Question 6. **Key 21**

2. What does your answer become as  $\ell \rightarrow \infty$  and  $Q/\ell$  remains constant? **Key 13**
3. Show that as  $\ell \rightarrow 0$ , your answer converges to that for a point charge.

## **Problem VI: Millikan's Oil Drop Experiment**

R. A. Millikan built an apparatus in which a tiny charged oil drop, placed in an electric field  $\mathbf{E}$ , could be “balanced” by adjusting  $\mathbf{E}$  until the electric force on the drop was equal and opposite to its weight. If the radius of the drop is  $1.59 \times 10^{-4}$  cm,  $\mathbf{E}$  at balance is  $1.92 \times 10^5$  N/C, and the density of the oil is  $\rho = 0.851$  g/cm<sup>3</sup>.

1. What charge is on the drop? **Key 16**
2. Why did Millikan not try to balance electrons in his apparatus instead of oil drops? (Millikan first measured the electronic charge in this way. He measured the drop radius by observing the limiting speed that the drops attained when they fell in air with the electric field turned off. He charged the oil drops by irradiating them with bursts of X-rays.) **Key 5**
3. In an early run (1911) Millikan observed that the following measured charges, among others, appeared at different times on a single drop:

$6.563 \times 10^{-19}$ C	$13.13 \times 10^{-19}$ C	$19.71 \times 10^{-19}$ C
$8.204 \times 10^{-19}$ C	$16.48 \times 10^{-19}$ C	$22.89 \times 10^{-19}$ C
$11.50 \times 10^{-19}$ C	$18.08 \times 10^{-19}$ C	$26.13 \times 10^{-19}$ C

What value for the elementary charge  $e$  can be deduced from these data? **Key 22**

## **Problem VII**

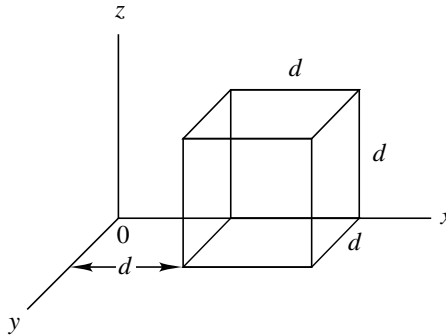
The following questions test your understanding of Gauss's law. If you have difficulty with any of them, study Tipler and Mosca -fifth edition, Chapter 22 again.

1. A point charge is placed at the center of a spherical Gaussian surface. Is the electric flux  $\Phi_E$  changed:
  - a) If the surface is replaced by a cube of the same volume? **Key 50**
  - b) If the sphere is replaced by a cube of one-third the volume? **Key 25**
  - c) If the charge is moved off center in the original sphere, still remaining inside? **Key 24**
  - d) If the charge is moved just outside the original sphere? **Key 31**
  - e) If a second charge is placed near, and outside, the original sphere? **Key 38**
  - f) If a second charge is placed inside the Gaussian surface? **Key 52**
- 2) A surface encloses an electric dipole of moment  $p$ . What can you say about  $\Phi_E$  for this surface? **Key 30**

- 3) Suppose that a Gaussian surface encloses zero *net* charge. Does Gauss's law require that  $\mathbf{E}$  equal zero for all points on the surface? Is the converse of this statement true? That is, if  $\mathbf{E}$  equals zero everywhere on the surface, does Gauss's law require that there be no net charge inside? **Key 20**
- 4) A plane surface of area  $A$  is inclined so that its normal makes an angle  $\alpha$  with a uniform field  $\mathbf{E}$ . Calculate  $\Phi_E$  for this surface. **Key 32**

### Problem VIII

The electric field components in the figure are  $E_x = a\sqrt{x}$  and  $E_y = E_z = 0$ .



Calculate

1. The electric flux  $\Phi_E$  through the cube. **Key 4**
2. The charge within the cube. **Key 35**

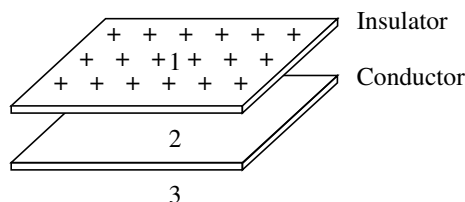
If you have trouble with part (1), see Helping Questions 7 and 8; with part (2), see Helping Question 9.

### Problem IX

A thin metallic spherical shell of radius  $R_1$  carries a charge  $q_1$ . Concentric with it is another thin metallic spherical shell of radius  $R_2$  ( $R_2 > R_1$ ) carrying charge  $q_2$ . Use Gauss's law to find the electric field at radial points  $r$  where

1.  $r < R_1$ . **Key 53**
2.  $R_1 < r < R_2$ . **Key 45**
3.  $r > R_2$ . **Key 36**
4. How is the charge on each shell distributed between the inner and outer surfaces of that shell? **Key 39**

If you have trouble with parts (1), (2), or (3), use as many of Helping Questions 10 through 13 as necessary. For part (4) use Helping Questions 14 to 16.

**Problem X**

A large plate of insulating material of area  $A$  carries a total charge  $Q_1$ , which is uniformly distributed over the area of the plate. Beside this plate and parallel to it is a plate of the same size and shape of a conducting material.

Find the electric fields for the regions 1 (above the insulator), 2 (between the plates), and 3 (below the conductor) for the following circumstances:

1. Conductor isolated and having zero net charge; **Key 12**

2. Conductor isolated and carrying total charge  $Q_2$  **Key 43**

Express your answers in terms of  $Q_1$ ,  $Q_2$ ,  $A$  and  $\epsilon_0$ . If you have difficulties, try Helping Questions 17 through 20.

**Problem XI**

Suppose that an electric field exists in a region of space where there are no charges; that is, suppose that the electric field is produced by charges that are outside the region considered. Furthermore, suppose that at a certain point  $P$  in this region, the electric field is zero. Therefore, if a test charge  $q_0 > 0$  is placed at  $P$ , there will be no force acting on it. If  $q_0$  is moved slightly away from  $P$  in a certain direction, it is found that the electric field exerts on  $q_0$  a force directed back toward  $P$ . Show that there *must* also exist a direction such that if  $q_0$  is moved slightly away from  $P$  in this new direction, then the electric field will exert on  $q_0$  a force directed *away from*  $P$ . (Try to use Gauss's law.) Thus, there can be no stable equilibrium in a static electric field.

If you've constructed a proof, then it must be right. You are finished. If not, look at Helping Questions 21 and 22.

**HELPING QUESTIONS**

1. How many electrons are in one coulomb? **Key 17**
2. How many electrons are in a penny? (It is useful to remember that a penny weighs about 3 g, and that  $A_{\text{Cu}} = 64$  and  $Z_{\text{Cu}} = 29$ .) **Key 8**
3. Find the  $y$ -component of force on  $+2q$  due to  $+q$  above it (and continue with the other two charges, and the  $x$ -components). **Key 9**
4. Using an analogue to gravitational potential energy, what is the magnitude of the energy barrier represented by this force? **Key 11**
5. What do the conservation of energy and the equipartition of energy say to help you here? **Key 14**
6. Take another look at Tipler and Mosca -fifth edition, Section 22-1.
7. What is the flux through the walls of the cube that are perpendicular to the  $z$ -axis? Through the walls of the cube that are perpendicular to the  $y$ -axis? **Key 28**
8. What is the flux through the wall of the cube that is perpendicular to the  $x$ -axis at  $x = d$ ? Through the wall of the cube that is perpendicular to the  $x$ -axis at  $x = 2d$ ? **Key 33**
9. How is the total flux through the cube related to the charge inside and outside the cube? **Key 49**
10. What is a convenient Gaussian surface to which to apply Gauss's law? **Key 27**
11. What is the total charge enclosed by your Gaussian surface? **Key 44**
12. What does the symmetry of the problem tell you about the electric field? **Key 51**
13. Express the flux through your Gaussian surface in terms of the electric field. **Key 41**
14. For each shell, consider a concentric spherical Gaussian surface which lies between the inner and outer surface of the given conducting shell. What is the flux through this Gaussian surface? **Key 29**
15. For the results of Helping Question 14, calculate the *total* charge enclosed by the Gaussian surface. **Key 34**
16. What does charge conservation tell you about the *total* charge on each shell? **Key 37**
17. With the insulator *alone* present, what is the field "above" and "below" the plate? **Key 46**

18. The “downward” field is intersected by the conductor. What induced charge density (i) terminates the field? (ii) Regenerates the field at the bottom face of the conductor? **Key 42**
19. Now take away the insulator, reisolate the conductor, and deposit charge  $Q_2$  on it. With the conductor *alone* present, what is the field “above” and “below” the plate? **Key 48**
20. Finally, superimpose these fields with those found previously (Key 46), taking direction into account.
21. In the absence of the test charge, what is the flux through a small sphere centered at  $P$ , according to Gauss’s law? **Key 47**
22. If the electric field at some region of the sphere points toward  $P$ , what is the sign of the flux through that region? **Key 40**

**ANSWER KEY**

1.  $7.5 \times 10^{-6}$  —i.e., about one in 133,000 electrons must be removed.
  2.  $8 \times 10^4$  N/C, away from point  $A$
  3.  $\sim 1.2 \times 10^{-13}$  J
  4.  $\Phi_E = d^{5/2}a(\sqrt{2} - 1)$
  5. Small  $m_e \Rightarrow$  too small  $\mathbf{E}$ ; how would you see electrons?
  6.  $9 \times 10^{11}$  N, or  $\sim 10^8$  tons, which is the weight of about 1500 battleships
  7.  $4 \times 10^{-3}$  N
  8.  $8 \times 10^{23}$  electrons
  - 9.
- $$F_y = -\frac{1}{4\pi\epsilon_0} \frac{2q^2}{a^2}$$
- 10.
- $$(F_x, F_y) = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} (4 + 1/\sqrt{2}, -2 + 1/\sqrt{2}).$$
- Resultant:  $|\mathbf{F}| = 0.176$  N
- Direction:  $\theta = 15.3^\circ$  clockwise from  $2q \rightarrow -2q$  line.
11.  $\sim 1.2 \times 10^{-13}$  J
  12.  $E_1 = Q_1/2A\epsilon_0$ ;  $E_2 = E_3 = -E_1$
  - 13.
- $$\mathbf{E} = \frac{Q}{4\pi\epsilon_0\ell y}(-\mathbf{i} + \mathbf{j})$$
14.  $U \simeq \frac{1}{2}\langle mv^2 \rangle \simeq \frac{3}{2}kT$  for each molecule.
  15.  $4 \times 10^{-3}$  N
  16.  $7.32 \times 10^{-19}$  C
  17.  $6 \times 10^{18}$  electrons

18.  $\mathbf{E} = \mathbf{F}/2q$ , where  $\mathbf{F}$  is given in Key 10.
19.  $T \simeq 10^{10}$  K
20. No. Gauss's law requires only that

$$\oiint \mathbf{E} \cdot d\mathbf{A} = 0$$

Yes, the converse is true.

21.

$$E_x = -\frac{Q}{4\pi\epsilon_0\ell} \left( \frac{1}{y} - \frac{1}{\sqrt{y^2 + \ell^2}} \right),$$

$$E_y = \frac{Q}{4\pi\epsilon_0\ell y} \sin \theta_0.$$

22.  $1.64 \times 10^{-19}$  C
23.  $2 \times 10^4$  N/C, away from point  $B$
24. No
25. No (if the charge is inside the cube)
26. Newton's third law requires  $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$ .
27. A concentric spherical surface of radius  $r$ , with  $r < R_1$  for part (a),  $R_1 < r < R_2$  for part (b), and  $r > R_2$  for part (c).

28.

$$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{A} = 0$$

for each of these four walls.

29.

$$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{A} = 0.$$

$\mathbf{E} = 0$  everywhere on the surface because the surface lies inside a conductor through which no current is flowing.

30.  $\Phi_E = 0$
31. Yes;  $\Phi_E = 0$ .



32.  $\Phi_E = |\mathbf{E}|A \cos \alpha$

33. For the wall at  $x = d$ ,

$$\iint \mathbf{E} \cdot d\mathbf{A} = -(a\sqrt{d})d^2;$$

and for the wall at  $x = 2d$

$$\iint \mathbf{E} \cdot d\mathbf{A} = +(a\sqrt{2d})d^2.$$

34.  $Q_{\text{enc}} = 0$

35.  $Q_{\text{inside}} = \epsilon_0 d^{5/2} a (\sqrt{2} - 1)$

36.  $E_r = (q_1 + q_2)/4\pi\epsilon_0 r^2$  for  $r > R_2$

37. Even though charges may separate, the total charge on the conductor of radius  $R_1$  must be  $q_1$ , and the total charge on the conductor of radius  $R_2$  must be  $q_2$ .

38. No, the flux through the surface depends only on the charge inside the surface.

39. Shell of radius  $R_1$ :  $q_1$  uniformly distributed on outer surface.

Shell of Radius  $R_2$ :  $-q_1$  uniformly distributed on inner surface;  $q_1 + q_2$  uniformly distributed on outer surface

40. Negative

41.

$$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{A} = E_r 4\pi r^2$$

42. (i)  $\sigma = -Q_1/2A$ ; (ii)  $\sigma = Q_1/2A$

43.

$$E_1 = \frac{Q_1 + Q_2}{2A\epsilon_0}; \quad E_2 = \frac{Q_2 - Q_1}{2A\epsilon_0};$$

$$E_3 = -E_1$$

$$44. Q_{\text{enclosed}} = \begin{cases} 0 & \text{for (1)} \\ q_1 & \text{for (2)} \\ q_1 + q_2 & \text{for (3)} \end{cases}$$

45.  $E_r = q_1/4\pi\epsilon_0 r^2$  for  $R_1 < r < R_2$ .

46.  $E_1 = Q_1/2A\epsilon_0$ ;  $E_2 = E_3 = -E_1$

47. Zero

48.  $E_2 = Q_2/2A\epsilon_0 = E_1$ ;  $E_3 = -E_2$

49. From Gauss's law

$$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} Q_{\text{inside}}.$$

50. No (if the charge is inside the cube)

51.  $\mathbf{E}$  is in the radial direction, and  $|\mathbf{E}|$  is the same for all directions at a given  $r$ .

52. Yes

53.  $E_r = 0$  for  $r < R_1$ .