

## LAB #4: ROLLING FRICTION

### BEFORE YOU COME TO LAB:

- **Read the writeup for this lab, which concerns rolling friction. This topic is not discussed in the Precepts, so the physics discussion in this writeup is more extensive than in the previous ones.**

### A. Introduction

In this lab, you will use your cameras and VideoPoint/Excel software to study rolling motion on an inclined plane.

The wheel is a great invention in that it avoids loss of mechanical energy due to sliding friction. When a wheel rolls without slipping, static friction is required to avoid sliding/slipping at the point of contact of the wheel with the surface on which it rolls. Static friction  $\mathbf{F}_s$  does no work, because the velocity  $\mathbf{v}$  of the point of contact (where the force of static friction applies) is zero, and hence  $W_s = \mathbf{F}_s \cdot \mathbf{v} = 0$ .

A rolling wheel is subject to energy loss associated with friction at the axle, deformation of the wheel (or the ball bearings in the wheel) and/or the surface on which the wheel rolls. In this Lab, rolling friction will be modeled as acting on being proportional to the normal force  $N$  on the wheel, directed opposite to the velocity of the center of the wheel,

$$\mathbf{F}_r = -\mu_r N \hat{\mathbf{v}}, \quad (1)$$

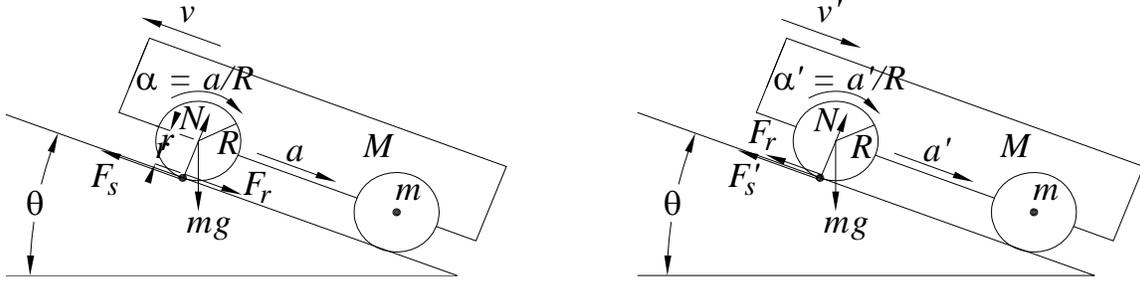
where  $\mu_r$  is the coefficient of rolling friction, and  $\hat{\mathbf{v}} = \mathbf{v} / v$  is the unit velocity vector. For other models of rolling friction, see [http://en.wikipedia.org/wiki/Rolling\\_resistance](http://en.wikipedia.org/wiki/Rolling_resistance)

In this Lab, you will study the motion of a 3-wheeled cart as it rolls up and back down a plane inclined at angle  $\theta$  to the horizontal, and you will determine the coefficient  $\mu_r$  of rolling friction by analysis of the acceleration of the cart, followed by analysis of its kinetic energy.

The rolling friction acts at some radius  $r$  (or several different radii) from the center of the wheel. In this Lab you will only determine the quantity  $\tilde{\mu}_r \equiv \mu_r (1 - r/R)$  where  $R$  is the radius of the wheel. *If the rolling friction acted precisely at the outer radius of the wheel, then like static friction it would do no work. Pump up your bicycle tires for a lower-friction (but bumpier) ride!*

## Acceleration

The acceleration  $a$  of the cart when it rolls up the inclined plane is different from the acceleration  $a'$  when it rolls down, although both vectors  $a$  and  $a'$  point down the slope. Each wheel of radius  $R$  has angular acceleration  $\alpha = a/R$  when rolling uphill, and  $\alpha' = a'/R$  when rolling downhill. In both cases the angular acceleration is clockwise in the figure above. This angular acceleration is due to a clockwise torque, which implies that the force of static friction (at the point of contact of the wheel with the inclined plane) points up the slope.



Considering the system as a whole, it has no acceleration normal to the slope, and acceleration of magnitude  $a$  or  $a'$  down the slope. The normal force on each wheel has magnitude  $N$  (assumed equal for the three wheels) related by

$$N = \frac{M_T}{3} g \cos \theta, \quad (2)$$

where  $M$  is the mass of the body of the cart and  $m$  is the mass of each of its wheels, and

$$M_T = M + 3m, \quad (3)$$

According to eq. (1) the force of rolling friction on each wheel has magnitude

$$F_r = \mu_r N = \mu_r \frac{M_T}{3} g \cos \theta. \quad (4)$$

The acceleration  $a$  of the cart when it moves up the slope is related by

$$M_T a = M_T g \sin \theta + 3F_r - 3F_s = M_T g (\sin \theta + \mu_r \cos \theta) - 3F_s, \quad (5)$$

When the cart moves down the slope with acceleration  $a'$ , the direction of rolling friction is reversed but its magnitude is the same, while the direction of static friction is the same but its magnitude  $F'_s$  is different. Thus,

$$M_T a' = M_T g (\sin \theta - \mu_r \cos \theta) - 3F'_s. \quad (6)$$

If the forces  $F_s$  and  $F'_s$  of static friction were equal the coefficient  $\mu_r$  of rolling friction would be simply related to the difference  $a - a'$ . But, they are not equal, and must be determined via a torque analysis of a wheel. The moment of inertia of each wheel is given by

$$I = kmR^2, \quad (7)$$

where  $k = 1$  for a hoop,  $k = 1/2$  for a solid disk, and  $k \approx 0.8$  for the wheels used in this Lab. The torque analysis (about the center of a wheel) when the cart rolls uphill is

$$\tau = RF_s - rF_r = I\alpha = kmR^2 \frac{a}{R}, \quad \text{so that} \quad F_s = kma + \frac{r}{R}F_r = kma + \mu_r \frac{M_T}{3} g \frac{r}{R} \cos \theta, \quad (8)$$

since gravity and the normal force  $N$  (and the internal force of the cart on the wheel) exert no torque about the center of the wheel. When the cart rolls downhill, the torque analysis is

$$\tau' = RF_s' + rF_r' = I\alpha' = kmR^2 \frac{a'}{R}, \quad \text{so that} \quad F_s' = kma' - \frac{r}{R}F_r' = kma' - \mu_r \frac{M_T}{3} g \frac{r}{R} \cos \theta. \quad (9)$$

Combining eqs. (5) and (8), and also eqs. (6) and (9), we obtain two equations for the two unknowns  $\mu_r$  and  $k$ ,

$$\mu_r \left(1 - \frac{r}{R}\right) M_T g \cos \theta - 3kma = M_T (a - g \sin \theta), \quad (10)$$

$$\mu_r \left(1 - \frac{r}{R}\right) M_T g \cos \theta + 3kma' = -M_T (a' - g \sin \theta). \quad (11)$$

*Note that if  $r = R$  then there is no effect of friction.* The solutions to eqs. (10)-(11) are

$$\tilde{\mu}_r \equiv \mu_r (1 - r/R) = \frac{a - a'}{a + a'} \tan \theta, \quad \text{and} \quad k = \frac{M_T}{3m} \left( \frac{2g \sin \theta}{a + a'} - 1 \right). \quad (12)$$

Under the possibly naïve assumption that the relative uncertainties in  $m$ ,  $M$ ,  $r$ ,  $R$  and  $\theta$  are negligible, the uncertainties on the measurements of  $\tilde{\mu}_r$  and  $k$  are

$$\sigma_{\tilde{\mu}_r} = \frac{2 \tan \theta}{(a + a')^2} \sqrt{a'^2 \sigma_a^2 + a^2 \sigma_{a'}^2}, \quad \text{and} \quad \sigma_k = \frac{M_T}{3m} \frac{2g \sin \theta}{(a + a')^2} \sqrt{\sigma_a^2 + \sigma_{a'}^2}. \quad (13)$$

## Energy

Another approach is to consider the work-energy relation of uphill and downhill motion of the cart while it travels distance  $s$  along the slope. In this case the change in potential energy is

$$\Delta PE = M_T g s \sin \theta. \quad (14)$$

When the cart has linear speed  $v$  the angular velocity of the wheel is  $\omega = v/R$ , so the total kinetic energy of linear plus rotational motion is

$$KE = \frac{M_T v^2}{2} + \frac{3I\omega^2}{2} = \frac{(M_T + 3km)v^2}{2}. \quad (15)$$

No work is done by static friction (assuming that the wheel rolls without slipping), but rolling friction does (negative) work on the cart. However, the work done by rolling

friction is not simply  $-F_r s$ , because the torque due to rolling friction acts to increase the angular velocity and does positive work  $\tau\phi = rF_r s / R$ . That is, the total work done by rolling friction of the three wheels has magnitude

$$W_r = 3F_r s \left(1 - \frac{r}{R}\right) = \mu_r M_T g s \cos \theta \left(1 - \frac{r}{R}\right). \quad (16)$$

If the cart is launched uphill with initial speed  $v_0$  and comes to rest after traveling distance  $s$  along the slope, the work-energy relation obtained from eqs. (14)-(16) is

$$W_r = KE_0 - \Delta PE, \quad \text{or} \quad \mu_r \left(1 - \frac{r}{R}\right) M_T g s \cos \theta - \frac{3kmv_0^2}{2} = M_T \left(\frac{v_0^2}{2} - gs \sin \theta\right). \quad (17)$$

(17)

Similarly, if the cart starts from rest and rolls distance  $s'$  down the slope to attain final speed  $v_f$ , then  $\Delta PE' = KE_f + W_r'$ , and

$$\mu_r \left(1 - \frac{r}{R}\right) M_T g s' \cos \theta + \frac{3kmv_f^2}{2} = -M_T \left(\frac{v_f^2}{2} - gs' \sin \theta\right). \quad (18)$$

From eqs. (17)-(18),

$$\tilde{\mu}_r \equiv \mu_r (1 - r/R) = \frac{1}{\cos \theta} \frac{s'v_0^2 - sv_f^2}{s'v_0^2 + sv_f^2}, \quad \text{and} \quad k = \frac{M_T}{3m} \left(\frac{4gss' \sin \theta}{s'v_0^2 + sv_f^2} - 1\right). \quad (19)$$

Assuming that the uncertainties on  $m$ ,  $M$ ,  $r$ ,  $R$  and  $\theta$  are negligible, the uncertainties on  $\mu_r$  and  $k$  are

$$\sigma_{\tilde{\mu}_r} = \frac{1}{\cos \theta} \frac{v_0 v_f}{(s'v_0^2 + sv_f^2)^2} \sqrt{v_0^2 v_f^2 (s'^2 \sigma_s^2 + s^2 \sigma_{s'}^2) + 4s^2 s'^2 (v_f^2 \sigma_{v_0}^2 + v_0^2 \sigma_{v_f}^2)}, \quad (20)$$

$$\text{and} \quad \sigma_k = \frac{4M_T g \sin \theta}{3m (s'v_0^2 + sv_f^2)^2} \sqrt{s'^4 v_0^4 \sigma_s^2 + s^4 v_f^4 \sigma_{s'}^2 + 4s^2 s'^2 (s'^2 v_0^2 \sigma_{v_0}^2 + s^2 v_f^2 \sigma_{v_f}^2)}. \quad (21)$$

## B. Things to Do

It is important that the optical axis of the camera is perpendicular to the vertical plane of the motion of the rolling cart. That vertical plane should be parallel to the black backdrop or your setup. The person who launches the cart should verify that it moves parallel to the backdrop. If not, try again.

You will find a ball hanging from a string at your station, which you can use to verify that the rotation of the camera about its optical axis is proper. It is more difficult to verify that the optical axis of the camera is perpendicular to the backdrop, but if it isn't, your later fits will have significantly nonzero cubic and quartic terms.

Weigh your cart using a balance on the center table. **Do not overload the digital balances.** There is a sample wheel on the center table, but its mass is not necessarily the same as that of the wheels on your cart. A survey of 5 wheels yielded  $m = 85.1 \pm 0.7$  g.

You can adjust the height of your ramp so that it takes about 1 sec for the cart to roll from the top to the bottom of the ramp. Use a meter stick to measure an appropriate height and horizontal distance to determine the angle  $\theta$ . Leave the meter stick in view, under the ramp along the center of the cart's path, so that you can determine the scale factor of your movie.

You only need one good movie for all the analysis in this lab.

However, you can take a short movie without the cart and digitize the ramp, transfer the data to **Excel** and make a linear fit of  $y$  vs.  $x$  to determine  $\tan\theta$ . *Compare with your result obtained using the meter stick. You could also use this movie to determine the scale factor of your later analysis*

## Main Data Analysis

In your main movie, launch the cart up the ramp, such that it comes to rest near the top of the ramp and then rolls back down the ramp. After using **VideoPoint** to digitize the trajectory of some point on your rolling cart, transfer the data  $(x, y, t)$  to **Excel**. Identify which points are on the upward part of the trajectory, and which are on the downwards. In general, you will not have digitized a point that corresponds to the cart being exactly at rest.

For the upward moving points, make plots of  $x$  vs.  $t$  and  $y$  vs.  $t$ , and then make quartic polynomial fits to these. If the cubic or quartic terms are large compared to their uncertainties (as reported by **WPTools**), consider realigning your camera and/or launching the cart on a path more nearly parallel to the backdrop.

If you haven't already done so, multiply your  $x$ - and  $y$ -data by your scale factor to convert them to cm (or m).

To find the factor  $k$  of the wheels and the coefficient  $\tilde{\mu}_r$  of rolling friction according to the formulae of part A, you need the acceleration  $a$ , the initial velocity  $v_0$ , and the distance the cart travelled up the ramp before stopping. And from fits to your data from the downward motion of the cart, you need to find  $a'$ ,  $v_f$  and  $s'$ .

To speed up the analysis of the motion along the inclined plane, divide your digitized data  $\{x_i, y_i\}$  into two sets, one for motion up the plane, say  $i = 1$  to  $m$ , and the other from motion down the plane, say  $i = m + 1$  to  $n$ . For the first set, use **Excel** to calculate  $s_i = \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}$ , and for the second set,  $s'_i = \sqrt{(x_i - x_n)^2 + (y_i - y_n)^2}$ .

Possible trick: subtract the final time  $t_n$  from the times  $t_i$  during the downward motion, to redefine  $t'$  to be zero at your last point. Then, the linear coefficient in your fit of  $s'_i$  vs.  $t'_i$  will be  $v_{f,x}$ . Of course, the acceleration  $a'$  is twice quadratic coefficient. The distance the cart travelled down the slope is then  $s' = \sqrt{v_f^2 / 2a'}$ , and the uncertainty on this measurement is given by  $\sigma_{s'}^2 = \frac{\sigma_{v_f}^2}{2a'} + \frac{v_f^2 \sigma_{a'}^2}{8a'^3}$ .

*Do your measurements of the accelerations  $a$  and  $a'$  differ by more than their uncertainties? If so, you have evidence for nonzero rolling friction. Equation (12) then gives a particular interpretation of this, which permits you to assign a value (and uncertainty!) to the coefficient  $\tilde{\mu}_r$  of rolling friction.*

In the energy analysis, you could also estimate the  $x$ - and  $y$ -coordinates of the point where the cart stopped by an interpolation between your upward and downward data points, and make a quick error estimate for  $s$  and  $s'$  as determined by this procedure. Which method is "better," in the sense of giving a smaller uncertainty?

*Did you get "better" results for  $\tilde{\mu}_r$  and  $k$  via your analysis of the acceleration or of the energy?*

### **C. Option: Rolling Other Stuff on the Ramp**

What happens when you roll a ball, a hollow "pipe," or a solid circular cylinder down the ramp? If you roll two of them down side by side, which one gets to the bottom first? Is mechanical energy conserved here? If not, where is it going?

If time permits, make movies of a couple of these objects rolling down the ramp, and look at the acceleration and energy graphs. Compare the acceleration of these objects to that of your cart. *What's going on here? -- There's more to consider than meets the eye!*