LAB #2: Forces in Fluids

- **Please do NOT attempt to wash the graduated cylinders once they have oil in them.**
- **Don't pour the oil in the graduated cylinders back into the big cylinder until the end of the lab.**
- **Please be sure that the small corks are well seated in the tubes when you are not measuring the fluid flow from them.**

Overview Comments:

In this lab, you will explore some basic effects of forces in fluids: viscous (frictional) forces, as well as the buoyant force.

Although the behavior of fluids is rather complicated in general, fluid motion obeys Newton's laws. A small element of fluid can be characterized by its volume, mass, and characteristic position, velocity and acceleration. But, the volume can change its shape, and in the case of compressible fluids, its magnitude can change as well.

In the first two parts of the Lab, you will consider an incompressible fluid, heavy machine oil, that is very viscous, and in the third part you will consider a compressible fluid, a gas, but at constant pressure so that its volume does not change.

Do you know that a cubic meter of air weighs almost three pounds? No wonder it takes strength to hold your arm out the window of a moving car – it takes force to make all that air get out of the way!

I. Flow of a Viscous Fluid in a Circular Pipe

It is a remarkable fact that fluid immediately adjacent to an immobile surface, such as the wall of a pipe, always has zero velocity. In order for fluid some distance y from the surface to flow at velocity v, a force must be applied:

$$F = \frac{\eta A v}{y}$$

where A is the area of the surface (or, equivalently, the area of the layer of fluid), and η is the coefficient of viscosity. Fluid flow through a circular pipe is slightly more complicated. Poiseuille's law states that for a circular pipe of radius R and length L, the pressure difference $\Delta P = \Delta F/A$ (where $A = \pi R^2$) between the two ends of the pipe required to maintain an average velocity $\overline{v}$ is of the fluid flow over the cross section of the pipe is related by

$$\Delta F = A\Delta P = \frac{4\eta A'\overline{v}}{R}, \qquad \text{or equivalently,} \qquad Q = A\overline{v} = \frac{\pi R^4 \Delta P}{8\eta L},$$

where, $A' = 2\pi RL$ is the surface area of the pipe, and Q is the volume rate of fluid flow. The R^4 dependence of Q is impressive (and implies that your heart must work very hard to pump blood through your arteries if they ``clog up" even a little).

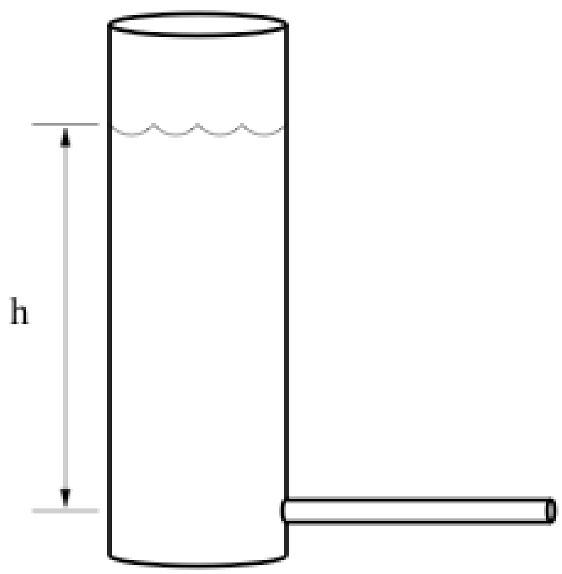

Figure 1: Apparatus for parts I and II of the Lab. The vertical cylinder is partly filled with oil. It is open to the atmosphere at the top.

Specific Instructions:

Use the apparatus shown in Figure 1 to test Poiseuille's law and to measure the viscosity of a fluid. The fluid is heavy machine oil, which fills the large vertical cylinder. Its weight produces the pressure at the bottom of the cylinder and, therefore, at one end of the small horizontal tube. The other end of the horizontal tube is at atmospheric pressure. Thus the pressure difference across the length of the small tube is $\Delta P = \rho\, g\, h$, where h is the height of the fluid above the tube.

Find the density of the oil using a scale and a graduated cylinder.

Measure the flow rate in each of the three available tubes (radii 0.319, 0.239 and 0.216 cm), using a stopwatch and a graduated cylinder.

Hints: Keep the small tube horizontal to minimize the effect of gravity on the flow. Measure the height of the fluid in the vertical cylinder before and after the oil flows out, and use the average value. From which point should the height be measured?

Analysis:

First use your data to test the assertion that Q is proportional to R^4. Although it isn't strictly true, assume that each tube has the same length L. Then you can reformulate Poiseuille's equation as:

$$Q = \text{Constant x } R^{\alpha}$$

Analyze your data to determine the exponent α.

Do this two ways, both using Excel.

After entering your data for R and Q, make a scatter plot of this using WPTools. Click on the horizontal axis, and then and Format Axis $\rightarrow$ Scale to check the box Lorgarithmic scale. Then, do the same for the vertical axis. This converts your plot to a log-log plot. On a printout of this plot, draw a "best fit" straight line, and measure its slope in units where each power of 10 on the plot counts as 1 unit. The numerical value of your slope is your measurement of α

You can get WPTools to do the equivalent of the above procedure by entering the log of your data for R and Q in your Excel sheet. As hinted in Appendix A, if a value of R is in cell A2, you can put its log in cell C2 by clicking on that cell and typing =log(A2) in the formula bar of the sheet. After typing Enter, the value should appear. Then, drag downwards on the little box in the lower right corner of the cell to take the log of your other values of R. After creating a column the values of log(Q) as well, use WPTools to make a scatter plot of log(Q) vs. log(R), and do a linear fit. Then parameter a1 is the value of α, and SE(a1) is an estimate of the uncertainty in your measurement of α

Next, find the viscosity η. For this part of the analysis, assume that the exponent $\alpha = 4$. Rework Poiseuille's equation to extract the value of the coefficient of viscosity, and use your three measurements of Q to calculate three values of η. Are the values close to each other? As mentioned in Appendix B, a simplified error analysis is to report the average of the 3 values of η as your best estimate, with an uncertainty of $|\eta_{\max} - \eta_{\min}|/2$.

II. Terminal Velocity

An object falling through a viscous fluid feels three forces. Gravity pulls the object downward:

$$F_{\text{grav}} = \rho V g$$

where ρ and V are the density and volume of the object, respectively, and g is gravitational acceleration. The buoyant force pushes the object upward:

$$F_{\text{buoy}} = \rho_f V g,$$

where ρ_f is the density of the fluid. Finally, there is a drag force opposing the motion of the object. Stokes' law gives the drag force on a spherical object of radius R moving with velocity v in a viscous medium:

$$F_{\text{drag}} = 6\pi \eta R v \quad \left(\approx \frac{A\eta}{R} \right),$$

where R is the radius of the sphere. When these three forces balance, no net force acts on the sphere, so it falls with constant velocity, called “terminal velocity”. Combine the expression of the three forces acting on the spherical object to derive the expression of the “terminal velocity”.

Specific Instructions and Analysis

Test the equation you just derived by measuring the terminal velocity of small lead spheres (of densityρ = 11.7 g cm^{-3}) that fall through the oil you analyzed in the first part of the Lab.

Measure the diameter of one of the spheres, taking an average of several measurements if it isn't really spherical. Measure the velocity of the sphere falling through the oil using a stopwatch. Repeat the experiment for at least three different spheres. Are the measured values close to the values predicted by your equation?

Assuming Stokes’ law to be correct, use your measurements of the terminal velocity to deduce another experimental value (and uncertainty) of the viscosity η of the fluid. Compare with your value from the first part of the Lab.

III. Buoyant Force

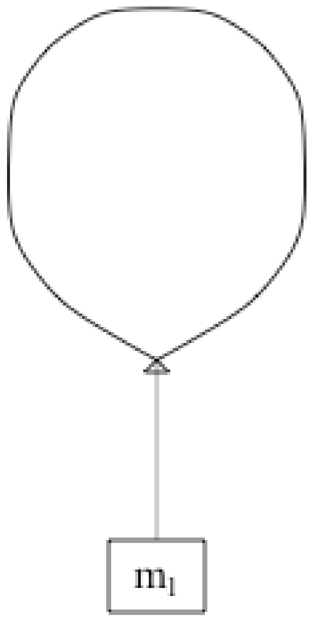

Figure 2: Apparatus for part III of the Lab.

The density of gas in a helium balloon is less than the density of the surrounding air, so the balloon feels a net upward force. The buoyant force (ρ_{air} = 1.29 kg m^{-3} at 1 atm pressure) can be balanced by hanging a mass below the balloon as in figure 2.

The total weight is:

$$W_{\text{total}} = \left(m_1 + m_{\text{string}} + m_{\text{balloon}} + m_{\text{He}}\right) g$$

where m_1 is the mass hanging below the balloon, m_{string} is the mass of the string, $m_{balloon}$ is the mass of the (empty) balloon, and m_{He} is the mass of the helium within the balloon.

The masses of the balloon, string, and hanging weight can be measured on scales, but for the mass of the helium you have to rely on measurements of volume and pressure. Given that the atomic mass of helium is 4, if there are *n* moles of helium in the balloon, the mass is $m_{He} = 4.00\ g \cdot n$.

The ideal gas law relates n to the pressure, volume, and temperature of the balloon (*P*, *V*, and *T*) and the universal gas constant: $P\ V = n\ R\ T$. Solving for *n* and substituting R = 8.3145 J mol^{-1} K^{-1} and T = 293K (approximate room temperature) allows you to calculate the mass.

Specific Instructions and Analysis

Measure the mass of the empty balloon. Fill it with helium, and after stopping the flow of helium, measure the pressure within the balloon before tying off the end of the balloon. You may need the following conversion factors: 1 psi = 6985 Pa, 1 atm = 1.013×10^5 Pa. Also, remember to add the atmospheric pressure to the "gauge pressure" reading on the pressure meter.

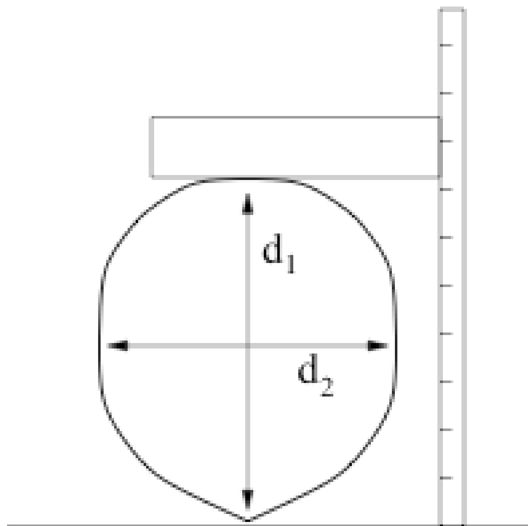

Figure 3: Measuring the dimensions of a balloon.

Next measure the volume of the balloon. One way of doing this is to put it on a table, hold a meter stick vertically next to it, and use a wooden board to help measure its size on the meter stick. (See figure 3.) You can estimate the size of the balloon from the dimensions d_1 and d_2.

Cut a piece of string a couple of feet long, measure its mass and tie it to the bottom of the balloon. Finally, tie a 5-g hanger to the string and keep adding weights to the hanger until the balloon is in equilibrium. To fine-tune the hanging weight, you may want to use small paper clips (about 0.3 g each) or pieces of tape. After you have achieved equilibrium, detach the hanger and its weights and measure their mass on a scale.

Now you have all the pieces of data you need to test the buoyancy formula. Calculate the buoyant force and the weight. *Are they close to each other?*