PRINT YOUR NAME: ___________SOLUTIONS___________

Please Circle your Section	
9am Olsen	10am McDonald
9am McDonald	10am Gregor
9am Gregor	10am Jones
9am Halyo	10am Wang
9am Jones	10am Klebanov
9am Wang	10am Meyers
9am Bernevig	

Problem	Score
1	/ 30
2	/ 30
3	/ 30
4	/ 30
5	/ 30
6	/ 30
Total	/ 180

Physics 103 – Fall 2009
Final Examination

Monday, January 18, 2010

180 minutes

Instructions: When you are told to begin, check that this examination booklet contains all the numbered pages from **2 through 14** (not counting the equation sheet).

Do not be discouraged if you cannot do all six problems, or all parts of a given problem. If a part of a problem depends on a previous answer you have not obtained, **assume** an answer and proceed. **Keep moving and finish as much as you can!**

Read each problem carefully. **You must show your work**—the grade you get depends on how well we can understand your solution even when you write down the correct answer. Unless otherwise stated, always write down analytic answers first and only then calculate numerical values. **Include correct units where appropriate**. For the purposes of this exam, **$g = 9.8 m/s^2$.** Please Box Your Answers.

THE ONLY MATERIALS ALLOWED DURING THE EXAM ARE THE EXAMINATION BOOKLET, PEN OR PENCIL, AND YOUR CALCULATOR. DO ALL THE WORK YOU WANT TO HAVE GRADED IN THIS EXAMINATION BOOKLET! YOU MAY USE THE BACK OF EACH SHEET, BUT YOU WILL NOT BE ALLOWED TO HAND IN ANYTHING ELSE.

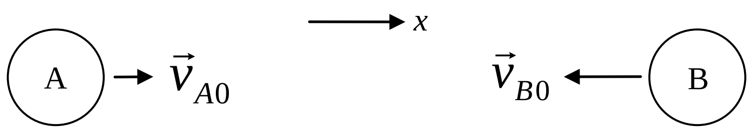
If you need to use the restroom briefly you may do so, but your exam booklet cannot leave the room. This is a timed examination. You will have 180 minutes to complete this exam. Two preceptors will be outside the room to answer questions during the exam.

____

REWRITE IN FULL AND SIGN THE PLEDGE IN THE SPACE ABOVE SIGNATURE

"I pledge my honor that I have not violated the Honor Code during this examination.

Problem 1. Collisions (30 pts)



Two solid spherical balls A and B, each with radius R and masses $m_A = 3m$ and $m_B = m$, are sliding along a frictionless surface with initial velocities $\vec{v}_{A0} = +v_0\hat{i}$ and $\vec{v}_{B0} = -v_0\hat{i}$ when they collide in a head-on **elastic** collision. Neither ball is rotating ($\omega_A = \omega_B = 0$) before or after the collision.

a) (4 pts) In terms of the positive constant v_0, what is the center-of-mass (CM) velocity $\vec{V}_{CM}$ of the balls before they collide?

$$\vec{V}_{CM} = \frac{m_A\vec{v}_{A0} + m_B\vec{v}_{B0}}{m_A + m_B} = \frac{+3mv_0 - mv_0}{4m}\hat{i} = \boxed{+\frac{1}{2}v_0\hat{i}}$$

b) (8 pts) In terms of v_0, what are the velocities $\vec{v}_A$ and $\vec{v}_B$ of each ball after the collision? Hint: it may be easier to work this problem in the CM reference frame.

First, transform velocities to the CM frame (resolving into x components):

$$v'_{A0} = v_{A0} - V_{CM} = +v_0 - \frac{1}{2}v_0 = +\frac{1}{2}v_0$$
$$v'_{B0} = v_{B0} - V_{CM} = -v_0 - \frac{1}{2}v_0 = -\frac{3}{2}v_0$$

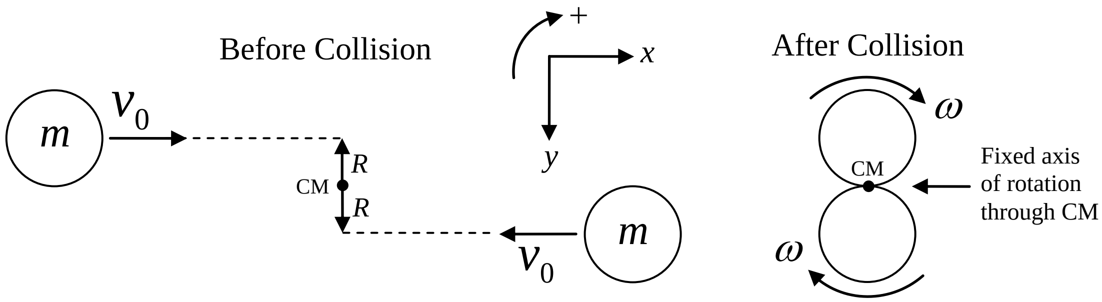
Now "do the collision," which means flip the velocities.

$$v'_A = -v'_{A0} = -\frac{1}{2}v_0\,;\; v'_B = -v'_{B0} = -\frac{3}{2}v_0$$

Now transform back to the fixed frame:

$$\vec{v}_A = \vec{v}'_A + \vec{V}_{CM} = \left(-\frac{1}{2}v_0 + \frac{1}{2}v_0\right)\boxed{= 0}$$
$$\vec{v}_B = \vec{v}'_B + \vec{V}_{CM} = \left(\frac{3}{2}v_0 + \frac{1}{2}v_0\right) = \boxed{+2v_0\hat{i}}$$

Problem 1, continued



Now two **identical** spherical balls of mass m and radius R are sliding along a frictionless surface with the same initial speed v_0, but are approaching each other such that their centers are separated laterally by a distance $2R$. When the balls hit they stick together in a **perfectly inelastic** collision. After the collision the balls are rotating together with constant angular speed ω around a fixed axis perpendicular to the page and passing through the contact point, which is also the CM of the system (filled circle in the diagram). Note that the CM is stationary ($V_{CM} = 0$) before **and** after the collision.

c) (5 pts) Before the collision, what is the initial total angular momentum $\vec{L}_0$ of the system around an axis perpendicular to the page and passing through the CM? Give both the magnitude (in terms of *m*, *R*, v_0) and direction of $\vec{L}_0$ in the given coordinate system.

$$\vec{L}_0 = \vec{r}_A \times \vec{p}_A + \vec{r}_B \times \vec{p}_B = \left(+mv_0R + mv_0R\right)\hat{k} = \boxed{+2mv_0R\hat{k}}$$

e) (8 pts) After the collision, what is the angular speed ω of the system?

The net torque on this system is zero, therefore, angular momentum is conserved:

$$\vec{L}_f = \vec{L}_0 \Rightarrow I\omega = 2mv_0R \Rightarrow \omega = \frac{2mv_0R}{I}.$$

The moment of inertia of the system the sum of the moments of inertia of each ball. We need to use the parallel axis theorem (twice), noting that $I_A = I_B$*:*

$$I = 2\left(\tfrac{2}{5}mR^2 + mR^2\right) = \frac{14}{5}mR^2$$

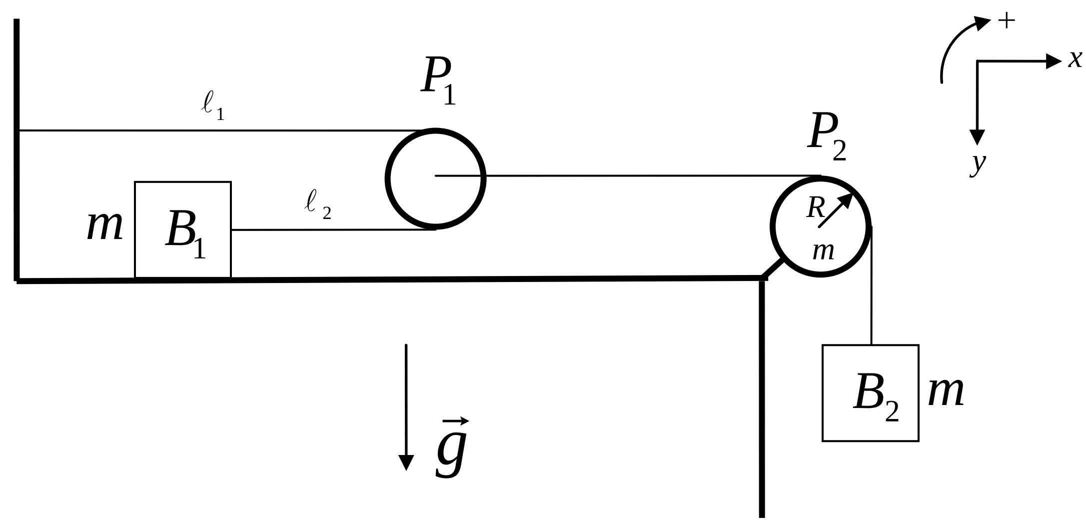
Solving for ω*, we find:*

$$\omega = \frac{2mv_0R}{\frac{14}{5}mR^2} = \boxed{\frac{5}{7}\frac{v_0}{R}}$$

e) (5 pts) What fraction of the initial energy is lost in collision?

$$\frac{E_o - E}{E_0} = \frac{2\left(\frac{1}{2}mv_0^2\right) - \frac{1}{2}I\omega^2}{2\left(\frac{1}{2}mv_0^2\right)} = \frac{2\left(\frac{1}{2}mv_0^2\right) - \left(\frac{1}{2}\right)\left(\frac{14}{5}mR^2\right)\left(\frac{5}{7}\frac{v_0}{R}\right)^2}{2\left(\frac{1}{2}mv_0^2\right)} = \boxed{\frac{2}{7}}$$

Problem 2. Blocks and Pulleys (30 pts)



Two identical blocks B_1 and B_2 with mass m are connected via two **massless** strings wound over pulleys P_1 and P_2, respectively. B_1 moves on a frictionless surface and is connected to the string that winds around the **massless** pulley P_1 (that is free to move) and connects on the other side to a rigid wall. B_2 is hanging from the string that winds over fixed pulley P_2 with mass m and radius R, and connects to pulley P_1 on the other side. When B_2 is released it begins to accelerate downward, which accelerates P_1 and B_1 to the right. Assume that the strings wind around the pulleys **without slipping**, and use a coordinate system where y is positive down, x is positive to the right, and positive rotation of P_2 is clockwise.

a) (8 pts) The accelerations a_1 and a_2 of blocks B_1 and B_2 are related by the equation $a_1 = ca_2$, where c is a constant. What is c? (Hint: if B_2 drops by a distance y, how far to the right does P_1 move? How far to the right does B_1 move?)

Define the displacements of B_2, P_1, and B_1, to be y, x_{P_1}, and x_{B_1}, respectively. It should be clear from the diagram that $x_{P_1} = y$. Measuring x_{P_1} between the wall and the center of P_1, and x_{B_1} between the wall and the right side of B_1, we have the following relationships (labeling the pieces of string ℓ_1 and ℓ_2):

$\ell_2 = x_{P_1}$; $L \equiv \ell_1 + \ell_2 = \text{constant}$

$$x_{B_1} = x_{P_1} - \ell_2 = x_{P_1} - (L - x_{P_1}) = 2x_{P_1} - L \Rightarrow \frac{d^2 x_{B_1}}{dt^2} = 2\frac{d^2 x_{P_1}}{dt^2} \Rightarrow \boxed{a_1 = 2a_2}$$

In the last step we used the fact that $y = x_{P_1}$.

b) (5 pts) Write down the relationship between the linear acceleration a_2 of block B_2 and the angular acceleration α of pulley P_2. *Answer:* $\boxed{a_2 = \alpha R}$

Problem 2, continued

c) (10 pts) In terms of g only, what is the acceleration a_2 of block B_2? If you could not get the answer to part a above, use $a_1 = ca_2$ and leave your answer in terms of g and c.

FBD for block 2 gives: $\Sigma F_y = mg - T_1 = ma_2$

FBD for pulley 2 gives: $\Sigma\tau = T_1R - T_2R = I\alpha = \frac{1}{2}mRa_2$

(we accepted solutions with pulley 2 being either a disk or a hoop)

FBD for pulley 3 gives: $\Sigma F = T_2 - T_3 - T_4 = 0 \Rightarrow T_2 = 2T_3$

(the tension in the string attached to pulley 1 must be the same on either side)

FBD for block 1 gives: $\Sigma F_x = T_3 = ma_1 = 2ma_2$

Putting it all together and solving for a_2*, we have:*

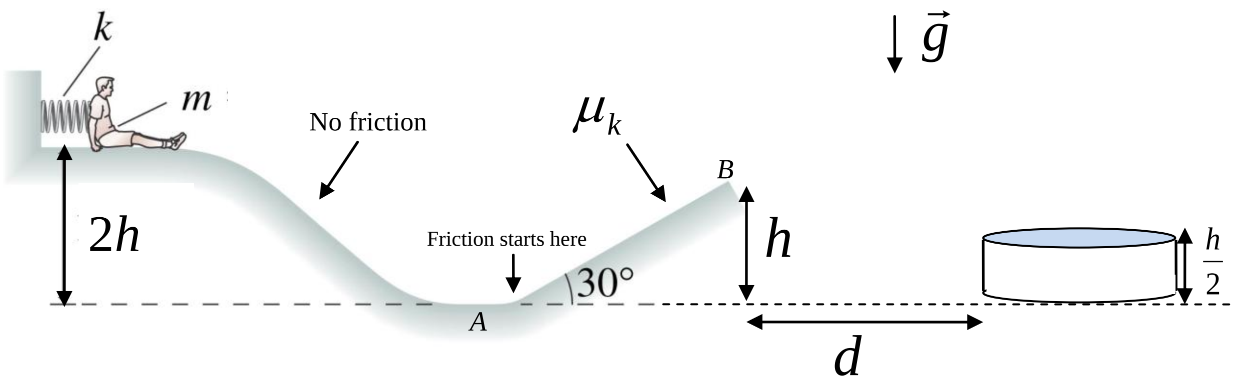
$$ma_2 = mg - T_1 = mg - \left(T_2 + \frac{1}{2}ma_2\right) = mg - \left(4ma_2 + \frac{1}{2}ma_2\right) = mg - \frac{9}{2}ma_2$$

$$\boxed{a_2 = \frac{2}{11}g}$$

d) (7 pts) What is the net torque $\vec{\tau}$ (magnitude and direction) on pulley P_2?

$$\Sigma\vec{\tau} = I\vec{\alpha} = +\frac{1}{2}mRa_2\hat{k} = \boxed{\frac{1}{11}mgR\hat{k}}$$

Problem 3. Don't try this at Home! Work, Energy, and Kinematics (30 pts)



A wild carnival ride consists of a massless spring with force constant k that is used to launch a fearless boy with mass m down a frictionless track of height $2h$ above its lowest point (A), and then up a 30° incline with coefficient of kinetic friction μ_k. The end of the incline (B) is at a height h above the ground and located a distance d from a tank of water with a height $h/2$. Neglect air resistance and use the following values for the given constants: $k = 1000\,\text{N/m}$, $m = 50\,\text{kg}$, $\mu_k = 1/\sqrt{3}$, and $h = d = 5\,\text{m}$.

Before attempting to launch himself into the water tank, our daredevil decides to take a practice run **without the spring**. He starts from rest at the top of the slide.

a) (6 pts) What is the boy's speed v_A at the bottom of the track (point A)?

$$E_f = E_i \Rightarrow \frac{1}{2}mv_A^2 = mg(2h) \Rightarrow v_A = \sqrt{4gh} = \boxed{14\,\text{m/s}}$$

b) (6 pts) How far up the ramp (i.e., the distance s along the ramp) does the boy reach?

$$\Delta K = W_{net}$$

$$\Delta K = -\frac{1}{2}mv_A^2 = W_{net} = -mgs\sin\theta - \mu_k mgs\cos\theta$$

$$s = \frac{2mgh}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{2h}{\frac{1}{2} + \frac{1}{\sqrt{3}}\frac{\sqrt{3}}{2}} = 2h = \boxed{10\,meters}$$

So he makes it just to the top of the ramp!

Problem 3, continued

Ok, no problem, now the hard part! The boy needs to compress the spring by a minimum amount $x_{\min}$ such that he lands in the water and not on the ground. (Note: ignore the diameter of the tank, he only needs to make it to the point a distance d beyond the end of the incline.)

c) (10 pts) What is the minimum speed v_B needed at the top of the incline such that the boy lands in the water?

Using the kinematic equations, we obtain:

$$d = v_B \cos\theta t \Rightarrow t = \frac{d}{v_B \cos\theta}$$

$$-\frac{h}{2} = v_B \sin\theta\left[\frac{d}{v_B \cos\theta}\right] - \frac{1}{2}g\left[\frac{d}{v_B \cos\theta}\right]^2 = d\tan\theta - \frac{gd^2}{2v_B^2 \cos^2\theta}$$

$$v_B = \sqrt{\frac{\frac{1}{2}gh}{\cos^2\theta\left(\tan\theta + \frac{1}{2}\right)}} = 5.5\,{}^{m}\!/\!{}_{s}$$

where we have used in the last step the fact that d = h.

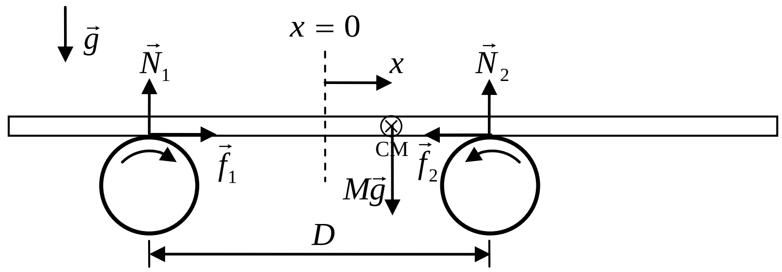
d) (8 pts) What is the minimum compression $x_{\min}$ of the spring that is required to achieve this speed at the top of the incline?

We know from part a that, without the spring, the boy reaches just to the top of the incline with zero speed. Thus, all of the extra energy needed to obtain a speed v_B at the top is provided by the spring. So we have:

$$\frac{1}{2}mv_B^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{m}{k}}v_B = \boxed{1.23 meters}$$

Problem 4. Purcell's Machine (30 pts)



In Lecture 9 you saw a demonstration based on Purcell's Machine, which consists of a uniform thin rigid plank of mass M resting on two rapidly counter-rotating disks. The disk on the left is rotating clockwise, while the one on the right is rotating counterclockwise. The coefficient of kinetic friction between the plank and the disks is μ_k, the midpoint between the two disks is located at $x = 0$, the disks are separated by a fixed distance D, and gravity points down in the diagram. At time $t = 0$, the center-of-mass of the plank is displaced a distance $x_0 = +A$ (with $A < D/2$) and released from rest ($v_0 = 0$). The plank then oscillates back and forth executing simple harmonic motion.

a) (5 pts) On the illustration above, draw a free-body diagram indicating all forces acting on the plank.

See diagram.

b) (15 pts) What is the frequency of oscillation ω of the plank as it moves back and forth? Show your work. (Note that no approximation is required.)

Summing the forces and torques gives the following three relations:

$$\Sigma F_x = f_1 - f_2 = Ma \Rightarrow \mu_k(N_1 - N_2) = Ma$$
$$\Sigma F_y = N_1 + N_2 - Mg = 0 \Rightarrow N_1 + N_2 = Mg$$
$$\Sigma\tau = N_1\left(\frac{D}{2} + x\right) - N_2\left(\frac{D}{2} - x\right) = 0 \Rightarrow N_1 - N_2 = -\frac{2}{D}(N_1 + N)x = -\frac{2Mg}{D}x$$

Substituting the last expression into the first expression we obtain:

$$Ma = -\frac{2\mu_k Mg}{D}x$$
$$\frac{d^2x}{dt^2} = -\left[\frac{2\mu_k g}{D}\right]x$$

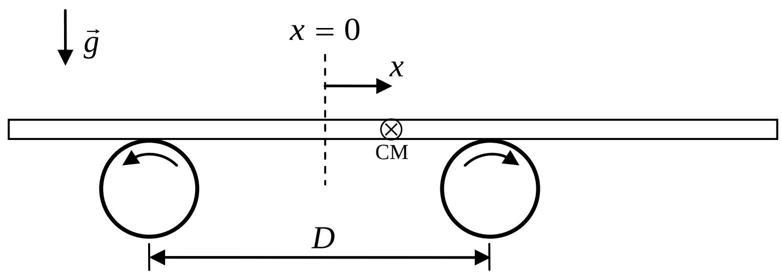
This is the equation of motion for a simple harmonic oscillator with frequency $\boxed{\omega = \sqrt{\frac{2\mu_k g}{D}}}$

Problem 4, continued

c) (5 pts) What is the velocity of the plank $v(t)$ in terms of A, ω, and t?

The solution to the equation of motion is $x(t) = A\cos\omega t$, and taking the derivative we find the velocity as a function of time:

$$v(t) = \frac{dx}{dt} = \boxed{-A\omega\sin\omega t}$$



Now suppose that the disks rotate in the opposite direction, so that the one on the right is rotating clockwise and the one on the left counterclockwise.

d) (5 pts) Box the answer that best describes the subsequent motion when the plank is displaced to the right and released from rest, as before.

i) No difference from before; it executes simple harmonic motion as in parts a-c.

ii) Moves a little to the right and then comes to rest.

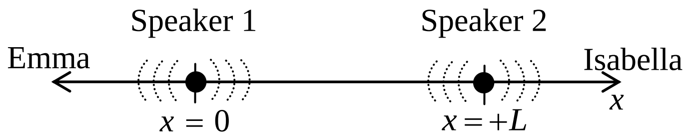
iii) Keeps moving to the right at constant speed.

iv) Moves to the right with constant acceleration ($da/dt = 0$).

v) Moves to the right with increasing acceleration ($da/dt > 0$). [boxed]

The net force is now to the right and continues to increase as x_{CM} increases. So the acceleration is increasing to the right. This continues until $x_{CM} > \frac{D}{2}$, at which point the plank will fall over the right disk (i.e., $\Sigma\tau \neq 0$).

Problem 5. Sound Interference: Emma and Isabella at Princeton. (30 pts)



Emma and Isabella are enjoying their first semester as roommates at Princeton (physics majors, of course). Isabella has an early schedule and needs to wake up before Emma, so the girls setup an alarm system consisting of two speakers (1 and 2) that emit the same tone, $f = 85\,\text{Hz}$, but are out of phase by an amount $\Delta\phi_0 = \phi_{20} - \phi_{10}$ and separated by a distance L. The idea is to have **destructive** interference to the left of speaker 1 (where Emma is sleeping) and **constructive** interference to the right of speaker 2 (where Isabella is sleeping). Assume the speed of sound in the room is $v = 340\,\text{m/s}$.

Each speaker emits traveling waves to the right (D_{1R}, D_{2R}) and to the left (D_{1L}, D_{2L}) that have the same amplitude a, wave number k, and frequency f, and whose displacements must match at the source. This last requirement leads to the following set of boundary conditions: $D_{1L}(x=0,t) = D_{1R}(x=0,t)$ and $D_{2L}(x=L,t) = D_{2R}(x=L,t)$, which must be satisfied **at all times** t.

a) (6 pts) Assume $D_{1R}(x,t) = a\sin[kx - \omega t + \phi_{10}]$ and $D_{2R}(x,t) = a\sin[k(x-L) - \omega t + \phi_{20}]$. What are the wave functions $D_{1L}(x,t)$ and $D_{2L}(x,t)$? Use the boxes provided.

$$D_{1L}(x,t) = a\sin[-kx - \omega t + \phi_{10}]$$

$$D_{2L}(x,t) = a\sin[-k(x-L) - \omega t + \phi_{20}]$$

b) (6 pts) In terms of k, L, and $\Delta\phi_0$, what is the condition that must be satisfied to have **constructive** interference to the right of speaker 2 ($x > L$)?

We need $\Delta\phi = m2\pi$, where m is an integer (1, 2, 3, …). Therefore,

$$k(x-L) - \omega t + \phi_{20} - [kx - \omega t + \phi_{10}] = m2\pi$$

$$\boxed{-kL + \Delta\phi_0 = m2\pi}$$

Problem 5, continued

c) (6 pts) In terms of k, L, and $\Delta\phi_0$, what is the condition that must be satisfied to have **destructive** interference to the left of speaker 1 ($x<0$)?

Here we need $\Delta\phi = \left(m+\tfrac{1}{2}\right)2\pi$ *, so we obtain:*

$$-k(x-L)-\omega t+\phi_{20}-\left[-kx-\omega t+\phi_{10}\right]=\left(m+\tfrac{1}{2}\right)2\pi$$

$$\boxed{kL+\Delta\phi_0=\left(m+\tfrac{1}{2}\right)2\pi}$$

d) (6 pts) What is the minimum separation L such that these conditions are satisfied simultaneously? Please give a quantitative result with proper units, and show your work for credit (correct answers with no work will receive zero points).

Subtracting the equation in part b from the equation in part c, we obtain:

$$2kL=\pi \Rightarrow \boxed{L=\frac{\lambda}{4}}$$

The frequency is $f=85\,Hz$*, so the wavelength is* $\lambda = v/f = 4\,m$*. Therefore,*

$$\boxed{L=1\,meter}$$

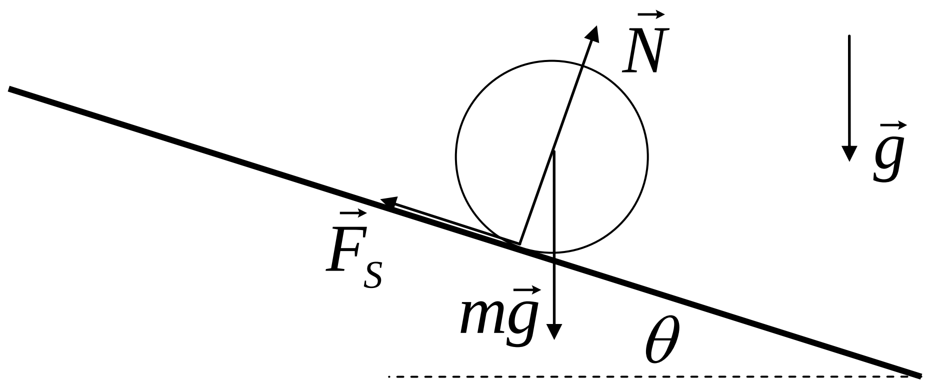
e) (6 pts) What are the possible values of $\Delta\phi_0$ for which this scheme will work?

Adding the equation in part b to the equation in part c and solving for $\Delta\phi_0$ *we obtain:*

$$\boxed{\Delta\phi_0=\left(2m+\tfrac{1}{2}\right)\pi}$$

Problem 6. Lab 5: Rolling Friction (30 pts)

In Laboratory 5 you explored the effects of rolling friction on a cart moving up and down an incline, while ignoring the rotational kinetic energy of the wheels. In this problem you will evaluate the impact of rotational kinetic energy on the overall description of the cart's motion, and derive the quantitative effect of this motion on the determination of the coefficient of rolling friction μ_r as obtained in the lab.



Before we investigate the laboratory setup, let's first consider a simpler problem consisting of a single wheel of mass m, radius r, and moment of inertia $I = \frac{1}{2}mr^2$ rolling up a ramp inclined at angle θ to the horizontal. The wheel **rolls without slipping**, which requires there to be a nonzero force of **static friction** F_s as it slows down while moving up the ramp. Ignore **rolling friction** for parts a and b below.

a) (5 pts) Using the diagram above, draw a free-body diagram for the wheel as it rolls up the ramp without slipping. *See diagram.*

b) (10 pts) What is the minimum coefficient of static friction μ_s such that the wheel will roll up the ramp without slipping? Your answer should depend on θ **only**.

Summing the forces and torques, we obtain the following equations (with x up the ramp, y down and perpendicular to the ramp, and counterclockwise positive rotation):

$$\Sigma F_x = F_s - mg\sin\theta = ma$$

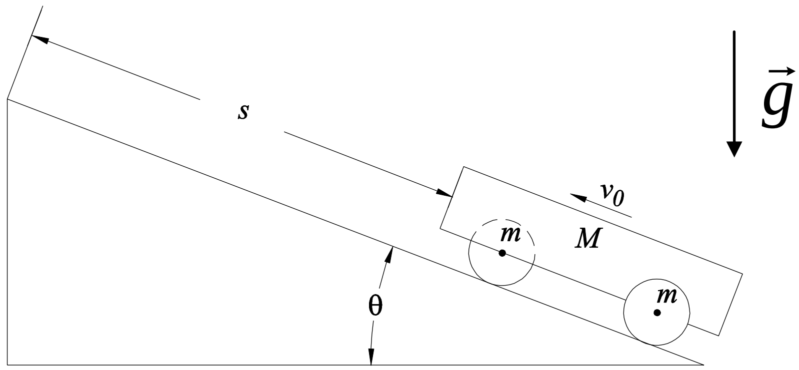
$$\Sigma F_y = N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$\Sigma\tau = -F_S r = I\alpha = \frac{1}{2}mra$$

Multiplying the last equation by $2/r$ *and subtracting from the first equation, we obtain:*

$$3F_s = mg\sin\theta \Rightarrow \boxed{\mu_s = \frac{1}{3}\tan\theta}$$

Problem 6, continued



Now let's analyze the laboratory setup. The cart used in the lab consisted of a block of mass M and *three* solid wheels each of mass m, radius r, and moment of inertia $I = \frac{1}{2}mr^2$ (the front wheel is in the center of the cart laterally). The cart was launched with initial velocity $\vec{v}_0$ up a plane that was inclined at angle θ to the horizontal, and was observed to travel a distance s up the slope before it stopped and reversed direction. In the lab you measured the speed v_0, the distance s, and the acceleration a while the cart moved up and down the slope.

We will now include the effects of both rolling friction and the rotational motion of the wheels. The force of rolling friction acts on the point of contact of a wheel with the inclined plane according to $F_r = \mu_r N$, where N is the normal force on the wheel at the point of contact. The direction of this force is **opposite** to the velocity of the center of the wheel. For an object with more than one wheel, like our cart, the **total force** of rolling friction is still $F_r = \mu_r N$, with N being the **total** normal force on the cart.

c) (7 pts) Consider the cart moving up the ramp with initial speed v_0 and ending with zero speed at distance *s* up the slope. Using the work-energy theorem, perform a correct analysis, including the effects of rolling friction and rotational kinetic energy, to show that the distance *s* is given by the following formula (with $\rho \equiv m/M$):

$$s = \frac{1+\frac{9}{2}\rho}{1+3\rho}\frac{v_0^2}{2g(\sin\theta + \mu_r\cos\theta)}.$$

$$\Delta K = W \Rightarrow -\frac{1}{2}(3m+M)v_0^2 - \frac{3}{2}I\omega_0^2 = -(3m+M)gs\sin\theta - \mu_r Ns$$

$$N = (3m+M)g\cos\theta$$

$$s = \frac{\frac{1}{2}(3m+M)v_0^2 + \frac{3}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_0}{r}\right)^2}{(3m+M)g\sin\theta + \mu_r(3m+M)g\cos\theta} = \frac{3m+M+\frac{3}{2}m}{3m+M}\frac{v_0^2}{2g(\sin\theta+\mu_r\cos\theta)} = \boxed{\frac{1+\frac{9}{2}\rho}{1+3\rho}\frac{v_0^2}{2g(\sin\theta+\mu_r\cos\theta)}}$$

Problem 6, continued

d) (4 pts) In the lab you calculated the coefficient of rolling friction from the measured values of s, v_0, and θ, but you ignored the contribution from the rotational kinetic energy of the cart wheels. Label this incorrect coefficient of friction μ_r' and use the work-energy theorem again, but this time **ignoring** rotational kinetic energy of the wheels, to obtain an expression for μ_r' that would be measured in this case. Your equation for μ_r' should be in terms of s, v_0, θ, and g only.

$$\Delta K = W \Rightarrow -\frac{1}{2}(3m+M)v_0^2 = -(3m+M)gs\sin\theta - \mu_r'(3m+M)gs\cos\theta$$

$$\sin\theta + \mu'\cos\theta = \frac{v_0^2}{2gs} \Rightarrow \boxed{\mu_r' = \frac{v_0^2}{2gs\cos\theta} - \tan\theta}$$

e) (4 pts) In the lab, this incorrect analysis led to some measured values of μ_r' that were negative. To see how this happens, substitute the equation for s in part c into your formula for μ_r' obtained in part d to derive an expression for μ_r' in terms of the true value μ_r. If $\theta = 10^\circ$ and the true coefficient of rolling friction is $\mu_r = 0.02$, for what values of the ratio $\rho = m/M$ would μ_r' be reported as negative?

$$\mu_r' = \frac{v_0^2}{2g\cos\theta}\frac{1+3\rho}{1+\frac{9}{2}\rho}\frac{2g(\sin\theta + \mu_r\cos\theta)}{v_0^2} - \tan\theta = \frac{(1+3\rho)(\tan\theta + \mu_r) - (1+\frac{9}{2}\rho)\tan\theta}{(1+\frac{9}{2}\rho)}$$

$$\boxed{\mu_r' = \frac{\mu_r + 3\rho\mu_r - \frac{3}{2}\rho\tan\theta}{1+\frac{9}{2}\rho}}$$

The righthand side of this equation is negative when the numerator is negative, thus, we need:

$$\tfrac{3}{2}\rho\tan\theta > \mu_r + 3\rho\mu_r$$

$$\rho > \frac{\mu_r}{3(\frac{1}{2}\tan\theta - \mu_r)}$$

Plugging in the numbers, we find that μ_r' *is negative when* $\boxed{\rho > 0.10}$