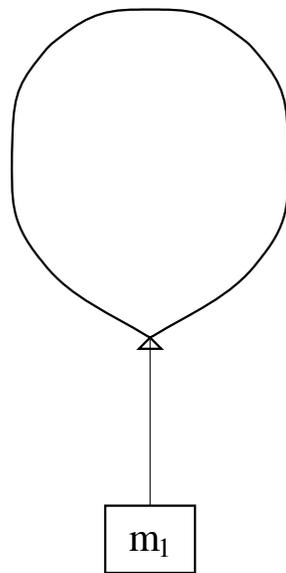


PRINCETON UNIVERSITY

Physics 101 Lab Manual

(Revised August 1, 2014)



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Lab 1: Newton's Laws: Motion in One Dimension

1.1 Background

In this lab you will study the motion of bodies moving in one dimension. To minimize unwanted forces on the test object, you will use an air track (see Fig. 1.1). The glider floats on a cushion of air above the track, eliminating most of the friction between the glider and the track. The glider can move freely with no horizontal force, or under the influence of a constant horizontal force when attached via a tape to a weight suspended off one end of the air track (see Fig. 1.2). By measuring the time, t , at which the glider passes several positions, x , you can test Newton's laws and measure g .

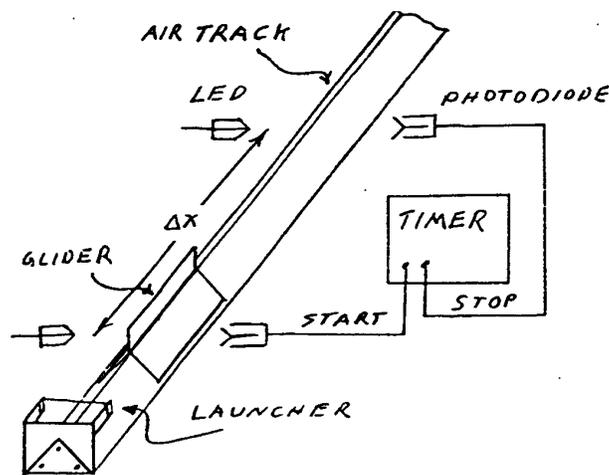


Figure 1.1: An air track with two photogates.

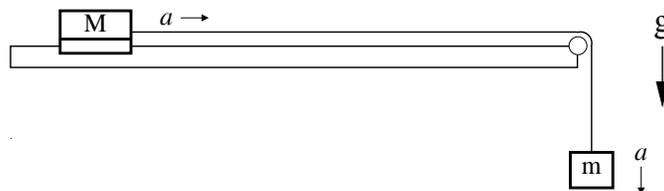


Figure 1.2: Glider of mass m on the horizontal air track, connected by a tape to mass M that falls vertically under the influence of gravity.

1.1.1 Newton's First Law

Newton's first law states that an object will move at constant velocity if no forces are imposed on it. Thus, defining the x axis to be along the direction of the velocity, its motion obeys:

$$x = x_0 + vt, \quad (1.1)$$

where x is the position of the object at time t , x_0 is its position at time t_0 , and v is its velocity. You can test this equation by giving the glider a push (so that it has some nonzero velocity) and seeing whether x increases linearly in time.

1.1.2 Newton's Second Law

Newton's second law states that the acceleration a of an object of mass m is proportional to the net force F applied to it,

$$\sum_i F_i = ma. \quad (1.2)$$

If the net force is constant, then the acceleration a is constant and the motion of the object obeys:

$$x = x_0 + v_0t + \frac{at^2}{2}, \quad (1.3)$$

with v_0 the initial velocity.

To test this, you need to impose a constant force on the glider. You can do this by attaching a hanging mass m to the glider (which itself has mass M) via a pulley, as in Fig. 1.2. Let T be the tension in the tape connecting the masses, and g be the acceleration due to gravity. The only horizontal force on the glider is the tension from the tape, so Newton's second law gives

$$T = Ma. \quad (1.4)$$

The hanging mass m feels both gravity (downwards) and tension T (upwards), so the second law for its vertical motion is

$$mg - T = ma, \quad (1.5)$$

noting that the downward vertical acceleration a of the hanging mass is the same as the horizontal acceleration of the glider. Substituting eq. (1.4) for T into eq. (1.5) gives

$$mg - Ma = ma, \quad (1.6)$$

or, upon rearranging terms,

$$g = a \frac{M+m}{m}. \quad (1.7)$$

By measuring the acceleration a and the masses M and m , you can find gravitational acceleration g .

1.2 Specific Instructions

1.2.1 Setup of the Air Track

Start air flowing into the track by plugging in the air pump and/or turning on the switch on the power strip.

The air track will probably already be set up, but you should check the following things:

- In the acceleration part of this lab, you will work with a glider, a hanging mass, and a tape, connected as shown in Fig. 1.2. The tape slides along the curved bit of the air track with minimal friction. Then, the glider will be accelerated by the hanging mass. Briefly set this up now, and make sure the tape is long enough that the glider can start behind photogate 1, and that the glider will get through photogate 4 before the hanging mass hits the floor.

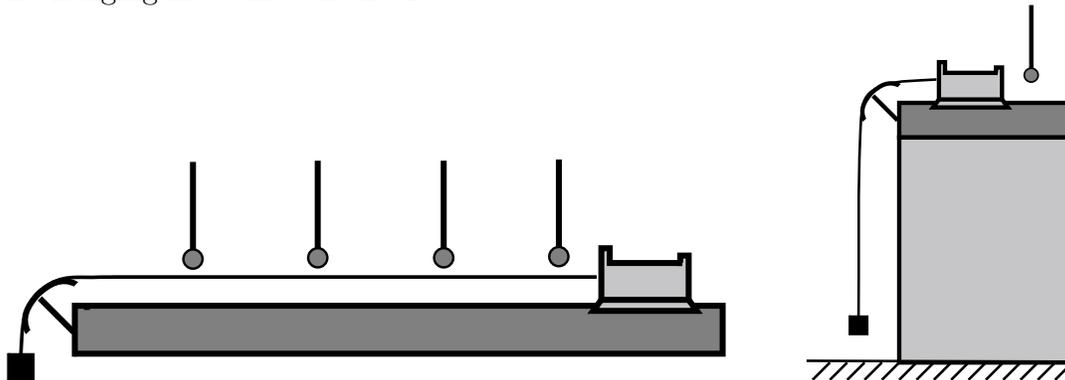


Figure 1.3: Photogates setup for the acceleration test.

- The four photogates (enclosed in U-shaped plastic tubing) should be set up so that each one is blocked (and unblocked) twice every time the glider passes by. The system will record the time when each gate is blocked, and not the times when they become unblocked. To check the operation and positioning of the photogates, watch the red lights on the photogate adapter box, which turn on when a gate is blocked.
- The distance between adjacent photogates should be longer than the length of the glider (so the glider blocks each photogate twice before it blocks the next photogate for the first time).
- **Shade the photogates from direct sunlight.**

1.2.2 Timing Software

The photogate timing is measured by a program called **Princeton Timer A**. Double click on its icon (in the Physics 101 folder of your lab computer) to open the program. When the window

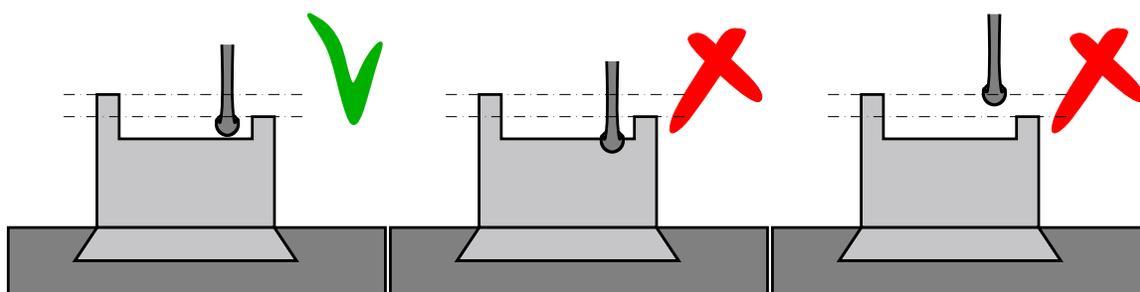


Figure 1.4: Correct and incorrect vertical positioning of the photogate.

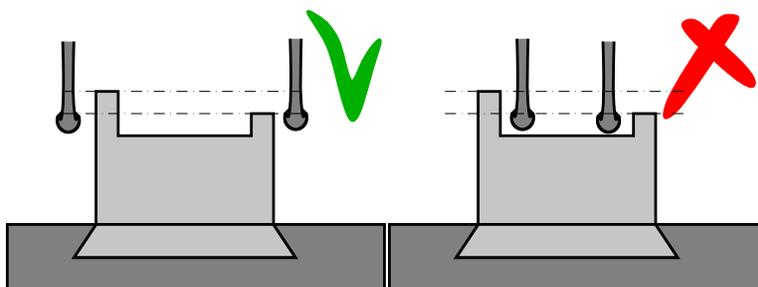


Figure 1.5: Correct and incorrect spacing between the photogate.

Sensor Confirmation pops up, click on **Connect** to activate the communication between the photogates and the program.

Once the program is open, you can click on **Collect** to start data collection, and on **Stop** to end a run. The system records the (eight) times of each successive blocking of a photogate. That's all there is to your data collection. Try the system a few times, blocking the optical beams with your hand repeatedly to watch data being collected.

Note: the data-collection system takes a second or so to organize itself after the **Collect** button is pushed. When the system is actually ready to record data the **Collect** button changes to **Stop**. If you get strange (generally negative) times in the first few data entries, it is probably because a photogate was interrupted before the system was ready to take data.

1.3 Events

Because of the way your measurements will be made, it will be useful to think of yourself as measuring the times and positions of eight “events” which occur as the glider moves from one end of the air track to the other:

- Event 1: the 1st photogate is blocked by the first “flag” on the glider .
- Event 2: the 1st photogate is blocked by the second “flag” on the glider .
- Event 3: the 2nd photogate is blocked by the first “flag” on the glider .

- Event 4: the 2nd photogate is blocked by the second “flag” on the glider .
- Event 5: the 3rd photogate is blocked by the first “flag” on the glider .
- Event 6: the 3rd photogate is blocked by the second “flag” on the glider .
- Event 7: the 4th photogate is blocked by the first “flag” on the glider .
- Event 8: the 4th photogate is blocked by the second “flag” on the glider .

You will measure the positions x and times t of each of these events, and fit these data to eqs. (1.1) and (1.3).

1.4 Measuring x Positions

You need to measure the position x of the front edge of the glider at each of the events described above. You can do this as follows:

- Turn the air off (this makes the measurements a bit easier).
- Note in which direction the glider will be moving, and move the glider to the point where its first “flag” is just starting to block photogate 1. By watching the lights on the photogate box to see when the photogate is blocked, you should be able to do this very accurately. Take your time - every millimeter counts here! Check that the photogate heights are set so that they are blocked by each of the two flags, and not blocked when the glider is at in-between positions.
- Once the glider is at this point, measure the position of the front edge of the glider using the scale attached to one side of the air track. Note, you could equally well use the back edge of the glider – just be consistent during these measurements. In fact, the absolute position of the scale on the air track in both eqs. (1.1) and (1.3) affects only the constant term x_0 , which is relatively unimportant; it does not affect the value of the constant velocity in your first measurement, or the value of the acceleration g in your second measurement.
- Make a measurement of the position of the front edge of the glider when its second “flag” is just starting to block photogate 1. Then, repeat this procedure for the other three photogates. Record all your measurements in your lab book.

Once you’ve measured the positions of the photogates, you should avoid touching them or even shaking the desk - this may dramatically affect the quality of your data!

1.4.1 Motion with Constant Velocity

As your glider moves through the four photogates, the system will record the times of the “events” 1-8 as discussed above. From your notebook, you know where the glider was at the time of each event. So you can make a plot of position *versus* time. To do this, use

the program Microsoft Office Excel 2003, whose icon is in your Physics 101 folder. (Excel is a standard spreadsheet and graphing program, and the version on your Lab computer includes WPtools (Workshop Physics Tools), an add-on capability which simplifies graphing and curve fitting. See Appendix A for additional details.)

To begin, let's test Newton's first law. If a tape is connected to the glider, disconnect it. Give the glider a push, and let it pass through all four photogates. *Remember: this is a constant velocity test, so you need to remove your pushing finger from the glider before it will block the first photogate. One way to assure this is to launch the glider in the opposite direction - it will nicely bounce off the air-track terminator and moves back in the "proper" direction.*

Start Collecting data before it gets to the first gate, and Stop after it leaves the fourth. *If it bounces back and retriggers some of the gates, just use the first eight times recorded and ignore the later ones.*

Without closing down the timer program, double click on Excel with WPtools to (re)open it. *If necessary, answer No to a question about something already being open, and then click on File and New and OK to bring up a blank spreadsheet.*

Transfer your eight time values from the timer program into a column in Excel, starting in row 2, rather than row 1.

Click on the first value on the data in the Timer program, and hold the mouse button down while "swiping" the cursor down to the last value. Then, click Edit/Copy in the timer program, and go back to Excel. Click on one Excel cell to highlight it, and use Edit/Paste to put the data in a column below the highlighted cell.

Enter your data for the eight x positions in the next Excel column, with each value adjacent to the corresponding value of t . Add a title to the cell above the data in each column, such as Time or t to one column, Position or x to the other.

To make a graph of position *versus* time, you have to tell Excel what to plot in the vertical and horizontal directions. Swipe your cursor vertically across the title and data cells corresponding to the horizontal variable (time). Then, while holding down the Control key, swipe the cursor across the title and data cells corresponding to the vertical variable (position).

With the horizontal and vertical data selected, click on WPtools / Scatter Plot to make your graph. *Print all "good" graphs, and tape them in your Lab book. Do the points lie close to a straight line?*

The slope of the line should be the velocity of the glider. Estimate the value of the slope from the printout of the graph (use of a ruler to draw a line on the graph is OK!), and record it in S.I. units. *Does the number seem reasonable?*

Next, use the computer to make a graph with a polynomial fit to the data. In terms of the general equation (1.3) for uniformly accelerated motion in one dimension, we expect position x to be a quadratic function of time t ,

$$x = a_0 + a_1t + a_2t^2. \quad (1.8)$$

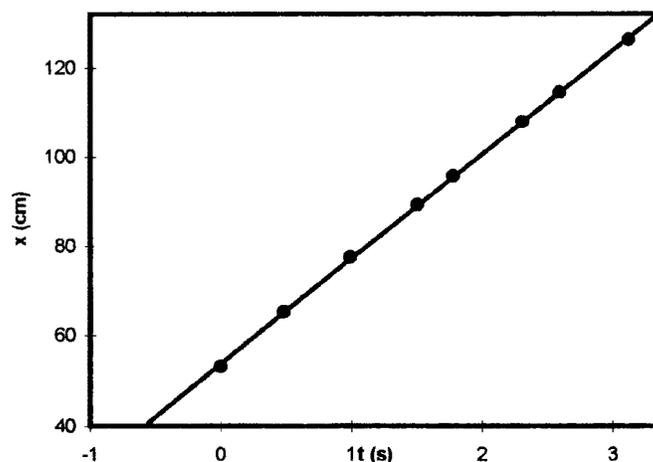


Figure 1.6: Sample plot of distance —it versus time for zero net force.

However, the value of the coefficient a_2 is expected to be zero for motion with constant velocity.

Highlight the two data columns as before, and click on **WPtools / Polynomial Fit**. Choose **Order 2** for the polynomial, and click **OK**. You get a graph equivalent to the previous one, but with lots of information added about the computer's best fit to your data.

You may want to resize the graph box, or to move some of the fit data boxes around or resize them, to get a good view of the new plot. Your instructor can also show you how to reset the scales on the graph. For now, let's focus on the meaning of the computer's information concerning its fit to your data.

The numbers a_0 , a_1 and a_2 are computed values for the coefficients of the fit equation. The **Standard Error (SE)** for each coefficient is the computer's estimate of how accurately these coefficients are known, based on how closely your data falls to the fitted line. R^2 is a measure of the goodness of the fit, which need not concern us.¹ The quantity symbolize by the Greek letter sigma σ is, however, of interest. It indicates how far a typical data point deviates from the fitted line.

If the horizontal force on the glider were zero as desired, the fitted value of the coefficient a_2 should be zero to within 1-2 times its **Standard Error**, $SE(a_2)$. If this is not the case, check that the air track is level, and that the air flow is sufficient.

If $a_2 < 2SE(a_2)$, then you can make a better measurement of the velocity, which corresponds to fit coefficient a_1 , by redoing the **Polynomial Fit** choosing **Order 1** for the polynomial. Does the (re)fitted value of a_1 agree with the velocity you calculated manually from the slope of your first graph to within 1-2 times the reported **Standard Error** $SE(a_1)$ on a_1 ? If not, recheck your procedures.

¹If you are curious as to the technical significance of the quantities R^2 and σ , see Appendix D.

1.4.2 Motion with Constant Acceleration: Measuring g

Attach a tape to the glider and suspend it **shiny side down** over the curved bracket so that a hanger can also be attached, as indicated in Fig. 1.2. Add weights to the hanger so that its mass m is between 15 and 25 g. *The hanger can be passed through the hole in the weights so they stay on the hanger.* Measure the masses of the glider and the hanging mass using a balance or digital scale, located on the tables in the center of the Lab.

Release the glider and record times using **Princeton Timer A** as described above. Transfer these times to **Excel** and enter the x measurements (if you didn't save them earlier). The displacement x should have a quadratic dependence on t (eq. (1.3)), but first use the **Polynomial Fit** option with **Order = 3** to fit the data to an equation of the cubic form

$$x = a_0 + a_1t + a_2t^2 + a_3t^3, \quad (1.9)$$

which should result in a value of a_3 consistent with zero if your data are good. If this is so, record the results of your cubic fit in your Lab notebook, but if not, check for possible reasons why your measurement might not be of good quality.

Once you are satisfied that the cubic fit gives no evidence for a significant cubic term in the acceleration, repeat the fit using **Polynomial Fit** option with **Order = 2**. Comparing eqs. (1.3) and (1.9) we see that the acceleration of the glider is $a = 2a_2$. So, calculate the acceleration g due to gravity from eq. (1.7) as

$$g = 2a_2 \frac{m + M}{m}. \quad (1.10)$$

The corresponding uncertainty σ_g in your measurement of g can be estimated as

$$\sigma_g = 2 \text{SE}(a_2) \frac{m + M}{m}. \quad (1.11)$$

Include a printout of the plot of x vs. t and the results of the fit of order 2 in your Lab notebook.

Is your measurement for g within 1-2 times the estimated uncertainty σ_g of the expected value?

How does the uncertainty $\text{SE}(a_2)$ on the fit coefficient a_2 obtained with the hanging mass compare with that obtained previously when the glider slid freely?

Run the experiment two more times, using different hanging masses, and calculate the value of g twice more. Are the values systematically high or low? If so, can you think of anything that might cause this experiment to have gone wrong?

From your three measurements of g , you can make another estimate the uncertainty σ_g according to

$$\sigma_g = \frac{\text{Max}(g) - \text{Min}(g)}{2}. \quad (1.12)$$

Compare this with your estimate based on eq. (1.11).

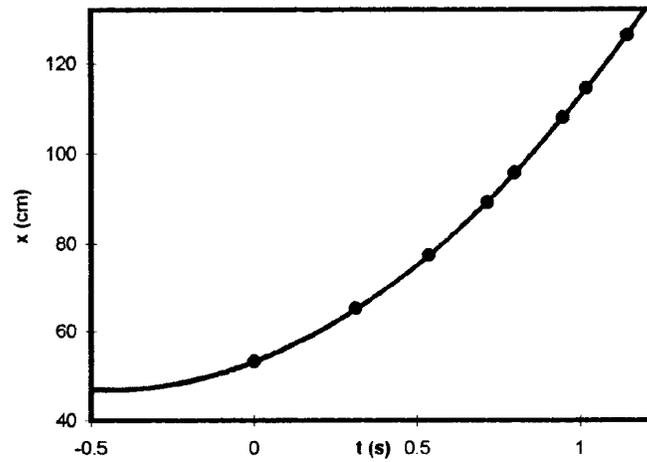


Figure 1.7: Sample plot of distance versus time for an accelerated glider.

The estimate (1.11) of the uncertainty in your measurement of g does not include the effect of the uncertainty in your measurements of the masses m and M . In fact, the uncertainty σ_m on the hanging mass m leads to an additional uncertainty on g of approximately

$$\sigma'_g = g \frac{\sigma_m}{m}. \quad (1.13)$$

Is this effect larger or smaller than the uncertainty (1.11) associated with the curve fitting?

Lab 2: Motion in Two Dimensions

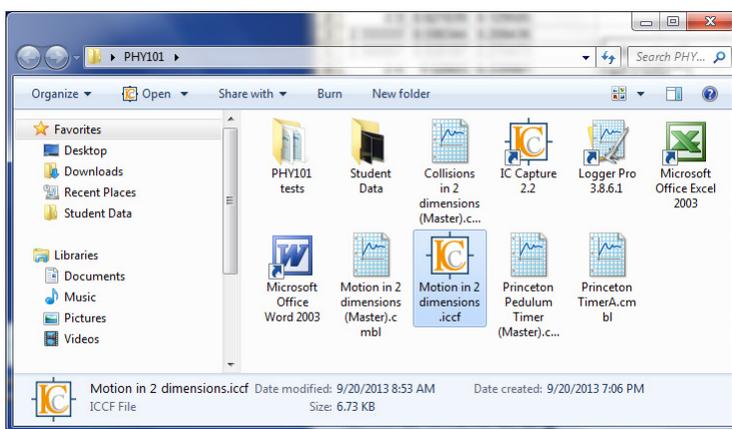
This lab extends your exploration of Newton's laws of motion to the case of two dimensions, as described in secs. 2.2 and 2.5. To accomplish this, you will use a video camera and supporting computer software to capture, edit and analyze images of two-dimensional motion. Sections 2.1, 2.3 and 2.4 introduce you to this infrastructure.

2.1 A New Tool

The first goal of this lab is to acquaint you with a new tool. You will use a video camera to make videos of moving objects, and a computer to digitize the locations of objects in your videos and analyze the motion. It is important that every member of your group learn how to use the computer and analysis software. Be sure that you all take turns operating the system and that, when you are the operator, your partners understand what you are doing.

When your computer is first turned on, the screen should show an icon for a folder named PHY101. Open this folder, and you will then see a screen with icons for the programs that we will use in this Lab.

These include:



- **IC Capture 2.3:** this program allows you to capture a series of images, called “frames”, from the signal produced by a video camera.
- **Logger Pro 3.8.6.1:** the program in which you will do your analysis. It allows you to determine the positions of objects in each of the video frames that you captured, and provides ways to generate and analyze graphs of your data.
- **Microsoft Office Excel 2003:** an extension of the common Excel spreadsheet program, which allows detailed calculations to be carried out on your data. The WP Tools (Workshop Physics) extensions, make it easy to use Excel's sophisticated graphing functions.
- **Student Data:** a folder with space for you to put your files.

2.1.1 Capturing a Single Still Image

Your first task is to capture a still picture to put at the front of your Lab notebook to help your Lab Instructor to help him or her remember your name and face.

1. Open **IC Capture**, by double-clicking with the left mouse button on its icon in the **Ph101** folder. If no video window opens, click on **File**, then **New** and **Click OK** in the **Select Device** window. The device size and color should be **RGB32 (640x480)**, and the **Codec** should be **RGB32**.
2. A window should appear that shows an image of whatever your camera is viewing at the moment. Confirm that the image window is showing live video by waving your hands in front of the camera. If the picture is too dark or too light, rotate the aperture ring on the camera lens to change the amount of light level reaching the camera's sensor. (The lens admits light through a circular opening, or aperture, whose diameter is controlled by rotating the ring. The larger the diameter, the more the light and the brighter the picture.)
3. Aim the camera at yourself and click the **Snap Image** icon to take a still picture. To take another picture, first click on the **Start/Stop Live Display** icon, and then again on **Snap Image**. Right click on the still image and **Save it**, say to the **Desktop** as a **.jpg** file.
4. To print your photo, first click on its icon on the **Desktop**, which opens it in **Windows Photo Viewer**.

2.1.2 Digital Video Images

Recall that the moving image on the monitor actually consists of a series of individual stationary, or still, pictures, appearing at a rate of N pictures, or *frames*, a second, where $N = 25$ for this Lab. Each image consists of an array of 640×480 pixels (*i.e.*, picture elements).

To determine the coordinates of an object in an image frame using **Logger Pro**, we will use the mouse to position a cursor over the object, and the system will count the pixel rows and columns to get to the location of the cursor, starting at the lower left of the image on the screen. Then, if we know how many pixels correspond to a meter long object, we can calculate real distances from the pixel counts. This is called “scaling the image”.

2.2 Physics Background

In this Lab you will analyze the trajectory of a bouncing ball, which will be accelerated by gravity in the vertical (y) direction, but not in the horizontal (x) direction. Its motion should obey the equations

$$x = x_0 + v_{0x}t, \quad (2.1)$$

$$y = y_0 + v_{0y}t + \frac{gt^2}{2}. \quad (2.2)$$

The experimental setup is sketched in Fig. 2.1. A ball rolls down a fixed rail (the “launcher”). After it leaves the launcher it falls freely, bouncing at least twice on the table

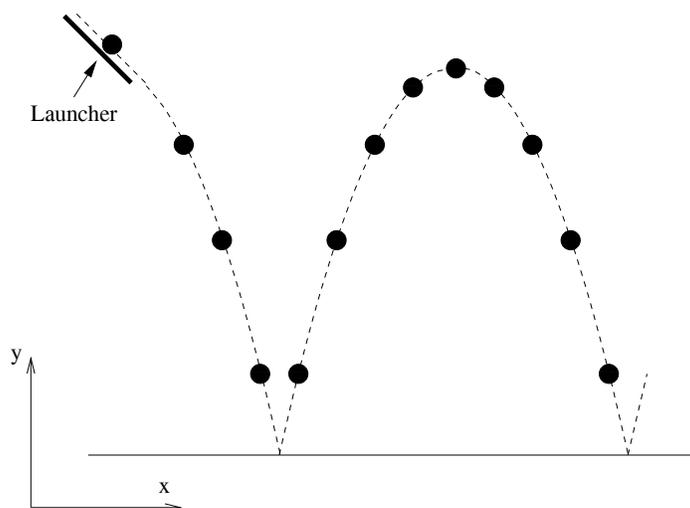


Figure 2.1: Setup of the experiment.

top. The camera takes a series of pictures at fixed time intervals during the motion. By measuring the position of the ball in each picture, you can test eqs. (2.1) and (2.2).

2.3 Acquiring the Data

You are now ready to record a video. Close **IC Capture**, and click on the icon **IC Capture: Motion in 2 dimensions** in the PHY101 window, and be sure the image is live. *If not, close the image window, click File/New and then OK in the Select Device window.*

You should not have to adjust the default parameters: **Video Size = RGB32 (640x480)**, **Codec = RGB32**, and **FPS = 30.00**. The **Exposure Time** should be **1/200 sec** or smaller; if the image is bright enough at **1/500 sec** to see the meter stick, use this setting.

The camera lens has adjustment for aperture, focus, and focal length (telephoto). *Don't partially unscrew the lens from the camera while making adjustment!* The aperture (ring closest to the camera) should be wide open. The focal length (knob farthest from the camera) should be chosen so that you can view most of the meter stick, the bottom end of the launcher, and the aluminum plate on which the ball will bounce). The focus adjustment (knob closer to the camera) does not have a large travel, so it is delicate to obtain the best focus. It is very important that the optical axis of the camera ends up perpendicular to the black backdrop of the bouncing ball.

Make a few trial runs, watching the image window to verify that the camera catches the motion properly. Launch the ball from a position against the rod in the v-groove of the launcher, so that this process is reproducible. Adjust the position of the launching track, if necessary, so that the arc between two bounces fills most of the view, and that the plane of the ball's trajectory is perpendicular to the camera axis.

Include a meter stick in the field of view, for scaling purposes; a black spring clip is attached to provide a clearer indication of a known position, such as 50 cm, on the meter stick. The meter stick should be the same distance from the camera as the bouncing ball will be; otherwise, your scale factor will be in error; *i.e.*, the meter stick should be above the trajectory of the bouncing ball.

Before recording a video, give it a unique name by first clicking on the icon at the right end of the **Video File** bar. Then, click on the similar icon at the right end of the **Video File** bar on the **Video** tab of the **Recording Settings** window that pops up. This should open a directory window for **PHY101/Student Data** where you can enter a filename, with a **.avi** extension. Click **Save** and the new filename should appear in the **Video File** bar of the **Recording Settings** window. Click **OK** and the new filename should appear in the **Video File** bar of the main **IC Capture** window.

To start recording, click the red oval **Start** button near the upper left corner of the **IC Capture** window. To stop recording, click the black rectangular **Stop** button near the red **Start** button, either in the **IC Capture** or in the **Record Video File** windows that pops up.

To replay your video, click on the **Check Video** icon close to the red **Start** button, and then click on the **Play** icon in the **AVI Check** window. You can also step through your video one frame at a time by clicking on the outermost arrow icons in the **AVI Check** window.

2.4 Image Analysis Using Logger Pro

You can record the pixel coordinates of the bouncing ball in your video using a customized version of **Logger Pro** software. In the **PHY101** window on the **Desktop**, click on the icon **Motion in 2 dimensions** to open the **Logger Pro** window. To bring your video into **Logger Pro**, Double click to browse for a movie, and then on **Browse** in the **Movie Options** window. Go to the **Student Data** directory, and double click on your video to open it. *In case of an error, it is best to close the Logger Pro window, and then restart it.*

The Movie Window

The **Movie** window in **Logger Pro** has video controls in the lower left corner with which you can step through your video frame by frame.

A vertical bar of analysis icons appears to the right of the video window. If these buttons are not visible, click on the icon **Enable/disable video analysis** in the lower right corner of the video window.

If you move the cursor within the video window, the pixel coordinates of the tip of the finger icon are displayed on the top bar of the video window, ranging between (1,1) and (648,496).

The cursor style can be chosen with the top two of the analysis button. The crosshair option is better for this Lab.

Setting the Scale

Logger Pro assumes the horizontal and vertical axes in pixel coordinates have the same scale. Set this by clicking on the **Set Scale** icon to the right of the video. Position the cursor on one end of the meter stick and drag it to the mark at a known position along the stick. A line appears on the image to show the scale, and a window asks you for the distance in m of the scale you have just identified.



If the meter stick is tilted, the scale line should follow that tilt. If you wish to reset the scale, just click on the **Set Scale** icon again and repeat the above.

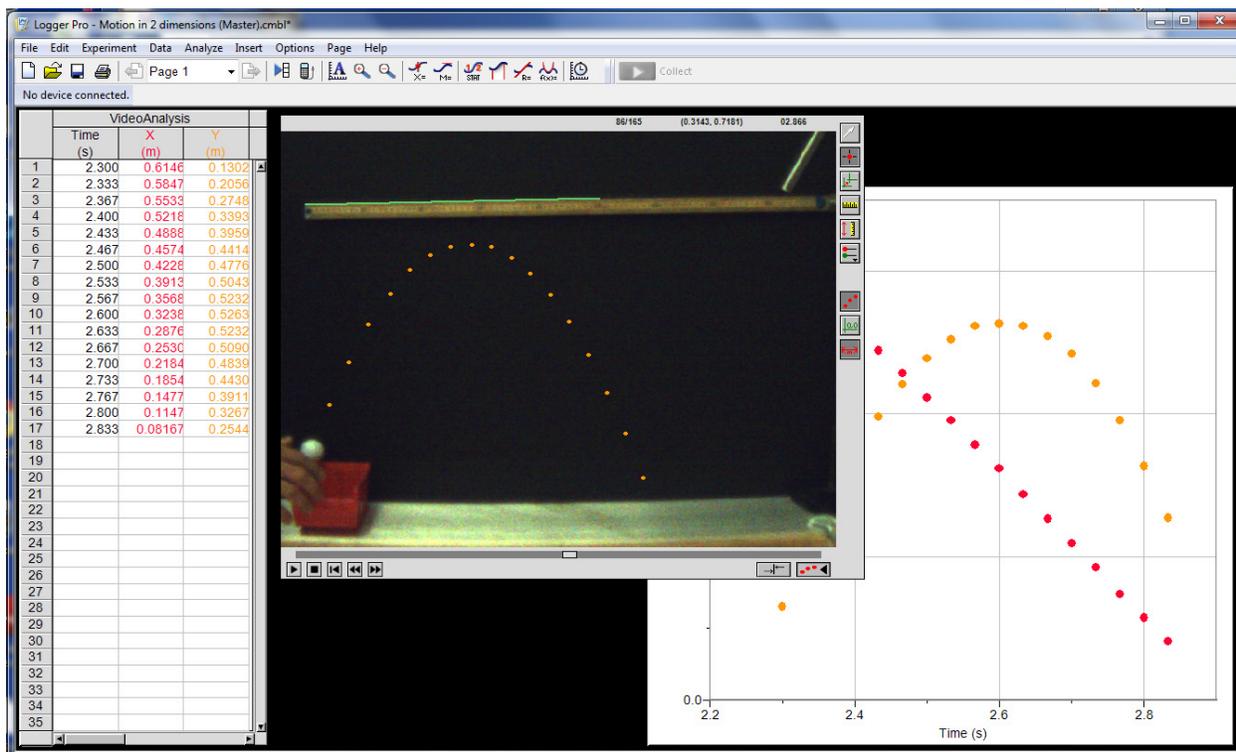
2.4.1 Digitizing Coordinates

Use the slider and/or the arrows at the bottom of the video screen to step through your video to find the earliest frame in which the ball is clearly rising after the first bounce. In case of doubt, skip the frame in which the ball might be either falling or rising. A frame counter is located near the center of the bar above the video image. *There may be an issue of occasional missing or mistimed frames in your video. If you notice this, make a new video after restarting IC Capture Motion in 2 dimensions.*

Step through your video to identify the last frame in which the ball is still falling before the second bounce. Then return to the first frame after the first bounce.

Click on the **Add Point** (crosshair) button to start the digitization process. Position

the crosshair over image of the ball and click. The video screen automatically advances one frame, and leaves a dot where you clicked on the ball, and automatically advances one frame. The digitized coordinates appear in VideoAnalysis window to the left of the Logger Pro screen. Position the crosshair over the ball in the new frame and click. Repeat until the last frame before the second bound has been digitized.



Your digitized data points also appear in a graph in the Logger Pro screen.

If it appears that you have digitized frames before the first bounce or after the second bounce, you can Delete the corresponding row(s) on your Excel worksheet; or exit Logger Pro and restart it, and repeat the entire process of scaling the video and digitizing the relevant frames.

2.5 Graphing and Fitting Your Data Using Excel

While Logger Pro has built-in graphing and curve-fitting features, it is preferable to transfer your table of values of x , y and t to an Excel spreadsheet.¹ Then, you can apply the methods you learned in Lab 1 to produce (and print out) plots of y vs. x , x vs. t , and y vs. t . A vs. B implies that A is plotted along the vertical, and B is plotted along the horizontal axis. First,

¹In Logger Pro, highlight your three data columns, and right click to Copy. Then, open an Excel workbook, click on cell A2, and right click on Paste. Add labels t , x and y in the cells above your data.

use **WTools** to fit the data of x vs. t to a **Polynomial of Order 2**, and to fit the data of y vs. t to a **Polynomial of Order 3**.

According to eqs. (2.1) and (2.2) the coefficient a_2 of the fit to x vs. t and the coefficient a_3 of the fit to y vs. t should be zero to within 1-2 times their respective **Standard Errors**.

2.5.1 A Common Problem

Did your graph of x vs. t have some curvature, rather than looking like a straight line as implied by eq. (2.1)? A common reason for this is that the plane of the trajectory of the golf ball was not perpendicular to the optical axis of the camera.²

Any/all of three angles could be poorly aligned:

1. The aluminum angle that initially guides the golf ball could be rotated about the vertical so the vertical plane of the ball's trajectory is not perpendicular to the optical axis of the camera.
2. The camera could be tilted up or down.
3. The camera could be tilted to the left or right.

If the coefficient a_2 of the fit to x vs. t or the coefficient a_3 of the fit to y vs. t is large, try adjusting one or more of the above angles. Repeat the experiment³ and check if the new fits have coefficient a_2 of the fit to x vs. t and the coefficient a_3 of the fit to y vs. t which are now more consistent with zero. Continue with such adjustments if these coefficients are still large compared to their **Standard Errors**.

2.5.2 Final Analysis

When you are convinced that your optical alignment is good enough, and the fit coefficients that should be zero are close enough to zero, refit your data of x vs. t to a **Polynomial of Order 1**, and refit the data of y vs. t to a **Polynomial of Order 2**.

Coefficient a_1 of the fit to x vs. t is the horizontal velocity of the bounding ball, and twice the coefficient a_2 of the fit to y vs. t should be the acceleration g due to gravity.

Report the x velocity and the y acceleration and their **Standard Errors**. Is the value of g

²If the horizontal angle between the optical axis and the perpendicular to the plane of the trajectory is θ_x in radians, then the coefficient a_2 in your fit to x vs. t is approximately $a_2 = v_{0x}^2 \theta_x / D$, where D is the distance from the camera (focal plane) to the plane of the trajectory. If the fit value for a_2 is larger (in absolute value) than twice the standard error $SE(a_2)$, your optical alignment may be poor.

A consequence of this is that your measurement of g via the y vs. t plot has a systematic error Δg roughly given by

$$\frac{\Delta g}{g} \approx \theta_x \left(1 - \frac{v_{0x} v_{0y}}{gD} \right). \quad (2.3)$$

This effect could well be a major source of error in your measurement of g , so you may wish to make new movies of the bouncing ball until you get one with a "small" value of θ_x .

³If you need to repeat the experiment, it is best to close both the **IC Capture** and the **Logger Pro** windows, and begin again at sec. 2.3

measured in this experiment within 1-2 times its **Error** of the expected value?

If not, it is very likely that something in your experiment has not been done well. For example, the digitization of the x - y positions of the ball might not have been done well; the scaling/conversion of pixels to m might not have been done well; the x and y scale factors have been assumed above to be the same, but they might be different; the optical alignment could still be poor.

does each part of the experiment once.

Is the technique of Lab 2 better or worse than that of Lab 1 in measuring the value of g ?

Lab 3: The Pendulum

3.1 Background

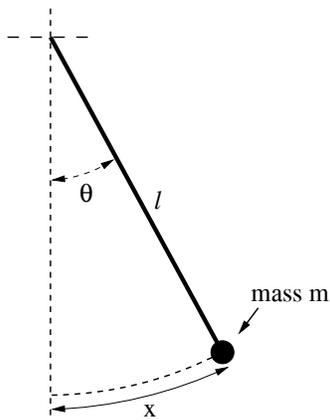


Figure 3.1: A simple pendulum, consisting of a mass m suspended from a string of length l .

The period T of a simple pendulum moving at small angles θ , as shown in Fig. 3.1, is related to its length l and the gravitational acceleration g by:

$$T = 2\pi\sqrt{\frac{l}{g}}. \quad (3.1)$$

This is derived in your textbook and in section 3.4 of this writeup. In this Lab you will confirm that the period T is proportional to $l^{1/2}$ and that T is independent of mass m . Also, you will use the pendulum to measure the gravitational acceleration g with precision of about 1 part in 1000.

3.2 Specific Instructions

3.2.1 Setup and Timing of the Pendulum

The pendulum is suspended on a string from a clamp with a slot that grips the string. Check that this slot is oriented vertically.

Set up a photogate at the bottom of your pendulum's motion. Use the program `Princeton Pendulum Timer (count)` in the `PHY101` folder to measure the pendulum's period. Start the ball swinging from a fixed angle $\theta \approx 5^\circ$. (Use the protractor near the pivot point to set this angle accurately.) Then click on `Collect`.

The program `Pendulum Timer` actually measures the time between every second blocking of the photogate by the swinging ball, which time is a full period of the pendulum swing. This makes the exact location of the photogate unimportant, but it should be more or less directly below the point of suspension of the pendulum.

Pendulum Timer continues to make measurements until you tell it to **Stop**. You should let it take at least a few dozen data points, and perhaps many more on your repeated runs. The program may not display more than the first 8 data points in the **Periods** column, but the number of data points collected is displayed in (every 4th row of) the **Number** column. When you click **Stop**, all the data collected will be displayed in the **Periods** column.

Transfer your **Periods** data from Pendulum Timer into an Excel spreadsheet, and calculate the average and standard deviations of a column of data using the **Average** and **STDEV** commands available on the menu obtained by clicking on the (solid) ∇ symbol next to the Σ symbol on the **Toolbar**. See Appendix A for further details. The standard deviation of your measurements of the period T is a good estimate of the uncertainty σ_T .

3.2.2 Measurement of g

Measure the period of the pendulum using a fairly long length l and the steel ball, and an initial angle of about 5° .

Measure the length l using the 2-meter-long scale, where relevant distance is from the bottom of the clamp that holds the string to the center of the ball.¹ To estimate the uncertainty of your measurement of l , you might have each group member take the measurement, and use the standard deviation of those measurements. From equation 3.1, you can calculate g as:

$$g = 4\pi^2 \frac{l}{T^2}. \quad (3.2)$$

Using the methods of Appendix B.3, one can show that:

$$\left(\frac{\sigma_g}{g}\right)^2 = \left(\frac{\sigma_l}{l}\right)^2 + \left(2\frac{\sigma_T}{T}\right)^2 \quad (3.3)$$

Using these formulae, estimate the measured value of g and the experimental uncertainty σ_g .

Appendix C reports the results of such an analysis based on data collected in Ph101 Lab 3 during a previous year. You may wish to read Appendix B to better appreciate this error analysis procedure.

A precise measurement of g , using a different technique, was made by J.E. Faller in Palmer Hall (now the Frist Student Center) room 130 for his 1962 Ph.D. Thesis. He found that:

$$g = 980.1604 \pm 0.0007 \text{ cm/s}^2. \quad (3.4)$$

How does your measured value compare with this? How does your uncertainty compare with the one obtained by Faller? If you repeated the experiment 100 times, what would be the uncertainty on your answer? In principle, could the uncertainty reach 10^{-6} m/s^2 ?

¹A possible strategy is to measure the diameter of the ball with digital calipers, and add the radius of the ball to the length of the string.

3.2.3 Dependence of the Period on Length

Measure the pendulum period with 5 different lengths l around 50, 75, 100, 125 and 150 cm (using an initial angle of about 5°). In each case, set up the pendulum, and then find the value of l , measuring from the pivot point at the top of the pendulum to the center of the ball. For each length, run Pendulum Timer for at least ten pendulum periods, record the average period T , and analyze your data for g and σ_g as in the previous part.

Enter your measurements of l and T into Excel, and use logarithmic plotting (described below) to find the exponent α in:

$$T \propto l^\alpha. \quad (3.5)$$

Does your measurement of the exponent α agree with the expected value of 0.5 to within the error reported by Excel?

Logarithmic Plots

You expect that T and l are related by an equation of the form:

$$T = cl^\alpha, \quad (3.6)$$

where c and α are fixed constants, and you want to find the value of the exponent α (which you expect to be 0.5 for the pendulum). Take the logarithm of both sides of eq. (3.6) to get:

$$\log T = \log(cl^\alpha). \quad (3.7)$$

The general logarithmic identities:

$$\log ab = \log a + \log b, \quad \log a^b = b \log a, \quad (3.8)$$

can be used to rewrite eq. (3.7) as:

$$\log T = \alpha \log l + \log c, \quad (3.9)$$

which is the equation for a straight line. For example, if you set $y = \log T$, $x = \log l$, $b = \log c$, and $m = \alpha$, then you obtain the standard form,

$$\underbrace{\log T}_y = \underbrace{\alpha}_m \underbrace{\log l}_x + \underbrace{\log c}_b. \quad (3.10)$$

Thus by fitting a straight line to your measurements of $\log T$ vs. $\log l$, you can find value and uncertainty of the exponent α . To do this with Excel:

- Enter your l and T measurements into two columns of a spreadsheet.
- Calculate $\log l$ and $\log T$. Use the LOG10 function in Excel (*e.g.*, cell B1 might contain the formula =LOG10(A1)).
- Graph and analyze the $\log l$ and $\log T$ columns using WPtools/Linear Fit.
- The slope a_1 and uncertainty $SE(a_1)$ reported from the fit are the value of α and its uncertainty σ_α .

3.2.4 Dependence of the Period on Mass

Prior to the insight of Galileo that all objects fall at the same rate (if air resistance can be neglected) one might have supposed that the period depended on the mass m of the hanging weight. Test this by measuring the period of the pendulum with a steel ball, an aluminum ball, and a wood ball, always using the same length of string (using an initial angle of about 2° in this part). Are the periods you measure the same within the measurement precision?² If not, comment on why the periods might not be the same. *Note that if you don't use the same length of string for the different masses, you might confuse the dependence of the period on length (sec. 3.2.3) with possible dependence of the period on mass.*

3.2.5 Optional: Dependence of the Period on the Initial Angle

For non-negligible angles, a more complete analysis of the simple pendulum tells us that the period T is

$$T = 2\pi\sqrt{\frac{l}{g}} \left(1 + \frac{1}{4} \sin^2 \frac{\theta_0}{2} + \frac{9}{64} \sin^4 \frac{\theta_0}{2} + \dots \right), \quad (3.11)$$

where θ_0 is the maximum angle to the vertical during the motion. For “small enough” launch angles θ_0 the terms $\sin^2(\theta_0/2)$, and $\sin^4(\theta_0/2)$ are very small so that we obtain eq. (3.1). For example, with $\theta_0 = 10^\circ$ the period would be increased by about 0.2% above the value predicted by eq. (3.1), and for $\theta_0 = 2^\circ$ the increase is only 0.008%.

Can such small effects be detected in your experiment?

To answer this, take data (for a steel sphere) with initial angles of $\theta_0 = 10^\circ$ and 20° and compare the period with that observed for $\theta_0 = 5^\circ$.

3.3 Optional: The Conical Pendulum

A conical pendulum is one in which the mass swings around at constant speed in a (horizontal) circle, as shown in Fig. 3.2, rather than swinging back and forth in a vertical plane as in Fig. 3.1.

Using what you have learned in class, derive a formula for the period T of a conical pendulum of length l at angle θ from the vertical. (Hint: first find a formula for the speed of the pendulum, then use that to find the time it takes to go around the circle). For small θ , your result should be the same as the simple pendulum formula (3.1).

You can study the conical pendulum with the apparatus set up in the center of the Lab. This will also be a low-precision experiment, and you need not perform an uncertainty analysis.

²Pendulum motion is a variant on falling motion, which doesn't depend on the mass of the falling object if air resistance can be neglected. As Galileo argued, if heavier objects fell faster, then two masses tied together should fall faster than either object by itself. But, if the lighter mass of the two fell more slowly, it would “pull back” the heavier object, and the tied pair would fall more slowly than heavier mass alone.

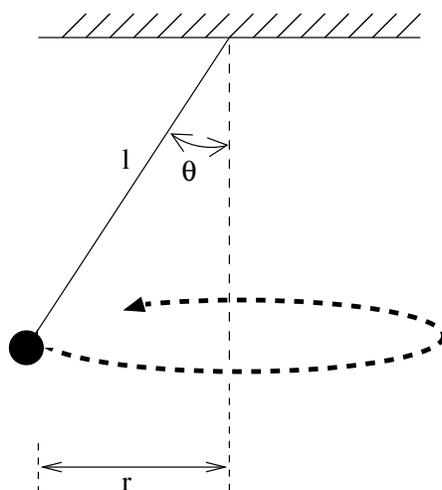


Figure 3.2: A sketch of a conical pendulum. The pendulum swings around in a horizontal circle.

Practice launching the ball so its motion is in a horizontal circle, rather than in a vertical one.

Test your formula for the pendulum period by measuring the period of circular motion (with a stopwatch) for at least three angles θ .

It is difficult to measure θ directly, but you can find it by measuring the diameter of the circular motion ($2r$ in Fig. 3.2) by operating the pendulum near a meter stick. This isn't very accurate, but you should be able to at least qualitatively verify the period formula. In particular, your formula should predict that either the period increases with θ or it decreases with θ . Does this prediction hold true?

3.4 Appendix: Derivation of the Pendulum Formula

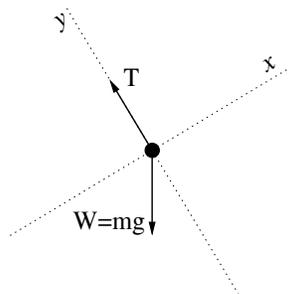


Figure 3.3: Free-body diagram of the pendulum mass.

A simple pendulum is sketched in Fig. 3.1, and a free-body diagram of its mass is given

in Fig. 3.3. Note that x is measured along the arc of motion of the mass; at the instant pictured, the x axis is at angle θ above the horizontal. The only force in the x direction is gravity:

$$F_x = -mg \sin \theta. \quad (3.12)$$

For small angles, and θ in radians, we can approximate $\sin \theta \simeq \theta$. Further, noting that $\theta = x/l$, eq. (3.12) becomes:

$$F_x = -mg \frac{x}{l}. \quad (3.13)$$

Newton's second law, combined with the definition of acceleration, says:

$$F_x = ma_x = m \frac{d^2 x}{dt^2}. \quad (3.14)$$

Using eq. (3.13) for F_x and canceling m on both sides of the resulting equation gives:

$$\frac{d^2 x}{dt^2} = -\frac{g}{l} x. \quad (3.15)$$

This differential equation has solutions like:

$$x(t) = x_0 \sin \left(\sqrt{\frac{g}{l}} t \right). \quad (3.16)$$

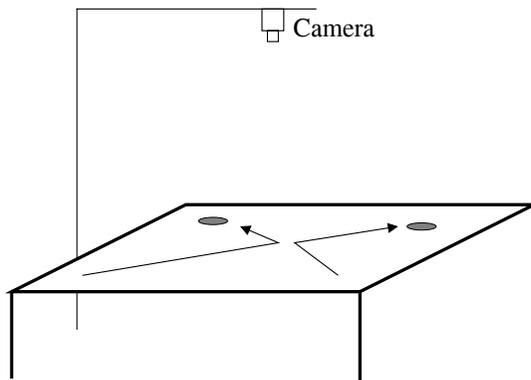
Note that this goes through one cycle when the argument to the sine function is 2π , that is, when:

$$t = 2\pi \sqrt{\frac{l}{g}} \quad (3.17)$$

so this is the period of the pendulum.

Lab 4: Collisions in Two Dimensions

In this Lab you will study collisions between two objects (pucks) on a horizontal, low-friction surface (an air hockey table), as sketched below.¹ Use the CCD camera to take a series of images before, during, and after the collisions, and then analyze the images to test conservation of momentum and energy and to evaluate the motion of the center of mass of the objects.



4.1 Background: Two-Body Collisions

A collision between two bodies is a brief interaction between them during which the forces are very strong (but equal and opposite according to Newton's third law).

This Lab involves collisions only between pucks of equal mass, which substantially simplifies the analysis.

4.1.1 Momentum Conservation

The total momentum \mathbf{P} of two masses m_1 and m_2 with velocities \mathbf{v}_1 and \mathbf{v}_2 is

$$\mathbf{P}_{\text{total}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2. \quad (4.1)$$

During a brief collision any external forces are small compared to the forces between the two masses, and the latter forces sum to zero. Newton's second law tells us that during the collision the rate of change of total momentum is equal to the total force, which is negligible. Hence, the total momentum is the same before and after the collision:

$$\mathbf{P}_0 = m_1\mathbf{v}_{0,1} + m_2\mathbf{v}_{0,2} = m_1\mathbf{v}_{f,1} + m_2\mathbf{v}_{f,2} = \mathbf{P}_f, \quad (4.2)$$

and we say that the total momentum is "conserved." In case of equal masses $m_1 = m_2 = m$ (as can be assumed in this Lab), eq. (4.2) simplifies to

$$\frac{\mathbf{P}_0}{m} = \mathbf{v}_{0,1} + \mathbf{v}_{0,2} = \mathbf{v}_{f,1} + \mathbf{v}_{f,2} = \frac{\mathbf{P}_f}{m}. \quad (4.3)$$

Since momentum is a vector, each component is independently conserved. Write down the x and y (horizontal) components of eq. (4.3), which component equations are to be tested

¹Please don't actually play air hockey with the lab equipment! The pucks break easily and are difficult to replace.

in this Lab. *The vertical velocities of the pucks are always zero in this Lab, so momentum is “obviously” conserved along the vertical direction.*

4.1.2 Center of Mass Motion

A concept related to the total momentum of a system is its center of mass velocity, \mathbf{v}_{cm} , defined by

$$\mathbf{v}_{\text{cm}} = \frac{\mathbf{P}_{\text{total}}}{m_{\text{total}}} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} \stackrel{\text{equal masses}}{\Rightarrow} \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}, \quad (4.4)$$

for a two-body system, where the final form holds for equal masses.

Since total momentum is conserved in a collision, the center of mass velocity is unchanged during a collision.

If in addition, the total external force on the system is zero, then the total momentum and the center of mass velocity are constant at all times.

A constant center of mass velocity implies that the position of the center of mass moves uniformly along a straight line. The center-of-mass position vector \mathbf{r}_{cm} , is defined by

$$\mathbf{r}_{\text{cm}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} \stackrel{\text{equal masses}}{\Rightarrow} \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \quad (4.5)$$

for a system of two masses whose position vectors are $\mathbf{r}_1 = x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}}$ and $\mathbf{r}_2 = x_2\hat{\mathbf{x}} + y_2\hat{\mathbf{y}}$, where the final form holds for equal masses.

4.1.3 Elastic and Inelastic Collisions

Mechanical energy may or may not be conserved during a collision (in contrast to the total momentum which is always conserved during free collisions).

If mechanical energy is conserved, we say that the collision is “elastic” (meaning that if the pucks deform slightly during the collision, they return to their original shape after the collision, so any mechanical energy that went into the deformation comes out on the recoil).

The mechanical energy of a two-body system is the sum to the kinetic energies of the two masses. Hence, in an elastic collision (without rotational motion),

$$KE_0 = \frac{1}{2}m_1v_{0,1}^2 + \frac{1}{2}m_2v_{0,2}^2 = \frac{1}{2}m_1v_{f,1}^2 + \frac{1}{2}m_2v_{f,2}^2 = KE_f, \stackrel{\text{equal masses}}{\Rightarrow} v_{0,1}^2 + v_{0,2}^2 = v_{f,1}^2 + v_{f,2}^2. \quad (4.6)$$

If mechanical energy is not conserved during a collision, we call this an “inelastic” collision. In particular, if the two pucks stick together after the collision (sec. 4.2.5), this collision is called “completely inelastic”, and

$$\mathbf{v}_{\text{final}} = \mathbf{v}_{\text{cm}} = \frac{m_1\mathbf{V}_{0,1} + m_2\mathbf{V}_{0,2}}{m_1 + m_2} \stackrel{\text{equal masses}}{\Rightarrow} \frac{\mathbf{V}_{0,1} + \mathbf{V}_{0,2}}{2}. \quad (4.7)$$

After a completely inelastic collision, the joined objects then spin about their (moving) center of mass position. Nonetheless, linear momentum is still conserved in such inelastic collisions.²

4.2 Specific Instructions

The camera and software are the same as you used in Lab 2 (the bouncing ball). If needed, see that writeup for camera and image analysis instructions. Begin by opening **IC Capture: Motion in 2 dimensions** in the **PHY101** folder.

4.2.1 Setup

Turn on the blower to supply air to the table. Make a few trial runs at colliding two simple pucks (no magnets, no Velcro bands). *For your measurements, the collisions should not be “head-on” (1-dimensional). But while practicing, try to arrange a head-on collision between a moving puck and one at rest. What is the simple result of such a collision?* Weigh each of the pucks and record the results in your notebooks.

Include a meter stick in your video; rest it on the wire frame around the edge of the air table, close to one of the longer sides. Add 2 black metal clips to the meter stick to make it easier to identify a known distance in your video.

Record a video (in the folder **Student Data**) of a collision between your two pucks when both have nonzero initial velocity. *It is preferable if none of the puck velocity vectors, nor \mathbf{v}_{cm} , is parallel to the edges of the air table, and that the collision is NOT “head-on.” The movie should contain several frames after you release the puck but before they collide.* You can replay your video using the **Check Video** button. If you need to remake the video, it may be simplest to close and reopen **IC Capture**.

Use the **Logger Pro** file **Collisions in 2 dimensions** in the **PHY101** folder to digitize the position of the pucks in your movie. Open your movie that was stored in the **Student Data** folder. If desired, drag the small square in the lower-right corner of the movie window to enlarge it.

Click the crosshair button to the upper right of the movie window if you wish the cursor to be of this type, rather than the default hand icon.

Use the **Set Scale** button and then the cursor to drag a line along the image of the meter stick to define the distance between the two black reference marks.

Step through your movie using the double-arrow buttons in the lower-left corner to find the first frame in which the pucks are in steady motion after their launch.

Verify that the **Set Active Point** button of the right side has **Point Series X:Y** checked

²There is another conservation law that applies to collisions, namely conservation of angular momentum. This law can be helpful in understanding the details of inelastic collisions, but we do not pursue this here.

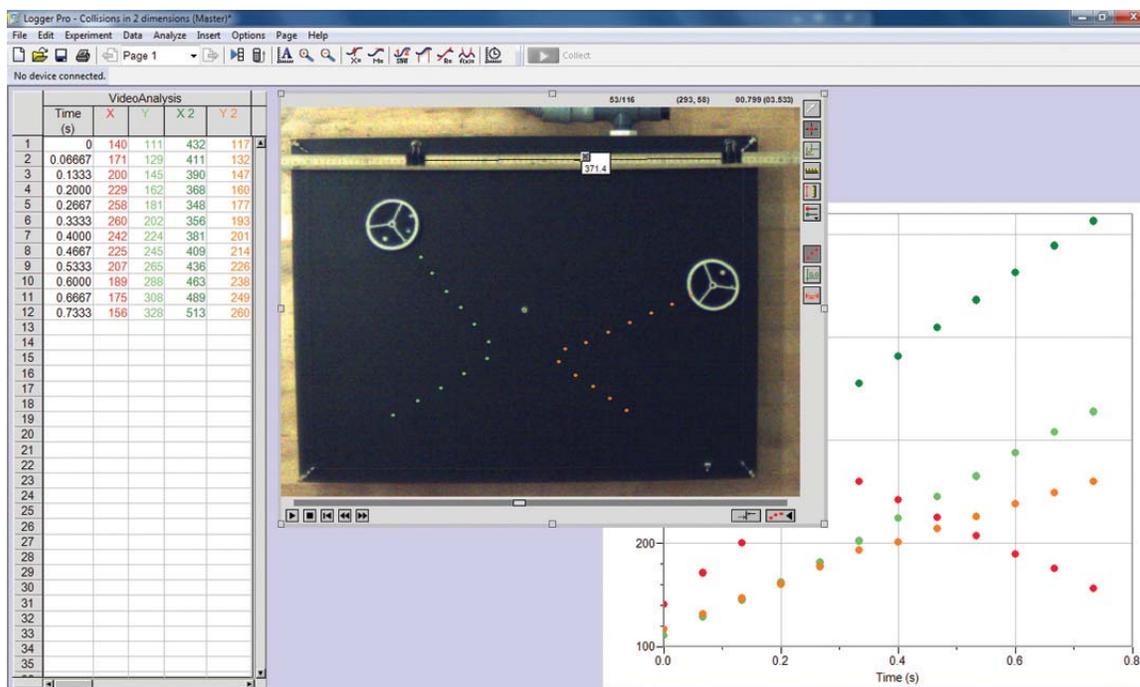
before you begin digitizing the positions of the center of the first puck.

Position the cursor on the center of the first puck and click. The values of time, and of the (scaled) x and y coordinates should appear in the table to the left, and on the graph (possibly under the movie window). The video should advance one frame, and a dot appear at the point you digitized in the previous frame.

The dots can be made to appear/disappear via the **Toggle Trails** button to the right of the movie window.

Continue digitizing the position of the first puck, both before and after its collision with the second puck, until the first puck hits something else (such as the wire frame around the edges of the air table). If you digitize frames after the second collision, this should appear as “kink” in the graph near the end of the data, and these point should be ignored in later analysis.

To digitize the positions of the second puck step your video back to the first “good” frame, and use **Set Active Point** to enable **Point Series X2:Y2**. Click on the position of the second puck, and proceed through your video as for the first puck.



4.2.2 Center of Mass Motion

Before the era of video cameras and computer analysis programs, the students took a Polaroid photo of the collision with strobe illumination. This permitted simple analyses based on connecting dots on the photo, as shown on the next page. You could still perform the graphic analysis on a printout of the movie window after all digitized points are shown. You do not need to do this here, although this technique will be used in sec. 4.2.5.

To perform a computer analysis, Copy your 5-column data table from **Logger Pro** to **Excel**, placing the top-left data point in cell A2. Enter column labels in the cells A1-E1.

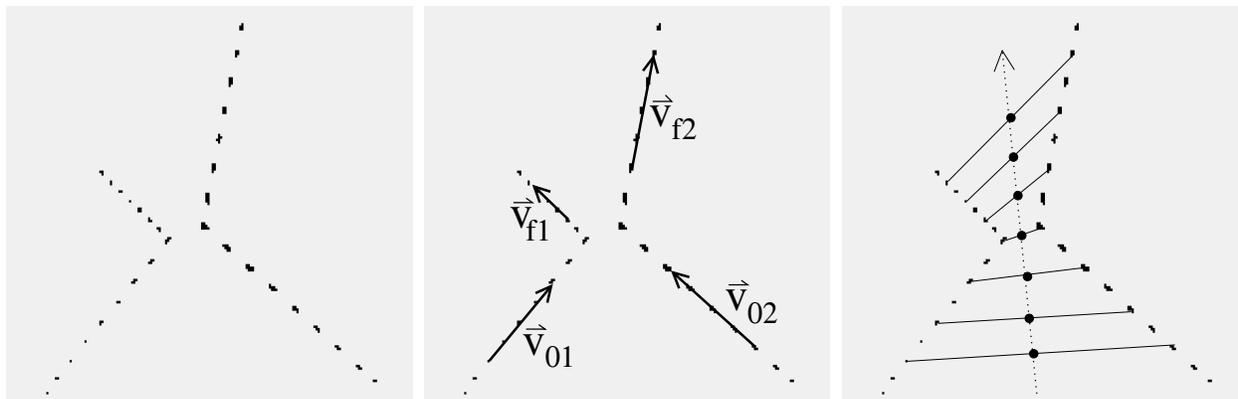


Image of a collision.

Calculating initial and final velocities.

The center of mass at a given time is highlighted by a dot.

Calculate the corresponding coordinates (x_{cm}, y_{cm}) of the center of mass of the system in each frame of the movie, using **Excel** to implement eq. (4.5) (See Appendix A.3). Use **WPTools** to make fits of a **Polynomial of Order 2** to each of x_{cm} vs. t and y_{cm} vs. t .

If there are no external forces (and there shouldn't be in this experiment), the center of mass will travel in a straight line at constant velocity. Do the positions of your center of mass follow a straight line, and does its speed appear constant? That is, are the fit coefficients a_2 equal to zero within 1-2 times their standard errors $SE(a_2)$?

4.2.3 Is Momentum Conserved?

Test conservation of momentum by checking that the equations you derived relating the total momentum before and after the collision hold true. To do this, you will need to find the x and y velocities of the two pucks before and after the collision. You can do this using the data you have already stored in an **Excel** worksheet, to make eight **Linear** fits (and plots, using **WPTools**) of the x - and y -components of the velocities of both pucks, before and after the collision. In each fit of position vs. time, coefficient a_1 represents the velocity component.

If momentum is conserved, eq. (4.3) implies that³

$$\Delta_x = \frac{\Delta P_x}{m} = v_{0x,1} + v_{0x,2} - v_{fx,1} - v_{fx,2}, \quad \text{and} \quad \Delta_y = v_{0y,1} + v_{0y,2} - v_{fy,1} - v_{fy,2}, \quad (4.8)$$

should be zero to within the experimental uncertainty, $\sigma_{\Delta_{x,y}} = [\sum_1^4 SE(a_1)^2]^{1/2}$, as can be computed from the 4 fits of x -position vs. time and from the 4 fits of y -position vs. time, respectively.

³Equation (4.8) assumes that the two pucks have equal mass. If you prefer to perform a more accurate analysis which includes the masses of the pucks, feel free to do so.

4.2.4 Is Energy Conserved?

Check energy conservation using the velocities you measured. According to eq. (4.6),⁴ if energy is conserved the ratio

$$\frac{KE_f}{KE_0} = \frac{v_{fx,1}^2 + v_{fy,1}^2 + v_{fx,2}^2 + v_{fy,2}^2}{v_{0x,1}^2 + v_{0y,1}^2 + v_{0x,2}^2 + v_{0y,2}^2} \quad (4.9)$$

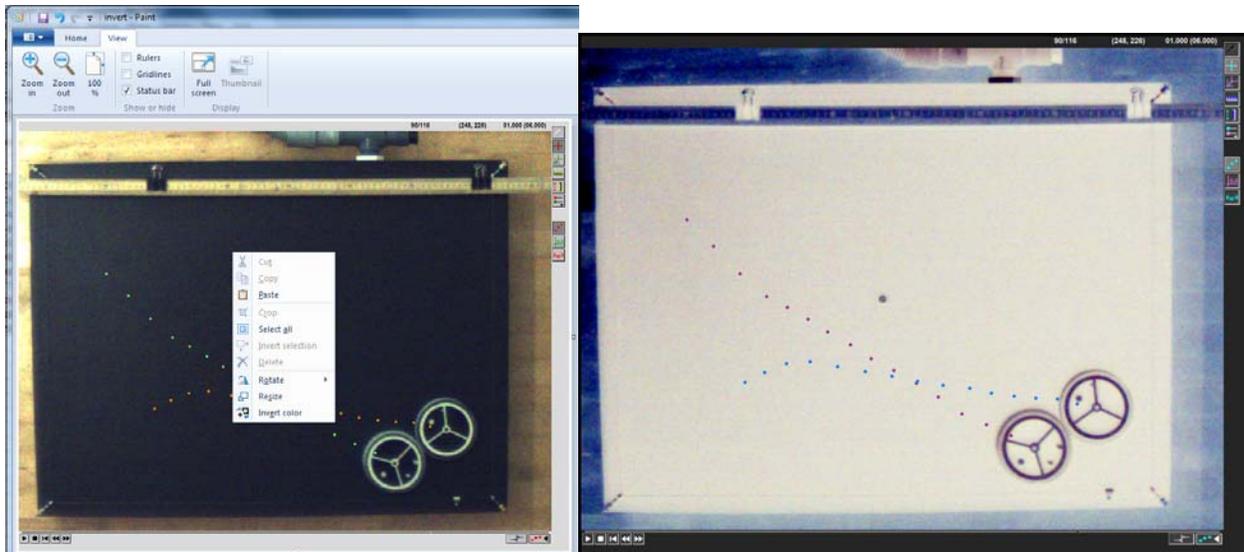
should be unity (*i.e.*, = 1).

Into what other forms of energy might the lost kinetic energy have been converted?

4.2.5 Velcro Pucks and Totally Inelastic Collisions

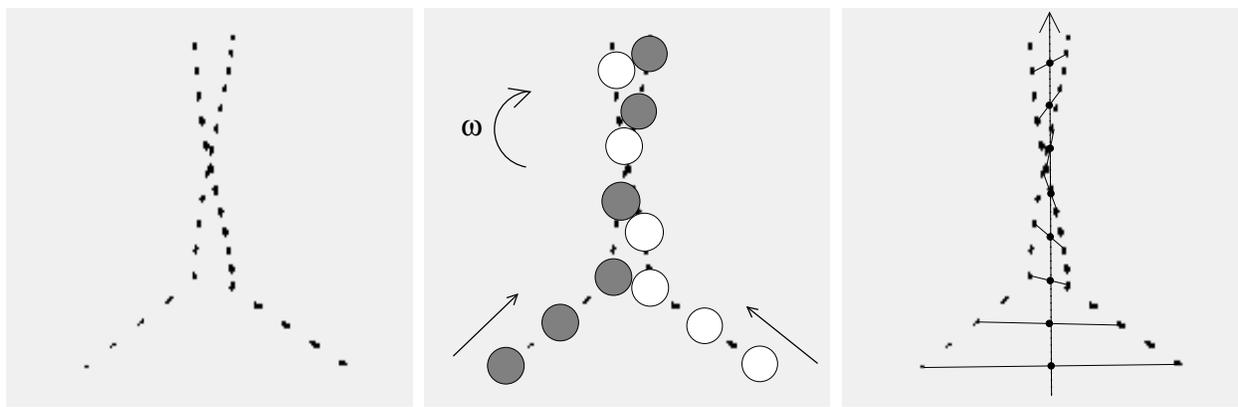
Put Velcro straps around a pair of (non-magnetic) pucks. Now when the pucks collide they will stick together. Make a video of a collision for which the pucks rotate slowly after colliding; it is better if the initial velocities of the puck are high, and again don't use a "head-on" collision. Digitize the positions of the centers of the two pucks before and after the collision, being sure that **Toggle Trails** shows the points that you have digitized on the movie screen.

Don't use Excel for data analysis in this part. Instead, **Copy** the movie screen from **Logger Pro** and **Paste** it into **Paint** (which program is on your Lab PC, if not on the Desktop). Right click on the image of the movie screen in **Paint** and click on **Invert Color** so your digitized dots will appear against a light background. Then, **Print** the image from **Paint** (which should occupy most of a sheet of paper).



⁴As does eq. (4.8), eq. (4.9) assumes that the two pucks have equal mass. If you prefer to perform a more accurate analysis which includes the masses of the pucks, again feel free to do so.

Even though the individual pucks aren't moving in straight lines after the collision, the center of mass should move in a straight line. Use a ruler and pen/pencil to find the center of mass in each frame (of each pair of dots) as sketched below. Do the points line up? Is momentum conserved, *i.e.*, is the spacing between successive positions of \mathbf{r}_{cm} the same before and after the collision? Is translational kinetic energy conserved, *i.e.*, does $v_{0,1}^2 + v_{0,2}^2 = 2v_{f,\text{cm}}^2$, where the speeds v are proportional to the spacings between successive locations of $\mathbf{r}_{0,1}$, *etc.*



Colliding Velcro pucks.

Diagram of puck motion.

After the collision, the pucks stick together (and slowly rotate in a clockwise direction in this example).

Center of mass of the Velcro pucks.

4.2.6 Optional Study: Magnetic Pucks

Experience in this Lab has been that a substantial fraction of energy is usually lost in collisions in which the pucks actually hit each other. This energy loss can be reduced if the pucks interact without contact. This can be done using magnetic pucks, which can repel each other without actually touching. *Note: the magnet pucks are fragile, and can be damaged if dropped or harshly handled.*

Practice launching the magnetic pucks so that they come close to each other, and their paths are changed, but they do not touch. You may need to use relatively low puck velocities, and to decrease the frame rate when taking your videos.

Once you've gotten a good video using the magnetic pucks, you could digitize the positions of the center of the pucks, and make a graphical analysis of the center-of-mass motion as in sec. 4.2.5 (or make computer analyses using Excel). Is momentum conserved for the case of magnetic pucks? *To address the issue of conservation of energy, you would need to include the kinetic energy of rotation of the two-puck system after the collision, which could be the topic of another Lab session.*

Lab 5: Rotational Motion

5.1 Experimental Apparatus

In this experiment you study rotational motion and its connection with translational motion. The experimental apparatus is sketched in Fig. 5.1. A large disk (radius r_1) and an attached smaller disk (radius r_2) rotate together on an axle. A string is wound around the smaller disk, and a mass m is attached. As the mass falls under gravity, the pulley undergoes angular acceleration.

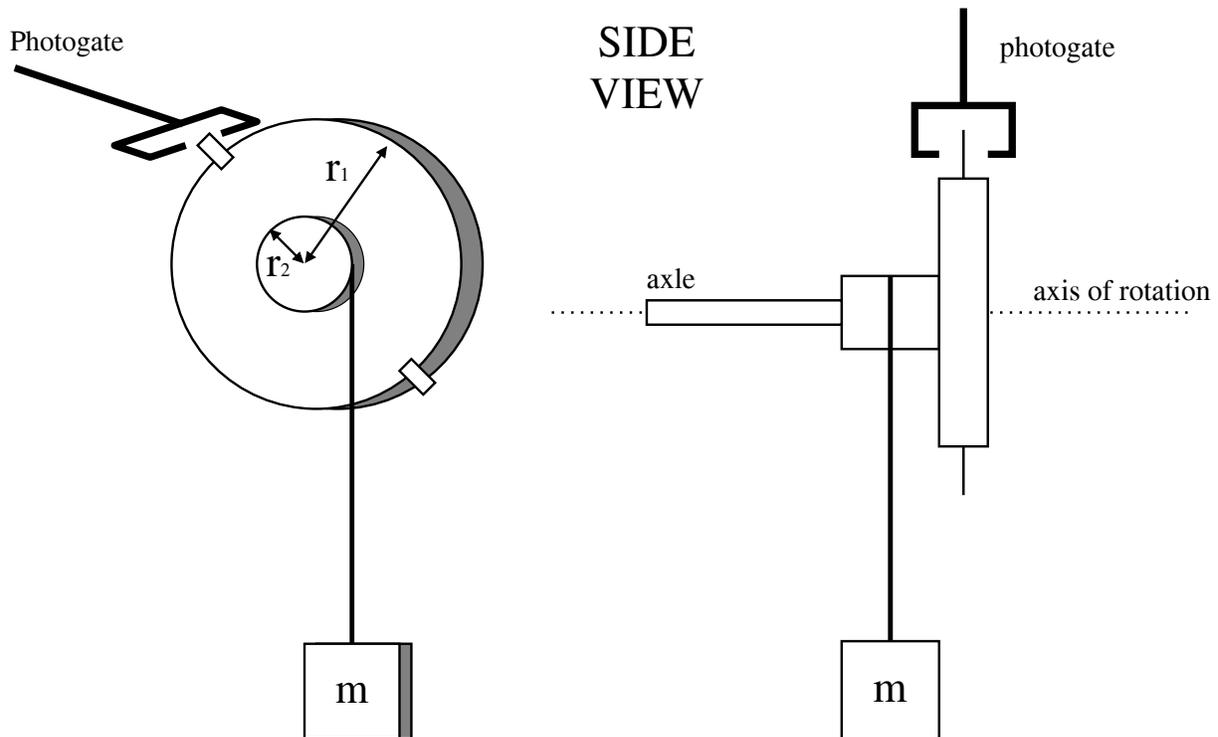


Figure 5.1: A sketch of the experimental apparatus.

In this type of apparatus, sometimes called *Atwood's machine*, the vertical acceleration a of mass m is much less than g , so the behavior of accelerated motion is easier for the human eye to discern.

5.2 Moment of Inertia of the Pulley

5.2.1 Background

When a body of mass m rotates about an axis, its motion can be analyzed using quantities analogous to those found useful for linear motion. Namely,

$$\text{position } x \rightarrow \text{angle } \theta (= s/r),$$

velocity v \rightarrow angular velocity ω ($= v_s/r$),
 acceleration a \rightarrow angular acceleration α ($= a_s/r$),
 mass m \rightarrow moment of inertia I ($= kmr^2$),
 force F \rightarrow torque τ ($= Fr \sin \theta$, where θ = angle between \mathbf{F} and \mathbf{r}),
 $F = ma$ \rightarrow $\tau = I\alpha$.

The moment of inertia I of a uniform disk is

$$I = \frac{1}{2}Mr^2 = \frac{\pi\rho r^4 h}{2}, \quad (5.1)$$

where $M = \pi\rho r^2 h$ is the mass of the disk of mass density ρ , radius r and thickness h (see Cutnell and Johnson Table 9.1). According to eq. (B.13) of Appendix B, the relative uncertainty σ_I/I in the moment of inertia I is given by

$$\left(\frac{\sigma_I}{I}\right)^2 = \left(\frac{\sigma_M}{M}\right)^2 + \left(2\frac{\sigma_r}{r}\right)^2 = \left(\frac{\sigma_\rho}{\rho}\right)^2 + \left(\frac{4\sigma_r}{r}\right)^2 + \left(\frac{\sigma_h}{h}\right)^2. \quad (5.2)$$

5.2.2 Specific Instructions

The actual pulley (Fig. 5.1) consists of two disks of masses M_1 and M_2 with radii r_1 and r_2 , respectively. The total moment of inertial I of this pulley is

$$I = I_1 + I_2 = \frac{1}{2}M_1r_1^2 + \frac{1}{2}M_2r_2^2 = \frac{\pi\rho}{2}(r_1^4h_1 + r_2^4h_2). \quad (5.3)$$

Measure the outer radius r_1 and thickness h_1 of the (aluminum) pulley (without dismounting it). Also measure the radius, r_2 , and thickness, h_2 , of the smaller disk, and then calculate the moment of inertial I using eq. (5.3), taking the density of aluminum to be

$$\rho = 2.70 \pm 0.01 \text{ g/cm}^3. \quad (5.4)$$

Which equipment you should use to make your measurements as accurate as possible? Does the moment of inertia of the smaller disk make a significant contribution to the total moment of inertia I of the pulley? Estimate the uncertainty σ_{I_1} of the larger disk using eq. (5.2). How does the moment of inertia I_2 compare with the uncertainty σ_{I_1} of the moment of inertia I_1 of the larger disk?

5.3 Measuring I from Rotational Dynamics

5.3.1 Background

An alternative method of measuring the moment of inertia I of the pulley uses its acceleration by a falling mass. The mass is attached to a string, which is wrapped around the smaller part of the pulley (radius r_2), as shown in Fig. 5.1.

Analysis of the system is straightforward. First consider the pulley. The tension T of the string exerts a torque¹

$$\tau = T r_2 \quad (5.5)$$

about the axle of the pulley, since the tension vector \mathbf{T} is at a right angle to the radius vector \mathbf{r}_2 . This torque leads to an angular acceleration α of the pulley, related to torque by

$$\tau = I\alpha. \quad (5.6)$$

Combining these gives

$$I = \frac{T r_2}{\alpha}. \quad (5.7)$$

Next consider the falling mass m . The (vertical) forces on mass m are the tension \mathbf{T} and gravity $m\mathbf{g}$, so Newton's second law says:

$$mg - T = ma, \quad (5.8)$$

where g is the gravitational acceleration constant and a is the acceleration of the mass. Since the string is wrapped around the pulley at radius r_2 :

$$a = \alpha r_2. \quad (5.9)$$

Combining these gives an expression for tension:

$$T = m(g - \alpha r_2). \quad (5.10)$$

Substituting this into eq. (5.7) yields:

$$I = \frac{mg r_2}{\alpha} - m r_2^2. \quad (5.11)$$

For an estimate of the uncertainty σ_I on the moment of inertia I , note that eq. (5.11) can be rewritten using eq. (5.9) as

$$I = m r_2^2 \left(\frac{g}{a} - 1 \right) \approx \frac{mg r_2^2}{a} = \frac{mg r_2}{\alpha}, \quad (5.12)$$

where the approximation holds since $a \ll g$. Then, using eq. (B.13) we have

$$\left(\frac{\sigma_I}{I} \right)^2 \approx \left(\frac{\sigma_m}{m} \right)^2 + \left(\frac{\sigma_\alpha}{\alpha} \right)^2 + \left(\frac{\sigma_{r_2}}{r_2} \right)^2. \quad (5.13)$$

¹Clockwise torques and downward translational motion are taken to be positive.

5.3.2 Specific Instructions

Use a falling mass of $m \approx 200$ g in this part, to measure the moment of inertia of the pulley. Measure the value of m with a scale. Wrap the string around the smaller disk in a single layer so that the rounds of the string do not overlap, avoiding the groove. For precision results, measure the diameter of the string with the digital caliper. and increase your previous measurement of the radius r_2 of the smaller disk by the radius of the string to obtain the lever arm through which the torque acts.

Use a photogate and the program **Princeton Timer A** to measure the angular acceleration α of the pulley. Two tabs are attached to the larger pulley; they will cause the photogate to be blocked at intervals of π radians. Make sure that both tabs block the timer gate.

The system will record the times when the photogate is blocked. Thus, each time reported by the computer is the time it takes the pulley to turn an integer multiple of π radians.

Start the **Timer** program before releasing the mass (recall sec. 1.2.2), and stop the **Timer** before the mass hits the ground. Leave the **Timer** on long enough to record at least 5 time intervals.

The angular motion of the pulley due to the constant angular acceleration α is described by

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2. \quad (5.14)$$

You can find both α and σ_α by fitting a curve to a plot of θ versus t . Use **Excel** and **WPTools** with a **Polynomial of Order 3**. The fit coefficient \mathbf{a}_3 should be 0 to within 1-2 times the reported error $\text{SE}(\mathbf{a}_3)$. Then, $\alpha = 2\mathbf{a}_2$ and $\sigma_\alpha = 2\text{SE}(\mathbf{a}_2)$.

Use your measurement of α and eq. (5.11) to calculate the moment of inertia I of the pulley. Note that this requires angle θ to have been measured in radians. Calculate the uncertainty σ_I using eq. (5.13).

Compare this value of I with the value (5.3) you calculated from the dimensions of the pulley. Do they agree within the larger of the uncertainties σ_I obtained from eqs. (5.2) and (5.13)?

5.4 The Block and Tackle

In this part of the lab, you analyze the block-and-tackle system sketched in Fig. 5.2. First examine the system in equilibrium, then analyze the kinetic and potential energies of the system in motion.

Before setting up the block-and-tackle system, wind the string used in the first part of this lab around the pulley and secure it with a piece of tape to the large disk. Please don't remove it.

Construct the system as shown in the figure. The “fixed” pulley is the one used in the first part of this lab. The “moving” pulley should have at least 100 g as a load (in addition

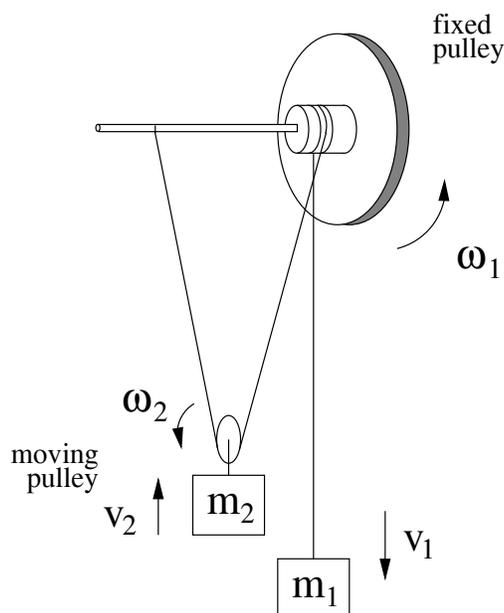


Figure 5.2: A sketch of the block-and-tackle system

to the hanger). Measure the mass m_2 of the moving pulley (including its hanger + load) before assembling the system. Tie a string to the axle of the fixed pulley near the hub, pass it around the moving pulley (in its groove), back up over the fixed pulley, and down to the hanging mass. The string should be long enough that when the hanging mass (m_1) is near the fixed pulley the moving pulley (m_2) is close to the floor. The “vertical” segments of the string should be nearly vertical, permitting the moving pulley and the hanging mass to pass close to one another. The string should loop around the groove in the hub of the fixed pulley, and it should not slip when the pulley and the masses move. It is important that there is no slippage, so loop the string around the hub ≈ 5 times. Also, be careful that the string freely winds and unwinds without tangling.

5.4.1 Static Equilibrium of the Block and Tackle

Adjust the mass of the hanging weight m_1 , using the available slotted masses, until the system is in static equilibrium. Find the largest and smallest masses $m_{1,l}$ and $m_{1,s}$ such that the system stays at rest. Remove the hanging mass and measure $m_{1,l}$ and $m_{1,s}$. What relation do you expect between m_1 , m_2 , and the tension in the string? Do your measured values of $\langle m_1 \rangle = (m_{1,l} + m_{1,s})/2$ and m_2 agree with this expectation?

5.4.2 Dynamics and Conservation of Energy of Block and Tackle

Background

Consider what happens if m_1 is increased, so the system is no longer in equilibrium. Mass m_1 will fall downwards, causing the moving pulley to move upwards and both pulleys to rotate. The motion can be analyzed in terms of forces and torques, but it turns out that an energy analysis is simpler.

The kinetic energy KE of the system at any instant is:

$$\text{KE} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2. \quad (5.15)$$

The moment of inertia of the moving pulley, I_2 , is sufficiently small that the last term may be ignored. The other terms can be simplified by noting that:

$$v_1 = \omega_1 r_2 \quad \text{and} \quad v_2 = \frac{1}{2}\omega_1 r_2, \quad (5.16)$$

where r_2 is the radius of the fixed pulley at the point the string is wrapped,² including the radius of the string as in sec. 5.3.2. (Why is there a factor of 1/2 in expression for v_2 ?) Inserting these expressions into eq. (5.15) and simplifying gives:

$$\text{KE} = \frac{1}{2} \left[\left(m_1 + \frac{1}{4}m_2 \right) r_2^2 + I \right] \omega^2 \quad (5.17)$$

Now consider the change in potential energy between two times. This is just the change in mgh for both masses, but here we will consider θ instead h . If the fixed pulley rotates by an angle of $\Delta\theta$ (in radians), mass 1 falls by height $h_1 = r_2\Delta\theta$, while mass 2 rises by height $h_2 = r_2\Delta\theta/2$ (do you see why?). The change in potential energy is then:

$$\Delta\text{PE} = m_1gh_1 - m_2gh_2 = \left(m_1 - \frac{1}{2}m_2 \right) g r_2 \Delta\theta. \quad (5.18)$$

If friction in the system can be ignored, then the kinetic energy (5.17) at any moment should be equal to the change in potential energy (5.18) since the beginning of the motion.

Specific Instructions

Add 150–200 g to the hanging mass m_1 and re-attach it to the system. Release the mass and use Princeton Timer A to record the times of successive half-turns of the fixed pulley, using the same procedure as in the first part of the Lab. Be sure that you start the Timer after the mass m_1 has started falling, and stop the Timer before the mass hits the floor.

²If you wrap the string in the groove in the hub of the “fixed” pulley, measure this radius/diameter, which is different from the radius/diameter of the hub.

To calculate the initial and final kinetic energies using eq. (5.17), you need to know the initial and final angular velocities of the pulley. The measurements reported are the times of the pulley when its angle θ is at successive multiples of π radians. Thus, the average angular velocity in any interval is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi}{\Delta t}. \quad (5.19)$$

Use the time measurement for the first time interval reported by **Princeton Timer** to calculate ω_0 , the average angular velocity during this interval, which is equal to the angular velocity at the midpoint of this time interval (if the angular acceleration α is constant). Then, use eq. (5.17), along with your measurements of m_1 , m_2 , r , and I from the first part of the lab to find the kinetic energy of the system at the middle of the first time interval. Similarly, use Δt from the last time interval to calculate ω_f and the kinetic energy at the midpoint of this time interval.

For a data set with N numbers, the pulley rotated a total of $\Delta\theta = (N - 2)\pi$ radians between the midpoints of the first and last time intervals (why?). Use this in eq. (5.18) to calculate the change ΔPE in potential energy between the midpoints of these time intervals.

Does your measurement of ΔPE agree with the difference between initial and final kinetic energy? You need not perform a detailed uncertainty analysis, but if the numbers don't agree within 5–10%, look for a gross error somewhere.

Lab 6: Springs and Simple Harmonic Motion

6.1 Background

Springs provide a useful model for many physical systems that exhibit oscillatory behavior. The key characteristic of springs is that, when stretched or compressed by an amount Δx from their relaxed position, they exert a force proportional to Δx :

$$F = -k\Delta x, \quad (6.1)$$

where k is the **spring constant**. If a mass, m , is attached to one end of the spring, it will oscillate with angular frequency:

$$\omega = \sqrt{\frac{k}{m}}. \quad (6.2)$$

The corresponding period is:

$$T = 2\pi\sqrt{\frac{m}{k}}. \quad (6.3)$$

In the first part of the experiment you will verify these equations. In later parts you will analyze systems with more than one spring.

6.2 Specific Instructions

The setup for the first part of the lab is shown in Fig. 6.1. A spring connects a glider of mass M to one end of an air track. A hanging mass, m , is attached to the other end of the glider via a piece of magnetic tape **shiny side down**.

6.2.1 Equilibrium

Analyze the system in equilibrium and find the equation which relates the mass, the spring constant and the displacement.

Turn on the air track. Measure the equilibrium positions, x , (in meters) of the glider with an empty hanger and also with at least four different weights. Make sure that you don't

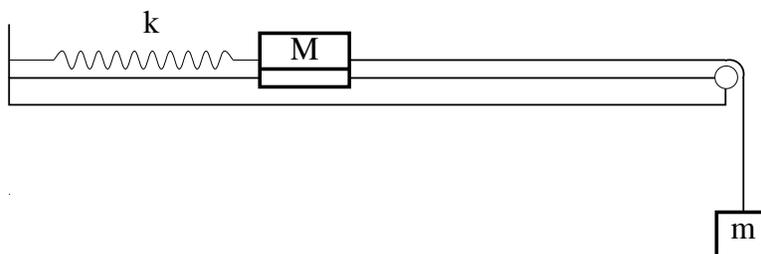


Figure 6.1: Equipment setup for the first part of the experiment.

stretch the spring too much such that the normal spring coefficient is affected. Also makes sure that the hanger is not touching either the ground or the end of the air track. Measure the masses of the hanging weights on a balance or digital scale for precision results.

Fit a line to a plot of weight mg versus x (Which fit order should you use?) Use the equation you previously derived to extrapolate k and its uncertainty, σ_k , from the fit parameters. Use $g = 9.8 \text{ m s}^{-2}$ where needed.

6.2.2 Oscillations of a Single Spring

If you displace the glider and release it, it will oscillate back and forth on the air track. Set up a photogate so that it is blocked twice per cycle (once for each direction of motion); only the taller tab on the glider should intercept the beam of the photogate. Now you can use **Logger Pro: Princeton Spring Timer** to measure the oscillation period. For each measurement you make, collect data for several cycles of the motion; then find the average period by clicking the **STAT** icon, which will open a statistics window.

The period of oscillation of the glider should be:

$$T = 2\pi\sqrt{\frac{m_{\text{total}}}{k}} = 2\pi\sqrt{\frac{m + M}{k}}, \quad (6.4)$$

independent of the amplitude of the oscillation (just as the period of a pendulum was independent of amplitude). Test this by recording the period of oscillation for glider displacements of around 8, 10 and 12 cm. Use larger displacements if necessary to avoid the effects of damping. Use the same hanging mass m for each of these trials. If you do not get enough numbers of full periods, adjust the hanging mass accordingly. Do you measure the same period within the experimental uncertainty?

Next test the dependence of period T on mass $m + M$ (for fixed amplitude of oscillation). Measure the period of oscillation with the hanger empty, then with 20 g of additional mass, and finally with 40 g of additional mass.

Compare these measured periods with periods calculated using eq. (6.4) and the spring constant measured in the previous part.

Another useful comparison can be made by rewriting eq. (6.4) as

$$k = 4\pi^2 \frac{m + M}{T^2}. \quad (6.5)$$

Use this to calculate three values of the spring constant k from your three measurements of period T as a function of mass M . For a simple statistical analysis, take your best estimate of k to be the average of the three values, and the uncertainty to be $\sigma_k = |\text{Max}(k) - \text{Min}(k)|/2$. Compare these results with those you obtained in part 6.2.1.

6.2.3 Two Springs

Set up the glider with two identical springs as shown in Fig 6.2. Connect the springs such that they are horizontal (not slanted downwards). What is the effective spring constant of the combination of springs? What is the expected period?

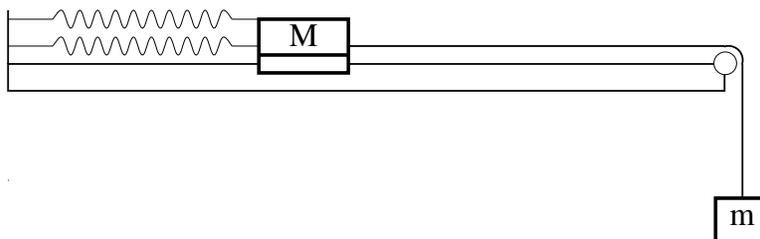


Figure 6.2: A glider connected to the air track via two springs.

Measure the period of the system. Does it agree with what you expected?

You need not do a detailed uncertainty analysis, but the numbers should be in the same “ball park.” If not, repeat sec. 6.2.1 to verify the spring constants of the springs that you used.

6.2.4 Three Springs, Two Masses

Set up two gliders and three identical springs as shown in Fig. 6.3. Again, connect the springs such that they are horizontal (not slanted downwards). Displace one of the gliders from its equilibrium position while holding the other fixed. Then, let them both go at the same time. The resulting motion can be quite confusing!

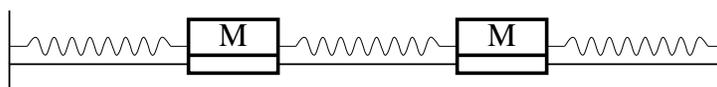


Figure 6.3: Two gliders and three springs.

You need not analyze the motion of the gliders in its full complexity. Instead, you will study two special cases of motion called **normal modes**, in which all movement takes place at a single frequency. It turns out that any one-dimensional system of springs that includes N masses has N such modes. Thus, the two-glider system has two normal modes.

The first normal mode occurs when the gliders move but the distance between them does not change. Displace both gliders by the same amount and in the same direction, and release them. The middle spring should keep a constant length, and the gliders should oscillate with the same period. Measure the period of oscillation of this mode (T_1) using the photogate aligned on one of the gliders.

The other normal mode occurs when the gliders move in opposite directions with equal amplitudes. Displace one glider to the right and the other to the left by the same amount, and release them. Again the two gliders should oscillate with the same period (but different from T_1). Measure this period (T_2). Does the center of mass of the gliders combination move?

Compare your measured periods with those deduced in sec. 6.2.5 below. You don't need to do a quantitative analysis of the uncertainty, but your measured periods should be close to those calculated in the analysis.

If you start the system by holding one glider fixed at its equilibrium position and displacing the other glider, you will create a motion that is a combination of the two modes studied above. With time, one or the other of the two modes may be more apparent, in an alternating pattern. However, if you are not attuned to the presence of the two modes, the motion may appear to be **chaotic** (a technical term meaning behavior that appears to be complex but which nonetheless obeys simple laws of physics).

Use a stopwatch to measure the longer period of the complicated general motion, and compare this with $2T_1T_2/(T_1 - T_2)$ based on your measured values for T_1 and T_2 . *The longer period is twice the time between "nodes" of the motion of one of the gliders.*

6.2.5 Analysis of Normal Modes for 3 Springs and 2 Masses

During the first normal mode the force from the middle spring never changes since its length never changes. Therefore, the changing force on, say, the left glider is only due to the change in length of the leftmost spring. If k is the spring constant of each of the springs, then the force on the left glider is:

$$F_L = -k\Delta x_L, \quad (6.6)$$

where Δx_L is the displacement of the left glider. Thus the period of oscillation in the first normal mode is:

$$T_1 = 2\pi\sqrt{\frac{M}{k}}. \quad (6.7)$$

During the second normal mode the left glider experiences changing forces from both the left and middle springs. After a displacement Δx_L the leftmost spring exerts force $-k\Delta x_L$ on the left glider. Meanwhile the middle spring exerts force $-k(2\Delta x_L)$ on the left glider since the middle spring is compressed (or stretched) twice as much as the leftmost spring is stretched (or compressed). Note that the direction of the force on the leftmost glider is the same for both springs, so the net force on the left glider is:

$$F_L = -3k\Delta x_L. \quad (6.8)$$

Thus the "effective" spring constant is $3k$ and the period of this mode is:

$$T_2 = 2\pi\sqrt{\frac{M}{3k}}. \quad (6.9)$$

The motion after starting with one glider at rest at its equilibrium point and the other displaced is of the form $A \cos \omega_1 t + A \cos \omega_2 t = 2A \cos(\omega_2 - \omega_1)t/2 \cos(\omega_2 + \omega_1)t/2$, for which the longer period is $4\pi/(\omega_2 - \omega_1) = 4\pi/(2\pi/T_2 - 2\pi/T_1) = 2T_1T_2/(T_1 - T_2)$.

Lab 7: Fluids

7.1 Flow of a Viscous Fluid in a Circular Pipe

7.1.1 Background

It is a remarkable fact that fluid immediately adjacent to an immobile surface, such as the wall of a pipe, always has zero velocity. For fluid some distance y from the surface to flow at velocity v , a force must be applied:

$$F = \frac{\eta Av}{y} \quad (7.1)$$

where A is the area of the surface (or, equivalently, the area of the layer of fluid), and η is the coefficient of viscosity.

Fluid flow through a circular pipe is slightly more complicated. The flow rate, Q (= volume of fluid passing any cross section of the pipe each second), through a circular pipe of radius R and length L is given by Poiseuille's law:

$$Q = \frac{\pi R^4 \Delta P}{8\eta L} \quad (7.2)$$

where ΔP is the difference in pressure at the ends of the pipe. The R^4 dependence is dramatic: a small reduction in R leads to a large reduction in the flow rate in the pipe (a blood vein, for example).

If v is the flow velocity at the center of the pipe, then the flow rate is $Q = \pi R^2 v/4$, where the factor of 4 arises due to the variation of velocity with distance from the wall of the pipe. From eq. (7.1), $v = FR/\eta A$, where the force F is the pressure difference times the cross-sectional area, $F = \pi R^2 \Delta P$, and the area A of the surface of a length L of the pipe is $A = 2\pi RL$. Combining these facts, we arrive at Poiseuille's law (7.2).

7.1.2 Specific Instructions

Use the apparatus shown in Fig. 7.1 to test Poiseuille's law and to measure the viscosity of a fluid. The fluid is a heavy machine oil, which fills the large vertical cylinder. Its weight produces the pressure at the bottom of the cylinder and, therefore, at one end of the small horizontal tubes. The other ends of the horizontal tubes (once uncorked) are at atmospheric pressure. Thus, the pressure difference across the length of the small tube is $\Delta P = \rho gh$, where h is the height of the fluid above the outlet of the (possibly curved) tube.

Measure the mass of an empty graduated cylinder

Measure the flow rate in each of the three available tubes (radii 0.349, 0.294 and 0.228 cm), using a stopwatch and a graduated cylinder. Measure the height of the fluid in the large vertical cylinder before and after the oil flows out. Use the average of these two values for h in your calculations. From which point should the height be measured?

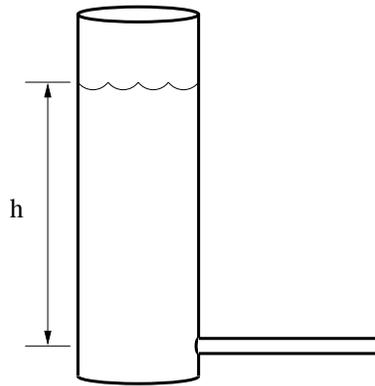


Figure 7.1: Apparatus for the study of Poiseuille's law. The vertical cylinder is partly filled with oil. It is open to the atmosphere at the top. The height h is to be measured between the liquid level and the outlet of the tube.

Measure the mass of the cylinder plus oil, and read the volume of the oil in the cylinder to determine the density ρ of the oil.

Hints: Keep the small tubes horizontal to minimize the effect of gravity on the flow. Try to position the end of the small tube over the center of the cylinder base so that the oil falls directly to the bottom of the cylinder. If oil adheres to the sides of the cylinder, the volume measurement will be inaccurate. Do not pour the oil from the small cylinder back into the big cylinder until the end of the Lab; so don't let more than 1/3 of the small cylinder fill with oil during each measurement.

7.1.3 Analysis

First use your data to test the assertion that Q is proportional to R^4 . Although it isn't strictly true, you may assume that each tube has the same length L . Then you can write eq. (7.2) as:

$$Q = \text{Constant} \times R^\alpha \quad (7.3)$$

You want to check that the exponent α is 4. Do this by analyzing your measurements of Q using logarithmic plotting, as discussed in sec. 3.2.3.

Next, determine the value of the viscosity η of the oil. For this part of the analysis, assume that the exponent $\alpha = 4$. Rework the Poiseuille's equation to extract the value of the coefficient of viscosity, and use your three measurements of Q to calculate three values of η . Are the values close to each other? Formal uncertainty analysis isn't required, but look for a procedural error if the measurements are grossly different.

7.2 Terminal Velocity

7.2.1 Background

An object falling through a viscous fluid feels three forces.

Gravity pulls the object downward:

$$F_{\text{grav}} = mg = \rho V g, \quad (7.4)$$

where ρ and V are the density and volume of the object, respectively, and g is gravitational acceleration.

The buoyant force pushes the object upward:

$$F_{\text{buoy}} = \rho_f V g, \quad (7.5)$$

where ρ_f is the density of the fluid. Finally, there is a drag force which acts to oppose the motion of the object.

The drag force on a spherical object of radius R moving with velocity v is given by Stokes' law:

$$F_{\text{drag}} = 6\pi\eta Rv, \quad (7.6)$$

where R is the radius of the sphere.

Stokes' law (7.6) is a variant on the basic rule (7.1). The area of the sphere is $A = 4\pi R^2$, its volume is $V = 4\pi r^3/3$, and the $y = 2R/3$ is the distance from the surface of the sphere at which the surrounding fluid is hardly perturbed by the motion of the sphere.

When these three forces balance, no net force acts on the sphere, so it falls with constant velocity, called **terminal velocity**. Combine eqs. (7.4)-(7.6) for the three forces acting on the sphere to deduce an expression for the terminal velocity.

7.2.2 Specific Instructions

Test the equation you just derived by measuring the terminal velocity of small lead spheres falling through the oil you analyzed in the first part of the experiment. The density of lead is $\rho = 11.4 \text{ g cm}^{-3}$.

Measure the diameter of one of the spheres, taking an average of several measurements if it isn't really spherical. *Don't squeeze the soft lead out of round during this measurement.* Measure the velocity of the sphere falling through the oil using a stopwatch. Repeat the experiment for at least three different spheres. Are the measured values close to the values predicted by your equation?

Solve for the viscosity η and compare with your result from sec. 7.1.

7.3 Buoyant Force

7.3.1 Background

The density of gas in a helium balloon is less than the density of the surrounding air ($\rho_{\text{He}} = 0.165 \text{ kg m}^{-3} * P$ for pressure P in atmospheres, $\rho_{\text{air}} = 1.29 \text{ kg m}^{-3}$ at 1 atm pressure), so the balloon feels a net upward force. The buoyant force (7.5) can be balanced by hanging a mass below the balloon, as shown in Fig. 7.2.

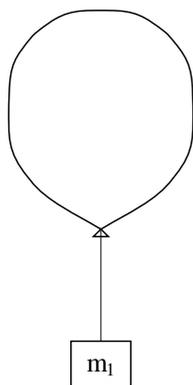


Figure 7.2: Apparatus for the study of buoyant force.

The total weight is:

$$W_{\text{total}} = (m_1 + m_{\text{string}} + m_{\text{balloon}} + m_{\text{He}}) g \quad (7.7)$$

where m_1 is the mass hanging below the balloon, m_{string} is the mass of the string, m_{balloon} is the mass of the (empty) balloon, and m_{He} is the mass of the helium within the balloon.

The masses of the balloon, string, and hanging weight can be measured on a precision scale, and $m_{\text{He}} = \rho_{\text{He}} V$ where V is the volume of the balloon.

7.3.2 Specific Instructions

Measure the mass of the empty balloon. Fill it with helium, and after stopping the flow of helium, measure the pressure within the balloon before tying off the end of the balloon. *You may need the following conversion factors: 1 atm = 15 psi = 1.013×10^5 Pa. Note that the gas regulator measures gauge pressure $P_g = P - P_{\text{atm}}$ rather than the absolute pressure P .*

Next measure the volume of the balloon. One way of doing this is to put it on a table, hold a meter stick vertically next to it, and use a wooden board to help measure its size on the meter stick (see Fig. 7.3). If d_1 is the size of the balloon in its longest dimension and d_2 is the size from side to side, its volume is approximately:

$$V = \frac{4}{3} \pi \left(\frac{d_1}{2} \right) \left(\frac{d_2}{2} \right)^2. \quad (7.8)$$

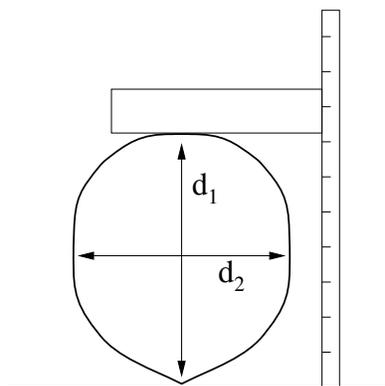


Figure 7.3: Measuring the dimensions of a balloon.

Cut a piece of string 1-2 feet long, measure its mass and tie it to the bottom of the balloon. Finally, tie a 5-g hanger to the string and keep adding weights to the hanger until the balloon is in equilibrium. To fine-tune the hanging weight, you may want to use small paper clips (about 0.3 g each) or pieces of tape. After you have achieved equilibrium, detach the hanger and its weights, and measure their total mass on a scale.

For a second measurement of the buoyant force, add enough weights to the hanger that the balloon + hanger + weights fall, and then use an electronic scale to determine the apparent mass of the system, as well as the mass of the hanger + weights.

Now you have all data you need to test the buoyancy formula. Calculate the buoyant force and the weight of the balloon + gas + string. Are they close to each other?

Lab 8: Speed of Sound and Specific Heats

8.1 Safety

Tanks of various gases are available in the lab. Choose just one of these for your measurement of the speed of sound and, of course, use the same gas for measurement of the density. The tanks contain gas at high pressure:

Do NOT try to move the tanks; if it is absolutely needed, ask the assistance of an instructor.

Every tank has three valves: the main valve on the throat of the bottle, which allows gas to enter a large reduction valve in the regulator, which bleeds the gas slowly out of a small needle valve into the rubber tubing.

You only need to use the needle valve. The shutoff valve and the regulator valve have already been adjusted: do NOT tamper with them.

The pressure has been set high enough to give a flow that will fill your flask in a reasonable time.

If someone should increase the pressure it could cause a flask to blow up! Do NOT touch the regulator valve.

As you open the needle valve check for a gentle breeze of gas flowing out of the rubber tube when held near your lips (the “lip test”). Be careful not to inhale the gas. Keep the needle valve closed except when you need some gas flow.

8.2 Velocity of Sound

In this Lab you will measure the velocity of sound in a gas, and you will weigh the gas in order to find its density. These measurements will lead to a determination of C_P/C_V , the ratio of specific heats at constant pressure and constant volume, and thus allow you to infer the molecular structure of the gas. Measurements of macroscopic lengths, masses, and forces provide insights on molecules of size $\approx 10^{-10}$ meters and mass $\approx 10^{-23}$ grams!

The speed v of longitudinal sound waves in gases is given by:

$$v = \sqrt{\frac{\gamma P}{\rho}}, \quad \text{where} \quad \gamma = \frac{C_P}{C_V}, \quad (8.1)$$

ρ is the gas density in kg/m^3 , P is the gas pressure in Pascals, and γ is the ratio of the specific heat capacity at constant pressure C_P to the specific heat capacity at constant volume C_V . The kinetic theory of gases predicts that $\gamma = \frac{5}{3}$ for an ideal monoatomic gas and $\gamma = \frac{7}{5}$ for an ideal diatomic gas. At standard temperature and pressure, $\gamma_{\text{Ar}} = 1.66$, $\gamma_{\text{CO}_2} = 1.29$ and $\gamma_{\text{N}_2} = 1.40$.

Using the ideal gas law,

$$PV = nRT, \quad (8.2)$$

where the gas constant is $R = 8.31 \text{ J/mole-K}$, we can express the gas density as

$$\rho = \frac{m}{V} = \frac{Mn}{V} = \frac{MP}{RT}, \quad (8.3)$$

where M is the molar mass. What density does the ideal gas law predict for gases Ar, CO₂ and N₂ at room temperature and atmospheric pressure? What is the velocity of sound for these gases according to eq. (8.1)?

If the sound wave has a pure frequency f and wavelength λ , then the velocity of sound is related by

$$v = f\lambda. \quad (8.4)$$

8.2.1 Resonance

The apparatus for measuring the speed of sound is a vertical cylinder of gas (see Fig. 8.1). The length of the column of gas is adjusted by changing the height of a reservoir of water. The gas volume is interconnected to the top of the water reservoir so that the gas volume does not change when the reservoir is raised or lowered.

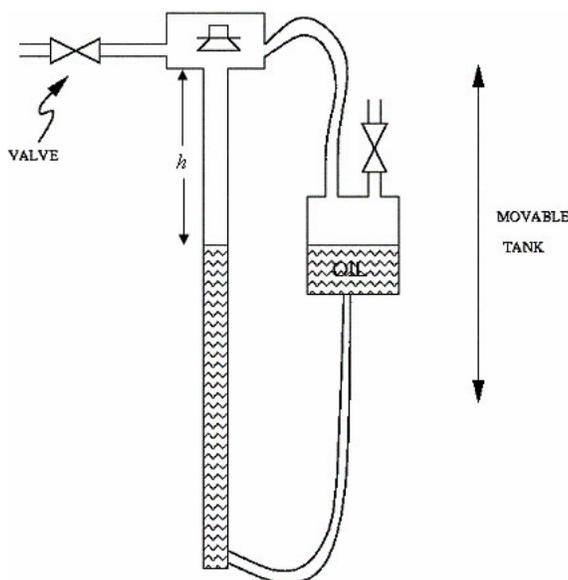


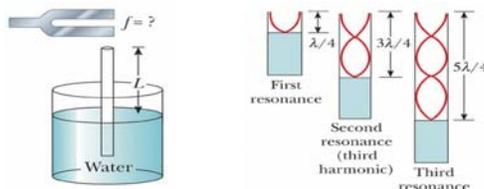
Figure 8.1: Apparatus for measuring the speed of sound.

A loudspeaker is attached to the top of the cylinder, and it generates sound from an electrical audio oscillator. The loudspeaker also responds to sound vibrations (like a microphone) and thus the sound waves set up in the cylinder of gas affect it also. This action is detected by a meter. When the cylinder is in “resonance” you will get a large reading on the meter (and you may be able to hear that the output of the loudspeaker is louder).

A resonant condition occurs when a pattern of longitudinal standing waves (sound waves) is established in the gas column when it is driven at certain frequencies. At the resonance,

the vertical vibration of the gas molecules is maximal (an antinode) at the tube end where the loudspeaker is located (and the vibration of whose “cone” drives the gas molecules). Of course, the vibration of the gas molecules is zero (a node) at the liquid surface in the tube.

For a column of height h , standing waves with a node at the bottom and an antinode at the top can have wavelengths λ related by $h = \lambda/4, 3\lambda/4, 5\lambda/4, \dots, (2n - 1)\lambda/4$. The spacing between neighboring resonances is $\Delta h = \lambda/2 = v/2f$.



For circuit experts: The impedance of the speaker is about $1\ \Omega$, which is small compared to the $50\text{-}\Omega$ output impedance of the wave generator. At acoustic resonance, the extra dissipation of energy by the loudspeaker increase its impedance by about 25%. This has negligible effect on the current through the speaker, but increases the voltage drop across it by 25%, which is detected by the AC voltmeter.

8.2.2 Specific Instructions

As a preliminary step, turn on the oscillator at a convenient frequency ≈ 800 Hz. Lower the water reservoir from an upper position to one about 60 cm below, while watching the meter and listening to the sound. You should be able to find several positions of the water level at which resonance occurs.

The meter reading should be near the maximum of its scale at resonance – if it goes off scale reduce the output level of the oscillator. You might find that a (10-20%) different frequency will give a stronger indication of resonance. Explore a bit.

You are now ready to fill the cylinder with gas from one of the tanks. Note that the gas can be fed into the apparatus in a way to flush out the gas previously there, as follows. Raise the water reservoir as high as possible during flushing to facilitate this. Be sure to open and close the valves in an order such that the pressure in the tube is not raised above atmospheric pressure.

Lower the water level from the top and note the approximate location of the first three resonances. Then raise the water slowly through each resonance. *Upward motion makes it easier to determine the level of water as you will discover on trying it both ways.* Find the position of resonance with the maximum possible accuracy.

Use the relation you previously derived to calculate the speed of sound from the spacing Δh between resonances and the frequency f of the audio oscillator.

Tip: A good way find a resonance position is for one person to slowly raise the water level while another records the meter readings. The maximum can be found by graphing the meter reading vs. position.

Also explore the resonances keeping the height h of the water constant while varying the

frequency f of the oscillator. What is the lowest resonant frequency for the height that you have chosen?

8.3 Density of the Gas

Find the gas density ρ weighing a known volume of the gas at atmospheric pressure. Two flasks of equal volume are provided, one coated with plastic and the other clear.

Determine the useful volume of the coated flask by filling the clear flask with water to the level of the rubber stopper in the coated flask. Then, measure the volume of water.

Evacuate the coated flask with the vacuum pump. You can tell when this is achieved when a sharp clacking sound of the pump replaces the initial slurping sound.

Weigh the evacuated flask. *Be careful not to drop it. The plastic coating is to minimize hazard of breakage.*

Then fill the coated flask with gas at one atmosphere pressure, using the procedure described below, and reweigh.

- Open the needle valve and find a gentle flow using the lip test (sec. 8.1). Allow the gas to flow through the tubing long enough to flush any air out. Attach the hose to your flask and open the stopcock. The flask is full to over one atmosphere pressure when the hissing noise at the needle valve stops. Shut the stopcock. Shut the needle valve and remove the tubing. Open the stopcock slightly and gas will hiss out allowing the pressure to drop to atmospheric. *If you leave the stopcock open too long, ordinary air will mix with the gas in the flask, thereby reducing its purity.*
- Weigh the filled flask, which may still contain gas above atmospheric pressure. Open the stopcock again briefly, reclose it, and reweigh the flask. If the weight has not changed again you are finished. If it has decreased, repeat again.

(Why does the weight change at all?)

8.4 What Gas Did You Use?

The tanks are labeled so that they can be distinguished, but the labels indicating what gases they contain are hidden. Sometime into the lab your AI will provide you with a list of gases in the tanks. Can you determine which gas you used only on the basis of your measurement of the gas density, or only on the basis of your measurement of the velocity of sound?

Use eq. (8.1) and your measurements of ρ and v to determine $\gamma = C_P/C_V$ for your gas. Was your measurement accurate enough to distinguish between monatomic, diatomic, and polyatomic gases?

Appendix A: Data Analysis with Excel

Computers are used for data analysis in any modern physics laboratory, and the Physics 101 lab is no exception. We have built our data analysis system around the program Excel, which is widely used on and off campus. We've added some Workshop Physics (WP) tools to make graphing data easier, and to let you do regression calculations with uncertainties, but otherwise we are using the standard, off-the-shelf software.¹

If you are already familiar with Excel, great! If not, we'll give brief instructions here. Like any software, it can be confusing at first, so don't hesitate to ask your instructors and your fellow students for help. Play around with the program a bit to get comfortable with it.

A.1 Starting things up

- If the computer isn't already on, turn it on and wait for it to boot up.
- If the Physics 101 window isn't already open, double click on the Physics 101 icon to open it
- Double click on Excel with WPtools (look for the X logo) to get the program running.

A.2 Entering Data: a Simple Example¹

When Excel is started up, you need to open a spreadsheet to work in. If you are asked if you want to reopen WPtools, click No. Then go to File→New and click on OK to open a new Workbook. (If you wanted to open a pre-existing spreadsheet, you would use the File→Open menu command; if you want to save the new spreadsheet, use the File→Save menu command. Since we'll be working with fairly small datasets, neither of these is really necessary for your lab work.)

Suppose you wish to record some (x, y) data pairs in two columns of a spreadsheet. Go to Excel, and start entering the data in the upper left most cell (called A1). To do this, move the cursor to this cell and click on it with the left mouse button. Enter the first x data value here, pressing Return when you are done. The cell below it (called A2) will automatically be selected next; enter the second x data value here. Work down the first data column in this way. If you need to go back and correct any of the numbers, simply move the cursor with the mouse, click on the relevant cell, and re-enter the number.

Once you've entered the first column of numbers, move the cursor to the top cell of the second column (cell B1) and click the left button to select it. Enter the first y data value. Press Return/Enter, and enter subsequent y data in the rest of the cells.

¹Some online documentation for WPtools is at http://physics.dickinson.edu/~wp-web/wp_resources/Documentation.html

¹Note: Menu commands are described as follows: File→Open means move the cursor to the word File on the line near the top of the screen, press *and hold* the left mouse button, drag the cursor down to the word Open, and release the button.

A.3 Calculations in Excel

Now that your two columns of data are in the computer, select them. Do this by moving the cursor to the top left cell, pressing *and holding* the **left** mouse button, dragging the cursor to the lowest filled cell in the second column, and then releasing the mouse button. The block of numbers you entered will now be “selected”, indicated by a blue-grey color.

Go to the **WPtools** pull-down menu and select **Linear Fit**. Immediately Excel will display a plot of your data, along with the values and uncertainties of the best-fit line. **Print** out a copy of the results if desired.²

Sometimes you will want to transform your raw data in some way before plotting it. For example, you may have entered two columns of data as above, but you want to convert the y values from inches to meters. This is where a spreadsheet program becomes really handy. Select a blank cell somewhere on the sheet (cell **C1** would be a good place). Instead of entering a number, enter the formula `=0.0254*B1`, and press **Return**. Excel will display the expected numerical value in cell **C1**, and it will also remember the formula. This is useful for two reasons, first, if you change the value in **B1**, the number in **C1** will be automatically updated. Second, you can copy the formula in **C1** to other cells, transforming the rest of column **B** using the same formula. To do this, first select cell **C1**. The cell becomes outlined, and note that there is a little square in the lower right corner of the outline. Move the cursor to this square, push and hold the **left** mouse button, drag the cursor down several cells, and release the mouse button. Voila! Excel will use the same formula to multiply all the cells in column **B** by 0.0254.

Excel can do much more complicated arithmetic. For example, you could use the formula `=sqrt(A1) * B1` to take the square root of the values of cells in column **A**, multiply them by the values in column **B**, and put the result in some other column.

You might also want to take differences between the successive items in your data list. If you type into cell **C2** the formula `=B2-B1`, and then use the little square to fill this formula into the cells **B3**, **B4**, *etc.*, then you will obtain the differences in column **C**.

If you do a transformation like this, and then you want to do a plot or a curve fit, the columns of data you want to plot may not be adjacent to each other. No problem. Say you want to plot cells **A1-A10** on the horizontal axis and cells **C1-C10** on the vertical axis. First select **A1-A10**. (Go to cell **A1**, hold down the left button, drag the cursor to **A10**, then release the mouse button). Then *hold down the Ctrl key* and select **C1-C10**. Now both cell groups **A1-A10** and **C1-C10** will be selected, but not **B1-B10**. Run the **WPTools**→**LinearFit** routine, and you will get the plot you want.

²You must first “grab” the plot by left-clicking on an open area inside it. If the plot legends are obscuring the graph, drag them aside with the mouse. You can add labels to your plot using the **Edit labels** option on the **WPtools** menu bar.

A.4 Accumulating Values via Excel Tricks

There will be times in the Physics 101 lab when you want to accumulate sums of a series of values. For example, you might have measured a series of time intervals,

Δt_1 = interval between event 1 and event 2,
 Δt_2 = interval between event 2 and event 3,
 Δt_3 = interval between event 3 and event 4,
etc.

You may wish to convert these into a continuous time scale. In other words, you may want to declare that $t = 0$ at the time of event 1, and then find

time of event 2 = Δt_1 ,
time of event 3 = $\Delta t_1 + \Delta t_2$,
time of event 4 = $\Delta t_1 + \Delta t_2 + \Delta t_3$,
etc.

This is easy to do. Say that Δt_1 , Δt_2 , *etc.*, are in cells **A1**, **A2**, *etc.*, and you want to put the accumulated times in column **B**. First put a 0 in cell **B1** (since $t = 0$ for the first event). Then go to cell **B2** and enter the formula =SUM(\$A\$1:A1). The SUM function simply adds up the cells in the range specified.

The usefulness of the \$ notation becomes apparent when you want to calculate the rest of the times. Select **B2**, move the cursor to the square in the lower righthand corner of the cell border, press and hold the left mouse button, drag the cursor down several cells, and release the button. The cells in column **B** are now filled with SUM functions, but in a special way: The \$A\$1 in the SUM function call remains the same in all the cells (because of the \$), but the second part of the function call changes from A1 to A2 to A3, ... In other words, cell **B3** now reads =SUM(\$A\$1:A2), cell **B4** reads =SUM(\$A\$1:A3), and so on. These are exactly the formulae we want for the event time calculations, so column **B** is now filled with calculated values of t .

A.4.1 Average and Standard Deviation

Excel provides shortcut procedures (macros) that can report various column statistics.

For example, one way to obtain the sum of a column of numbers is to highlight that column by dragging the mouse over it, and then click on the Σ symbol on the toolbar. The sum of the column entries will be entered in the cell below the column (without a label to that effect; if you want a label, type it in, say, the cell to the right of that where the sum just appeared).

To obtain the average of a column of numbers, highlight the column, click on the (solid) ∇ symbol next to the Σ , and then click on **Average**. The average of the column entries will be entered in the next open cell below the column.

If you prefer the sum or average to appear in some other cell, you can follow a different procedure. First, click on the desired cell, which should turn its boundary into a bold rectangle. Then, click on the (solid) ∇ symbol next to the Σ , and click on **Sum** or **Average**. This should enter a temporary formula in the desired cell. Now drag the mouse over the column of numbers you wish to **Sum** or **Average**, which should outline the column in a dashed line. Hit **Enter** and the **Sum** or **Average** should appear in the desired cell.

Similarly, the standard deviation of a column of numbers can be entered in a desired cell by first clicking on it, then on the (solid) ∇ symbol next to the Σ , then on **More Functions**. Highlight **STDEV** in the menu box (scrolling down if necessary) and click **OK**. Again, this should enter a temporary formula in the desired cell. Now drag the mouse over the column of numbers you wish to analyze, which should outline the column in a dashed line. Hit **Enter** and the standard deviation about the mean/average should appear in the desired cell.

A.5 Further Notes about Workshop Physics Routines

- Use the **WPtools**→**Polynomial Fit** menu command, and set **Order=2** to fit lines of the form $y = c_0 + c_1x + c_2x^2$.
- If you enter non-numerical text in the cell above each column of data, it will be used to label the horizontal and vertical axes on the plot.
- The fitting routines always use the first selected column for the horizontal (x) points, and the second selected column for the vertical (y) points.
- Empty rows are usually ignored (but partially-empty rows may corrupt the fit).
- To delete a plot, select it (move cursor to it and click once), then press the **Delete** key. To delete a column of the sheet, select the entire column (by clicking on the letter at the top) and use **Edit**→**Delete**.
- If data are modified after running a fit, the associated plot will be automatically updated, but the fit parameters will not be re-calculated. Usually it is best to delete both the old plot and fit parameters after updating data.

Appendix B: Estimation of Errors

While the subject of error analysis can become quite elaborate, we first emphasize a basic but quite useful strategy, discussed in secs. B.1-2. Then, we distinguish between **random** (or **statistical**) uncertainties and **systematic** uncertainties in sec. B.3. Random uncertainties follow the famous bell curve, as sketched in secs. B.4-5. The important distinction between the uncertainty on a single measurement, and the uncertainty on the average of many repeated measurements is reviewed in secs. B.7-7. The subject of **propagation of errors** on measured quantities to the error on a function of those quantities is discussed in sec. B.8.

B.1 67% Confidence

Whenever we make a measurement of some value v , we would also like to be able to say that with 2/3 probability the value lies in the interval $[v - \sigma, v + \sigma]$. We will call σ the **uncertainty** or **error** on the measurement. That is, if we repeated the measurement a very large number of times, in about two thirds of those measurements the value v would be in the interval stated.

B.2 A Simple Approach

Repeat any measurement three times, obtaining a set of values $\{v_i\}$, $i = 1, 2, 3$. Report the average (mean),

$$\bar{v} = \frac{1}{N} \sum_{i=1}^N v_i \quad (\text{for } N = 3), \quad (\text{B.1})$$

as the best estimate of the true value of v , and the uncertainty σ as

$$\sigma = \frac{v_{\max} - v_{\min}}{2}. \quad (\text{B.2})$$

If you take more than three measurements, you can still implement this procedure with the aid of a **histogram**. Divide the range of observed values of v into 5-10 equal intervals (called **bins**). Located the bin that contains each measurement, and draw a box one unit high above that bin. Stack the boxes on top of one another if more than one measurement falls in a bin. To estimate the error, determine the interval in v that contains the central 2/3 of the measurements, *i.e.*, the central 2/3 of the boxes you just drew, and report the error as 1/2 the length of this interval.

B.3 Random and Systematic Uncertainties

The uncertainty in a measurement of a physical quantity can be due to intrinsic random uncertainty (colloquially: error) as well as to systematic uncertainty.

Random uncertainties lead to difference in the values obtained on repetition of measurements. Systematic uncertainties cause the measurement to differ from its ideal value by the same amount for all repetitions of the measurement.

Random uncertainties can arise from vibrations of the components of a set-up driven by random thermal fluctuations, random noise in the electronics, and/or many other small but uncontrolled effects including quantum fluctuations.

In principle, the effect of random uncertainties can be made as small as desired by repetition of the measurements, such that the dominant uncertainty is due to systematic effects (which can only be reduced by designing a better measurement apparatus).

B.4 The Bell Curve

In many cases when a measurement is repeated a large number of times the distribution of values follows the bell curve, or **Gaussian distribution**:

$$P(v) = \frac{e^{-(v-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}, \quad (\text{B.3})$$

where $P(v)dv$ is the probability that a measurement is made in the interval $[v, v + dv]$, μ is true value of the variable v , and σ is the **standard deviation** or uncertainty in a single measurement of v . See Figure B.1.

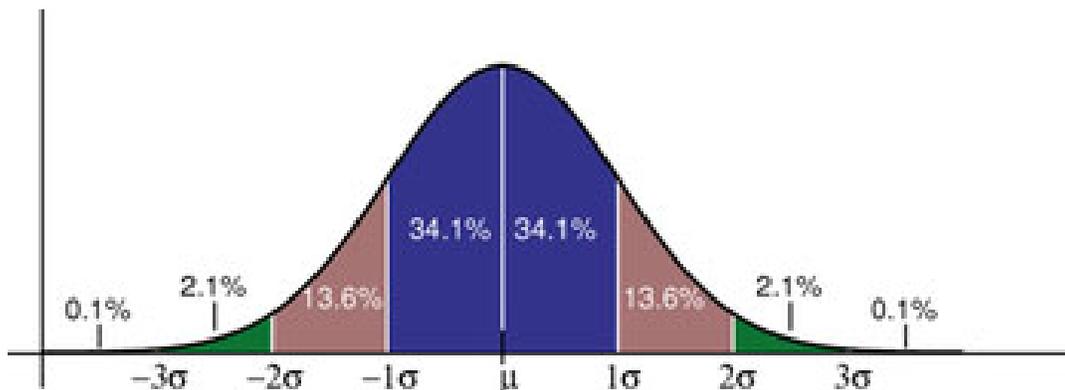


Figure B.1: The probability distribution measurements of a quantity with true value μ and Gaussian uncertainty σ of a single measurement. About 68% of the measurements would fall in the interval between $\mu - \sigma$ and $\mu + \sigma$, and 95% would fall in the interval $\mu \pm 2\sigma$.

The Table lists the confidence that a single measurement from a Gaussian distribution falls within various intervals about the mean. If the 100 students in Ph101 each make 100 measurements during these lab sessions, then 10,000 measurements will be taken in all. The Table tells us that if those measurements have purely Gaussian ‘errors’, then we expect one of those measurements to be more than 4σ from the mean.

Table B.1: The probability (or confidence) that a measurement of a Gaussian-distributed quantity falls in a specified interval about the mean.

Interval	Confidence
$\pm\sigma$	68%
$\pm 2\sigma$	95%
$\pm 3\sigma$	99.7%
$\pm 4\sigma$	99.994%

B.5 Estimating Uncertainties When Large Numbers of Measurements Are Made

One can make better estimates of uncertainties if the measurements are repeated a larger number of times. If N measurements are made of some quantity resulting in values v_i , $i = 1, \dots, N$ then the mean is, of course,

$$\bar{v} = \frac{1}{N} \sum_{i=1}^N v_i, \quad (\text{B.4})$$

and the standard deviation of the measurements is

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (v_i - \bar{v})^2}. \quad (\text{B.5})$$

Calculus experts will recognize that the operation $(1/N) \sum_{i=1}^N$ becomes $\int P(v) dv$ in the limit of large N . Then, using the Gaussian probability distribution (B.3) one verifies that

$$\bar{v} = \langle v \rangle = \int_{-\infty}^{\infty} v P(v) dv, \quad \text{and} \quad \sigma^2 = \langle (v - \bar{v})^2 \rangle = \int_{-\infty}^{\infty} (v - \bar{v})^2 P(v) dv. \quad (\text{B.6})$$

B.6 The Uncertainty on Mean of a Uniformly Distributed Quantity

Not all measurable quantities follow the Gaussian distribution. A simple example is a quantity with a uniform distribution, say with values v equally probable over the interval $[a, b]$. It is clear that the average measurement would be $(a + b)/2$, but what is the uncertainty of the measurement? If we adopt the simple prescription advocated in secs. B.2 we would report the uncertainty as $(b - a)/3$ since $2/3$ of the time the measurement would fall in an interval $2(b - a)/3$ long. If instead we use the calculus prescription for σ given in eq. (B.6) we find that

$$\sigma = \frac{b - a}{\sqrt{12}} = \frac{b - a}{3.46}, \quad (\text{B.7})$$

which result is often used by experts.

B.7 The Uncertainty in the Mean

Thus far we have considered only the uncertainty or spread in measured values of some quantity v . A related but different question is: what is the uncertainty on our best estimate of v (which is just the mean value of our measurements, $\bar{v} = (1/N) \sum v_i$)?

The uncertainty on the mean \bar{v} is surely less than the uncertainty, σ , on each measurement v_i . Indeed, the uncertainty on the mean is given by

$$\sigma_{\bar{v}} = \frac{\sigma}{\sqrt{N}}, \quad (\text{B.8})$$

where σ is our estimate of the measurement error obtained from one of the methods sketched previously.

Appendix C illustrates eq. (B.8) using measurements of g from past Ph101 labs.

B.8 The Uncertainty on a Function of Several Variables (Propagation of Error)

In many cases we are interested in estimating the uncertainty on a quantity f that is a function of measured quantities a, b, \dots, c . If we know the functional form $f = f(a, b, \dots, c)$ we can estimate the uncertainty σ_f using some calculus. As a result of our measurements and the corresponding ‘error analysis’ we know the mean values of a, b, \dots, c and the error estimates $\sigma_a, \sigma_b, \dots, \sigma_c$ of these means. Our best estimate of f is surely just $f(a, b, \dots, c)$ using the mean values.

To estimate the uncertainty on f we note that the change in f due to small changes in a, b, \dots, c is given by

$$\Delta f = \frac{\partial f}{\partial a} \Delta a + \frac{\partial f}{\partial b} \Delta b + \dots + \frac{\partial f}{\partial c} \Delta c. \quad (\text{B.9})$$

If we just averaged this expression we would get zero, since the ‘errors’ $\Delta a, \dots, \Delta c$ are sometimes positive, sometimes negative, and average to zero. Rather, we square the expression for Δf , and then average.

$$\Delta f^2 = \left(\frac{\partial f}{\partial a} \right)^2 \Delta a^2 + \dots + \left(\frac{\partial f}{\partial c} \right)^2 \Delta c^2 + \dots + 2 \frac{\partial f}{\partial a} \frac{\partial f}{\partial c} \Delta a \Delta c + \dots \quad (\text{B.10})$$

On average the terms with factors like $\Delta a \Delta c$ average to zero (under the important assumption that parameters a, b, \dots, c are independent). We identify the average of the squares of the changes relative to the mean values as the squares of the errors: $\langle \Delta a^2 \rangle = \sigma_a^2$, etc. This leads to the prescription

$$\sigma_f^2 = \left(\frac{\partial f}{\partial a} \right)^2 \sigma_a^2 + \dots + \left(\frac{\partial f}{\partial c} \right)^2 \sigma_c^2 + \dots \quad (\text{B.11})$$

Some useful examples are

$$f = a \pm b \pm \dots \pm c \quad \Rightarrow \quad \sigma_f = \sqrt{\sigma_a^2 + \sigma_b^2 + \dots + \sigma_c^2}, \quad (\text{B.12})$$

and

$$f = a^l b^m \dots c^n \quad \Rightarrow \quad \frac{\sigma_f}{f} = \sqrt{l^2 \left(\frac{\sigma_a}{a}\right)^2 + m^2 \left(\frac{\sigma_b}{b}\right)^2 + \dots + n^2 \left(\frac{\sigma_c}{c}\right)^2}, \quad (\text{B.13})$$

where l , m and n are constants that may be negative.

For more detailed and rigorous analyses one can consult, for example:

- P.R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Science*, 2nd ed. (McGraw-Hill, New York, 1992).
- J.R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, 1997).

Appendix C: Standard Deviation of the Mean of g

Suppose you make N repeated measurements of a quantity g , such as the acceleration due to gravity. How well is the value of g determined by these measurements?

For example, during the 2006 sessions of Ph101 Lab 3 a total of 37 different measurements of g were made, as shown in the histogram Fig. C.1.

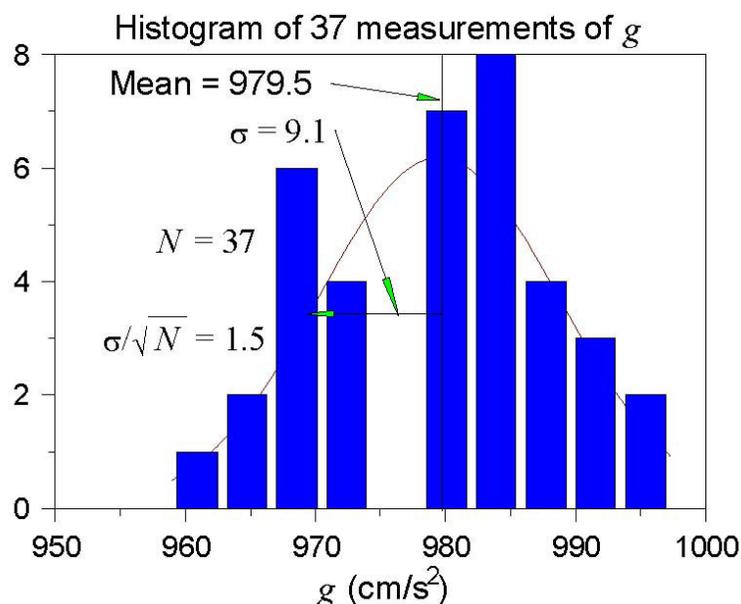


Figure C.1: Histogram of the values of g measured in the 2006 Ph101 Lab 3. The horizontal axis is g , and the vertical axis is the number of times a value of g was reported to lie with the range of g corresponding to the width of a vertical bar.

A histogram is a graph containing M vertical bars in which the height of a bar indicates the number of data points whose value falls within the corresponding “bin”, i.e., within the interval $[g_j - \Delta/2, g_j + \Delta/2]$, where $g_j, j = 1, M$ and the centers of the M bins and Δ is the bin width. One can make a histogram of a data set $\{g_i\}$ using Excel/Tools/Data Analysis/Histogram. Enter the data $\{g_i\}$ in one column of an Excel spreadsheet. Click on the Input Range: box of the Histogram window; then click and hold the left mouse button on the first data point, and drag the mouse to the last data point to enter the cell addresses of the data. Click on Chart Output and then OK to create a basic histogram. If the number/spacing of “bins” chosen by Excel is awkward, fill a new column with a linear series of 5-10 steps that begins near the lowest g_i and ends near the highest; create a new histogram with the Excel addresses of the first and last elements of the bin list in the box Bin Range:.

The mean value \bar{g} is calculated according to

$$\bar{g} = \frac{\sum_{i=1}^N g_i}{N}, \quad (\text{C.1})$$

and was found to be $\bar{g} = 939.5$ cm/s² for the data shown in Fig. C.1.

The distribution of the value of g is approximately Gaussian, and the standard deviation of this distribution is calculated according to

$$\sigma_g = \sqrt{\frac{\sum_{i=1}^N (g_i - \bar{g})^2}{N - 1}}, \quad (\text{C.2})$$

with the result that $\sigma_g = 9.1 \text{ cm/s}^2$.

The standard deviation σ_g is a good estimate of the uncertainty on a **single** measurement of g . However, after 37 measurements of g , the uncertainty on the mean value \bar{g} is much smaller than σ_g .

An important result of statistical analysis is that the standard deviation (*i.e.*, the uncertainty) of the mean of the N measurements is related to the standard deviation of the distribution of those measurements by,

$$\sigma_{\bar{g}} = \frac{\sigma_g}{\sqrt{N}}. \quad (\text{C.3})$$

For the data shown in Fig. C.1, where $N = 37$, we obtain

$$\sigma_{\bar{g}} = \frac{9.1}{\sqrt{37}} = 1.5 \text{ cm/s}^2. \quad (\text{C.4})$$

That is, we can report the result of all 37 measurements of g as

$$g = 979.5 \pm 1.5 \text{ cm/s}^2. \quad (\text{C.5})$$

As a check that eq. (C.3) is valid, we can analyze the data another way. Namely, we can first calculate the means \bar{g}_i for the 5 different sessions of Ph101 Lab 3. Then, we can make a histogram of these 5 values, as shown in Fig. C.2.

The mean of the 5 means is 979.6 cm/s^2 , which is essentially identical to the mean of the 37 individual measurements of g . The standard deviation of the 5 means shown in Fig. C.2 is calculated to be 1.6 cm/s^2 , which is essentially identical to the previous calculation (C.4) of the standard deviation of the mean.

Concluding Remarks: If N were much larger than what we have here, the histogram C.1 would approach the Gaussian distribution (the bell-curve) shown in Appendix B. The peak in the histogram would be very close to the mean value \bar{g} of the measurements, which represents the best estimate of g from the data. The standard deviation $\sigma_g \approx \text{width}/2$ is a measure of the uncertainty of a single measurement,¹ while σ_g/\sqrt{N} is the uncertainty on the best estimate \bar{g} .

¹Strictly speaking, the full width at half maximum of a Gaussian distribution is $2.35\sigma_g$.

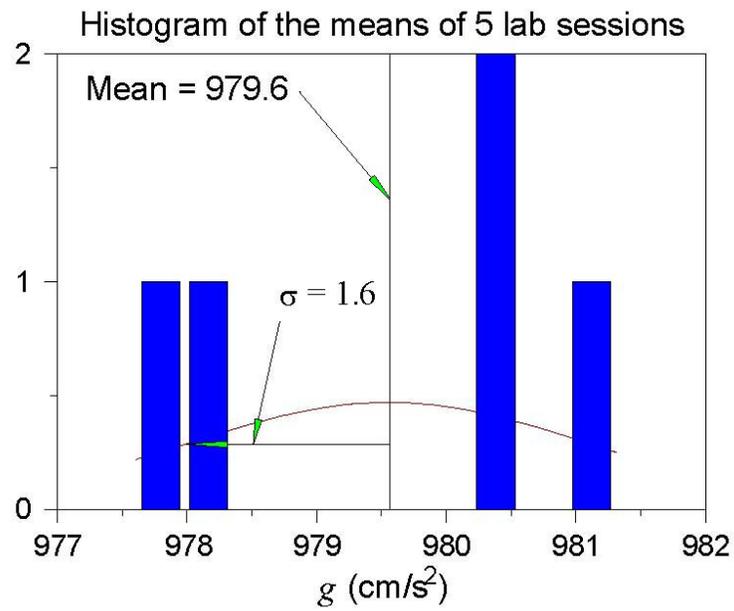


Figure C.2: Histogram of the mean values of g measured in the 5 sessions of Ph101 Lab 3 in 2006.

Appendix D: Polynomial Fits in WPtools

D.1 Polynomial Regression

In this technical Appendix we sketch the formalism used in the polynomial regression method for fitting data. This is a generalization of the method of linear regression.

We start with a set of data (x_j, y_j) , $j = 1, \dots, m$, and we wish to fit these data to the n th-order polynomial

$$y(x) = \sum_{i=0}^n a_i x^i. \quad (\text{D.1})$$

In general each measurement y_j has a corresponding uncertainty σ_j . That is, if the measurements were repeated many times at coordinate x_j the values of y_j would follow a gaussian distribution of standard deviation σ_j . We indicate in sec. D.2 how the program WPtools proceeds in the absence of input data as to the σ_j .

Because of the uncertainties in the measurements y_j we cannot expect to find the ideal values of the coefficients a_i , but only a set of best estimates we will call \hat{a}_i . However, we will also obtain estimates of the uncertainties in these best-fit parameters which we will label as $\sigma_{\hat{a}_i}$.

The best-fit polynomial is then

$$\hat{y}(x) = \sum_{i=0}^n \hat{a}_i x^i. \quad (\text{D.2})$$

The method to find the \hat{a}_i is called least-squares fitting as well as polynomial regression because we minimize the square of the deviations. We introduce the famous chi square:

$$\chi^2 = \sum_{j=1}^m \frac{[y_j - \hat{y}(x_j)]^2}{\sigma_j^2} = \sum_{j=1}^m \frac{(y_j - \sum_{i=0}^n \hat{a}_i x_j^i)^2}{\sigma_j^2}. \quad (\text{D.3})$$

Fact: $\exp(-\chi^2/2)$ is the (un-normalized) probability distribution for observing a set of variables $\{y_j(x_j)\}$ supposing the true relation of y to x is given by eq. (D.2).

A great insight is that $\exp(-\chi^2/2)$ can be thought of another way. It is also the (un-normalized) probability distribution that the polynomial coefficients have values a_i when their best-fit values are \hat{a}_i with uncertainties due to the measurements $\{y_j\}$. Expressing this in symbols,

$$\exp(-\chi^2/2) = \text{const} \times \exp\left(-\sum_{k=0}^n \sum_{l=0}^n \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{2\sigma_{kl}^2}\right), \quad (\text{D.4})$$

or equivalently

$$\chi^2 = \text{const} + \sum_{k=0}^n \sum_{l=0}^n \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{\sigma_{kl}^2}. \quad (\text{D.5})$$

The uncertainty on \hat{a}_k is σ_{kk} in this notation. In eqs. (D.4) and (D.5) we have introduced the important concept that the uncertainties in the coefficients \hat{a}_k are correlated. That is, the quantity σ_{kl}^2 is a measure of the probability that the values of a_k and a_l both have positive

fluctuations at the same time. In fact, σ_{kl}^2 can be negative indicating that when a_k has a positive fluctuation then a_l has a correlated negative one.

One way to see the merit of minimizing the χ^2 is as follows. According to eq. (D.5) the derivative of χ^2 with respect to a_k is

$$\frac{\partial \chi^2}{\partial a_k} = \sum_{l=0}^n \frac{\hat{a}_l - a_l}{\sigma_{kl}^2}, \quad (\text{D.6})$$

so that all first derivatives of χ^2 vanish when all $a_l = \hat{a}_l$. That is, χ^2 is a minimum when the coefficients take on their best-fit values \hat{a}_i . A further benefit is obtained from the second derivatives:

$$\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} = \frac{1}{\sigma_{kl}^2}. \quad (\text{D.7})$$

In practice we evaluate the χ^2 according to eq. (D.3) based on the measured data. Taking derivatives we find

$$\frac{\partial \chi^2}{\partial \hat{a}_k} = \sum_{j=1}^m \frac{(y_j - \sum_{i=0}^n \hat{a}_i x_j^i) (-x_j^k)}{\sigma_j^2} = \sum_{i=0}^n \sum_{j=1}^m \frac{\hat{a}_i x_j^i x_j^k}{\sigma_j^2} - \sum_{j=1}^m \frac{y_j x_j^k}{\sigma_j^2}, \quad (\text{D.8})$$

and

$$\frac{\partial^2 \chi^2}{\partial \hat{a}_k \partial \hat{a}_l} = \sum_{j=1}^m \frac{x_j^k x_j^l}{\sigma_j^2} \equiv M_{kl}. \quad (\text{D.9})$$

To find the minimum χ^2 we set all derivatives (D.8) to zero, leading to

$$\sum_{i=0}^n \sum_{j=1}^m \frac{x_j^i x_j^k}{\sigma_j^2} \hat{a}_i = \sum_{j=1}^m \frac{y_j x_j^k}{\sigma_j^2} \equiv V_k. \quad (\text{D.10})$$

Using the matrix M_{kl} introduced in eq. (D.9) this can be written as

$$\sum_{i=0}^n M_{ik} \hat{a}_i = V_k. \quad (\text{D.11})$$

We then calculate the inverse matrix M^{-1} and apply it to find the desired coefficients:

$$\hat{a}_k = \sum_{l=0}^n M_{kl}^{-1} V_l. \quad (\text{D.12})$$

Comparing eqs. (D.7) and (D.9) we have

$$\frac{1}{\sigma_{kl}^2} = M_{kl}. \quad (\text{D.13})$$

The uncertainty in best-fit coefficient \hat{a}_i is then reported as

$$\sigma_{\hat{a}_i} = \sigma_{ii} = \frac{1}{\sqrt{M_{ii}}}. \quad (\text{D.14})$$

D.2 Procedure When the σ_j Are Not Known

This method can still be used even if the uncertainties σ_j on the measurements y_j are not known. When the functional form (D.1) correctly describes the data we claim that on average the minimum χ^2 has value $m - n - 1$.¹ To take advantage of this remarkable result we suppose that all uncertainties σ_j have a common value, σ . Then

$$\chi^2 = \sum_{j=1}^m \frac{[y_j - \hat{y}(x_j)]^2}{\sigma^2} \approx m - n - 1, \quad (\text{D.15})$$

so that

$$\sigma_j = \sigma = \sqrt{\frac{\sum_{j=1}^m [y_j - \sum_{i=0}^n \hat{a}_i x_j^i]^2}{m - n - 1}}. \quad (\text{D.16})$$

In practice it appears that the error estimates from this procedure are more realistic if a fit is made using a polynomial with one order higher than needed for a ‘good’ fit to the data.

Using eq. (D.16) as the estimate of the uncertainty σ on each of the measurements y_j , the matrix M_{kl} of eq. (D.9) becomes

$$M_{kl} = \frac{m - n - 1}{\sum_{j'=1}^m [y_{j'} - \sum_{i'=0}^n \hat{a}_{i'} x_{j'}^{i'}]^2} \sum_{j=1}^m x_j^k x_j^l. \quad (\text{D.17})$$

The estimate (D.14) of the uncertainty on the fit coefficient \hat{a}_i is now given by

$$\sigma_{\hat{a}_i} = \frac{1}{\sqrt{M_{ii}}} = \sqrt{\frac{\sum_{j'=1}^m [y_{j'} - \sum_{i'=0}^n \hat{a}_{i'} x_{j'}^{i'}]^2}{(m - n - 1) \sum_{j=1}^m x_j^{2i}}}. \quad (\text{D.18})$$

When WPtools performs a polynomial regression it generates a plot of the data points and the best-fit curve, along with numerical values of various parameters associated with the fit. Figure D.1 gives an example of a fit to a set of 8 data points of the form $y = x^2$. The fit is to the form $y = a_0 + a_1 x + a_2 x^2$. The fit coefficients are $a_0 = -0.4107$, $a_1 = -0.3274$ and $a_2 = 1.1964$. The uncertainties (standard errors) on the fit coefficients are reported as $\text{SE}(a_0) = 4.0070$, $\text{SE}(a_1) = 2.0429$ and $\text{SE}(a_2) = 0.2216$, as calculated according to eq. (D.18). *Note that the uncertainties on coefficients a_1 and a_1 are larger than the coefficients themselves, which tells us that these coefficients are indistinguishable from zero.*

Also indicated on the plot are the values $R^2 = 0.9915$ and $\sigma = 2.8721$. The latter is the uncertainty in the data points $\{y_j\}$, calculated according to eq. (D.16) with $m = 8$ and $n = 2$. The quantity R^2 is defined by

$$R^2 = \frac{\sum_{j=1}^m [\hat{y}(x_j) - \bar{y}]^2}{\sum_{j=1}^m [y(x_j) - \bar{y}]^2}, \quad (\text{D.19})$$

¹The whole fitting procedure does not make sense unless there are more data points (m) than parameters ($n + 1$) being fitted.

where the average $\bar{y} = \sum_{j=1}^m y(x_j)/m$. This is a measure of the “goodness of fit”. If the fit is perfect then $\hat{y}_j = y_j$ for all j and $R^2 = 1$. It is not obvious, but $R^2 \leq 1$ always. The extreme case of $R^2 = 0$ occurs when the fit has the trivial form $\hat{y}(x) = \bar{y}$ for all x , which in general is a bad fit. The qualitative conclusion is that if R^2 is not close to 1, the fit results are to be regarded with suspicion.

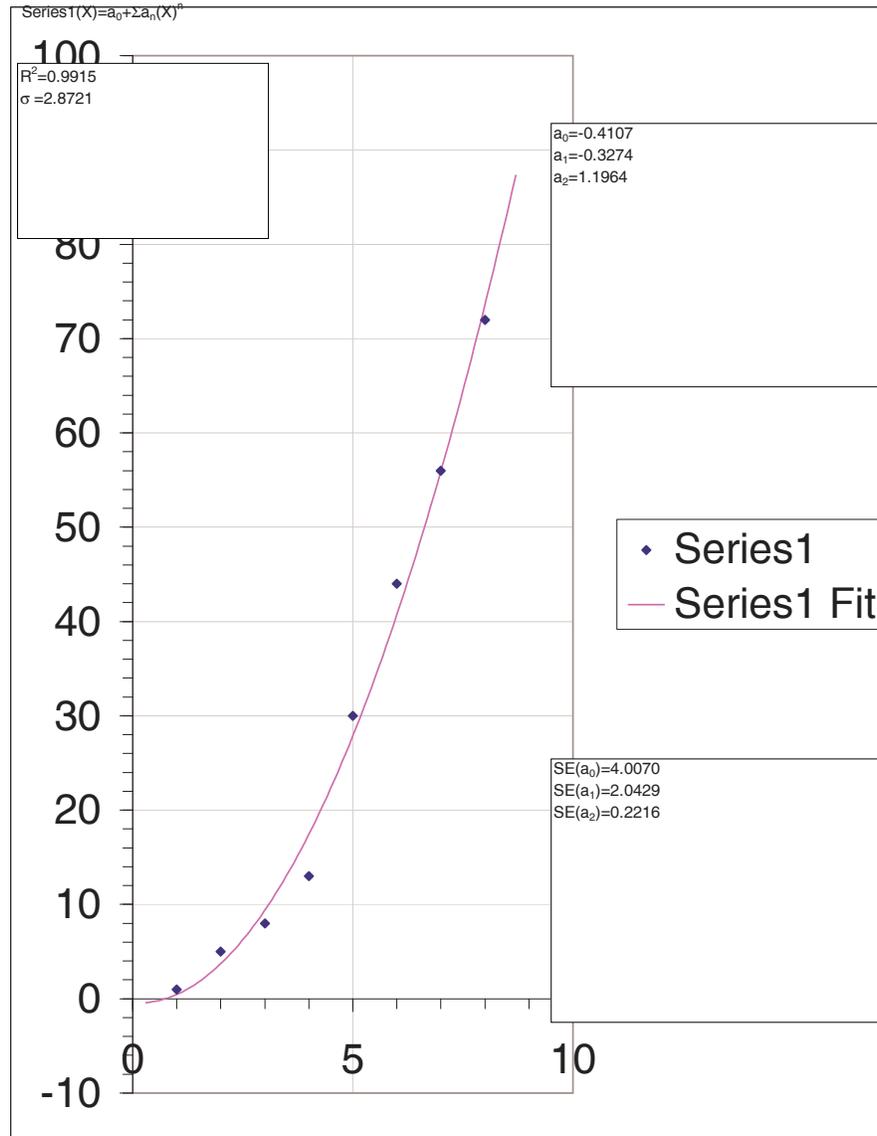


Figure D.1: Sample plot from WPtools Polynomial Fitting.