Solutions

NAME________________________

Please Circle Your Section's Time/Instructor

9:00am	Sohn		
10:00 am	Nice	10:00 am	Hasan
11:00 am	Nice	12:30 pm	Gubser
12:30 pm	Shutt	12:30 pm	Rastelli

Problem	Score
1	/19
2	/7
3	/18
4	/10
5	/11
6	/14

PHYSICS 101 MIDTERM EXAM

October 23, 2002 2 Hours

Instructions: When you are told to begin, check that this examination booklet contains all the numbered pages from 2 through 12. The exam contains 6 problems. Read each problem carefully. Box your final answer. Do not panic or be discouraged if you cannot do every problem; there are both easy and hard parts in this exam. **If part of a problem depends on a previous answer you have not obtained, pick a reasonable value and proceed.** Keep moving and finish as much as you can.

$L = I\omega$
$\mathrm{PE} = mgh$
$\omega = \omega_0 + \alpha t$
$v = v_0 + at$
$F = \mu_k N$
$\Sigma\tau = I\alpha$
$a_c = v^2/r$
$W_{nc} = \Delta\mathrm{KE} + \Delta\mathrm{PE}$
$\mathbf{p} = m\mathbf{v}$
$\mathrm{KE}_f + \mathrm{PE}_f = \mathrm{KE}_0 + \mathrm{PE}_0 + \mathrm{W}_{nc}$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$
$\mathrm{KE} = \frac{1}{2}I\omega^2$
$P = F\overline{v}$
$\overline{\mathbf{F}}\Delta t = \Delta\mathbf{p}$
$s = r\theta$
$v_t = r\omega$
$W = Fs\cos\theta$
$a_t = r\alpha$
$I = \frac{1}{2}mr^2$ [disk]

$P = W/t$
$\mathrm{KE} = \frac{1}{2}mv^2$
$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$F = \frac{GMm}{r^2}$
$\tau = F\ell\sin\theta$
$\Delta x = v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a\Delta x$
$I = \Sigma m_i r_i^2$
$F \leq \mu_s N$
$\mathbf{P}_f = \mathbf{P}_0 + \overline{\mathbf{F}}_{\mathrm{ext}}\Delta t$

Rewrite and sign the Honor Pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

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Signature________________________

1. Quickies

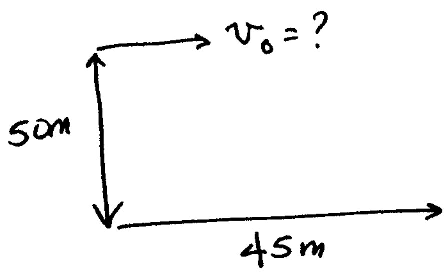
(a) A helicopter blade starts from rest and reaches an angular velocity $\omega = 40$ rad/s in 10 seconds. Assuming constant angular acceleration, how many revolutions does the blade make in these 10 seconds? [3 points]

$\omega_i = 0$
$\omega_f = 40$ rad/s
$t = 10s$
$\theta = ?$

$\omega_f = \cancelto{0}{\omega_i} + \alpha t$
$40 = 10\alpha$
$\alpha = 4\ \text{rad/s}^2$

$\omega_f^2 = \cancelto{0}{\omega_i^2} + 2\alpha\theta$
$40^2 = 2(4)\theta$
$\theta = 200\text{rad}\left(\frac{1\text{rev}}{2\pi\text{rad}}\right) \approx \boxed{32\text{rev.}}$

(b) A ball is thrown horizontally from the roof of a building 50 m tall and lands 45 m from the base. What was the ball's initial speed? [5 points]



$\hat{x}$

$45 = v_0 t$

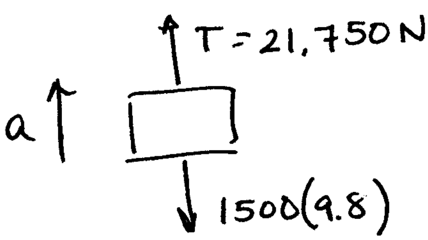
$v_0 = \frac{45}{3.2}$
$= \boxed{14\text{m/s}}$

$\hat{y}$

$d = -50\text{m}$
$v_i = 0$
$a = -9.8$
$t = ?$

$d = \frac{1}{2}at^2 + \cancelto{0}{v_i t}$
$-50 = \frac{1}{2}(-9.8)t^2$
$t = 3.2S$

(c) The cable supporting a 1500 kg elevator has a maximum strength of 21,750 N. What maximum upward acceleration can it give the elevator without breaking? [3 points]



$T - mg = ma$

$a = \frac{T - mg}{m}$

$= \frac{21{,}750 - 1500(9.8)}{1500}$

$= \boxed{4.7\ \text{m/s}^2}$

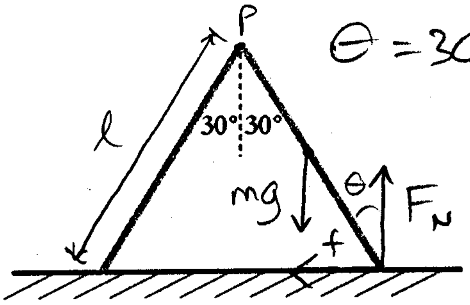
(d) An 80 g arrow is fired from a bow whose string exerts an average force of 95 N on the arrow over a distance of 80 cm. What is the speed of the arrow as it leaves the bow? [3 points]

Work-energy theorem: $W = Fs = \Delta(KE) = \frac{1}{2}mv^2$

$$V = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(95N)(0.8m)}{(0.08\,kg)}} = 44\,m/s$$

$$\boxed{v = 44\,m/s}$$

(e) An inverted "V" is made of uniform boards and weights 356 N. Each side has the same length and makes a 30° angle with vertical, as shown below. Find the magnitude of the static frictional force that acts on the lower end of each leg of the "V". [5 points]



$\theta = 30°$

$$mg = \frac{356\ N}{2} = 178N$$

① $\sum F_x = 0$ ② $\sum F_y = 0$ ③ $\sum \tau = 0$ around P

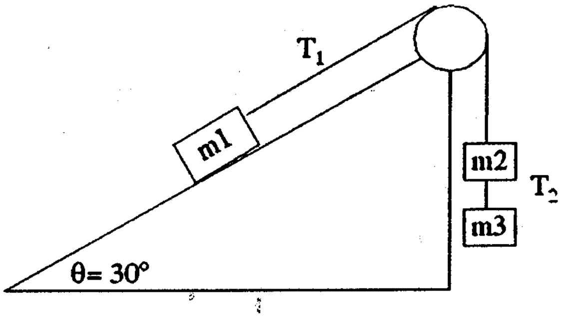
② $\Rightarrow F_N = mg$ ① we don't need

③ reads $\sum \tau_p = \ell F_N \sin\theta - \frac{\ell}{2} mg \sin\theta - \ell f \cos\theta = 0$

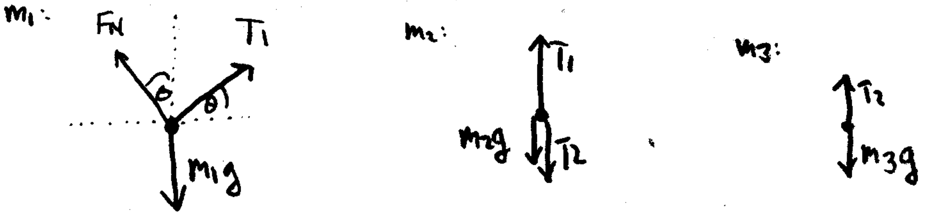
$\frac{1}{2} mg \sin\theta = f\cos\theta$ so $f = \frac{1}{2} mg \tan\theta$

$$\boxed{f = 51.4\,N}$$

2. Mass m_1 sits on a frictionless ramp at an angle of $\theta = 30°$. It is attached to mass m_2 via a rope winding over a pulley. Mass m_2 is attached, via another rope, to mass m_3, as shown. The ropes and the pulley are massless, and the pulley is frictionless.



(a) Draw a free body diagram for each of the three masses. [3 points]



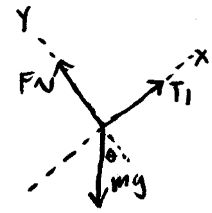
(b) The masses are in equilibrium—when released, they do not move. Masses m_2 and m_3 are each 5.0 kg. Find the tensions in the ropes, T_2, T_1, and the mass, m_1. [4 points]

Forces on mass 3: $\Sigma F_y = 0 \Rightarrow T_2 - m_3 g = 0 \Rightarrow T_2 = m_3 g = \boxed{49N} \leftarrow T_2$

Forces on mass 2: $\Sigma F_y = 0 \Rightarrow T_1 - T_2 - m_2 g = 0 \Rightarrow T_1 = T_2 + m_2 g = \boxed{98N} \leftarrow T_1$

Forces on mass 3:

Easy Way

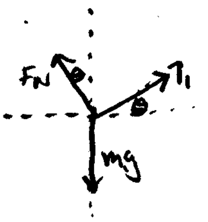


$\Sigma F_x = 0$

$T_1 - mg \sin\theta = 0$

$m_1 = \frac{T_1}{g \sin\theta} = \boxed{20 kg} \leftarrow m_1$

Hard Way



$\Sigma F_x = 0 \Rightarrow T_1 \cos\theta - F_N \sin\theta = 0$

$F_N = T_1 \frac{\cos\theta}{\sin\theta}$

$\Sigma F_y = 0$ plug in

$T_1 \sin\theta + F_N \cos\theta - m_1 g = 0$

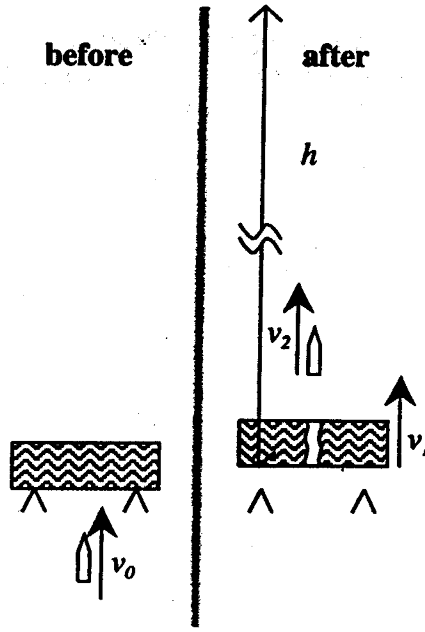
$T_1 \sin\theta + T_1 \frac{\cos\theta}{\sin\theta} \cdot \cos\theta - m_1 g = 0$

$T_1 \left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right) = m_1 g$

$m_1 = \frac{T_1}{g}\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right) = \boxed{20 kg} \leftarrow m_1$

[Note: the trig can be simplified: $\sin + \frac{\cos^2}{\sin} = \frac{\sin^2}{\sin} + \frac{\cos^2}{\sin} = \frac{\sin^2 + \cos^2}{\sin} = \frac{1}{\sin}$]

3. A bullet of mass m=0.01 kg is fired at a wooden block of mass M=1.00 kg as shown. The velocity of the bullet just before it strikes the block is v_0=850 m/s. The bullet passes through the block, and rises to a maximum height h=1200 m. Ignore air resistance.



(a). What is the velocity, v_2, of the bullet just after it exits the block? [3 points]

Apply E cons. after collision to bullet:

$$E_i = E_f$$

$$\frac{1}{2} m V_2^2 = mgh$$

$$V_2 = \sqrt{2gh} = 153 \text{ m/s}$$

(b) What is v_1, the velocity of the block, just after the bullet has left it? [4 points]

P cons. applies during collision:

$$m V_0 = M V_1 + m V_2$$

$$V_1 = \frac{m V_0 - m V_2}{M} = 6.97 \text{ m/s}$$

(c) What is the kinetic energy that is lost in the collision)? [3 points]

$$KE \text{ lost} = KE_i - KE_f$$

$$= \frac{1}{2} m V_0^2 - \left(\frac{1}{2} M V_1^2 + \frac{1}{2} m V_2^2\right)$$

$$= 3470 \text{ J}$$

(d) The bullet passes through the block in 3 x 10^{-5} s. What is the average force (magnitude and direction) that the block exerts on the bullet? [4 points]

$$\bar{F}\Delta t = \Delta p$$

Apply this to bullet:

$$\bar{F} = \frac{p_{f,bullet} - p_{i,bullet}}{\Delta t}$$

$$= \frac{mV_2 - mV_0}{\Delta t} = -2.32 \cdot 10^5\ N$$

The negative sign shows that the force is downwards.

(e) Now the whole incident is repeated, except that the bullet is not aimed directly at the center of the block, and the block spins with ω=30 rad/s as it flies into the air. The block is L=30.0 cm long. The exit velocity of the bullet v_2 is the same. For a block $I=(1/12)ML^2$. What is v_1 in this case? [4 points]

Linear momentum is still conserved:

$$mV_0 = MV_1 + mV_2$$

and, because V_2 is the same, so will be V_1:

$$V_1 = \frac{mV_0 - mV_2}{M}$$

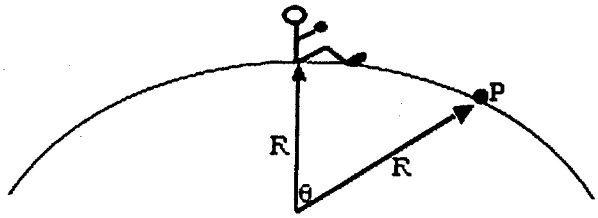
$$= 6.97\ m/s$$

same as before.

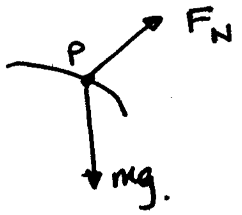
(The energy situation does change because the block spins: the translational KE is the same, but, since there is now rotational KE, less energy must be lost in the collision to heat + deformation than in the case with no rotation.

- But this doesn't change v.)

4. A boy of mass M is seated on top of a hemispherical mound of ice of radius R as shown below. He starts to slide down the ice and eventually flies off the mound of ice. The ice is frictionless.



(a) Draw a free body diagram for the boy when he is at point P. [2 points]



(b) At angle θ, what is the boy's velocity? Express your answer in terms of R, g, and θ. [4 points]

Conserv. of Energy:

$$(PE + KE)_i = (PE + KE)_f$$

$$\cancel{m}gR = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gR\cos\theta$$

$$2gR = v^2 + 2gR\cos\theta$$

$$v^2 = 2gR - 2gR\cos\theta$$

$$\boxed{v = \sqrt{2gR(1-\cos\theta)}}$$

(c) What is θ_o, the angle at which the boy flies off the ice mound? [4 points]

Boy flies off when $F_N = 0$

From (a) :

$$\cancelto{0}{F_N} - mg\cos\theta = -\frac{mv^2}{R}$$

$$\cancel{m}g\cos\theta = \frac{v^2}{R}$$

$$v^2 = R\cancel{m}g\cos\theta$$

Plug in v^2 from (b) :

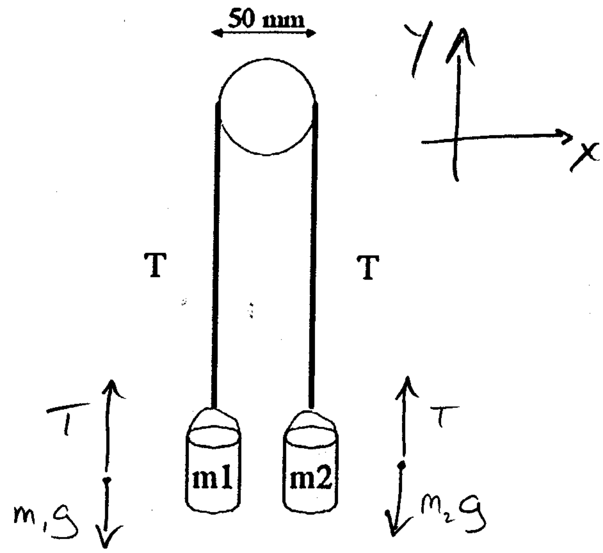
$$2g\cancel{R} - 2g\cancel{R}\cos\theta = \cancel{m}g\cancel{R}\cos\theta$$

$$2 - 2\cos\theta = \cos\theta$$

$$3\cos\theta = 2$$

$$\boxed{\begin{array}{c}\cos\theta = 2/3 \\ \theta = 48^\circ\end{array}}$$

5. Two buckets full of nails are connected by a light cord which pass over a light, frictionless pulley with a diameter of 50 mm. Initially, the two bodies are held at the same level. The buckets including the nails, have mass m_1=500g and m_2=600g.



(a) The buckets are released from rest and at the same height at t= 0. Calculate the tension of the string and the accelerations of the buckets after they are released. [4 points]

$\vec{F} = m\vec{a}$ applies to each mass

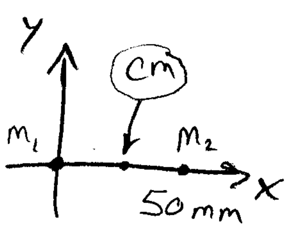
$m_1: \; T - m_1 g = m_1 a_1$ $\qquad$ $m_2: \; T - m_2 g = m_2 a_2$

but $a_1 + a_2 = 0$ so $T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - 2g = 0$ $\qquad$ $T = \frac{2m_1 m_2}{m_1 + m_2} g$

$a_1 = -a_2 = \frac{T}{m_1} - g = \frac{m_2 - m_1}{m_1 + m_2} g$ $\qquad$ $\boxed{a_1 = -a_2 = 0.89 \frac{m}{s^2} \qquad T = 5.3\ N}$

(b) Where is the center of mass at the instant the buckets are released? [3 points]

$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$ $\qquad$ $y_{cm} = 0$, level with both masses.



$$x_{cm} = \frac{m_2 (50\ mm)}{m_1 + m_2} = 27\ mm$$

$\boxed{\text{27 mm to the right of the cm of mass 1}}$

(c) Where is the center of mass after they are released? Your answer should be written in terms of time, t. What is the acceleration of the center of mass? [4 points]

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \qquad Y_1 = \frac{1}{2} a_1 t^2 \qquad Y_2 = \frac{1}{2} a_2 t^2$$

X_{cm} = const, same as in part b

$$Y_{cm} = \frac{1}{2} \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} t^2 = \frac{1}{2} \frac{m_1 - m_2}{m_1 + m_2} a_1 t^2$$

$$= -\frac{1}{2} \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g t^2 = -\left(0.040 \frac{m}{s^2}\right) t^2$$

$$\boxed{Y_{cm} = -\left(0.040 \frac{m}{s^2}\right) t^2} \qquad a_{cm} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

$\boxed{a_{cm} = -0.081 \frac{m}{s^2}}$, ie in the downward direction

6. On a playground, there is a small carousel of radius r=2.00 m and mass M= 300 kg. Its mass is uniformly distributed in a disk shape. (There are handles which children can grab, but they have negligible mass). For a solid disk, $I=MR^2/2$.

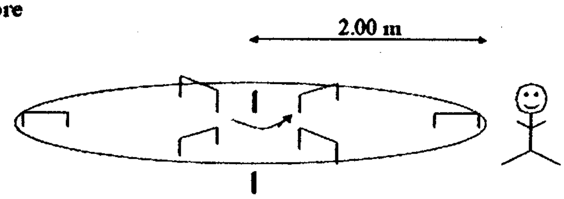
(a) If the carousel rotates once every 3.0 sec, what is its angular velocity in radians? [2 points]

$$\omega = \frac{\Delta\theta}{T} = \frac{2\pi}{3\,\text{sec}} = \boxed{2.09\ \frac{\text{rad}}{\text{sec}}}$$

(b) Mercedes, whose mass is m=15.0 Kg, has been standing at rest next to the carousel. Suddenly, she grabs one of the handles and pulls herself up onto the carousel. She stays at the edge of the carousel, at radius 2.0 m. What is the carousel's angular velocity now? [4 points]

1 = carousel   2 = Mercedes

Before



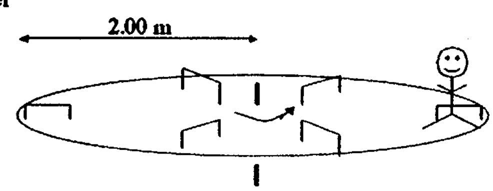
carousel is a solid disk, so

$I_1 = \frac{1}{2}Mr^2 = \frac{1}{2}(300\text{kg})(2\text{m})^2 = 600\,\text{kgm}^2$

Mercedes's mass is all at $r = 2\text{m}$, so

$I_2 = mr^2 = (15\text{kg})(2\text{m})^2 = 60\,\text{kgm}^2$

After



Conserve angular momentum, noting that

$\omega_{1o} = 2.09\ \frac{\text{rad}}{\text{sec}}$ $\omega_{2o} = 0$ $\omega_{1f} = \omega_{2f} = \omega_f$

$$L_f = L_o + \cancel{\tau_{ext}\Delta t}^{\,0}$$

$$(I_1 + I_2)\,\omega_f = I_1\,\omega_{1o}$$

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_o = \frac{600}{600+60}\left(2.09\,\frac{\text{rad}}{\text{s}}\right) = \boxed{1.90\,\frac{\text{rad}}{\text{s}}}$$

(c) What is the change in energy when Mercedes gets on the carousel? [4 points]

In general, $KE = \frac{1}{2} I \omega^2$ for a rotating object

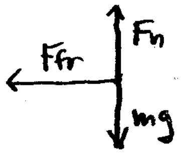
$$\Delta E = KE_f - KE_0 = \frac{1}{2}(I_1 + I_2)\,\omega_f^2 - \frac{1}{2} I_1\, \omega_{10}^2$$

$$= \frac{1}{2}(600\ kgm^2 + 60\ kgm^2)(1.90\ \tfrac{1}{sec})^2 - \frac{1}{2}(600\ kgm^2)(2.09\ \tfrac{1}{sec})^2$$

$$= -120\ \frac{kgm^2}{s^2}$$

$$\boxed{\Delta KE = -120\ J}$$

(d) Mercedes lets go of the handle. What is the minimum coefficient of static friction between her shoes and the carousel such that she doesn't slide off the carousel? [4 points]



The diagram shows the Forces on Mercedes when she is on the right side of the carousel, as pictured in part b. She must be accelerated towards the center of the carousel, and friction is the only force that can do it.

$$\Sigma F_y = 0$$
$$F_n - mg = 0$$
$$F_n = mg$$

$$\Sigma F_x = m a_c$$
$$-F_{fr} = m\left(-\frac{v^2}{r}\right) \leftarrow \text{centripetal acceleration to the left.}$$
$$F_{fr} = \frac{mv^2}{r}$$

But $F_{fr} \le \mu F_n = \mu mg$, so

$$\cancel{m}\frac{v^2}{r} \le \mu \cancel{m} g$$

$$\frac{v^2}{gr} \le \mu \qquad v = \omega r$$

$$\frac{\omega^2 r^{\cancel{2}}}{g\cancel{r}} \le \mu$$

$$\mu \geq \frac{\omega^2 r}{g} = \left(\frac{1.90}{sec}\right)^2 \frac{(2m)\ sec^2}{9.8m} = \boxed{0.74}$$