approximately 8.5 kHz at 0.0 s delay to a minimum of approximately 4.5 kHz at 0.75 s delay.

Difficulty in obtaining DSFS with distinct cutoff frequencies arises when eye movements occur during the measurement. When movements carry the retinal vessel out of the path of the incident beam, the scattered light is collected either entirely from the choroidal vasculature, located immediately behind the retina, or partially from the retinal vessel and partially from the choroid. In these cases, the spectra exhibit a rapidly decreasing amplitude at frequency shifts that are low compared to the expected cutoff frequency.

CONCLUSION

Our measurements show that DSFS of the light scattered from RBC’s flowing individual human retinal vessels can be obtained rapidly and with relative ease. Furthermore, the spectra exhibit distinct cutoff frequencies that are directly related to the maximum RBC speeds by the basic Doppler formula. We expect that this technique will become a very valuable tool for the diagnosis and management of circulatory disorders of the eye. In addition, because measurements can be made in less than a second, this technique will enable us to monitor fast changes in the retinal blood flow induced by controlled physiological conditions.

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I. IMAGE-FORMING AND NON-IMAGE-FORMING CONCENTRATORS

In conventional optical technology, problems of light concentration are usually attacked by using image-forming optical systems, of which the microscope condenser of high numerical aperture is typical. Occasionally non-image-forming systems such as hollow reflecting cones have been used for such purposes as concentrating light onto small detectors. In fact it turns out that neither image-forming systems nor cones come anywhere near the theoretical limit in concentration. In the last decade a new class of non-image-forming concentrators has been applied to other purposes requiring efficient light concentration. It has been shown that the new collectors approach very closely to the theoretical limit and that for a two-dimensional geometry, i.e., a cylindrical shape with generators indefinitely extended by means of plane end mirrors, the maximum theoretical concentration \((\sin \theta_1)^{-1}\) for this case is actually reached; however, the three-dimensional form, obtained as a surface of revolution from the two-dimensional form, is not quite ideal: it is found that a small proportion of rays at and near the edge of the exit aperture of correct diameter and the result should be an ideal concentrator.

This principle is used implicitly in many designs, for example, in the earliest systems the profile is a portion of a parabola but the axis of symmetry is not the parabola axis (Fig. 2); in two dimensions, i.e., with straight generators perpendicular to the diagram, it can be shown that this is an ideal concentrator. However, the 3D system obtained by revolving the profile of Fig. 2 about the axis of symmetry is not quite ideal: it is found that a small proportion of rays at and near the extreme angle are turned back to emerge from the entry end after multiple reflections inside the concentrator.

We can show that fulfillment of the edge-ray principle is a necessary condition for a concentrator to be ideal. Suppose the ratio of exit to entrance apertures is \(\sin \theta_1\). First let a ray incident at an angle \(\theta_1\) to the axis not be transmitted by the concentrator; then if we decrease the angle of incidence to \(\theta_1\) until the ray is just transmitted, we have found a set of rejected rays incident at angles between \(\theta_1\) and \(\theta_1\) which are outside the phase space volume \(H\) which ought to be transmitted if the concentrator is ideal. Secondly suppose a ray incident at angle \(\theta_1\) is transmitted but emerges clear of the edge of the exit aperture; then we can increase the angle of incidence to \(\theta_1\) until the ray just grazes the edge of the exit aperture and we have found a set of accepted rays which are outside the phase space volume \(H\) which ought to be transmitted if the concentrator is ideal; but since no greater phase space volume than \(H\) can

II. GARWIN'S THEOREM

In a certain sense the question, can the theoretical limit be achieved for a 3D system, has already been answered. Garwin\(^7\) showed that a light pipe of any initial cross section could in general be continued and changed in shape and cross-sectional area in such a way as to achieve any concentration up to the theoretical limit; unfortunately the deformation of the pipe must proceed adiabatically, i.e., there must be a very large number of reflections of each ray over any length of pipe in which an appreciable change in cross section takes place. In other words, the concentrator would have to be infinitely long and the reflections would attenuate the light power to zero. Thus in considering what is possible we should add a restriction to systems of finite length.

III. MINIMUM REQUIREMENT FOR AN IDEAL CONCENTRATOR: THE EDGE-RAY PRINCIPLE

When we admit the notion of non-image-forming concentrators we see that the requirements are much less stringent than for image-forming systems. Let \(PP'\) in Fig. 1 be the entry aperture of a concentrator to be designed to accept rays up to angles \(\pm \theta_1\); if it is an ideal concentrator then all these rays must emerge from the exit aperture and this must have diameter \(QQ' = PP' \sin \theta_1\). Suppose that the optical system inside the concentrator is such that rays at \(\theta_1\), i.e., extreme angle rays, emerge just grazing the edge of the exit aperture of this size. Then it is a reasonable conjecture that with any moderately simple optical system in the concentrator any ray entering at an angle less than \(\theta_1\) will get through the exit aperture. Thus our edge-ray principle is to design the concentrator to make all rays at the extreme angle \(\theta_1\) just cut the edge of the exit aperture of correct diameter and the result should be an ideal concentrator.

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be transmitted even by an ideal concentrator it follows that some rays inside the wanted phase space volume must be rejected. Thus all rays incident at $\theta_i$ must emerge at the edge of an ideal concentrator, i.e., the edge-ray principle is a necessary condition.

**IV. EDGE-RAY PRINCIPLE AND IMAGE-FORMING SYSTEMS**

J. Clerk Maxwell showed that a perfect optical system in the broadest sense is impossible but the requirements for an ideal concentrator seem to be much less stringent, according to the edge-ray principle, so it is reasonable to ask whether an image-forming system could be designed to fulfill them.

First, if an image-forming system is to have the maximum theoretical concentration ratio it must, from Sec. III, have a relative aperture $F/0.5$ and should have perfect correction at a field angle $\theta_i$. This seems on the basis of classical optical design experience to be impossible; the only systems approaching this requirement are certain microscope objectives of very high numerical aperture and then only for $\theta_i$ less than $3^\circ$ and a narrow wavelength range.

If we relax the requirement for $F/0.5$ and ask whether an image-forming system can fulfill the edge-ray principle alone, certain possibilities suggest themselves but these turn out to be systems with concentration ratios much below the theoretical maximum. Suppose we have an axisymmetric optical system with $N$ surfaces, all of which may be aspheric, and let the $j$th surface be as in Fig. 3; we show in the figure some meridian rays of the extreme off-axis pencil, i.e., the pencil of which the rays must just graze the edge of the exit aperture. If this condition is fulfilled we can express it either in terms of optical path lengths along the rays or in terms of the angles of incidence and refraction; either way there will be an equation of the form

$$F(Y_1, z_1(y_1), Y_2, z_2(y_2), \ldots, Y_N, z_N(y_N)) = 0. \quad (1)$$

In this equation we may take $y_1$ as a parameter governing the rays of the pencil and then $y_2, y_3, \ldots, y_N$ will be functions of $y_1$ and of the angle $\theta_i$ of the extreme off-axis pencil. Since Eq. (1) must be satisfied for all $y_1$ within a certain range it must be an identity and thus be equating the coefficients of powers of $y_1$ to zero we obtain certain relations between the coefficients in the power series for the $z_i$. But these coefficients are for even power series, since the optical system is axisymmetric, and if we change the sign of $y_1$ we shall get a different set of relations which are inconsistent. That is, we are trying to make a set of even functions satisfy conditions which are not even.

We can express this another way by considering points such as $P_1$ and $P_3$ on rays 1 and 3 in Fig. 3; let these points be equidistant from the axis of symmetry. Ray 3 has in general a different angle of incidence from ray 1 so that the functional form of the identity, Eq. (1), for rays near $P_3$ will be different from that for $P_1$. Then if the surfaces $z_j(y_j)$ are expanded as power series we see that they have to satisfy two different identities,

$$F_1 = 0 \quad \text{and} \quad F_2 = 0, \quad (2)$$

one of these for rays incident above the axis and the other for rays incident below the axis, and this is impossible. Thus in general it is not possible to correct a system with a finite number of surfaces for meridian rays to one off-axis object point, much less for skew rays. There are well-known exceptions to this, namely, systems of spherical symmetry such as Maxwell’s fish-eye lens and the Luneburg lens, since in such systems the symmetry ensures that off-axis pencils of rays are identical to the axial pencil; also, if we do not demand that both object and image shall be real, a single spherical surface can be used at the aplanatic conjugates. The figure gives us a clue to another solution: if we have a surface in the system so placed that all the rays meet it on one side only of the axis, i.e., very remote from the aperture stop in ordinary lens design language, then this surface does not have to satisfy contradictory conditions and the edge-ray condition can be satisfied. This is in fact how the system of Fig. 2 and others derived from it work. Nevertheless, there is still no correction guaranteed for skew rays.

There has been much discussion in the literature about the possibility of perfect optical systems according to different definitions. This has taken the form of deriving conditions which the aberrations of the system must fulfill if it is to be perfect in the sense under discussion and generally the aberrations have been terms of an expansion starting from the axis of symmetry, rather than at a finite field angle. If the question of existence was discussed it was always in terms of correction near the axis to a certain approximation in the field angle, as for example in the many solutions for two mirror aplanats. Of course, in a practical sense perfect optical systems are possible if the aberrations fall within suitable tolerance ranges and if the tolerances are set according to physical optics criteria such systems are as perfect as they can be over the chosen aperture and field, but it is well known that the tolerances will always be exceeded if the linear scale of the system is sufficiently increased.

Thus we conclude that conventional image-forming systems are not suitable for concentrators, even if we relax the requirement for maximum theoretical concentration and ask only for a well-defined collecting angle, i.e., fulfillment of the edge-ray condition.

**V. NON-IMAGE-FORMING SYSTEMS**

By comparison it is trivially simple to design a non-image-forming concentrator in two dimensions which will have the maximum theoretical concentration; the basic solution was
shown in Fig. 2 and many elaborations and variations have since been published. The use of reflection to operate on the extreme pencils with the part of an optical surface on one side only of the axis of symmetry seems to be essential to the success of these designs. We should also remark that we distinguish them from image-forming systems not merely because, considered as image-forming elements, they would have absurdly large aberrations but because the different rays of a single pencil traverse qualitatively different paths through the concentrator in that they have different numbers of reflections (including zero), whereas in an image-forming system all wanted rays have the same number of reflections and refractions. Another distinction is that performance in an image-forming system can be assessed, in the geometrical optics approximation, by the accuracy with which rays of a single pencil come together in a simple image point, whereas in a non-image-forming concentrator the important thing is simply that all rays of an accepted pencil should emerge somewhere at the exit aperture: non-ideal performance is manifested as turning back of some of the rays after multiple reflections and their reemergence from the entry aperture.

However, Fig. 2 shows a construction which is ideal only for two-dimensional systems (it is easily seen that it is then ideal also for rays out of the plane of the diagram). It can be seen also that this is a unique solution subject to the condition that rays entering at the extreme angle emerge after at most one reflection, since the edge-ray principle determines the profile completely; if more than one reflection is permitted for extreme rays the solution of Fig. 2 is not unique; for example, one could place any odd number of such systems in tandem alternately reversed in direction. If we convert the system to one of axial symmetry in three dimensions by rotating it about the center line we find that some rays inside the nominal entry angle \( \theta_i \) are rejected and others outside \( \theta_i \) are transmitted. This result, obtained by rather tedious raytracing, is illustrated in Fig. 4; the ordinate represents the proportion of rays transmitted at a given entry angle; the full line is for a two-dimensional system, the broken line for a three-dimensional system with the same entry angle \( \theta_i \) but with rotational symmetry.

But if we want to improve the performance of the three-dimensional system we have no more degrees of freedom: the profile of an axial section is completely determined by the need to ensure that meridian rays satisfy the edge-ray condition and in producing the figure of rotation we have gained no more parameters. We therefore conclude that if we restrict ourselves to systems of finite length, i.e., excluding Garwin systems, no three-dimensional concentrator can be ideal, although they can approach much more nearly to the ideal condition than an image-forming system.

M. Ploke, Optik (Stuttg.) 25, 31–43 (1967).
For a useful survey giving several of these variations see A. Rabl, Solar Energy 18, 93–111 (1976).