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For short coils we have also the alternative series given in (18). We find then

\[ L = 2\pi dN^2 \left[ 2 \left( 1 + \frac{1}{8} \left( \frac{h}{a} \right)^2 - \frac{1}{64} \left( \frac{h}{a} \right)^4 \right) \log \frac{4a}{h} - 1 \right. \]

\[ + \frac{1}{16} \left( \frac{h}{a} \right)^2 + \frac{1}{48} \left( \frac{h}{a} \right)^4 \]. \quad \ldots \quad (32) \]

By using an expression for \( L \) in terms of elliptic integrals and expanding, Coffin * has obtained a series for \( L \) with which (32) agrees; Coffin's series was evaluated up to terms in \((h/a)^n\). Instead of using one such complicated series, it seems that the two series (31) and (32) should cover between them all the cases that occur in practice.

XXXIII. Effect of a Prism on Newton's Rings.

By Lord Rayleigh, O.M., Pres.R.S.†

When Newton's rings are regarded through a prism (or grating) several interesting features present themselves, and are described in the "Opticks." Not only are rings or arcs seen at unusual thicknesses, but a much enhanced number of them are visible, owing to approximate achromatism—at least on one side of the centre. The first part of the phenomenon was understood by Newton, and the explanation easily follows from the consideration of the case of a true wedge, viz. a plate bounded by plane and flat surfaces slightly inclined to one another. Without the prism, the systems of bands, each straight parallel and equidistant, corresponding to the various wave-lengths (\( \lambda \)) coincide at the black bar of zero order, formed where the thickness is zero at the line of intersection of the planes. Regarded through a prism of small angle whose refracting edge is parallel to the bands, the various systems no longer coincide at zero order, but by drawing back the prism, it will always be possible so to adjust the effective dispersive power as to bring the \( n \)th bars to coincidence for any two assigned colours, and therefore approximately for the entire spectrum.

In this example the formation of visible rings at unusual thicknesses is easily understood; but it gives no explanation of the increased numbers observed by Newton. The width of the bands for any colour is proportional to \( \lambda \), as well after the displacement by the prism as before. The manner of

† Communicated by the Author.
overlapping of two systems whose \( n \)th bars have been brought to coincidence is unaltered; so that the succession of colours in white light, and the number of perceptible* bands, is much as usual.

"In order that there may be an achromatic system of bands, it is necessary that the width of the bands near the centre be the same for the various colours. As we have seen, this condition cannot be satisfied when the plate is a true wedge; for then the width for each colour is proportional to \( \lambda \). If, however, the surfaces bounding the plate be curved, the width for each colour varies at different parts of the plate, and it is possible that the blue bands from one part, when seen through the prism, may fit the red bands from another part of the plate. Of course, when no prism is used, the sequence of colours is the same whether the boundaries of the plate be straight or curved."

In the paper † from which the above extracts are taken, the question was further discussed, and it appeared that the bands formed by cylindrical or spherical surfaces could be made achromatic, so far as small variations of \( \lambda \) are concerned, but only under the condition that there be a finite separation of the surfaces at the place of nearest approach. If \( a \) denote the smallest distance, the region of the \( n \)th band may form an achromatic system if

\[
a = \frac{1}{4} n \lambda.
\] (1)

At the time pressure of other work prevented my examining the question experimentally. Recently I have returned to it and I propose now to record some observations and also to put the theory into a slightly different form more convenient for comparison with observation.

For the present purpose it suffices to treat the surfaces as cylindrical, so that the thickness is a function of but one coordinate \( x \), measured along the surfaces in the direction of the refraction. The investigation applies also to spherical surfaces if we limit ourselves to to points lying upon that diameter of the circular rings which is parallel to the refraction ‡. If we choose the point of nearest approach as the origin of \( x \), the thickness may be taken to be

\[
t = a + bx^2, \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)
\]

* Strictly speaking the number of visible bands is doubled, inasmuch as they are now formed on both sides of the achromatic band.
‡ In the paper referred to the general theory of curved achromatic bands is considered at length.
where $b$ depends upon the curvatures. The black of the $n$th order for wave-length $\lambda$ occurs when

$$\frac{1}{2}n\lambda = a + bx^2, \quad \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

or

$$x = \sqrt{\left(\frac{1}{2}n\lambda - a\right)/b}, \quad \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

so that

$$\frac{dx}{d\lambda} = \frac{\frac{1}{2}n}{\sqrt{b} \cdot \sqrt{\left(\frac{1}{2}n\lambda - a\right)}}. \quad \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

The $n$th band, formed actually at $x$, is seen displaced under the action of the prism. The amount of the linear displacement ($\xi$) is proportional to the distance $D$ at which the prism is held, so that we may take approximately

$$\frac{d\xi}{d\lambda} = -\beta \cdot D, \quad \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

$\beta$ representing the dispersive power of the prism, or grating. The condition that the $n$th band may be achromatic (for small variations of $\lambda$) is accordingly

$$d(x + \xi) = 0, \quad \ldots \ldots \ldots \ldots \ldots \ldots (7)$$

or

$$\frac{1}{16} \beta^2 D^2 b = \frac{1}{2}n\lambda - a, \quad \ldots \ldots \ldots \ldots \ldots \ldots (8)$$

a quadratic in $n$. The roots of the quadratic are real, if

$$\beta^2 D^2 b > a/\lambda^2. \quad \ldots \ldots \ldots \ldots \ldots \ldots (9)$$

If $a$ be zero, the condition (9) is satisfied for all values of $D$, so that at whatever distance the prism be held there is always an achromatic band. And if $a$ be finite, the condition can still always be satisfied if the prism be drawn back far enough.

From (8) if $n_1, n_2$ be the roots,

$$\frac{1}{n_1} + \frac{1}{n_2} = \frac{\lambda}{2a} \ldots \ldots \ldots \ldots \ldots (10)$$

Again, if $a=0$, that is if the plates be in contact, $n_1=0$, and

$$n_2 = 8\lambda \beta^2 D^2 b. \quad \ldots \ldots \ldots \ldots \ldots (11)$$

The order of the achromatic band increases with the dispersive power of the prism and with the distance at which it is held. The corresponding value of $x$ from (4) is

$$x = 2\lambda \beta D. \quad \ldots \ldots \ldots \ldots \ldots (12)$$
If \( a \) be finite, there is no achromatic band so long as \( D \) is less than the value given in (9). When \( D \) acquires this value, the roots of the quadratic are equal, and

\[
\frac{1}{n_1} = \frac{1}{n_2} = \frac{\lambda}{4a},
\]

or

\[
n_1 = n_2 = \frac{4a}{\lambda}. \quad \ldots \quad (13)
\]

This is the condition formerly found for an achromatic system of bands. If \( D \) be appreciably greater than this, two values of \( n \) satisfy the condition, viz. there are two separated achromatic bands, though no achromatic system. From (8)

\[
n_1n_2 = 16ab\beta^2D^2, \quad \ldots \quad (14)
\]

Thus if \( D \) be great, one of the roots, say \( n_2 \), becomes great, while the other, see (10), approximates to \( 2a/\lambda \), that is to half the value appropriate to the achromatic system (13).

There is no particular difficulty in following these phenomena experimentally, though perhaps they are not quite so sharply defined as might be expected from the theoretical discussion, probably for a reason which will be alluded to presently. It is desirable to work with rather large and but very slightly curved surfaces. In my experiments the lower plate was an optical "flat" by Dr. Common, about six inches in diameter and blackened behind. The upper plate was wedge-shaped with surfaces which had been intended to be flat but were in fact markedly convex. In order to see the bands well, it is necessary that the luminous background, whether from daylight or lamp-light, be uniform through a certain angle, and yet this angle must not be too large. Otherwise it is impossible to eliminate the light reflected from the upper surface of the upper plate, which to a great extent spoils the effects. In my case it sufficed to use gas-light diffused through a ground-glass plate whose angular area was not so great but that the false light could be thrown to one side in virtue of the angle between the upper and lower surfaces of the wedge.* It will be understood that these precautions are needed only in order to see the effects at their best. The most ordinary observation and appliances suffice to exhibit the main features.

Another question which I was desirous of taking the opportunity to examine was one often propounded to me by my lamented friend Lord Kelvin, viz. the nature of the

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obstruction usually encountered in trying to bring two surfaces nearly enough together to exhibit the rings of low order. In favour of the view that the obstacle is merely dust and fibres, I remember instancing the ease with which a photographic print, enameled by being allowed to dry in contact with a suitably prepared glass plate, could be brought back into optical contact after partial separation therefrom. My recent observations with the glass plates point entirely in the same direction. However carefully the surfaces are cleaned by washing and wiping—finally with a dry hand, the rings of low order can usually be attained only at certain parts of the surface*. If we attempt to shift them to another place chosen at random, they usually pass into rings of higher order or disappear altogether. On the other hand, when rings of low order have once been seen at a particular place, it is usually possible to lift the upper glass carefully and to replace it without losing the rings at the place in question. I have repeatedly lifted the glass when the centre of the system was showing the white of the first order or even the darkening (I do not say black) corresponding to a still closer approximation, and found the colour recovered under no greater force than the weight of the glass. Some time is required, doubtless in order that the air may escape, for the complete recovery of the original closeness; but in the absence of foreign matter it appears that there is no other obstacle to an approximation of say \( \frac{1}{8} \lambda \).

In making the observations it is convenient to introduce a not too small magnifying lens of perhaps 8 inches focus and to throw an image of the source of light upon the pupil of the eye. With the glasses in contact it is easy to trace the rise in the order of the achromatic band as the eye and prism are drawn back. As regards the latter a directvision instrument of moderate power (three prisms in all) is the most suitable. An interval between the glasses may be introduced by stages. When the approximation is such as to show colours of the 3rd or 4th orders at the centre, it becomes apparent that the best achromatic effects are attained when the prism is at a certain distance, and that when this distance is exceeded the more achromatic places are separated

* The plates are here supposed to be brought together without sliding. By a careful sliding together of two surfaces, the foreign matter may be extruded, as in Hilger's echelon gratings, where optical contact is attained over considerable areas.

by a region where the bands are fringed with colour. This feature becomes more distinct as the interval is still further increased, so that without the prism only faint rings or none at all can be perceived. For the greater intervals the interposition of a piece of mica at one edge is convenient. In judging of the degree of achromatism, I found that narrow coloured borders could be recognized as much more easily by one of my eyes than by the other, and the difference did not seem to depend on any matter of focussing.

In observing bands of rather high order, the question obtruded itself as to whether the achromatism was anywhere complete. It will have been remarked that the theoretical discussion, as hitherto given, relates only to a small range of wave-length and that no account is taken of what in the telescope is called secondary colour. So long as this limitation is observed, the character of the dispersive instrument does not come into play. It appeared, however, not at all unlikely that even with gaslight the range of wave-length included might be too great to allow of this treatment being adequate; and with daylight, of course, the case would be aggravated. It is thus of interest to examine what law of dispersion is best adapted to secure compensation and in particular to compare the operation of a prism and a grating.

As to the law of dispersion to be aimed at, we have from (4), if \( \lambda = \lambda_0 + \delta \lambda \),

\[
x = \left( \frac{\frac{1}{2} n \lambda_0 - a}{b} \right)^{\frac{1}{2}} \left\{ 1 + \frac{\frac{1}{2} n \delta \lambda}{\frac{1}{2} n \lambda_0 - a} - \frac{1}{8} \left( \frac{\frac{1}{2} n \delta \lambda}{\frac{1}{2} n \lambda_0 - a} \right)^2 + \ldots \right\} (15)
\]

If \( \xi \) be the displacement due to the instrument, \( \xi \) should be a similar function of \( \delta \lambda \). In this matter the constant terms (independent of \( \delta \lambda \)) are of no account, and the terms in \( \delta \lambda \) may be adjusted to one another, as already explained, by suitably choosing the distance \( D \). In pursuing the approximation, what we are concerned with is the ratio of the term in \( \left( \delta \lambda \right)^2 \) to that in \( \delta \lambda \). And in (15) this ratio is

\[
\frac{1}{8} \frac{n \delta \lambda}{\frac{1}{2} n \lambda_0 - a} : \ldots \ldots \ldots (16)
\]

thus in the particular cases

\[
a = 0, \quad \frac{1}{4} \frac{\delta \lambda}{\lambda_0} : \ldots \ldots \ldots (17)
\]

\[
a = \frac{1}{2} n \lambda_0, \quad \frac{1}{2} \frac{\delta \lambda}{\lambda_0} : \ldots \ldots \ldots (18)
\]
Corresponding expressions are required for the dispersive instruments. In any particular case they could of course be determined; but no very simple rules are available in general. If the intrinsic dispersion be small—the necessary effect being arrived at by increasing $D$, we may make the comparison more easily. Thus in the case of the grating the variable part of $\xi$ is proportional to $\delta \lambda$ simply, so that the ratio of the second and third terms, corresponding to (16), is zero. And in the case of the prism if we assume Cauchy's law of dispersion, viz. \( \mu = A + B\lambda^{-2} \), we get in correspondence with (16)

\[
\frac{3 \, \delta \lambda}{2 \, \lambda_0} \ldots \ldots \ldots \ldots \ldots (19)
\]

So far as these expressions apply, it appears that the dispersion required is between that of a grating and of a prism, and that especially when $a = 0$ the grating gives the better approximation. It would be possible to combine a grating and a prism in such a way as to secure an intermediate law, the dispersions cooperating although the deviations (in the case of a simple prism) would be in opposite directions.

I have made observations with a grating, using for the purpose a photographic reproduction upon bitumen*. This contains lines at the rate of 6000 to the inch and gives very brilliant spectra of the first order. I thought that I could observe the superior achromatism of the most nearly achromatic bands as compared with those given by the prism, but the conditions were not very favourable. The dispersive power was so high that the grating had to be held very close, and the multiplicity of spectra was an embarrassment. If it were possible to prepare a grating with not more than 3000 lines to the inch, and yet of such a character that most of the light was thrown into one of the spectra of the first order, it might be worth while to resume the experiment and, as suggested, to try for a more complete achromatism by combining with the grating a suitable prism.

Terling Place, Witham,
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