Reciprocity in optics

R J Potton

Joule Physics Laboratory, School of Sciences, University of Salford, UK

Received 21 July 2003
Published 26 April 2004
Online at stacks.iop.org/RoPP/67/717
DOI: 10.1088/0034-4885/67/5/R03

Abstract

The application of reciprocity principles in optics has a long history that goes back to Stokes, Lorentz, Helmholtz and others. Moreover, optical applications need to be seen in the context of applications of reciprocity in particle scattering, acoustics, seismology and the solution of inverse problems, generally. In some of these other fields vector wave propagation is, as it is in optics, of the essence. For this reason the simplified approach to light wave polarization developed by, and named for, Jones is explored initially to see how and to what extent it encompasses reciprocity. The characteristic matrix of a uniform dielectric layer, used in the analysis of interference filters and mirrors, is reciprocal except when the layer is magneto-optical. The way in which the reciprocal nature of a characteristic matrix can be recognized is discussed next. After this, work on the influence of more realistic attributes of a dielectric stack on reciprocity is reviewed. Some of the numerous technological applications of magneto-optic non-reciprocal media are identified and the potential of a new class of non-reciprocal components is briefly introduced. Finally, the extension of the classical reciprocity concept to systems containing components that have nonlinear optical response is briefly mentioned.

(Some figures in this article are in colour only in the electronic version)
Contents

1. Introduction 719
2. Reciprocity in the context of particle scattering, electrical network theory and Green’s functions 720
3. Time-reversal symmetry and reciprocity 722
   3.1. Maxwell’s equations and time-reversal symmetry 722
   3.2. Phase-conjugation of time-reversed waves 722
4. Reciprocity in optics 724
   4.1. Optical applications of reciprocity 726
5. Polarizing stacks 726
   5.1. de Hoop Reciprocity 729
   5.2. Unitarity 729
   5.3. Hermiticity 730
6. Stratified media 732
   6.1. Reciprocity 732
   6.2. Time-reversal 735
   6.3. Generalized reciprocity of stratified media 736
   6.4. Reciprocity failure 736
7. Reasons for reciprocity failure 737
   7.1. Magneto-optical gyrotropy 738
   7.2. Nonlocal photorefractive response 740
   7.3. Noncentrosymmetric media 741
8. Coherence and insertion loss and modal properties 741
9. Magneto-optic non-reciprocal components and their applications 742
10. Nonlinear response and Onsager reciprocity in optics 746
11. Conclusions 749
    References 749
1. Introduction

In volume II of his treatise on sound Rayleigh (1878) refers to the principle of reciprocity in the following terms:

‘On his extension of Green’s theorem Helmholtz founds his proof of the important theorem contained in the following statement:

If in a space filled with air, which is partly bounded by finitely extended fixed bodies and is partly unbounded, sound waves be excited at any point A, the resulting velocity potential at a second point B is the same both in magnitude and phase, as it would have been at A, had B been the source of sound.’

Born and Wolf (1999) formulate a reciprocity theorem for optical systems treated in the scalar wave approximation in terms of scattering amplitudes of waves incident and scattered in alternate directions. However, the transverse character of electromagnetic waves adds considerably to the complications of analysis (Perrin 1942). It requires that the exact meaning of exchanging the positions of source and detector is spelt out in a particular experimental configuration. It can be argued that there is more than one way of doing this and that the various alternatives may provide genuine insights that are complementary to one another. By keeping close to the source equations, taken to be Maxwell’s equations in linear form, and restricting attention to reciprocal propagation of the scattering of a polarized incident wave, a reciprocity principle of extreme generality is obtained (de Hoop 1959). According to this formulation, devices that are non-reciprocal in their properties are so because of asymmetry of the dielectric tensors of the media they contain or because of nonlinearity. This classification is in accord with the predominant nature of components such as magneto-optic based devices. On the other hand, there is an obvious sense in which certain stacks of polarizing elements might be described as non-reciprocal, in the sense that their polarizing effects, irrespective of the polarization of the incident radiation, are different according to the direction of passage of light through the stack. These stacks may, nevertheless, be reciprocal according to de Hoop’s formulation of reciprocity. An alternative formulation is required if it is felt that they should be distinguished, as far as reciprocity is concerned, from stacks that have the same effect on polarization state irrespective of the direction of passage of light. In this review the scope of reciprocity in optics, its basis in theory and its engineering significance will be examined. As part of this examination the connection with time-reversal symmetry will be explored so as to better understand the basics as well as some more recent developments.

The topicality of this review arises from the increasing requirement in optical fibre systems for isolators and circulators (or gyrators) similar to those that have long been in use in microwave systems. Specifically, isolators have a role in blocking destabilizing reflections into sources to prevent the generation of phase noise. Circulators are crucial to the efficient routing of power in channel dropping filters for wavelength division multiplexing and dispersion compensating components.

Following Rayleigh many advances in the exploitation of the reciprocity principle have been made in acoustics including seminal works by Gerjuoy and Saxon (1954) and by Saxon (1955). In the next section other types of physical systems, apart from acoustic and optical ones, in which reciprocal propagation can be recognized are identified and the general notion of what is meant by reciprocity is thereby elaborated.

An electromagnetic characterization consistent with Maxwell’s equations of sources and detectors as well as of propagating radiation is the approach taken by Landau and Lifshitz (1960) and later by Afanasiev (2001). Section 3 places reciprocity in the context of Maxwell’s equations and their time-reversal symmetry. The relevance of phase-conjugation to reverse
propagation of electromagnetic waves and their polarization is explained. A connection between reciprocity, however defined, linear superposition and insertion loss is identified.

In section 4, the general scattering formalisms of Born and Wolf (1999) and of de Hoop (1959) are briefly summarized as giving the most accessible axiomatic statements of what optical reciprocity, in its classical form, amounts to.

The properties of Jones and Mueller matrices, used to characterize transmission through stacks of polarizers and wave-plates, are discussed in section 5. Except when they include magneto-optic components, such stacks are reciprocal in the sense of de Hoop. The use of Stokes vectors (Stokes 1852) to represent the polarization states of incident and scattered waves permits changes in the degree of polarization of the light to be analysed. Reciprocity plays a major part in such an analysis as was shown in great detail by Perrin. Finally, an alternative, but unconventional, definition of reciprocity might classify a sandwich of linear polarizer and quarter-wave plate as being a non-reciprocal component. This possibility is discussed in section 5 using references to studies by workers who adopt this point of view.

The important topic of the dielectric stack forms the subject matter of section 6. A great deal of effort has been expended on elucidating the reciprocal features of reflection and transmission from layered structures, with or without corrugated or rough surfaces. The fact that reciprocal propagation admits evanescent components greatly complicates this area of study. This topic specifically has been addressed by Nieto-Vesperinas and Wolf (1986) and by Carminati et al (1998, 2000). Moreover, excitation of surface waves is a recognized feature of configurations of this type. Nevertheless, the generality of de Hoop reciprocity must ensure that, in most cases (magneto-optic media excepted) its predictions can be relied upon.

Apart from magneto-optic effects that are discussed in section 7.1, another possible fundamental source of non-reciprocal behaviour is the two-wave mixing phenomenon that occurs in photovoltaic electro-optic crystals. For this, consideration of the conditions that lead to reciprocity of Green’s functions can prove illuminating in a way that is discussed in section 7.2.

From an engineering standpoint, device performance, whether of a reciprocal or of a non-reciprocal device, usually requires the specification of insertion losses. The connection of these constructs with the superposition principles introduced in section 3 is the subject of section 8.

Section 9 draws on the considerable pool of knowledge relating to ferrite based non-reciprocal microwave components to introduce the analogous (infrared) optical components that are, and will increasingly be, used in the development of optical fibre networks.

Finally, section 10 introduces some more sophisticated work that seeks to relax the conditions that de Hoop spelt out in introducing the reciprocity principle in the context of vector waves.

2. Reciprocity in the context of particle scattering, electrical network theory and Green’s functions

Reciprocal situations that share common features with those described in this paper are well known in the theory of elasticity (Love 1944), acoustics (Rayleigh 1945), seismological and sonar surveys (Tarantola 1987), particle scattering experiments (Bohr and Mottelson 1969) and electrical circuit theory (Bleaney and Bleaney 1976). Mathematical methods for the solution of inhomogeneous partial differential equations have been applied to reciprocal and non-reciprocal situations and the distinction between these cases is well recognized in texts on scattering theory (Schiff 1968) and on Green’s functions (Barton 1989). In 1857, Bunsen and Roscoe referred to the eventuality that, in photography, the optical density of a
developed emulsion depended only on the product of illuminance and exposure time, by the term reciprocity. This usage is obviously unrelated to the present topic.

In the study of elasticity the theorem that states that ‘the displacement at B caused by a force applied at A is equal to the displacement at A caused by an equal force at B’ is ascribed to James Clark Maxwell. This theorem has been applied in the study of ground displacements in earthquake zones for the purpose of inferring stress concentrations from surveying data. Betti reciprocity is described by Love (1994), who, at the same time cites reciprocity in elasticity theory as an example of the more general principle reviewed by Lamb (1889). More recently, Gerjuoy and Saxon (1954), Saxon (1955) and Gangi (2000) have made major contributions to the acoustic applications of scattering theory.

A major review of time-reversed acoustics (Fink et al. 2000) shares much of the conceptual framework of the present topic but dwells comparatively little on the subject of reciprocity as defined by Rayleigh (1945). It is difficult to see why this should be, since Stokes reciprocity looms large in the application of acoustical surveys to practical problems in seismology (Aki and Richards 1980). The field of remote sensing (de Hoop and de Hoop 2000) also finds a major role for ideas about reciprocity with intense concentration of published work in works devoted to the solution of inverse problems (Tarantola 1987).

Partial wave analysis of scattering and introduction of the scattering matrix leads to the reciprocity theorem as well as the optical theorem and the generalized optical theorem of particle scattering (Schiff 1968). These theorems relate the scattering amplitudes of scalar waves from a given scatterer for different scattering geometries. Unitarity is in evidence when the Hamiltonian is Hermitian (and the potential real) whereas, in contrast, the optical theorem centres on the imaginary part of the scattering potential. To derive a reciprocity theorem the imaginary part of the scattering potential is eliminated from the mathematical formulation. Scattering with the possibility of bound states becoming populated is relevant to photonic bandgap materials and optical trapping.

The reciprocity principle is well known in electrical network theory (Bleaney and Bleaney 1976). Analogues of reciprocal optical components such as dielectric waveguides can be recognized but, in addition, the spectra of non-reciprocal networks (Figotin and Yitebsky 2001) suggest the existence of novel phenomena in non-reciprocal photonic bandgap media (Dowling 1998). On the other hand, it may be worth remarking at this point that active components such as amplifiers which, in an electrical context are usually thought of as intrinsically non-reciprocal components, may be, in an optical context, essentially reciprocal in response (Oh et al. 2001).

Dyadic Green’s functions provide a powerful tool for solving vector partial differential equations (Morse and Feshbach 1953) though applications are usually confined to scalar, rather than vector, wave equations (Barton 1989). As with scattering theory (Schiff 1968, Berger 1990), reciprocity appears as a fundamental property of the mathematical entities used to construct the theory. However, it is also possible to view it as a consequence of a spatial symmetry combined with time translation symmetry. This is particularly interesting in view of the considerations about reciprocity failure in photorefractive media discussed in section 7.2. Photorefractive media are those that require the use of hereditary integrals in the formulation of their constitutive relations. A distinction to be made in modelling optical systems is that between vector and scalar wave representations and, in this respect, the usually encountered Green’s function analyses are not fully general. According to Gerlach (2003) the Green’s functions used in solving problems with self-adjoint boundary conditions parallel what, in linear algebra, would be Hermitian matrices.

Several of the fields, described above, in which the relevance of the reciprocity principle has been recognized, have been brought together in a study of transmission and reception by a prototype antenna: the flanged waveguide (Kriegsmann 1999). The study alludes to
acoustic, electromagnetic and oceanographic problems to which solutions provided might be relevant.

Most of the approaches referred to below depend on linearity of the optical response and, as a consequence, use various matrix methods to solve optical problems of different types. When this is done the condition for reciprocal, or indeed time-reversible, propagation can be expressed as a condition on the matrices employed. Examples of matrix methods employed in polarization optics and in thin film optics that may lead to unitary, unimodular or Hermitian matrices are referred to in sections 5 and 6. In the next section, the role that time-reversed light waves play in phase-conjugate reflection and reciprocal propagation is elaborated on.

3. Time-reversal symmetry and reciprocity

The power of the reciprocity principle lies largely in the fact that it is applicable even in the presence of absorption when satisfactory mathematical treatments lack time-reversal symmetry. The link between reciprocity, time-reversal symmetry and a third construct, that of optical phase-conjugation, has been discussed in detail by Carminati et al (2000). Gangi (2000) has shown that, in elastic media, reciprocity can be exploited not only in the presence of viscoelastic constitutive relations but even when fictitious forces are employed.

3.1. Maxwell’s equations and time-reversal symmetry

In conducting media charge carriers move apart under their mutual repulsion and thereby adopt stable configurations. The production of these configurations is an irreversible process that involves the dissipation of electromagnetic energy as heat. A sufficient condition for such effects to be absent is the vanishing of free currents represented by $\mathbf{J}$ in Maxwell’s equations (Lipson et al 1995):

\begin{align}
\text{curl } \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \\
\text{div } \mathbf{B} &= 0, \\
\text{curl } \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\text{div } \mathbf{D} &= \rho.
\end{align}

In dielectric media $\mathbf{J}$ is zero and Maxwell’s equations describing the behaviour of real, physical fields are symmetric under the symmetry of time-reversal (Gangi 2000). A special, but widely exploited, case occurs when $\mathbf{J}$ is proportional to the optical electric field (for more general cases see section 7.2). In the special case, and for monochromatic light, the effects of conductivity can be incorporated into a complex dielectric function.

3.2. Phase-conjugation of time-reversed waves

In the paraxial approximation, harmonic solutions of the wave equation for the vector potential are found as the real part of a complex wave:

\[ \Delta = \hat{\mathbf{p}} \cdot u(x, y) \exp[i(kz - \omega t)], \]

(3.2)

where $\hat{\mathbf{p}}$ is the polarization vector and $\hat{\mathbf{p}} \cdot \exp(ikz)$ satisfies the Helmholtz equation. Since the time dependent factor is understood to be $\exp(-i\omega t)$, the time-reversal operation is alternatively expressed as:

\[ T : u \mapsto u^*, \quad \hat{k} \mapsto -\hat{k}. \]
This is possible because of an extension of the property of Fourier transforms pointed out by Fink et al (2000). In the present case: if real $E(r, t)$ has spatial and temporal Fourier transform $u(k, \omega)$ then $E(r, -t)$ has Fourier transform $u^*(-k, \omega)$.

This pair of operations, conjugation of complex amplitudes and inversion of wavevectors, reverses the sign of the two parts of the Poynting vector that can be expressed as (Allen et al 1992):

$$S = \frac{1}{2}(E^* \times H + E \times H^*) = \frac{i\omega}{\mu}(u^* \nabla u - u \nabla u^*) + \frac{\omega k}{\mu} |u|^2 \hat{k}, \quad (3.3)$$

where $\nabla u = \hat{i}(\partial u/\partial x) + \hat{j}(\partial u/\partial y)$.

The time-reversal operator $T$ may be seen to reverse the direction of energy flow provided that, at the same time as reversing $k$, the complex amplitude of the wave is conjugated. The association of phase-conjugation with the operation $k \mapsto -k$ suggests the possibility of extending reciprocity statements derived for plane waves to finite beams formed as linear superpositions of paraxial waves. This possibility has been explored by Sheppard and Gu (1993). On the other hand, Vigoureux and Giust (2000) have introduced the operation $k \mapsto -k$ divorced from complex conjugation in their study of reciprocity of stratified media.

The need to reverse the phase of the time-reversed version of a monochromatic light wave can be illustrated by reference to the counter-propagation of a wavefront or, alternatively, to the state of polarization of a reversed polarized wave. The invariance of Maxwell’s equations under time-reversal symmetry means that time-reversed versions of solutions also satisfy the equations. In some cases, plane wave solutions provide a point of contact with experimental observations. More generally, linear superpositions of plane waves travelling in different directions and having appropriate amplitudes and phases give a more realistic description of real field distributions. In such cases there is a more obvious connection between the original solution and a time-reversed solution in which the amplitudes of the constituent plane waves are the phase-conjugates of the original ones. It may be shown (Yariv 1997) that after phase-conjugate reflection the wavefront retraces the evolution that it formerly underwent, even to the extent that distortions introduced by a scattering medium are reversed. In the plane perpendicular to the direction of propagation of an aberrated wave, points at which the phase of the field is locally advanced are the points at which the phase should be locally retarded by the same amount in the reversed wave if that wave is to preserve the wavefront shape of the original. This means that the complex amplitude of the reversed wave must be conjugate to that of the forward wave. In a similar technique, that of spin echoes in magnetic resonance, dephasing of superposition states is reversed in a quite surprising way to resurrect an original state that might have been thought to have been irretrievably lost. By the same token, the time-reversal operation in mechanics involves the reversal of the velocities of all particles at a given instant and also converts leading to lagging phase relationships. Phase-conjugation is the equivalent process in optics.

In the context of polarization optics, reflection from a normal (metallic or dielectric stack) mirror reverses the handedness of polarization of a circularly polarized incident wave. In contrast, the circular polarization of a phase-conjugately reflected wave is the same as that of the incident wave. A polarized wave, propagating in the $z$-direction, in which the $x$-component of the $E$-field leads the $y$-component has the same handedness as a reversed wave in which the $y$-component leads the $x$-component. This convention about what constitutes the time-reversed version of a circularly, or elliptically, polarized wave will be shown, in section 5, to lead to the conventional view that chiral media are reciprocal in their response, whereas, Faraday magneto-optic media are non-reciprocal (Nye 1969).
The time-reversed version of an optics experiment has time-reversed, phase-conjugate versions of all of the light waves in the original experiment and can be realized if the propagation medium is lossless and is itself time-symmetric. Reflection/refraction at a dielectric interface is a time-reversible phenomenon. It requires time-reversed reflected and refracted waves to be brought together with the correct phase relationship to generate the time-reversed version of the incident wave only. Though this does not automatically come about, it is, in fact, exactly what is achieved in the beam combiner of an interferometer. The steering of the output of a fibre optic Mach–Zehnder interferometer to one arm only demonstrates the degree of precision that can be achieved in beam combining.

However, reciprocity is not the same as time-reversal symmetry since reciprocity relates input and output waves in pairs irrespective of the presence or absence of other waves. Reciprocity recognizes that there is utility in specifying transfer between input and output waves of optical components in a way that is insensitive to the polarization or degree of coherence of the incident light. In an engineering context the transfer is characterized by insertion losses between a pair of ports of an optical device. In isolators and circulators high insertion loss is achieved between particular ordered pairs of ports in such a way that it does not depend upon the presence or absence of inputs at other ports. This property makes non-reciprocal components complementary to other linear components such as directional couplers.

In considering the practical applications of non-reciprocal components in section 9 of this review, attention will be drawn to routing of signals through a system. This topic is coming to assume a greater importance with the development of agile sources to be used in conjunction with dichroic couplers. This allows the insertion loss between a particular pair of ports in a directional coupler to be manipulated by a programmed change of the carrier wavelength (forming a so-called reconfigurable system). In a linear system re-routing can in principle also be achieved by using a coherent local source to produce constructive or destructive interference in the required paths. Were it not for the inconvenience of having to implement optical phase locked loops at many points in a system, this possibility of ‘controlling light with light’ might have proved attractive. In coherent microwave systems the routing of signals using the linear principle has been well used for many years as, for example, by the waveguide ‘magic tee’. In the optical domain, the phrase ‘controlling light with light’ is usually reserved (Gibbs 1985) for systems that exploit nonlinear optical phenomena.

4. Reciprocity in optics

Electromagnetic wave propagation in graded index media admits both wave guiding (normal mode) and scattering treatments. In each case the wave equation for the electric field is the same and is augmented with an index gradient term:

\[ \nabla^2 E = \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} + \nabla \left( \frac{\nabla n^2 \cdot E}{n^2} \right), \]  

(4.1)

The arena within which the reciprocity theorem and other optical theorems exist is scattering theory. Subject to many assumptions and simplifications (monochromatic illumination, finite size of scattering region, unique illumination and detection directions) scattering can be characterized by a scattering amplitude derived from a scattering potential \( F(r) \).

Born and Wolf (1999) set out detailed arguments based on the scalar Helmholtz equation:

\[ \nabla^2 E_i(r) + k^2 E_i(r) = -4\pi F(r) E_i(r) \]  

(4.2)
Reciprocity in optics

Figure 1. Reciprocal configurations (after Born and Wolf (1999)).

with scattering potential $F(r)$ to establish a rather general reciprocity theorem complementary to the optical theorem that depends on the imaginary part of $F(r)$. The solution:

$$E_1(r) = \exp(i k \cdot r) + f(\hat{s}_2, \hat{s}_1) \frac{\exp(ikr)}{r},$$  \hspace{1cm} (4.3)

for the total field from a wave incident with wave vector $k = k \hat{s}_1$, which involves the scattering amplitude $f(\hat{s}_2, \hat{s}_1)$, is reciprocal to a field incident in the direction $k = k \hat{s}_2$ when the scattering amplitudes satisfy

$$f(\hat{s}_2, \hat{s}_1) = f(-\hat{s}_1, -\hat{s}_2).$$  \hspace{1cm} (4.4)

Such reciprocity implies the existence of the two scattering configurations shown in figure 1.

The formulation of the principle of reciprocity for general states of polarization was addressed by Perrin (1942) and, more comprehensively, by Chandrasekhar (1950) using a real representation of components of fields. The latter treatment was directed especially towards understanding the scattering of light in planetary atmospheres. The type of invariance identified with reciprocity was that of the scattering and transmission matrices acting on Stokes vectors under simultaneous transposition and exchange of polar and azimuthal angles of incident and scattered waves (Chandrasekhar 1950, pp 95 and 172).

In the same vein Mishchenko (1992) formulates a complex amplitudes scattering matrix $F(\hat{n}, \hat{n})$ to which the reciprocal relation:

$$F(-\hat{n}', \hat{n}) = Q F^t(\hat{n}', \hat{n}) Q,$$  \hspace{1cm} (4.5)

with $Q = \text{diag}(1, -1)$ and $t$ denoting matrix transposition, applies. In doing this he cites Saxon (1955) who finds a reciprocity relation:

$$q' \cdot A_q(n, n_0) = q \cdot A_q(-n_0, -n),$$  \hspace{1cm} (4.6)

relating incident and scattered wave polarization vectors $q$ and $q'$ and vector scattering amplitudes $A_q(n, n_0)$ and $A_q(-n_0, -n)$.

de Hoop (1959) gives:

$$B \cdot F_\alpha(\beta) = A \cdot F_\beta(\alpha)$$  \hspace{1cm} (4.7)

for the reciprocal connection between incident and scattered complex vector wave amplitudes, $A$ and $B$, and complex vector scattering amplitudes, $F_\alpha(\beta)$ and $F_\beta(\alpha)$, $\alpha$ and $\beta$ being unit vectors in the illumination and detection (observation) directions.

Reciprocity that depends only on the symmetry of, permittivity, permeability and conductivity tensors (de Hoop 1987, Raab and de Lange 2001, de Lange and Raab 2003) may be expressed in words as follows:

The scattering amplitude for a $B$ polarized scattered wave in the direction $\beta$ arising from an $A$ polarized incident wave in the direction $\alpha$ is equal to the amplitude for an $A$ polarized scattered wave in the direction $-\alpha$ from a $B$ polarized incident wave in the direction $-\beta$. 


Particularization of these notions of reciprocity to examples of transmission through stacks of polarizers and wave plates is given in the next section. The Jones matrix for transmission through the stack in the reverse direction is the transpose of the matrix for forward transmission when the two directions of propagation are reciprocal to one another. For stacks that contain non-magnetic components only, or for those in which the change from forward to backward propagation is accompanied by reversal of magnetization and currents, the conditions for reciprocal propagation will be seen to be satisfied.

In section 6 the application of reciprocity to beam splitting by division of amplitude is discussed. Subject to the absence of absorption, a number of connections between amplitude transmission and reflection coefficients can established. Starting with such relations for a single interface (Stokes 1849) a number of authors have extended the scope of this type of relation to multilayer stacks (Vigoureux and Giust 2000), continuously varying stratified media (Jacobsson 1966) and dielectric scatterers of arbitrary shape (Nieto-Vesperinas and Wolf 1986). Like other forms of symmetry, reciprocity can be used to simplify the analysis of light propagation. However, by relating different configurations of scatterer and field rather than of the scatterer on its own, its expression is somewhat different from that for normal spatial symmetries. The interplay of time and spatial symmetries is discussed by Shelankov and Pinkus (1992) and by other authors and is another aspect of reciprocity alluded to in section 6. The question of how illumination from a source is characterized, which has been mentioned in section 3, is illuminated by the major article of Afanasiev (2001) relating to selection of sources of different types.

The generality of de Hoop’s reciprocity theorem means that, in most circumstances, some form of symmetry exists for reverse energy flow in an optical system. As a consequence of reflections from defects and from boundaries of components, light can generally return to the source. This is frequently an undesirable occurrence and, for this reason, non-reciprocal components that block reflections have a significant role to play in the design of optimal systems.

4.1. Optical applications of reciprocity

Reciprocal response underlies the behaviour of optical components such as polarizers, wave-plates and beam splitters, as will be explained in sections 5, 6 and 8. Its failure, by reason of magneto-optic or nonlocal response leads to new families of components that are described in sections 7.1 and 7.2. The properties of these two distinct sets of components are, in many respects, complementary.

Apart from antenna theory (Kriegsmann 1999), of concern especially in microwave optics and radio, optical reciprocity has been used to reinforce the fundamental bases of thermal physics. Specifically, Kirchoff’s Law, a fundamental phenomenological law of thermal physics, has been derived from the reciprocity of the scattering operator (Greffet and Nieto-Vesperinas 1998).

The recent interest in near-field imaging (Greffet and Carminati 1997) has led to further developments in which the reciprocity principle takes a pivotal position (Carminati and Saenz 2000, Porto et al. 2000).

5. Polarizing stacks

The simplest illustration of the scope of optical reciprocity is provided by the transmission at normal incidence of collimated light through a stack of polarization changing elements (Shurcliff 1962, O’Neill 1963, Gerrard and Burch 1975). When the incident light is coherent,
the transmission through each element can be expressed in terms of its Jones matrix (Jones 1941, Han et al 1997). The Mueller matrix (Mueller 1948) provides a corresponding linear description of transmission for incoherent illumination (McMaster 1954, Azzam 1981, Kats and Spevak 2002). In using these representations the effect of reflections on the overall transmission is neglected. This conceit has significant implications when a distinction is made between reciprocity and time-reversibility of a stack because the transmission of the stack is expressed as a matrix that incorporates in some way the boundary conditions that are satisfied by incident and reflected waves and the wave within the stack. When using either Jones or Mueller matrices the distinction between reciprocity and time-reversibility is reflected in different algebraic properties of the transmission matrix in question. In more complicated situations, where a stack is neither reciprocal nor time-reversible in its transmission properties, determination of the attributes of individual elements may lead to a clearer understanding of the total polarization transformation achieved. Table 1 shows the Jones and Mueller matrices of a number of polarizing elements and waveplates.

As may be seen from column 3 in the table the matrix that describes transmission in the reverse direction through the element is, in most cases, the transpose of the matrix for forward transmission. The identification of transmission matrices for the reverse direction of propagation relies on the identification of reversed input and output waves as phase-conjugate versions of the originals. It must not be supposed that the symmetry of the transmission matrix directly determines the reciprocity of transmission. Nor does it alone determine time-reversibility of transmission. That transposition of the forward transmission matrix should yield the transmission matrix for reverse propagation may be inferred from Landau and Lifshitz’s (1960) formulation of reciprocity in linear media with symmetric dielectric tensors.

A familiar example of the transposed character of the Jones matrix for reverse propagation is the circular polarizer formed when a linear polarizer is combined with a quarter wave plate with the polarizing direction at 45˚ to the principal axes of the wave plate. When the linear polarizer is on the input side, the pair acts as a circular polarizer with Jones matrix:

\[
M = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & i \\ 1 & i \end{vmatrix}.
\]

When the wave plate is on the input side, the same pair of elements act as a linear polarizer with Jones matrix:

\[
M' = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 1 \\ i & i \end{vmatrix}.
\]

The Jones matrix of an optically active medium that introduces a phase angle of \(\delta\) between right and left circularly polarized waves is

\[
M = \begin{vmatrix} \cos \frac{\delta}{2} & \sin \frac{\delta}{2} \\ -\sin \frac{\delta}{2} & \cos \frac{\delta}{2} \end{vmatrix}.
\]

The same circular birefringence is maintained for reciprocal propagation when the Jones matrix is replaced by the transpose:

\[
M' = \begin{vmatrix} \cos \frac{\delta}{2} & -\sin \frac{\delta}{2} \\ \sin \frac{\delta}{2} & \cos \frac{\delta}{2} \end{vmatrix}.
\]
### Table 1. Jones and Mueller matrices of selected idealized polarizing elements and wave plates (linearly polarized basis).

<table>
<thead>
<tr>
<th>Action of element</th>
<th>Jones matrix forward-propagating</th>
<th>Jones matrix counter-propagating</th>
<th>Mueller matrix</th>
<th>Hermitian</th>
<th>Unitary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Polarizers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear polarizer</td>
<td>$\begin{pmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Vertical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear polarizer</td>
<td>$\begin{pmatrix} 0 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; -1 &amp; 0 &amp; 0 \ -1 &amp; 1 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Linear polarizer at 45°</td>
<td>$\begin{pmatrix} 1 &amp; 1 \ 1 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 1 \ 1 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Homogeneous right circular polarizer</td>
<td>$\begin{pmatrix} 1 &amp; i \ -i &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; i \ -i &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Wave plates</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarter-wave plate—fast axis vertical</td>
<td>$\exp\left(\frac{i\pi}{4}\right)\begin{pmatrix} 1 &amp; 0 \ 0 &amp; -i \end{pmatrix}$</td>
<td>$\exp\left(\frac{i\pi}{4}\right)\begin{pmatrix} 1 &amp; 0 \ 0 &amp; -i \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Right circular retarder delay $\delta$</td>
<td>$\begin{pmatrix} \cos\frac{\delta}{2} &amp; \sin\frac{\delta}{2} \ -\sin\frac{\delta}{2} &amp; \cos\frac{\delta}{2} \end{pmatrix}$</td>
<td>$\begin{pmatrix} \cos\frac{\delta}{2} &amp; -\sin\frac{\delta}{2} \ \sin\frac{\delta}{2} &amp; \cos\frac{\delta}{2} \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Faraday cell: right circular retardation $\delta$</td>
<td>$\begin{pmatrix} \cos\frac{\delta}{2} &amp; \sin\frac{\delta}{2} \ -\sin\frac{\delta}{2} &amp; \cos\frac{\delta}{2} \end{pmatrix}$</td>
<td>$\begin{pmatrix} \cos\frac{\delta}{2} &amp; \sin\frac{\delta}{2} \ -\sin\frac{\delta}{2} &amp; \cos\frac{\delta}{2} \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

On the other hand the Jones matrix for a Faraday cell:

$$M = \begin{pmatrix} \cos\frac{\delta}{2} & \sin\frac{\delta}{2} \\ -\sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{pmatrix}$$

has a companion matrix, $M'$, that describes the effect of the same cell on a counter-propagating wave:

$$M' = \begin{pmatrix} \cos\frac{\delta}{2} & \sin\frac{\delta}{2} \\ -\sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{pmatrix}.$$

Schiff (1968) distinguishes different varieties of reciprocity in the presence or absence of auxiliary symmetries (viz inversion symmetry, $V(r) = V(-r)$, of a scattering potential). In
Reciprocity in optics, it may be appropriate to classify polarizing elements according to whether their effect on an incident wave with a given state of polarization is the same for either direction of propagation. It happens that de Hoop reciprocity, as described in section 4, and as applied to polarization optics in section 5.1, is not entirely fitted to performing this function. Consequently, an alternative formulation of reciprocity, which in contrast to de Hoop’s, classifies transmission through linear analyser/waveplate pairs (Glaser 2002) as non-reciprocal, is discussed in section 5.3.

5.1. de Hoop Reciprocity

The essence of reciprocal behaviour may be understood by reference to figure 2. In figure 2(a) a polarizing stack is illuminated with an input wave represented a Jones or Stokes vector depending on whether the light is coherent or incoherent. In order to simplify the description of complex wave reciprocity, the remainder of this section is confined to completely polarized radiation. The polarization state of the light is represented by a two-component complex Jones vector, $a$. The transmitted wave that emerges from the stack is analysed by projection on some polarization state represented by Jones vector $b$.

![Figure 2. Reciprocity of a polarizing stack in terms of its Jones matrix.](image)

The reciprocal situation illustrated in figure 2(b) has the time-reversed version of $b$, namely $b^*$, as the incident wave from the right and the transmitted wave is analysed by projection on the polarization state $a^*$.

In the context of Jones (or Mueller) matrices de Hoop reciprocity takes the form:

$$b^\dagger Ma = (a^*)^\dagger M^r b^*,$$

(5.1)

where $M^r$ is the Jones matrix for reverse propagation and $b^\dagger$ is the adjoint of $b$ etc. This equation is identically satisfied if $M^r = M^t$, the transpose of $M$, since the right-hand side of the equation is then the transpose of the left-hand side and the equation is scalar. Thus, the condition that the polarization states observed experimentally in configurations 2(a) and (b) conform with equation (5.1) is the condition that the matrix for reverse transmission is the transpose of that for forward transmission for the particular stack under investigation. All those components that contain only media whose permittivity, permeability and conductivity tensors are symmetric satisfy this condition. This includes linear and circular polarizers and linearly and circularly birefringent plates. Faraday and magneto-optic Kerr effect based devices do not, in general, satisfy the reciprocity condition as their dielectric tensors are asymmetric.

5.2. Unitarity

The phase-conjugate companions discussed by Yariv (1997) correspond, for polarized waves, to the configurations shown in figure 3.
The conditions for such companions to exist are more restrictive, since, in contrast to the situation for de Hoop reciprocity, the medium must be lossless. This requires Jones matrices to be unitary (or Mueller matrices to be orthogonal). Figure 3(a) is expressed algebraically as

\[ b = Ma, \]

so that:

\[ M^{-1}b = a \]

and

\[ (M^{-1})^*b^* = a^*. \]

If \( M \) is unitary with \( M^{-1} = M^T = (M^T)^* \), then,

\[ a^* = M^*b^*. \]

This last equation expresses the situation shown in figure 3(b), the phase-conjugate companion of figure 3(a).

Since a unitary transformation is norm preserving, this formulation brings out clearly the often remarked feature that time-reversal symmetry precludes absorption (Tang and Kwok 2001) whilst reciprocity can be recognized and exploited even in the presence of absorption. The inner product of Jones vectors is also preserved under unitary transformation, so it seems that unitarity is required for the preservation of polarization orthogonality addressed by Van de Venter (1992).

5.3. Hermiticity

A further type of propagation/counter-propagation pair may be analysed by reference to figures 4(a) and (b).

In this case the input wave on the left in figure 4(a) is reversed to form the input wave from the right in figure 4(b). This pair of configurations will exist when the transmission matrix is Hermitian (symmetric in the case of Mueller matrices).
Algebraically this situation is expressed in the following way:

\[ b = Ma. \]  

(5.3)

Therefore,

\[ b^* = M^* a^* \]

and, if \( M \) is Hermitian and \( M^* = M' \),

\[ b^* = M' a^* \]

(5.4)

which expresses the situation shown in figure 4(b).

The above considerations show that stacks that have different effects on waves incident from the left and right may, nevertheless, be reciprocal in the sense of de Hoop. For example, the combination of linear polarizer and quarter wave plate mentioned earlier produces a linearly polarized output for one direction of illumination and circularly polarized output for the other direction of illumination, irrespective of the polarization state of the incident light.

The Stokes vector representation of partially polarized light dispenses with the explicit representation of phase, and the Stokes vectors themselves as well as the Mueller matrices are real. However, the specialization of reciprocal transmission matrices as symmetric and of time-reversal symmetric ones as orthogonal are just as one would expect for consistency with what has been set out above. When changes in the degree of polarization result from scattering in a medium (di Stasio 2002), Perrin (1942) has shown that, in forward scattering, transmitted and scattered components must be distinguished. In making extensive use of the reciprocity principle Perrin sets out clearly the extent of its applicability. Excluded from its scope are systems containing magnetic gyrotropic media (unless movements, currents and magnetic fields are reversed at the same time as the directions of propagation of waves). Moreover, irreversible changes in frequency such as those in fluorescence and in the Raman effect are also not covered. Some of the many attributes of scatterers considered by Perrin in enumerating the coefficients that can relate scattered to incident Stokes vectors are particle shape, size and rotatory power. In addition to the simplification in the set of scattering coefficients that follows from reciprocity, the symmetry of the scattering medium has an influence. Sixteen scattering coefficients are reduced to ten independent elements by reciprocity and further reduced to six for media with inversion symmetry (i.e. not having optically active scatterers). Graham (1980) considered further the combined restrictions imposed by spatial symmetries and reciprocity on Mueller matrices for scattering in fluids. Perrin’s paper also alludes to the possible consequences of multiple scattering (Kokhanovsky 2002) and incident beam convergence (Sheppard and Gu 1993). Multiple scattering is a major interest for astronomers studying reflection and transmission of light by planetary atmospheres and its study, including the phenomenon of enhanced back-scattering, has resulted in considerable advances based on the exploitation of reciprocity (Mishchenko 1992).

Polarization transformation by resonance diffraction has been investigated more recently by Kats and Spevak (2002) in the expectation of being able to design more effectively optical devices that are selective with respect to polarization, wavelength and orientation.

In table 1, it is apparent that retardation plates, at least as idealized lossless components, are characterized by unitary Jones matrices and orthogonal Mueller matrices. The other principal type of idealized component, the polarizer, turns out to be lossy with a nonunitary but Hermitian Jones matrix or a nonorthogonal but symmetric Mueller matrix. Since a general Jones matrix will be neither unitary nor Hermitian, and a general Mueller matrix neither orthogonal nor symmetric, it is interesting that favoured components that are not lossless nevertheless form a rather special class, i.e. those that have reciprocal response even in a sense more restricted than that of de Hoop.
6. Stratified media

Reciprocity of reflection and transmission at a plane interface, a topic originally broached by Stokes (1849) has subsequently been extended to stratified media with arbitrarily many layers (Vigoureux and Giust 2000). Reflection from films with continuously varying refractive index was earlier the subject of a review by Jacobsson (1966). Figure 5 shows how this topic can be related to de Hoop’s general result.

![Figure 5.](image)

It should be noted that, for a highly idealized stack of uniform, isotropic, non-gyrotropic layers with flat interfaces, the polarization subtleties of de Hoop reciprocity are simplified by the resolution of incident, reflected and transmitted radiation into S- (TE) and P- (TM) components. Moreover, the laws of reflection and refraction mean that the scattering amplitude is non-zero in only two directions: those of the reflected and of the transmitted waves. Even this simplification is subject to modification when ‘non-geometric’ reflection occurs as a result of coupling to surface excitations (Imbert 1972, de Beauregard et al 1977). de Hoop reciprocity amounts to the statement that, for the two geometries (b) and (c), each reciprocal to (a) in figure 5, the amplitude reflectance \( r' \) should be equal to \( r \) and the amplitude transmittance \( t'' \) should be equal to \( t \). Moreover, these equalities will hold for both TE and TM components of illumination.

6.1. Reciprocity

It is apparent from the discussion above that de Hoop reciprocity has nothing directly to say about the equality or otherwise of \( t \) and \( t' \), nor of \( r \) and \( r'' \), in figure 5. However, following Stokes (1849) other authors including Nieto-Vesperinas and Wolf (1986) and Carminati et al (2000) have developed relations that connect these quantities. In nonabsorbing media connections between \( t \) and \( t' \) and between \( r \) and \( r'' \) exist by virtue of the relations (Ou and Mandel 1989, Dupertuis et al 1994):

\[
|t|^2 + |r|^2 = 1 \tag{6.1a}
\]

\[
|t'|^2 + |r'|^2 = 1 \tag{6.1b}
\]

and

\[
|t''|^2 + |r''|^2 = 1 \tag{6.1c}
\]

and the equality of \( t \) and \( t'' \) and of \( r \) and \( r' \).

In the presence of absorption, it is only necessary to consider reflection from an interference filter back coated with an absorbing layer to see that that \( r \) and \( r'' \) will not be equal. High reflectance from the uncoated side will be little affected by the absorbing layer, whilst
reflectance from the coated side will be low. The interesting feature of such a structure is the reciprocal transmittance on illumination from either side. A further example is provided by asymmetrically located exciton absorption in coupled semiconductor microcavities (Armitage et al. 1998, Agarwal and Gupta 2002).

The conclusions about reciprocity arrived at above accord with what Jacobsson (1966) has shown, that, under quite general conditions, transmission through a stratified medium is reciprocal. However, he has also shown that, in most cases, reflection is not reciprocal, at least in the sense of Stokes (1849) \( r = r'' \) in figure 5. The different status of transmission and reflection in stacks containing absorbing layers is well known to designers of optical coatings (Bass 1995). However, by virtue of unitarity (norm conservation), the sum of the intensity transmittance and reflectance is unity. Therefore, the lack of reciprocity in reflection in nonabsorbing stacks is a matter of phase change only and since the stack necessarily lacks mid-plane symmetry there is no automatic choice of reference plane at which the phase of incident and reflected waves should be measured. It is, in fact, possible to choose a reference plane in such a way that the phase-conjugation associated with time-reversal of the respective waves in the reciprocal situation renders reflection as well as transmission reciprocal. As explained above this is not the case when absorption is introduced.

In what follows, it will be shown that interesting algebraic properties of the combined characteristic matrix of such stacks are associated with the reciprocity described above.

The matrix methods that are used are not always equivalent to the matrix formulation summarized below. Nevertheless, reciprocal relationships may often be shown to be with the algebraic properties of transfer matrices of one type or another. By paying attention to such features and introducing a transformation in which wave vectors are changed in sign without at the same time taking the complex conjugate of wave amplitudes, Vigoureux and Giust (2000) are able to establish a connection between reflection coefficients on opposite sides of a stack. The argument does not require the dielectric properties of the bounding layers to be the same, in contrast to what is discussed below and by Aronson (1997). However, the connections that result are somewhat involved, which may make their practical exploitation rather difficult.

The most satisfactory way of analysing the transmission of waves through a stratified medium makes use of the characteristic matrix of each layer. This method, due to Abeles, is set out in section 1.6 of their text by Born and Wolf (1999). It is analogous to the use of characteristic impedance in analysing the electrical properties and of transmission lines having distributed capacitance and inductance. As with Jones and Mueller matrices the matrix for a stack is obtained by multiplying the matrices for the individual layers in the stack in the correct order. However, there is a difference since the characteristic matrix of a layer relates the values of the vector, the components of which are the electric and magnetic fields in an electromagnetic wave, at the two surfaces of that layer. Transposition does not play the same part in the theory of the characteristic matrix as it does in the theory of the Jones and Mueller matrices. The characteristic matrix is generally asymmetric. Its elements depend on the angle of propagation of the light and on the permittivity (and permeability) of the layer only. Unless the layer is gyrotropic its characteristic matrix is unchanged when the direction of propagation is reversed; that is to say when the Poynting vector (Dupertuis et al. 1994, Vigoureux and Giust 2000) is reversed. In using the characteristic matrix to compute transmission through and reflection from a stack, the boundary conditions at input and output faces are handled explicitly in a way that they are not in using Jones and Mueller matrices. Time-reversal symmetry may be shown to follow from a characteristic matrix that is unimodular. In general, there is no reciprocity of transmission through, or reflection from a dielectric stack but, exceptionally, a mid-plane symmetric stack does obviously exhibit reciprocal behaviour.
The reason that it is most instructive to couch consideration of reciprocity, in the context of reflection and transmission of light through a stack of dielectric layers, in terms of the characteristic matrix of the stack is because the characteristic matrix encapsulates the optical transfer through the stack itself as distinct from the configuration of external fields that are applied to it or that propagate from it. The characteristic matrix is the matrix that connects the complex amplitudes of the electric and magnetic fields at the top and bottom surfaces of the stack for a particular polarization state of the obliquely incident light:

\[
\begin{bmatrix}
E_i + E_r \\
H_i + H_r
\end{bmatrix} = M_1 \begin{bmatrix}
E_t \\
H_t
\end{bmatrix}.
\]  

The characteristic matrix of a uniform, isotropic dielectric layer is:

\[
M_1 = \begin{bmatrix}
\cos(k_1 h) & i \sin(k_1 h) \\
i \gamma_1 \sin(k_1 h) & \cos(k_1 h)
\end{bmatrix},
\]

where for TE modes gamma is given by

\[
\gamma_1 = \sqrt{\frac{\varepsilon_0}{\mu_0}} n_1 \cos(\theta_1)
\]

and for TM modes it is given by

\[
\gamma_1 = \sqrt{\frac{\varepsilon_0}{\mu_0}} n_1 \cos(\theta_1).
\]

The amplitude transmission and reflection coefficients \(t\) and \(r\) of the combined characteristic matrix \(M = M_1 M_2 M_3 \ldots\) of a stack are given by

\[
t = \frac{2\gamma_0}{\gamma_1 m_{11} + \gamma_1 \gamma_m 12 + m_{21} + \gamma_1 m_{22}},
\]

and

\[
r = \frac{\gamma_1 m_{11} + \gamma_1 \gamma_m 12 - m_{21} + \gamma_1 m_{22}}{\gamma_1 m_{11} + \gamma_1 \gamma_m 12 + m_{21} + \gamma_1 m_{22}},
\]

respectively.

A reciprocal version occurs when the time-reversed versions of incident and reflected or of incident and transmitted waves satisfy the same boundary conditions as the original external fields. The condition for this to happen is that the waves within the stack are time-reversible. Once again, the connections between time-reversed situations and the condition for them to be possible can be expressed algebraically.

In the context of the type of dielectric stack mentioned at the beginning of this section, the reciprocal configurations shown in figures 5(b) and (c) are justified by rather different arguments. The equality of \(r\) and \(r'\) follows from the symmetry of equations (6.1)–(6.6) under the reflection:

\[
R_x : \theta \mapsto -\theta,
\]

that maps the incident wave direction to the reflected wave direction.

The equality of \(t\) and \(t''\) follows from a more involved argument. Something to look out for is that unless a stack is bounded by semi-infinite media with the same optical properties the line of reasoning in relation to reciprocity of transmission below cannot be followed. Aronson (1997) has explored the consequences of this observation in some detail. This restriction of the scope of reciprocity does not apply to time-reversal symmetry where time-reversed versions of reflection/refraction of the boundary between dissimilar semi-infinite media can meaningfully
be compared. By its very symmetry, a mid-plane symmetric stack must exhibit reciprocal reflection and transmission properties. The challenge is to find whether a dielectric stack that is not mid-plane symmetric might, nevertheless, possess reciprocal attributes.

Given the restriction $\gamma_i = \gamma_t$, the expression for the amplitude transmittance in terms of the characteristic matrix elements of a stack becomes:

$$t = \frac{2}{m_{11} + m_{22} + \gamma_t m_{12} + (m_{21}/\gamma_t)}.$$  \hfill (6.8)

The implication is that if $M_1M_2$ differs from $M_2M_1$ by a traceless diagonal matrix then reciprocal transmission is assured. The argument for this proceeds as follows.

If $M_1$ is as given above and $M_2$ is given by

$$M_2 = \begin{bmatrix} \cos(k_2h) & \frac{i\sin(k_2h)}{\gamma_2} \\ i\gamma_2 \sin(k_2h) & \cos(k_2h) \end{bmatrix},$$ \hfill (6.9)

it follows that

$$M_1M_2 = M_2M_1 + X,$$ \hfill (6.10)

where

$$X = \begin{bmatrix} \left(\frac{\gamma_1}{\gamma_2} - \frac{\gamma_2}{\gamma_1}\right) \sin(k_1h) \cos(k_2h) & 0 \\ 0 & \left(\frac{\gamma_2}{\gamma_1} - \frac{\gamma_1}{\gamma_2}\right) \sin(k_1h) \cos(k_2h) \end{bmatrix}.$$ \hfill (6.11)

For a three layer stack the difference between the product of layer characteristic matrices in reverse order, $M_3M_2M_1$, and $M_1M_2M_3$ for forward propagation can be expressed in terms of commutators of $M_1$, $M_2$ and $M_3$ taken in pairs:

$$M_1M_2M_3 - M_3M_2M_1 \equiv \frac{1}{2}([M_1, [M_2, M_3]] - [M_2, [M_3, M_1]] + [M_3, [M_1, M_2]]),$$ \hfill (6.12)

where $[M_1, [M_2, M_3]] \equiv M_1[M_2, M_3] + [M_2, M_3]M_1$, etc are anticommutators.

This difference, like the commutator of $M_1$ and $M_2$ is diagonal and traceless. Consequently, equation (6.8) confirms the reciprocity of transmission under the stipulated conditions.

6.2. Time-reversal

For the purpose of considering the time-reversal, figure 5(a) is reproduced in figure 6(a) and its time-reversed configuration is shown in figure 6(b).

![Figure 6](image-url)
The condition for time-reversed propagation to be possible is that the characteristic matrix is unimodular since then the situation shown in figure 6(a) expressed as:

\[
\begin{bmatrix}
E_i + E_r \\
H_i + H_r
\end{bmatrix} = M_1 M_2 \begin{bmatrix}
E_t \\
H_t
\end{bmatrix},
\]

implies

\[
\begin{bmatrix}
E_i^* + E_r^* \\
H_i^* + H_r^*
\end{bmatrix} = M_1^* M_2^* \begin{bmatrix}
E_t^* \\
H_t^*
\end{bmatrix}. \tag{6.14}
\]

For a stack of layers each with a unimodular characteristic matrix:

\[
M_2 M_1 \begin{bmatrix}
E_i^* + E_r^* \\
H_i^* + H_r^*
\end{bmatrix} = M_2 M_1 M_1^* M_2^* \begin{bmatrix}
E_t^* \\
H_t^*
\end{bmatrix} = \begin{bmatrix}
E_t^* \\
H_t^*
\end{bmatrix}. \tag{6.15}
\]

This last equation expresses the time-reversed version of figure 6(a) as shown in figure 6(b). A characteristic matrix of the form given in equation (6.3) can be seen to be unimodular only if the wave number \(k_1\) is real.

Insofar as figure 6(a) can be identified as a beam splitter of the type that might be used in a conventional two-beam interferometer, figure 6(b) represents the process of beam combination in the same component for a particular relative phase of the two beams that are being combined. What this shows is that the two configurations illustrated in figure 6 and related to one another by time-reversal can both, in practice, be realized.

### 6.3. Generalized reciprocity of stratified media

Most authors on reciprocity are at pains to point out that their results are independent of linear loss and hence apply to evanescent waves equally as to travelling waves (Carminati et al 1998). The exposition given above can also be generalized in this direction as might be expected, time-reversibility being seen to be a different phenomenon with a separate formulation.

Reciprocity of reflection or transmission when the interfaces between the layers of a stack are corrugated or rough has attracted a great deal of attention (Brown et al 1984, Lewis et al 1998, Blumberg et al 2002, Gigli et al 2001). One comparatively recently discovered phenomenon here is that of enhanced back-scattering; this does not appear to have been explicitly linked with reciprocity though the connection would seem to be an obvious one.

Optically anisotropic layers introduce major complications not least as a result of double refraction. The very much simplified approach by way of Jones or Mueller matrices outlined in section 5 provides some sort of general framework. Reciprocity of reflection at oblique incidence from anisotropic layered structures assumes some importance in the context of liquid crystal display technology. Extended Jones matrix methods (Yeh 1982, Gu and Yeh 1993) can be applied to such problems but certain inferences are possible based on the spatial symmetry of particular devices and on reciprocity (Yeh and Gu 2000). Waves guided in layered structures are obviously important in being the underlying entities of integrated optical devices.

The reciprocity of Fresnel reflection from non-centrosymmetric media has been confirmed by Graham and Raab (1996) in a number of different relative orientations of crystal symmetry axes and illumination directions.

### 6.4. Reciprocity failure

The characteristic matrix given in equation (6.3) depends on the wave number of light in the layer. According to the magneto-optic Faraday effect, when a magnetic field is applied normal to the layer, this wave number is different according to the direction of propagation of the
light. An analysis is most easily carried out when the light is normally incident, in which case a circularly polarized incident wave is reflected and transmitted without polarization conversion. Similarly polarized radiation incident from the other side travels with a different phase speed with the result that, for this direction of propagation the characteristic matrix is different. In a paramagnetic or diamagnetic layer the resulting failure of reciprocity can be traced to an asymmetric, though still Hermitian, dielectric tensor with off-diagonal elements proportional to the applied magnetic field.

In more general cases, non-reciprocal propagation arising from an asymmetric dielectric tensor (Wettling 1976) in a magneto-optic layer of some structure or device provides a major challenge to modelling. This challenge has been met, in part, in works such as those of Visnovsky et al (2001).

Further discussion of magneto-optic gyrotropy as a source of reciprocity failure is given in section 7.1, where reference is made to the symmetry considerations that distinguish the phenomenon from the optical activity of chiral systems. In section 9 methods of modelling magneto-optical phenomena in the context of component design are cited.

7. Reasons for reciprocity failure

Afanasiev (2001) has exposed in detail the analytical complexities of fully characterizing source and detector in order to be able correctly to state electromagnetic reciprocity theorems. He gives conditions under which the Lorentz lemma (1896):

$$\int \vec{j}_1 \cdot \vec{E}_2 \, dV = \int \vec{j}_2 \cdot \vec{E}_1 \, dV,$$

between source and detector current densities and fields will hold. This lemma was reiterated by Landau and Lifshitz (1960) but the corresponding Feld–Tai lemma for the magnetic fields:

$$\int \vec{j}_1 \cdot \vec{H}_2 \, dV = \int \vec{j}_2 \cdot \vec{H}_1 \, dV,$$

was formulated later (Feld 1992, Tai 1992). In this context, only when sources are small in extent (compared with wavelengths) and current densities are given by separable functions of space position and time does Lorentz reciprocity necessarily hold. Otherwise reciprocity failure is a consequence of certain types of unusual medium response as explained below.

Reciprocity of scattering of a plane incident wave, analysed in terms of the electromagnetic field rather than in some scalar wave approximation (Born and Wolf 1999), has been explicitly restricted to bounded scattering media with symmetric and linear permittivity, conductivity and permeability (de Hoop 1959). The sort of non-reciprocal propagation that is most often exploited in passive optical components is that associated with the gyrotropy of magneto-optic media (Agranowitz and Ginzburg 1966). Microwave and optical isolators and circulators are usually based on the phenomenon of gyrotropy and are capable of producing high ratios of reverse to forward insertion loss irrespective of the degree of polarization or coherence of inputs. However, another type of reciprocity failure (Gu and Yeh 1991, Yeh 1983, Jones and Cook 2000) is intrinsic to the wave-mixing phenomena studied in photorefractive adaptive optics. Photorefractive media are nonlocal in the sense that the gradients of fields appear in their constitutive relations and, therefore, such material lie outside the scope of the theories formulated by de Hoop and others (Beldyugin et al 1992). Until now, signal processing applications of adaptive response of optical materials have been few and far between but the developments of all-optical components for use in optical fibre systems may lead, in the medium term, to roles for non-reciprocal as well as reciprocal devices. In questioning claims to have observed reciprocity failure in reflection from rough
surfaces, Venable (1985) pointed out different circumstances in which such failure was to be expected. For example, changes in the spectral distribution of propagating light easily leads to non-reciprocal behaviour in layered structures. Thus, both inelastic light scattering and nonlinear frequency changing can lead to non-reciprocal behaviour in spatially unsymmetric media.

7.1. Magneto-optical gyrotropy

For what has been said up to now, it is evident that the reciprocity principle is extremely wide in its scope. The order in which the layers are traversed only exceptionally influences the transmission along a given path. However, the presence of reflections on the incidence side and their absence on the transmission side mean that the reciprocity theorem can be applied in different ways to the same experimental set-up. In this section, the possibility of reversible propagation within a uniform dielectric medium is considered. Once again, time-reversed versions of propagating waves must be explicitly considered for an appreciation of the nature of reciprocal behaviour to be obtained.

Media in which the normal modes of propagation of em-waves are circularly or elliptically polarized are those having asymmetric permittivity tensors. Special cases are (i) optically active media and (ii) magneto-optic media in which the permittivity tensor is asymmetric only in the presence of spontaneous magnetization or an externally applied magnetic field.

Consider the circularly polarized modes that are the eigenmodes of an optically active medium such as a quartz crystal. A forward propagating, right circularly polarized mode is transformed by the operation of time-reversal to a backward propagating mode that is also right circularly polarized. As explained in section 3, this follows when the time-reversal operation is defined to include phase-conjugation in accordance with the notion that time-reversal exchanges a leading for a lagging relationship between a pair of entities including the Cartesian components of the electric field of an electromagnetic wave.

In generating one (backward) mode from another (forward) one the symmetry operation introduces degeneracy. This is an example of a quite general association between symmetry and degeneracy (Heine 1960). Thus, in an optically active medium the forward and backward right circularly polarized modes travel with the same speed—slow or fast as the case may be.

In a magneto-optic medium there is a similar connection between time-reversal and degeneracy. However, in this case time-reversal of a fast, forward mode produces a fast backward mode only if the magnetization of the medium is simultaneously reversed. This means that time-reversed waves in the same medium (with magnetization in a fixed direction) are not degenerate and, consequently, that non-reciprocal propagation exists already within a single homogeneous part of a system.

Just as in an optically active medium, the normal modes of propagation in a gyrotrropic medium are circularly or elliptically polarized. However, counter-propagating waves of the same handedness in a magneto-optically gyrotrropic medium, unlike those in an optically active medium have different refractive indices. It is this type of non-reciprocity, at the level of material dielectric response that makes possible Faraday isolators, etc.

In a magnetic gyrotrropic medium the handedness of an eigenmode does not determine the effective refractive index of that mode. The direction of propagation, relative to the magnetization of the medium also plays a part. However, it is the optically active medium that is nonlocal in its dielectric response and in which the dielectric tensor depends on the wavevector of the light wave (Agranowitz and Ginzburg 1966).
Consider a solution of the wave equation:
\[ \nabla^2 E_i = \varepsilon_0 \mu_0 \sum_j \varepsilon_{ij} \frac{\partial^2 E_j}{\partial t^2}, \]  
(7.1)
of the form
\[ E_i(z, t) = E_i \exp[i(kz - \omega t)]. \]  
(7.2)
In the absence of absorption the dielectric tensor is Hermitian. For a nonabsorbing linearly
magneto-optic cubic crystal \( \varepsilon_{11} = \varepsilon_{22} \) and \( \varepsilon_{12} = -\varepsilon_{21} \propto M \) (Wettling 1976). Its imaginary
off-diagonal elements couple the transverse components of the field \( E_1 \) and \( E_2 \) of a wave
travelling in the z- (3) direction according to:
\[ (\varepsilon_{11} - n^2)E_1 + E_2 = 0, \quad \varepsilon_{21}E_1 + (\varepsilon_{22} - n^2)E_2 = 0, \]  
(7.3)
where
\[ \frac{k^2}{\varepsilon_0 \mu_0 \omega^2} = \frac{k^2 c^2}{\alpha^2} = \frac{k^2}{k_0^2} = n^2. \]  
(7.4)
The polarization eigenstates are:
\[ E_2 = \pm i E_1 \]
with eigenvalues
\[ (n_{\pm})^2 = \varepsilon_{11} \pm i \varepsilon_{12} \]  
(7.5)
with the upper sign associated with right and the lower with left circularly polarized light.
The time-reversed version of the right circularly polarized eigenmode is
\[ E^*_r(z, t) = E^*_i \exp[i(-kz - \omega t)] \]  
(7.6)
with
\[ E^*_2 = -i E^*_1 \]
and is also right circularly polarized. However, it is the eigen-mode associated with the eigen-
value \( \varepsilon_{11} - i \varepsilon_{12} \) rather than with \( \varepsilon_{11} + i \varepsilon_{12} \); i.e. for \( i \varepsilon_{12} > 0 \) it is a fast mode rather than a slow
mode.

To distinguish between the different polarizabilities of enantiomorphic units of opposite
handedness in a chiral medium, field gradient terms are required (Landau and Lifshitz 1960,
Agranowitz and Ginzburg 1966). The asymmetric elements of the dielectric tensor are
proportional to odd powers of the wave vector and the tensor takes a different form according to
the direction of propagation of the radiation. As a result, the circular birefringence is the same
irrespective of the direction of propagation in contrast to what happens in a magneto-optically
gyrotropic medium.

A general second rank tensor can be decomposed into the sum of a Hermitian tensor and
an anti-Hermitian tensor. Absorption is frequently modelled by the anti-Hermitian part of the
permittivity tensor. However, to derive the reciprocity theorem, property tensors are required
to be symmetric. This is achieved in lossless magneto-optic media by associating reversal of
the sign of the magnetization with permutation of tensor indices (Onsager 1931). In absorbing
media the anti-Hermitian part of \( \varepsilon_{ij} \) can be subsumed into a symmetric conductivity tensor
thereby retaining the property tensor symmetry required for reciprocity.

Magneo-optic effects, include those named for Faraday et al; the Faraday effect consisting
of dispersion between circularly polarized waves of the same handedness propagating parallel
and antiparallel to the magnetization of the optical medium; the Kerr magneto-optic effect
in which the dispersion between right and left circularly polarized waves influences, through the boundary condition, the reflection from a magnetized medium; the Voigt effect in which double refraction arises from magnetization perpendicular to the direction of propagation of a light wave. Magneto-optic effects can be divided into those that are even in components of the magnetization and those that are odd (Wettling 1976). Even (quadratic and higher) magneto-optic effects only usually arise when radiation is symmetrically incident on a magneto-optic medium. They are of little interest for the fabrication of non-reciprocal devices because their contribution to property tensors is symmetric even without the need to reverse magnetization direction. Propagation is reciprocal even in a permanently magnetized medium. Apart from the symmetry of illumination geometry, crystal symmetry, as indicated above, exerts an influence on the range of phenomena that present themselves in magneto-optic media. Detailed analysis of the symmetry restrictions requires consideration of coloured symmetry groups (Shubnikov et al 1964). In the context of microwave engineering Dimitriyev (1997) has analysed the interplay between device symmetry and crystal symmetry that underlies magneto-optic device design.

In the microwave regime, ferromagnetic resonance can be exploited in circular polarizers. The corresponding effect in the visible or infrared spectrum is magnetic circular dichroism (Stephens 1970). It is well known as a natural phenomenon (Shurcliff 1962) but does not appear to have attracted the same attention for device development as its microwave counterpart.

7.2. Nonlocal photorefractive response

Photovoltaic electro-optic crystals (Gunter 1982) exhibit extremely interesting and potentially useful wave-mixing phenomena as a result of the dynamic grating that can be formed in them when they are subjected to optical fringe patterns formed by light of an appropriate wavelength. Non-reciprocity in these materials takes the form of two-wave mixing and requires the formation of a phase grating that is displaced from the fringe pattern generated by the two waves that are being mixed. This ‘nonlocal’ material response is the result of trapping of charges at sites remote from the peaks of the bright fringes where they are generated. The adaptive nature of the material response enables self-forming and self-aligning optical components to be constructed without the need to provide feedback by way of an external electronic circuit. Applications that have been investigated include tracking filters to convert double sideband modulated optical carriers to single sideband (Saffman et al 1991, Vourc'h et al 2002) and diffractive couplers to allow a beam of low spatial coherence to pump an oscillator of high beam quality (Kaczmarek and Eason 1998).

The question of whether the asymmetric wave mixing phenomena that are evident in photovoltaic, photorefractive materials are, or are not, reciprocal has been addressed by Yeh (1983), Gu and Yeh (1991) and by Jones and Cook (2000). The hereditary nature of the constitutive relations of the photorefractive material is central to the resolution of this issue. Apart from magneto-optic photorefractive media the dielectric properties of the materials so far investigated seem to give rise to scattering phenomena in which the scattering amplitude is reciprocal \( f(\hat{s}_2, \hat{s}_1) = f(-\hat{s}_1, -\hat{s}_2) \) (the notation is that of Born and Wolf (1999)). What is unusual about these materials is that \( f(\hat{s}_2, \hat{s}_1) \) changes in time in response to light exposure, a phenomenon indicated by the inequality:

\[
\frac{d}{dt} f(\hat{s}_2, \hat{s}_1) \neq 0.
\] (7.7)
There are theoretical and experimental reasons for believing this secular change to be non-reciprocal in the following sense:

$$\frac{df(\hat{s}_2, \hat{s}_1)}{dt} \neq \frac{df(-\hat{s}_1, -\hat{s}_2)}{dt}. \quad (7.8)$$

Figure 7 shows the simplest type of wave mixing stable states that can be expected to be non-reciprocal in this sense (MacDonald et al 1984, Zha and Gunter 1985).

Whether this behaviour can be exploited to produce new families of self-forming and self-healing non-reciprocal components is yet to be determined. Hendricks et al (2001) describe a multimode fibre non-reciprocal resonator aimed at producing non-reciprocal elements to be used in high coherence gain grating lasers.

Malinowski et al (1996) ascribe the experimental observation (Zheludev et al 1994) that propagation in a noncentrosymmetric medium is only approximately reciprocal to nonlocality expressed by gradient terms in the linear dielectric constitutive relation. In the next section further studies of the connection between centrosymmetry, time-reversibility and locality are reviewed.

### 7.3. Noncentrosymmetric media

The influence of more general discrete space-time symmetries than those considered so far is examined by Svirko and Zheludev (1995). Basing their argument on the validity of relativistic reversality (the simultaneous inversion of space and time) in optics, they conclude that media that lack a centre of inversion may exhibit departure from reversality. There is evidence (Zheludev et al 1994, 1995), albeit disputed in some quarters (Etchegoin et al 1994, Lakhtakia 2002), that zinc-blende structure semiconductors are time-nonreversible in their interaction with light.

Chiral media have also been found to exhibit non-reciprocal (Bennett et al 1996) or time-odd (Schwancke et al 2003) phenomena as a result of their lack of centrosymmetry.

### 8. Coherence and insertion loss and modal properties

The emphasis on linear analysis in what has been described above pays dividends in allowing engineering devices to be considered from the point of view of reciprocity. Those optical elements that have roles in routing signals through optical systems can be divided into two categories. There are those that depend upon interference, for which the light input must exhibit a sufficient degree of coherence. The theory of such devices exploits, whenever possible, the linear superposition of electric and magnetic fields. On the other hand, magneto-optically non-reciprocal components such as Faraday isolators and circulators may function irrespective of the degree of coherence of the light signal being routed. When coherence is lacking, to the extent that light signals when superposed do not interfere, a linear analysis may still be provided as, for example, by the use of Stokes vectors.
Figure 8. A microwave magic tee steers waves from a source to an antenna and from the antenna to the detector.

The spatial coherence required for components that exploit interference for beam steering is usually achieved in waveguides with only one, or at most a few, guided modes. Optical components based on integrated optical guides include electro-optic Mach–Zehnder modulators and switches and magneto-optic Mach–Zehnder isolators that will be referred to in the next section. Microwave components include also the passive magic tee that can route excitation to the arms of a Michelson interferometer and away from the detector arm (figure 8).

This component performs the same function as the beam splitter/combiner in a conventional optical interferometer. By shaping the electric field profile of the guided wave in the input arm, counter-propagating waves are directed towards the measurand and reference arms with no power reaching the detector directly. However, for certain relative phases of the measured and reference reflections a signal does reach the detector. In radar applications using the same antenna for transmission and reception the antenna is placed in the measurand arm. Optical fibre directional couplers perform the same function as the magic tee in a microwave system and, once again, their operation depends on coherent excitation in the various arms.

The engineer’s preference (Faria 2002) is to treat passive optical components as n-ports characterized by insertion losses between each ordered pairs of ports. Magneto-optics provides the means of achieving non-reciprocal insertion losses between pairs of ports. This can be implemented by interference that, as explained above, requires a degree of coherence, or by polarization. In the latter case light coherence is not essential. This is because each of the component waves in an incoherent superposition can experience the same insertion loss irrespective of the other components, provided that each component has a frequency within the stop band of the device. From what has been said above about coherent beam combining, it is clear that, under coherent illumination from more than one direction the insertion losses between pairs of ports of an optical n-port provides at best a partial description of the total system response. In practice, the most useful non-reciprocal devices are those that are insensitive in their transmission between a pair of ports to the conditions prevailing at any other port. These are usually, but not invariably, those devices that are based on non-reciprocal transmission associated with a magneto-optic effect.

Coherent routing of signals by mode interference is extremely important in both microwave and optical systems. It has sometimes been treated as a form of non-reciprocal propagation but, on the criteria set out in section 4 it is clearly reciprocal in nature. Nevertheless, references to non-reciprocal behaviour crop up in connection with such arrangements as antireflection stacks (Glaser et al 2002) and splices between single mode and multimode optical fibres (Kweon and Park 1999).

9. Magneto-optic non-reciprocal components and their applications

Infrared waveguiding techniques have followed in the wake of microwave electromagnetic guides though with a time lag of nearly thirty years. After the development of sources
Reciprocity in optics

Figure 9. The effect of reflections on the spectrum of a laser diode (Giallorenzi et al. 1982): (a) isolated laser; (b) 0.04% feedback; (c) 0.06% feedback; (d) 0.3% feedback; (e) 1.5% feedback.

and low-loss waveguides, each area has spawned specialized components including some non-reciprocal ones. Microwave gyrators are mostly based on ferrite loading of metallic waveguides (Krupka 1991, Dmitriyev 1997, Adams et al. 2002) and are used in isolators, circulators and phase modulators. Such devices are needed in phase array radar systems for the efficient generation, routing and processing of microwave signals. The infrared analogues of these devices, that are usually also based on non-reciprocal transmission through magneto-optic media, are needed for somewhat different engineering purposes in optical fibre systems (Auracher and Witte 1975).

The spectral purity of the output of a semiconductor laser diode is susceptible to major degradation by inadvertent reflection from outside the laser. In any application that is sensitive to phase noise from the source including interferometry and coherent detection this can cause serious problems. Figure 9 shows the affect of small amounts of reflection on the spectrum of a double-heterostructure laser diode similar to those used in optical fibre communications (Miles et al. 1980, Giallorenzi et al. 1982).

The increase in spectral width that results from unwanted reflections increases pulse spreading in propagation of pulses along a fibre and can cause cross-talk between bands in a wavelength division multiplexed system. The need to suppress reflections in optical fibre systems has led to a market for pigtailed fibre isolators based on the Faraday effect. A device consisting of a linear polarizer, with Jones matrix $M_1$, a 45° Faraday rotator, with Jones matrix $M_2$, and a linear polarizer, with Jones matrix $M_3$, has high transmission in the forward direction but blocks reverse propagation. When the combination of elements with Jones matrices:

\[
M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ -\frac{1}{\sqrt{2}} & 1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} \frac{1}{2} & i/2 \\ i/2 & \frac{1}{2} \end{bmatrix}
\]

and
that act on the Jones vector \( \begin{bmatrix} E_x \\ E_y \end{bmatrix} \) is used in a stack with Jones matrix \( M = M_3 M_2 M_1 \), the action is that of an isolator. In this case,

\[
M = M_3 M_2 M_1 = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & \frac{1}{\sqrt{2}}
\end{bmatrix}.
\]

For reciprocal components, the Jones matrix for reverse propagation would, as shown in section 5, be

\[
M' = M_3' M_2' M_1',
\]

which gives

\[
M' = \begin{bmatrix}
0 & 0 \\
1 & 1
\end{bmatrix}.
\]

This would be the form to use if the Faraday rotator was replaced by a reciprocal optically active medium such as a sugar solution. On the other hand, for a general Faraday rotator the Jones matrix, \( M \), is given by

\[
M = \begin{bmatrix}
\cos(VBl) & \sin(VBl) \\
-\sin(VBl) & \cos(VBl)
\end{bmatrix},
\]

where \( V \) is the Verdet constant, \( B \) is the permanent magnet’s field acting in the direction of propagation and \( l \) is the path length. The relation between propagation direction and field direction is changed for reverse propagation and \( M'_2 = M_2 \) not \( M'_2 \). Therefore,

\[
M' = M_3' M_2' M_1' = \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
1 & \frac{1}{2} \\
\frac{1}{2} & 1
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}.
\]

In this direction the insertion loss of a single stage Faraday isolator can exceed 30 dB.

The operational principle of a Mach–Zehnder isolator is shown in figure 10 (Bahlmann et al 1998). As a result of the non-reciprocal Faraday effect, rightward propagating waves in
Reciprocity in optics

A different principle of operation uses non-reciprocal dispersion and a diffractive structure to phase match to radiative modes for only one sense of propagation (Shintaku 1995, 1998). Integrated optic circulators have also been developed based on the Faraday effect. These have also taken the form of non-reciprocal Mach–Zehnder interferometers with wave plates and polarization rotators incorporated asymmetrically in the two arms (Shirasaki et al 1981, Mizumoto et al 1990, Sugimoto et al 1999).

Lumped dispersion compensation is required for erbium doped fibre amplified systems using standard single mode silica fibres. This is because these fibres have a zero dispersion wavelength, at which waveguide dispersion and material dispersion cancel, at 1300 nm, whilst the erbium amplifiers operate around a wavelength of 1550 nm. Chirped fibre Bragg gratings can provide lumped dispersion compensation in reflection but efficient routing of signals through them creates a need for optical fibre, three-port circulators (see figure 11). Different optical frequencies are reflected at different positions in the grating, thereby compensating for dispersion accumulated over many kilometres of optical fibre cable.

Wavelength division multiplexing (WDM) provides a means of increasing the bit rate in a single mode optical fibre from tens to hundreds of gigabytes per second but creates a need for precise channel dropping filters for separating one of the multiplexed signals from all of the others at a receiver. Once again, as shown in figure 12, a combination of fibre Bragg grating and circulator allows this to be done in a way that retains the advantages of optical fibre based components.

The materials requirements for isolators and circulators fall into two areas: the magneto-optic Faraday medium and materials for wave plates. Yttrium iron garnet (YIG) with substitutions of lanthanum, gallium or cerium as a film on a gadolinium gallium garnet (GGG) serves in the former capacity (Sugimoto et al 1993) whilst polymer (polyimide) films (Inoue et al 1997) or inorganic crystal slices (Radojevic et al 2000) are good for the latter. In addition,
permanent magnets of samarium cobalt are suitable (Shirasaki et al 1981, Sugimoto et al 1999) for providing the magnetic fields that make devices non-reciprocal. One recognized feature is that antiferromagnetic elements, even in the absence of an applied magnetic field can lead to reciprocity failure (Graham and Raab 1991, Dumelow and Camley 1996).

Consideration of reciprocity plays a part in the design of measuring devices as well as of communications components (Akhavan Leilabady et al 1986, Wilson and Reinholtz 1992, Kung et al 1996, Lin et al 1999, Ruan et al 2000). Optical gyroscopes exploit non-reciprocal propagation of relativistic origin in a Sagnac interferometer (Post 1967). They are required to discriminate between rotation with respect to inertial frames and other sources of reciprocity failure. Dielectric mirrors containing a single magneto-optic layer have been investigated with a view to their application in this type of instrument (Andrews and King 1994). However, the literature on optical gyroscopes and their reciprocal and non-reciprocal properties is too extensive in some ways to include in this report.


The extension of modelling to nonlinear devices has also attracted much attention (Boardman et al 1991a, b). However, in contrast to microwave technology where parametric frequency conversion and gain have assumed a significant practical role, nonlinear optical and, more significantly infrared devices have yet to find significant market niches. The reason for this may be that nonlinear microwave processing is usually based on the very strong nonlinearity of varicap diodes rather than the weak distributed nonlinearity (second-order or otherwise) of dielectric media. Optical fibres differ from microwave guides in having much lower losses, with the result that the thresholds for processes such as stimulated Raman gain are within reach of signal powers that can readily be achieved. Optical fibre distributed nonlinear processes that have been intensively investigated in the laboratory but have been slow to be adopted in field systems include, in addition to Raman gain (under consideration for use in conjunction with fibre lasers for gain flattened optical amplification), soliton propagation. Possible ultimate application of these technologies means that the extension of the reciprocity principle to nonlinear systems may be of more than theoretical interest.

10. Nonlinear response and Onsager reciprocity in optics

Nonlinearity, together with asymmetry of property tensors, was explicitly excluded by de Hoop in his formulation of the reciprocity principle. It is easy to see (Venable 1985) why a nonlinear material response might be thought to lead to scattering that vitiates the reciprocity principle. Consider, for example, a layered medium in which frequency changing or self-focusing is asymmetrically located and in which there is also nonuniform dichroism. The order in which the nonlinear and dichroic layers are encountered by incident light will significantly influence the balance between nonlinear and absorptive effects. A specific ordering of such effects is
indeed exploited in such measuring instruments as pulse autocorrelators for the purpose of enhancing signal-to-noise ratios by dichroic filtering of changed frequency spectra.

The nonlinear constitutive relation:

$$P_i = \varepsilon_0 (\chi^{(1)}_{ij}(-\omega; \omega) E_j(\omega) + \chi^{(2)}_{ijk}(-\omega_j; \omega_1, \omega_2) E_j(\omega_2) E_k(\omega_1) + \cdots)$$ (10.1)

contains the linear susceptibility $\chi^{(1)}_{ij}$ and the second-order susceptibility $\chi^{(2)}_{ijk}$. It has been pointed out (Butcher and Cotter 1990) that, even in the presence of resonant enhancement, the susceptibility tensors $\chi^{(n)}_{ijk}$ are intrinsically symmetric in the pairs $(j, \omega_2)$, $(k, \omega_1)$, etc. For the second-order susceptibility this might be thought to justify the contracted notation (Landolt-Börnstein 1984):

$\chi_{11}^{i} = \chi_{11}^{i}$,
$\chi_{22}^{i} = \chi_{22}^{i}$,
$\chi_{33}^{i} = \chi_{33}^{i}$,
$\chi_{23}^{i} = \chi_{32}^{i} = \chi_{4}^{i}$,
$\chi_{31}^{i} = \chi_{13}^{i} = \chi_{5}^{i}$,
$\chi_{12}^{i} = \chi_{21}^{i} = \chi_{6}^{i}$,

in accordance with the expectation that $\chi^{(2)}_{ij}$ is symmetric in $j$ and $k$. Away from resonance total symmetry under perturbation of $(i, -\omega_0)$, $(j, \omega_2)$, $(k, \omega_1)$ is held to apply and to justify the symmetry of $\chi^{(2)}_{ijk}$ under all permutations of its indices (Kleinman 1962). There has been an experimental investigation (Crane and Bergman 1976) of the extent to which this symmetry actually holds in a particular material (quartz). The above symmetries reduce the number of independent tensor elements of $\chi^{(2)}_{ijk}$ to 18 and 10, respectively, before crystal symmetry restrictions are taken into account.

Time-reversal symmetry expressed as

$$\chi^{(1)}_{ij}(-\omega; \omega) = \chi^{(1)}_{ji}(-\omega; -\omega)$$ (10.2)

has been combined with total permutation symmetry of $\chi^{(1)}_{ij}$ to infer its symmetry (Nye 1969) thus:

$$\chi^{(1)}_{ij}(-\omega; \omega) = \chi^{(1)}_{ji}(-\omega; -\omega) = \chi^{(1)}_{jj}(-\omega; \omega).$$ (10.3)

A similar line of reasoning, applied to nonlinear processes, allows Naguleswaran and Stedman (1998) to infer the following symmetry of $\chi^{(3)}_{ijkl}$. They have restricted their attention to self-conjugate processes in order to be able to extend reciprocity to the nonlinear realm. By ‘self-conjugate’ they mean processes involving even numbers of photons in which input and output photons can be associated in degenerate pairs. An example of the type of reciprocal configurations that they expect to exist is shown in figure 13.

The reasoning used above gives, in this case,

$$\chi^{(3)}_{ijkl}(-\omega_1; -\omega_2, \omega_1, \omega_2) = \chi^{(3)}_{ijkl}(\omega_1; \omega_2; -\omega_1, -\omega_2) = \chi^{(3)}_{iklj}(-\omega_1; -\omega_2, \omega_1, \omega_2).$$ (10.4)
There is a debate about whether examples of pairs of processes related as in figure 13 might exhibit reciprocity failure as a result of spin–orbit interactions or other perturbations (Zheludev et al 1994, Naguleswaran and Stedman 1998).

Naguleswaran and Stedman interpret the reciprocal pair shown in figure 13 as an example of Onsager reciprocity (Gabrielli et al 1999). Onsager reciprocity, just like the type of reciprocity discussed earlier, is a type of symmetry that can manifest itself even when losses are present. Its origin goes back to the thermodynamic relationships among thermoelectric coefficients that were discussed by Lord Kelvin. If a matrix connecting forces and currents (albeit of generalized type) is made, there may exist certain symmetries of the coefficients relating different Cartesian components of force and current. This has in common with source/detector reciprocity the symmetry of a response matrix that is evidently not purely an expression of time-reversal symmetry. Unlike source/detector reciprocity, Onsager reciprocity relates to the local response of the optical medium. In this respect, it is reminiscent of the symmetry conditions that were imposed by de Hoop to ensure linear reciprocity.

As in linear reciprocity, where the symmetry of property tensors

\[ \varepsilon_{ij} = \varepsilon_{ji}, \]

generalizes (Onsager 1931, Landau and Lifshitz 1960) to

\[ \varepsilon_{ij}(M) = \varepsilon_{ji}(-M), \]  \hspace{1cm} (10.5)

so

\[ \chi^{(3)}_{ijkl}(M) = \chi^{(3)}_{klij}(-M) \]  \hspace{1cm} (10.6)

for the self conjugate four photon process described earlier.

The intrinsic symmetry used by Butcher and Cotter implies that, in second harmonic generation, the nonlinear polarization is given by

\[ P_s(2\omega) = \varepsilon_0 \chi^{(3)}_{ijkl}(-2\omega; \omega, \omega)(E_j(\omega)E_k(\omega) + E_k(\omega)E_j(\omega)). \]  \hspace{1cm} (10.7)

In other words, \( \chi^{(3)}_{ijkl}(-2\omega; \omega, \omega) \) is symmetric in its last two indices, justifying the contracted notation given above. The argument depends on the degeneracy of the driving fields and does not hold for sum frequency generation. In that case the sum frequency polarization can depend on antisymmetrized products of field components like

\[ E_x(\omega_1)E_y(\omega_2) - E_y(\omega_1)E_x(\omega_2) \]  \hspace{1cm} (10.8)

and, apart from crystallographic symmetry restrictions, \( \chi^{(3)}_{ijkl}(-\omega_2; \omega_1, \omega_2) \) has no permutation symmetry with respect to its indices. Given that the simultaneous permutation of tensor indices and wave frequencies referred to as total permutation symmetry relates elements of different property tensors, there must be scope for considering more restricted permutation symmetries. It may be that a time-reversed process, with the same degree of resonant enhancement, occurs with the same amplitude as the original process. The processes that give rise to higher order dielectric susceptibilities are cyclic and, therefore, cyclic permutations of indices and frequencies have a special significance. Anti-cyclic permutations are associated with time-reversed versions of nonlinear processes. A question arises as to whether the terms in the development of the nonlinear polarization, as products of components of the electric field, are restricted to particular representations of the permutation group (Hamermesh 1962). Sequences of atomic states as well as of states of the radiation field will need to be classified according to representations of the symmetric group in order that invariants of this group can be constructed and the number of independent tensor elements for a given nonlinear process enumerated. In this process, one-dimensional representations of the symmetric group will certainly not exhaust the possibilities.
All in all, and not surprisingly, recent developments in the application of reciprocity principles to nonlinear optics are less transparent than what has been found for linear response. However, one of their key features seems to be that reciprocal relationships are based on microreversibility through explicit consideration of quantum amplitudes for processes of restricted types. One of the things that is unclear is the role that spatial symmetry plays in nonlinear reciprocity and, in particular, whether the symmetry of nonlinear local response is sufficient to ensure some kind of reciprocity of scattering amplitudes in the same way that linear local response does in the classical case. Frequency changing is only one aspect of nonlinear dielectric response and it is probable that other nonlinear phenomena such as self-focusing and soliton propagation can equally well lead to reciprocity failure.

11. Conclusions

The idea of interchanging cause and effect that underlies the reciprocity principle is distinct from time-reversibility. This has been shown to be because, for a given position of the source, there are alternative positions for the detector. Each of these alternatives leads to a distinct reciprocal configuration. Predictions based on reciprocity can be made under less restrictive conditions than those that apply to time-reversibility.

In systems with linear response, matrix representations of fields and transfer operators naturally arise. However, the exact forms of such representations are various and the algebraic conditions associated with reciprocal or time-reversible propagation are different in different cases. For example, the norms of Jones vectors used to represent the polarization states of waves are unchanged by lossless elements which, in consequence, are represented by unitary matrices. By way of contrast, the field vectors acted on by the characteristic matrices of dielectric stacks do not admit the definition of a norm as their elements are of different dimensional character. In this case, losslessness of a stack appears in the unimodularity of the characteristic matrix.

Magneto-optic media are well known from their device applications in microwave and optical technologies. In many cases these devices are based on the non-reciprocity of wave propagation within the magneto-optic medium. It is this that allows circulators and isolators to be fabricated. Magneto-optic non-reciprocal devices are characterized by being capable of giving zero output at a particular port irrespective of the level or degree of coherence of excitation at all input ports.

Apart from magneto-optic media, the other main type of medium that is clearly non-reciprocal in its response is the two-wave mixing medium. For such a medium the insensitivity of particular outputs to the degree of coherence of input excitations mentioned above does not apply. Nevertheless, reciprocity of transmission, as formulated by de Hoop, fails when the stable states established by forward and reverse configurations are compared.

Finally, a clear exposition of what might be meant by ‘reciprocity’ in the presence of nonlinear material response is likely to be based on relations between quantum amplitudes for symmetrically related nonlinear processes.

References

Reciprocity in optics

751

di Stasio S 2002 Experiments on depolarized optical scattering to sense in situ the onset of early agglomeration between nano-size particles J. Quant. Spectrosc. Radiat. Transfer 73 423–32
Dmitriyev V A 1997 Symmetry of microwave devices with gyrotropic media-complete solution and their applications IEEE Trans. Microwave Theory Tech., 45 394–401
Gangi A F 2000 Constitutive equations and reciprocity Geophys. J. Int. 143 311–18
Gerjuoy E and Saxon D S 1954 Variational principles for the acoustic field Phys. Rev. 94 1445–58
Gibbs 1985 Optical Bistability: Controlling Light with Light (London: Academic)
Gigli M L, Depine R A and Valencia C I 2001 Reciprocity relations for uniaxial gratings Optik 112 567–72
Hamermesh M 1962 Group Theory and its Applications to Physical Problems (Reading, MA: Addison-Wesley)
Heine V 1960 Group Theory in Quantum Mechanics (Oxford: Pergamon)
Imbert C 1972 Calculation and experimental proof of the transverse shift induced by total internal reflection of a circularly polarized light beam Phys. Rev. D 5 787–96
Jones R C 1941 J. Opt. Soc. Am. 31 488
Jones R C 1941 J. Opt. Soc. Am. 46 126
Kweon G and Park I 1999 Splicing losses between dissimilar optical waveguides J. Lightwave Technol. 17 690–703
Lamb H 1889 Lond. Math. Soc. Proc. 19 144
Landolt-Börnstein 1984 New Series vol 18 ed K H Hellwege and A M Hellwege (Heidelberg: Springer) (also 1, 2 and 11)
Lerner L 1993 Reception of radiation by bent dielectric wave-guides Opt. Lett. 18 1627–9
McMaster W H 1954 Polarization and the stokes parameters Am. J. Phys. 22 351–62
McIntyre P R 1991 Mode orthogonality in reciprocal and non-reciprocal wave-guides IEEE Trans. Microwave Theory Techn. 39 1808–16
McMaster W H 1954 Polarization and the stokes parameters Am. J. Phys. 22 351–62
Reciprocity in optics


Morse P M and Feshbach H 1953 Methods of Theoretical Physics (New York: McGraw-Hill) section 7.5


O’Neill E L 1963 Introduction to Statistical Optics (Reading, MA: Addison-Wesley) p 133

Ou Z Y and Mandel L 1989 Derivation of reciprocity relations for a beam splitter from energy balance Am. J. Phys. 57 66–7

Perrin F 1942 Polarization of light scattered by isotropic opalescent media J. Chem. Phys. 51 415–27


Rayleigh J W S 1878 Treatise on Sound vol II (London: Macmillan) p 131


Saxon D S 1955 Tensor scattering matrix for the electromagnetic field Phys. Rev. 100 1771–5


Sheppard C J R and Gu M 1993 Imaging by a high aperture optical system J. Mod. Opt. 40 1631–51


Shintaku T 1995 Integrated optical isolator based on nonreciprocal higher-order mode conversion Appl. Phys. Lett. 66 2789–91

Shintaku T 1998 Integrated optical isolator based on efficient nonreciprocal radiation mode conversion Appl. Phys. Lett. 73 1946–8


Shubnikov A V and Belov N V 1964 Coloured Symmetry (Oxford: Pergamon)


Stokes G G 1849 Cambridge Dublin Math. J. 4 1

Stokes G G 1852 On the composition and resolution of streams of polarized light from different sources Trans. Camb. Phil. Soc. 9 399–416

Sugimoto N, Katoh Y and Tate A 1993 Magneto-optic buried channel waveguides for 45° non-reciprocal waveguide rotator Appl. Phys. Lett. 63 2744–6


Tang S T and Kwok H S K 2001 3×3 matrix for unitary optical systems JOSA 18 2138–45
Tarantola A 1987 Inverse Problem Theory (Amsterdam: Elsevier) p 548
Yamaguchi J, Ikegaya M and Nakano H 1992 Analysis of bent step-index optical fibres by the beam propagation method IEE Proc.—J. Optoelectron. 139 201–7
Yeh P and Gu C 2000 Symmetry of viewing characteristics of liquid crystal displays with compensators Displays 21 31–8