Quantum Heat Engines and Refrigerators: Continuous Devices

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Keywords
quantum thermodynamics, quantum tricycle, quantum amplifier, laser cooling, absolute zero temperature

Abstract
Quantum thermodynamics supplies a consistent description of quantum heat engines and refrigerators up to a single few-level system coupled to the environment. Once the environment is split into three (a hot, cold, and work reservoir), a heat engine can operate. The device converts the positive gain into power, with the gain obtained from population inversion between the components of the device. Reversing the operation transforms the device into a quantum refrigerator. The quantum tricycle, a device connected by three external leads to three heat reservoirs, is used as a template for engines and refrigerators. The equation of motion for the heat currents and power can be derived from first principles. Only a global description of the coupling of the device to the reservoirs is consistent with the first and second laws of thermodynamics. Optimization of the devices leads to a balanced set of parameters in which the couplings to the three reservoirs are of the same order and the external driving field is in resonance. When analyzing refrigerators, one needs to devote special attention to a dynamical version of the third law of thermodynamics. Bounds on the rate of cooling when $T_c \to 0$ are obtained by optimizing the cooling current. All refrigerators as $T_c \to 0$ show universal behavior. The dynamical version of the third law imposes restrictions on the scaling as $T_c \to 0$ of the relaxation rate $\gamma_c$ and heat capacity $c_V$ of the cold bath.
1. INTRODUCTION

Our cars, refrigerators, air conditioners, lasers, and power plants are all examples of heat engines. The trend toward miniaturization has not skipped the realm of heat engines, leading to devices on the nano- or even on the atomic scale. Typically, in the practical world, all such devices operate far from the maximum efficiency conditions set by Carnot (1). Real heat engines are optimized for powers sacrificing efficiency. This trade-off between efficiency and power is the focus of the field of finite-time thermodynamics. The field was initiated by the seminal paper of Curzon & Ahlborn (2). From everyday experience, the irreversible phenomena that limit the optimal performance of engines can be identified as losses due to friction, heat leaks, and heat transport (3). Is there a unifying fundamental explanation for these losses? Is it possible to trace the origin of these phenomena to quantum mechanics? The tradition of thermodynamics is followed by the study of hypothetical quantum heat engines and refrigerators to address these issues. Once understood, these models serve as a template for real devices.

Gedanken heat engines are an integral part of thermodynamical theory. In 1824, Carnot (1) set the stage by analyzing an ideal engine. His analysis preceded the systematic formulation that led to the first and second laws of thermodynamics (4). Amazingly, thermodynamics was able to keep its independent status despite the development of parallel theories dealing with the same subject matter. Quantum mechanics overlaps with thermodynamics in that it describes the state of matter. However, quantum mechanics additionally includes a comprehensive description of dynamics. This suggests that quantum mechanics can originate a concrete interpretation of the word dynamics in thermodynamics, leading to a fundamental basis for finite-time thermodynamics.

Quantum thermodynamics is the study of thermodynamical processes within the context of quantum dynamics. Historically, consistency with thermodynamics led to Planck’s law, the basics of quantum theory. Following the ideas of Planck on blackbody radiation, Einstein (5) quantized the electromagnetic field in 1905. Einstein’s paper is the birth of quantum mechanics together with quantum thermodynamics.

Quantum thermodynamics is devoted to unraveling the intimate connection between the laws of thermodynamics and their quantum origin (6–34). The following questions come to mind: How do the laws of thermodynamics emerge from quantum mechanics? What are the requirements of a theory to describe quantum mechanics and thermodynamics on a common ground? What are the fundamental reasons for a trade-off between power and efficiency? Do quantum devices operating far from equilibrium follow thermodynamical rules? Can quantum phenomena affect the performance of heat engines and refrigerators?

We address these issues below by analyzing quantum models of heat engines and refrigerators. Extreme care has been taken to choose a model that can be analyzed from first principles. We consider two types of models: continuous operating models resembling turbines and discrete four-stroke reciprocating engines. The present review focuses on continuous devices.

2. THE CONTINUOUS ENGINE

An engine is a device that converts one form of energy to another: heat to work. In this conversion, part of the heat from the hot bath is ejected to the cold bath, limiting the efficiency of power generation. This is the essence of the second law of thermodynamics.

A continuous engine operates in an autonomous fashion, attaining a steady-state mode of operation. The analysis therefore requires an evaluation of steady-state energy currents. The operating part of the device is connected simultaneously to the hot, cold, and power output leads. The primary macroscopic example is a steam or gas turbine. The primary quantum heat engine is
Figure 1

The quantum three-level amplifier as a Carnot heat engine. The system is coupled to a hot bath with temperature \( T_h \) and to a cold bath with temperature \( T_c \). The output \( P \) is a radiation field with frequency \( \nu \).

(In the figure, \( h = 1 \).) Power is generated, provided there is population inversion between level \( \epsilon_2 \) and \( \epsilon_1 \): \( p_2 > p_1 \). The hot bath equilibrates levels \( \epsilon_0 \) and \( \epsilon_2 \) via the rates \( \kappa \uparrow \) and \( \kappa \downarrow \) such that \( \kappa \uparrow / \kappa \downarrow = \exp(\hbar \omega_{20}/k_B T_h) \). The efficiency \( \eta = \nu/\omega_{20} \leq 1 - T_c/T_h \). Reversing the direction of operation using power to drive the population from level \( \epsilon_1 \) to \( \epsilon_2 \) generates a heat pump; then \( p_2 < p_1 \).

2.1. The Quantum Three-Level System as a Heat Engine

A contemporary example of a Carnot engine is the three-level amplifier. The principle of operation is to convert population inversion into output power in the form of light. Heat gradients are employed to achieve this goal. Figure 1 shows its schematic construction. A hot reservoir characterized by temperature \( T_h \) induces transitions between the ground state \( \epsilon_0 \) and the excited state \( \epsilon_2 \). When equilibrium is reached, then the population ratio between these levels becomes

\[
p_2/p_0 = e^{-\omega_{20}/k_B T_h},
\]

where \( \omega_h \equiv \omega_{20} = (\epsilon_2 - \epsilon_0)/\hbar \) is the Bohr frequency, and \( k_B \) is the Boltzmann constant. The cold reservoir at temperature \( T_c \) couples level \( \epsilon_0 \) and level \( \epsilon_1 \), meaning that

\[
p_1/p_0 = e^{-\omega_{10}/k_B T_c},
\]

where \( \omega_c \equiv \omega_{10} = (\epsilon_1 - \epsilon_0)/\hbar \). The amplifier operates by coupling the energy levels \( \epsilon_1 \) and \( \epsilon_2 \) to the radiation field, generating an output frequency with an on resonance of \( \nu = (\epsilon_1 - \epsilon_2)/\hbar \). The necessary condition for amplification is positive gain or population inversion, defined by

\[
G = p_2 - p_1 \geq 0.
\]

From this condition, by dividing by \( p_0 \), we obtain

\[
e^{-\omega_{10}/k_B T_c} - e^{-\omega_{20}/k_B T_h} \geq 0,
\]

which leads to

\[
\omega_c/\omega_h = \omega_{10}/\omega_{20} \geq T_c/T_h.
\]
The efficiency of the amplifier is defined by the ratio of the output energy $\hbar \nu$ to the energy extracted from the hot reservoir, $\hbar \omega_{20}$:

$$\eta_o = \frac{\nu}{\omega_{20}} = 1 - \frac{\omega_c}{\omega_h}. \quad (3)$$

Equation 3 is termed the quantum Otto efficiency (23). Inserting the positive gain condition given in Equations 1 and 2, one finds that the efficiency is limited by Carnot efficiency (1):

$$\eta_o \leq \eta_c \equiv 1 - \frac{T_c}{T_h}. \quad (4)$$

This result connecting the efficiency of a quantum amplifier to the Carnot efficiency was first obtained by Scuville and colleagues (6, 7).

The above description of the three-level amplifier is based on a static quasi-equilibrium viewpoint. Real engines that produce power operate far from equilibrium conditions. Typically, their performance is restricted by friction, heat transport, and heat leaks. A dynamical viewpoint is therefore the next required step.

### 2.2. The Quantum Tricycle

A quantum description enables one to incorporate dynamics into thermodynamics. The tricycle model is the template for almost all continuous engines (Figure 2). This model is employed to incorporate the quantum dynamics of the devices. Surprisingly, very simple models exhibit the same features of engines generating finite power. Their efficiency at operating conditions is lower than the Carnot efficiency. In addition, heat leaks restrict performance, meaning that reversible operation is unattainable.

The tricycle engine has the generic structure displayed in Figure 2. The basic model consists of three thermal baths: a hot bath with temperature $T_h$, a cold bath with temperature $T_c$, and a

![Figure 2](image)

The quantum tricycle, a quantum device coupled simultaneously to a hot, cold, and power reservoir. A reversal of the heat currents constructs a quantum refrigerator.
work bath with temperature $T_w$. Each bath is connected to the engine via a frequency filter, which we model by three oscillators:

$$\hat{H}_F = \hbar \omega_h \hat{a} \hat{a}^\dagger + \hbar \omega_c \hat{b} \hat{b}^\dagger + \hbar \omega_w \hat{c} \hat{c}^\dagger,$$

where $\omega_h$, $\omega_c$, and $\omega_w$ are the filter frequencies on resonance $\omega_w = \omega_h - \omega_c$. The device operates as an engine by removing an excitation from the hot bath and generating excitations on the cold and work reservoirs. In second quantization formalism, the Hamiltonian describing such an interaction becomes

$$\hat{H}_I = \hbar \epsilon (\hat{a} \hat{b}^\dagger \hat{c}^\dagger + \hat{a}^\dagger \hat{b} \hat{c}),$$

where $\epsilon$ is the coupling strength.

The device operates as a refrigerator by removing an excitation from the cold bath as well as from the work bath and generating an excitation in the hot bath. The term $\hat{a} \hat{b}^\dagger \hat{c}^\dagger$ in the Hamiltonian of Equation 6 describes this action (see Section 3).

One can employ different types of heat baths, including bosonic baths comprising phonons or photons and fermionic baths comprising electrons. From the continuous spectrum of the bath, the frequency filters select the working component to be employed in the tricycle. These frequency filters can also be constructed from two-level systems or can be formulated as qubits (35–37).

The interaction term is strictly nonlinear, incorporating three heat currents simultaneously. This crucial fact has important consequences. A linear device cannot operate as a heat engine or refrigerator (38). A linear device is constructed from a network of harmonic oscillators with linear connections of the type $\hbar \mu_{ij} (\hat{a}_i \hat{a}^\dagger_j + \hat{a}^\dagger_i \hat{a}_j)$ with some connections to harmonic heat baths. In such a device, the hottest bath always cools down and the coldest bath always heats up. Thus, this construction can transport heat but not generate power because power is equivalent to transporting heat to an infinitely hot reservoir. Another flaw in a linear model is that the different bath modes do not equilibrate with each other. A generic bath should equilibrate any system Hamiltonian irrespective of its frequency.

Many nonlinear interaction Hamiltonians of the type $\hat{H}_I = \hat{A} \otimes \hat{B} \otimes \hat{C}$ can lead to a working heat engine. One can reduce these Hamiltonians to the form of Equation 6, which captures the essence of such interactions.

The first law of thermodynamics represents the energy balance of heat currents originating from the three baths and collimating on the system:

$$\frac{dE_i}{dt} = \mathcal{J}_h + \mathcal{J}_c + \mathcal{J}_w.$$  \hspace{1cm} (7)

At steady state, no heat is accumulated in the tricycle; thus $(dE_i)/(dt) = 0$. In addition, in steady state, the entropy is generated only in the baths, leading to the second law of thermodynamics:

$$\frac{d}{dt} \Delta S_b = - \frac{\mathcal{J}_h}{T_h} - \frac{\mathcal{J}_c}{T_c} - \frac{\mathcal{J}_w}{T_w} \geq 0.$$  \hspace{1cm} (8)

This version of the second law is a generalization of Clausius’s statement; heat does not flow spontaneously from cold to hot bodies (39). When the temperature $T_w \rightarrow \infty$, no entropy is generated in the power bath. An energy current with no accompanying entropy production is equivalent to generating pure power: $\mathcal{P} = \mathcal{J}_w$, where $\mathcal{P}$ is the output power.

The evaluation of the currents $\mathcal{J}_i$ in the tricycle model requires dynamical equations of motion. The major assumption is that the total system is closed, and its dynamics is generated by the global Hamiltonians:

$$\hat{H} = \hat{H}_0 + \hat{H}_{SB}.$$  \hspace{1cm} (9)
This Hamiltonian \( \hat{H}_0 \) includes the bare system and the heat baths:

\[
\hat{H}_0 = \hat{H}_s + \hat{H}_H + \hat{H}_C + \hat{H}_W,
\]

where the system Hamiltonian \( \hat{H}_s = \hat{H}_I + \hat{H}_h + \hat{H}_c + \hat{H}_w \) consists of three energy-filtering components and an interaction part with an external field. The reservoir Hamiltonians are the hot \( \hat{H}_H \), cold \( \hat{H}_C \), and work \( \hat{H}_W \). The system-bath interaction Hamiltonian, \( \hat{H}_{SB} \), is equal to \( \hat{H}_sH + \hat{H}_sC + \hat{H}_sW \).

A thermodynamical idealization assumes that the tricycle system and the baths are uncorrelated, meaning that the total state of the combined system becomes a tensor product at all times (31):

\[
\hat{\rho} = \hat{\rho}_s \otimes \hat{\rho}_H \otimes \hat{\rho}_C \otimes \hat{\rho}_W.
\]

Under these conditions, the dynamical equations of motion for the tricycle become

\[
\frac{d}{dt} \hat{\rho}_s = \mathcal{L} \hat{\rho}_s,
\]

where \( \mathcal{L} \) is the Liouville superoperator described in terms of the system Hilbert space, where the reservoirs are described implicitly. Within the formalism of a quantum open system, \( \mathcal{L} \) can take the form of the Gorini-Kossakowski-Sudarshan-Lindblad (GKS-L) Markovian generator (40, 41).

Thermodynamics is notorious for employing a very small number of variables. In equilibrium conditions, the knowledge of the Hamiltonian is sufficient. When deviating from equilibrium, additional observables are added. This suggests presenting the dynamical generator in Heisenberg form for arbitrary system operators \( \hat{O}_s \):

\[
\frac{d}{dt} \hat{O}_s = \mathcal{L}^* \hat{O}_s = \frac{i}{\hbar} [\hat{H}, \hat{O}_s] + \sum_k \hat{V}_k \hat{O}_s \hat{V}_k^\dagger - \frac{1}{2} \{ \hat{V}_k \hat{V}_k^\dagger, \hat{O}_s \}.
\]

The operators \( \hat{V}_k \) are system operators still to be determined. The task of evaluating the modified system Hamiltonian \( \hat{H}_s \) and the operators \( \hat{V}_k \) is made extremely difficult owing to the nonlinear interaction in Equation 6. Any progress from this point requires a specific description of the heat baths and approximations to deal with the nonlinear terms.

### 2.3. The Quantum Amplifier

The quantum amplifier is the most elementary quantum continuous heat engine converting heat to work. Its purpose is to generate power from a temperature difference between the hot and cold reservoirs. The output power is described by a time-dependent external field. The general device Hamiltonian is therefore time dependent:

\[
\hat{H}_s(t) = \hat{H}_0 + \hat{V}(t).
\]

When the system is coupled to the hot and cold baths, the Markovian master equation in Heisenberg form for the system operator \( \hat{O}_s \) is

\[
\frac{d}{dt} \hat{O}_s = \frac{i}{\hbar} [\hat{H}_s(t), \hat{O}_s(t)] + \mathcal{L}_H^* (\hat{O}_s(t)) + \mathcal{L}_L^* (\hat{O}_s(t)).
\]

One can obtain the change in energy of the device replacing \( \hat{O}_s \) with \( \hat{H}_s \):

\[
\frac{d}{dt} E_s = \langle \mathcal{L}_H^* (\hat{H}_s) \rangle + \langle \mathcal{L}_L^* (\hat{H}_s) \rangle + \left\langle \frac{d\hat{H}_s}{dt} \right\rangle.
\]
Equation 16 can be interpreted as the time derivative of the first law of thermodynamics (8, 31, 42, 43) based on the Markovian GKS-L generator. Power is identified as

$$\mathcal{P} = \frac{\partial H}{\partial t}(t)$$

and the heat current as

$$\mathcal{J}_b = \langle \mathcal{L}_b(\dot{\mathbf{H}}) \rangle, \quad \mathcal{J}_c = \langle \mathcal{L}_c(\dot{\mathbf{H}}) \rangle.$$  

The partition between the Hamiltonian and the dissipative part in the GKS-L generator is not unique (31). A unique derivation of the master equation based on the weak coupling limit leads to dynamics consistent with the first and second laws of thermodynamics (44).

The template for the tricycle model is employed to describe the dynamics of the amplifier. The interaction Hamiltonian is modified to become

$$\hat{\mathbf{H}}_I(t) = \hbar \epsilon \mathbf{b}^\dagger e^{-i\omega t} + \mathbf{a}^\dagger \mathbf{b} e^{-i\omega t},$$

where $v = \omega_c$ is the frequency of the time-dependent driving field, and $\epsilon$ is the coupling amplitude. The modification of Equation 6 eliminates the nonlinearity; it amounts to replacing the operator $\mathbf{c}$ and $\mathbf{c}^\dagger$ by a c-number. The amplifier output power becomes

$$\mathcal{P} = \hbar \epsilon \nu (\mathbf{a}^\dagger \mathbf{b}) e^{i\omega t} - (\mathbf{a}^\dagger \mathbf{b}) e^{-i\omega t}.$$  

After the nonlinearity has been eliminated, one can derive the quantum master equation for the amplifier from first principles based on the weak system-bath coupling expansion. This approximation is a thermodynamic idealization equivalent to an isothermal partition between the system and baths (31).

The interaction with the baths is given by

$$\hat{\mathbf{H}}_{SB} = \lambda_s (\mathbf{a} + \mathbf{a}^\dagger) \otimes \hat{\mathbf{R}}_s + \lambda_b (\mathbf{b} + \mathbf{b}^\dagger) \otimes \hat{\mathbf{R}}_b,$$  

where $\hat{\mathbf{R}}$ are reservoir operators and $\lambda$ is the small system-bath coupling parameter. A crucial step has to be performed before this procedure is applied. The system Hamiltonian has to be rediagonalized with the interaction before the system is coupled to the baths. This diagonalization modifies the frequencies of the system, resulting in a splitting of the filter frequencies. The prediagonalization is crucial for the master equations to be consistent with the second law of thermodynamics (9, 37, 45).

The operators in the diagonal form $\hat{\mathbf{d}}_s = (\mathbf{a} + \mathbf{b})/(\sqrt{2})$, $\hat{\mathbf{d}}_c = (\mathbf{a} - \mathbf{b})/(\sqrt{2})$, are determined from the interaction representation: $\hat{\mathbf{a}}(t) = \hat{\mathbf{U}}_b(t, 0) \hat{\mathbf{a}} \hat{\mathbf{U}}_b(t, 0)$ and $\hat{\mathbf{b}}(t) = \hat{\mathbf{U}}_b(t, 0) \hat{\mathbf{b}} \hat{\mathbf{U}}_b(t, 0)$. The main ingredients of the derivation of the master equation can be seen in the Supplemental Appendix (follow the Supplemental Material link from the Annual Reviews home page at http://www.annualreviews.org).

2.3.1. Solving the equations of motion for the engine. In thermodynamic tradition, the engine is well described by a small set of observables. They in turn are represented by operators defining the heat currents in the engine. To exploit this property, we transform the Hamiltonian to the interaction frame:

$$\hat{\mathbf{H}}_I(t) = \hat{\mathbf{U}}_b(t, 0) \hat{\mathbf{H}}(t) \hat{\mathbf{U}}_b(t, 0) = \hbar \frac{\omega_b + \omega_s}{2} \hat{\mathbf{W}} + \hbar \frac{\omega_c - \omega_s}{2} \hat{\mathbf{X}} + \hbar \epsilon \hat{\mathbf{Y}},$$

where the operators are closed to commutation relations, forming the SU(2) Lie algebra:

$$\hat{\mathbf{W}} = (\mathbf{d}^\dagger_+ \mathbf{a}_+ + \mathbf{d}^\dagger_- \mathbf{a}_-), \quad \hat{\mathbf{X}} = (\mathbf{d}^\dagger_+ \mathbf{d}_- e^{2i\omega t} + \mathbf{d}^\dagger_- \mathbf{d}_+ e^{-2i\omega t}),$$

$$\hat{\mathbf{Y}} = i(\mathbf{d}^\dagger_+ \mathbf{d}_- e^{2i\omega t} - \mathbf{d}^\dagger_- \mathbf{d}_+ e^{-2i\omega t}), \quad \hat{\mathbf{Z}} = (\mathbf{d}^\dagger_+ \mathbf{a}_+ - \mathbf{d}^\dagger_- \mathbf{a}_-).$$
The dynamical description of the engine is completely determined by the expectation values of the operators constituting the SU(2) algebra and can be found in the Supplemental Appendix. One finds that the commutator $[\hat{Y}, \hat{H}] \neq 0$; therefore, $\hat{Y}$ is related to the nondiagonal elements of the Hamiltonian that define the coherence between energy levels.

The solution of the equation of motion leads to the thermodynamical observables at steady-state conditions; the power and heat flows become

$$\mathcal{P} = \hbar \epsilon v (\hat{Y}) = -\frac{2\hbar(\omega_b - \omega_c)\epsilon^2 G_1}{4\epsilon^2 + \kappa_b \kappa_c},$$

$$\mathcal{J}_b = \mathcal{L}^b_\delta (\hat{H}_b) = \left\langle \frac{\hbar \epsilon G_2}{2} + \frac{2\hbar \omega_b \epsilon^2 G_1}{4\epsilon^2 + \kappa_b \kappa_c} \right\rangle, \quad \kappa_b \kappa_c \equiv \kappa_b + \kappa_c,$$

$$\mathcal{J}_c = \mathcal{L}^c_\delta (\hat{H}_c) = -\left\langle \frac{\hbar \epsilon G_2}{2} + \frac{2\hbar \omega_c \epsilon^2 G_1}{4\epsilon^2 + \kappa_b \kappa_c} \right\rangle,$$

where the generalized gain is

$$G_1 = (N_b^+ + N_c^-) - (N_b^+ + N_c^-),$$

$$G_2 = (N_b^- - N_c^+) - (N_b^- + N_c^+),$$

and

$$N_{\pm}^{\delta(\epsilon)} = \left\lfloor \exp\left(\frac{\hbar \omega_{\pm}^{\delta(\epsilon)}}{\kappa_b T_{\delta(\epsilon)}}\right) - 1\right\rfloor^{-1},$$

with $\omega_{\pm}^{\delta(\epsilon)} = \omega_{\delta(\epsilon)} \pm \epsilon$. The effective heat conductivities $\kappa_{\pm}^{\delta}$ are defined in the Supplemental Appendix.

The first law of thermodynamics is satisfied such that $\mathcal{J}_b + \mathcal{J}_c + \mathcal{P} = 0$, as well as the second law: $-\mathcal{J}_b / T_b - \mathcal{J}_c / T_c \geq 0$. The power $\mathcal{P}$ is proportional to the expectation value of the coherence $\langle \hat{Y} \rangle$. As a consequence, additional pure dephasing originating from external noise will degrade the power. Such noise generated by a Gaussian random process is described by the generator $\mathcal{L}_D = -\frac{\hbar}{2} \{\hat{H}_c, \{\hat{H}_b, \bullet\}\}$ (46). In the regime where the external driving amplitude is larger than the heat conductivity ($\epsilon^2 \gg \kappa_b \kappa_c$), Equation 23 converges to the results of Reference 35 when the tricycle operates as a heat engine.

Optimal power is obtained when the heat conductances from the hot and cold baths are balanced, $\Gamma \equiv \kappa_b = \kappa_c$; then the power and the heat flows from the reservoirs become

$$\mathcal{P} = \hbar \epsilon v (\hat{Y}) = \frac{\hbar \epsilon \Gamma G_1}{4\epsilon^2 + \Gamma^2},$$

$$\mathcal{J}_b = \mathcal{L}_{\delta} (\hat{H}_b) = \frac{\hbar \epsilon \Gamma G_2}{4} + \frac{\hbar \omega_b \epsilon^2 \Gamma G_1}{4\epsilon^2 + \Gamma^2},$$

$$\mathcal{J}_c = \mathcal{L}_{\delta} (\hat{H}_c) = -\frac{\hbar \epsilon \Gamma G_2}{4} - \frac{\hbar \omega_c \epsilon^2 \Gamma G_1}{4\epsilon^2 + \Gamma^2}.$$

Figure 3 shows the power as a function of the heat conductivity coefficient $\Gamma$ and the coupling to the external field $\epsilon$. A clear global maximum is obtained for $\Gamma = 2\epsilon$.

The efficiency of the amplifier is defined as $\eta = -\mathcal{P} / \mathcal{J}_b$. For the present model, it becomes

$$\eta = \frac{\nu}{\omega_b + \frac{(\Gamma^2 + \epsilon^2) G_1}{4\epsilon^2 \Gamma}},$$

In the limit of $\epsilon \to 0$ and $\Gamma \to 0$, the efficiency approaches the Otto value, $\eta_o = 1 - \omega_c / \omega_b$, and for zero gain, $\omega_b / \omega_c = T_c / T_b$, we obtain the Carnot limit, $\eta_c = 1 - T_c / T_b$. 

Kosloff • Levy
The normalized power $P/P_{\text{max}}$ as a function of the coupling amplitude to the external field $\epsilon$ and the heat conductivity $\Gamma$. A clear maximum is obtained for $\Gamma = 2\epsilon$.

For finite fixed $\Gamma$, the efficiency in the limit $\epsilon \to 0$ becomes

$$\eta = \frac{\nu}{\omega_b + \frac{\nu^2(T_b(1-\cosh(\frac{\epsilon}{\omega_h}))-T_c(1-\cosh(\frac{\epsilon}{\omega_c})))}{4T_b T_c(\sinh(\frac{\epsilon}{\omega_c})-\sinh(\frac{\epsilon}{\omega_h}))}}.$$  

(28)

By examining Equation 28, one can see that the Carnot limit is unattainable. When the Otto efficiency approaches the Carnot limit $\omega_c/\omega_h = T_c/T_b$, the amplifier’s efficiency becomes zero. This comparison between the two limits can be seen in Figure 4.

The efficiency of Equation 27 can be either smaller or greater than the Otto efficiency depending on the sign of $G_2$. In the low-temperature limit, $G_2$ is less than zero; thus $\eta_o \leq \eta \leq \eta_c$. Increasing $\epsilon$ will increase the efficiency up to a critical point $\epsilon_{\text{crit}}$, at which both $G_1 \to 0$ and $G_2 \to 0$ because $N_h^+ \to 0$ and $N_h^- \to 0$ and $N_c^+ \sim N_c^-$. At this point, the engine operates at the Carnot limit, and all currents vanish, such that the process becomes isoentropic (Figure 5). In the high-temperature limit for harmonic oscillators, while increasing $\epsilon$, the gain $G_2$ will change sign and become positive; thus $\eta \leq \eta_o$ (Figure 6).

### 2.3.2. Optimizing the amplifier’s performance.

Further optimization for power is obtained when the pumping rate $\Gamma$ is optimized for maximum power; then $\Gamma_{\text{max}} = 2\epsilon$, and the power becomes

$$P = -\frac{1}{2}h\nu|G_1|.$$  

(29)

At the limit of high temperature and small $\epsilon$, the gain becomes $G_1 \approx (K_b T_b)/(h\omega_b)-(K_b T_c)/(h\omega_c)$. Then optimizing the power given in Equation 29 with respect to the output frequency $\nu$ leads to

$$\frac{\omega_c}{\omega_b} = \sqrt{\frac{T_c}{T_b}}.$$
Figure 4
The normalized efficiency $\eta/\eta_c$ versus the normalized power $P/P_{\text{max}}$ for fixed $\epsilon$, $\Gamma$, and $\omega_c$ while varying $\omega_h$. The blue line is for finite $\Gamma$, and the dashed red line is optimized at each point $\Gamma = 2\epsilon$. In this case, $\epsilon \ll 1$.

Figure 5
(a) The heat $J_h$ and $J_b$ and power $P$ currents as a function of $\epsilon$. (b) Efficiency $\eta$ as a function of $\epsilon$. (c) Entropy production as a function of $\epsilon$. (d) The gain $G_1$ and $G_2$ as a function of $\epsilon$. The harmonic tricycle engine operates in the limit of low temperature. $k_B T_1 = 0.1$ and $k_B T_2 = 0.3$, $\hbar \omega_b = 4$ and $\hbar \omega_b = 6$, and $\Gamma = 0.05$. 
resulting in the efficiency at maximum power:

\[ \eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}, \]

which is the well-established endoreversible result of Curzon & Ahlborn (2, 8). The optimal power is obtained when all the characteristic parameters are balanced: \( \epsilon \sim \Gamma \sim \kappa_c \sim \kappa_h \). Figure 7 shows a trajectory of efficiency with respect to power with changing field coupling strength \( \epsilon \) for different frequency ratios \( \omega_c/\omega_h = (T_c/T_h)^{\alpha} \). The power vanishes with the coupling \( \epsilon = 0 \), and then when \( \epsilon = \epsilon_{crit} \), the splitting in the dressed energy levels nulls the gain.

### 2.4. Dynamical Model of a Three-Level Engine

The dynamical description of the three-level engine is closely related to the tricycle model (9). The model is a template for the three-level laser shown in Figure 1 with the inclusion of a dynamical description (47, 48).

The Hamiltonian of the device has the form

\[
\hat{H}_0 = \hat{H}_0^0 + \hat{V}(t) = \begin{pmatrix}
\epsilon_0 & 0 & 0 \\
0 & \epsilon_1 & \epsilon^{i\omega t} \\
0 & \epsilon^{-i\omega t} & \epsilon_2
\end{pmatrix},
\]

where \( \hat{H}_0^0 = \sum \epsilon_i \hat{P}_{ii}, \hat{P}_{ij} = |i\rangle\langle j| \), and \( \hat{V}(t) = \epsilon (\hat{P}_{12} e^{i\omega t} + \hat{P}_{21} e^{-i\omega t}) \). The state of the three-level system is fully characterized by the expectation values of any eight independent operators, excluding the identity operator.
The normalized efficiency $\eta / \eta_c$ versus the normalized power $P / P_{\text{max}}$ for different optimal values of $\Gamma = 2\epsilon$. $P_{\text{max}}$ is obtained for the Curzon-Ahlborn ratio $\omega_c / \omega_h = \sqrt{T_c / T_h}$ shown in blue. The orange line is for $\omega_c / \omega_h = (T_c / T_h)^{1/4}$, and the green line is for $\omega_c / \omega_h = (T_c / T_h)^{3/4}$. The red line is for the Carnot ratio $\omega_c / \omega_h \sim T_c / T_h$, where the power is multiplied by $10^3$.

The final equations of motion for these observables are given in the Supplemental Appendix. The expressions for the power and efficiency become up to numerical factors identical to the expressions obtained for the driven quantum tricycle. Figure 8 shows a schematic view of the splitting of the energy levels of the three-level system owing to the driving field. As a result, the engine also splits into two parts, the upper and lower manifolds.

One can evaluate the steady-state power and heat flows by solving the generalized Lamb equations. The results are represented by Equation 23 in which the definition of the population is changed from that of a harmonic oscillator to a two-level system:

$$N^{\pm}_{\hbar} = \frac{1}{e^{\beta \hbar \omega_{\pm}} + 1}.$$ 

The numerator of the expressions for the power and heat flows consists of a sum of two contributions: One, proportional to the gain $N^\pm_{\hbar} - N^\pm_{\epsilon}$, is associated with the population inversion.

The three-level amplifier. The interaction with the external field splits the two upper levels. As a result, the heat transport terms are modified. Two parallel engines emerge, which can operate in opposite directions, producing zero power for a certain choice of parameters.
in the upper manifold, whereas the other, proportional to the gain $N^+ - N^-$, is associated with the population inversion in the lower manifold.

At low temperatures, the three-level amplifier model is equivalent to that of the harmonic tricycle. At the high-temperature limit, the gain $G_2$ is positive for all driving conditions $\epsilon$. The maximum power point is the result of competition between the upper and lower manifolds. As $\epsilon$ increases, $N^+_h - N^+_c$ increases, whereas $N^-_h - N^-_c$ decreases. Hence, the power production of the upper manifold increases (i.e., becomes more negative), whereas that of the lower manifold decreases (i.e., becomes less negative).

From a thermodynamic point of view, each manifold is associated with a separate heat engine. As the coupling with the work reservoir ($\epsilon$) increases, the engine associated with the upper manifold operates faster, while that associated with the lower one operates slower. In addition, the energy is leaking from the former to the latter, thereby diminishing the net power production.

Not only does the power decrease as a function of $\epsilon$, it also changes sign at a certain finite value of the field amplitude, denoted by $\epsilon_{\text{crit}}$. This results from the fact that at some point, the lower manifold starts operating backward as a heat pump, rather than a heat engine. At $\epsilon = \epsilon_{\text{crit}}$, the power consumption by the lower manifold is exactly balanced by the power production of the upper manifold such that the net power production is zero.

An examination of the steady-state heat fluxes reveals that they do not vanish at zero-power operating conditions. The entropy generated exclusively by the heat leak from the upper to lower manifold is obtained by eliminating the power in Equation 26:

$$\frac{d}{dt}S' = \frac{h\epsilon G_2}{4} \left( \frac{1}{T_c} - \frac{1}{T_h} \right) \geq 0.$$  \hspace{1cm} (32)

Zero-power operating conditions correspond to the short-circuit limit. Heat is effectively transferred from the hot bath into the cold bath such that no net work is involved.

An interesting limit is that of low temperatures such that $h\epsilon/k_B \gg T_c, T_h$. For example, this limit is realistic in the optical domain where $\nu \gg T_c, T_h$. In such a case, $N^+_h - N^+_c$ is negligible relative to $N^-_h - N^-_c$, and one needs to account for only the lower manifold.

The value of $\epsilon_{\text{crit}}$ in this limit, denoted by $\epsilon_{\text{crit}}^{\text{LT}}$, is that for which $N^-_h - N^-_c = 0$. It is generally given by

$$\epsilon_{\text{crit}}^{\text{LT}} = \frac{T_h \omega_c - T_c \omega_h}{T_h - T_c}.$$  \hspace{1cm} (33)

The condition $T_h \omega_c - T_c \omega_h > 0$ is necessary and sufficient for lasing in the limit of weak driving fields. Population inversion becomes increasingly more difficult in this manifold as the field intensifies. It is therefore no longer a sufficient condition for lasing in intense driving fields. Finally, zero-power operating conditions in the low-temperature limit, obtained at $\epsilon = \epsilon_{\text{crit}}^{\text{LT}}$, asymptotically imply zero heat flows and hence the reversible limit. Indeed, substituting $\epsilon_{\text{crit}}^{\text{LT}}$ from Equation 33 for $\epsilon$ reduces to the Carnot efficiency, $\eta_c = 1 - T_c/T_h$.

The difference between a three-level amplifier and the tricycle with harmonic oscillator filters at the high-temperature limit is observed in Figure 9. The source of the difference is the saturation of finite levels. The efficiency at maximum power is lower than the Curzon-Ahlborn efficiency, approximated by

$$\eta_{\text{max}} \approx \eta_c - \frac{k_B T_c}{h \omega_h} \eta_c^2.$$  \hspace{1cm} (34)
Figure 9

The normalized efficiency $\eta/\eta_c$ versus the normalized power $P/P_{\text{max}}$ for $\omega_c/\omega_h \sim T_c/T_h$ for varying $\Gamma = 2\epsilon$. The high-temperature limit is shown where $\hbar \omega \ll k_B T$. The red line is the three-level amplifier, and the blue line is the harmonic tricycle engine. Saturation limits the performance of the three-level engine. Abbreviations: HO, harmonic oscillator; TLS, two-level system.

2.5. The Four-Level Engine and Two-Level Engine

The four-level engine is a dynamical model of the four-level laser (49). In the four-level engine, the pumping excitation step is isolated from the coupling to the external field by employing an additional cold reservoir. Figure 10 shows a schematic view of the engine’s structure. Analyzing

Figure 10

The four-level engine. The interaction with the external field splits the two upper levels. As a result, the heat transport terms are modified. Two parallel engines emerge, which can operate in opposite directions, producing zero power for a certain choice of parameters. Coherence between levels 0 and 1 can enhance the performance by synchronizing the engines. $\omega_h = \omega_{30}, \omega_{11} = \omega_{12}$, and $\omega_{12} = \omega_{10}$.
a static viewpoint, one can see that positive gain is obtained when
\[ G = p_3 - p_2 = p_0 \left( e^{\frac{\hbar \omega_{12}}{k_B T_h}} e^{\frac{\hbar \omega_{21}}{k_B T_c}} - e^{\frac{\hbar \omega_{10}}{k_B T_h}} \right) \geq 0, \] (35)
where \( \omega_{10} = \omega_{12} \text{ and } \omega_{21} \). The optimal output frequency at resonance is
\[ \nu = \omega_h - \omega_{12} \] therefore, the efficiency becomes \( \eta = \nu / \omega_h \). If the additional cold reservoir is assumed to have temperature \( T_c \), then the Carnot restriction is obtained: \( \eta \leq \eta_c \).

A dynamical analysis reveals a splitting of the energy levels driven by the external field, leading to a similar performance as the three-level amplifier. The advantage of the four-level engine is that the hot bath thermal pumping is isolated from the coupling to the power extraction, thus replacing the decoherence associated with the hot bath with a quieter operation associated with the cold bath. In addition, one can optimize performance by balancing the rates between the upper and lower manifold by \( \omega_{12} = \omega_{12} \). The four-level engine has been employed to study the influence of initial coherence on the engine's performance (50–56). In this case, the power output is connected to the two upper levels, and the coherence is generated by splitting the ground state. The basic idea is that the initial coherence present between energy levels associated with the entangling operator \( \hat{Y} \) can be exploited to generate additional power, even from a single bath. There is no violation of the second law because the initial coherence reduces the initial entropy. Contact with a single bath will increase this entropy. A unitary manipulation can then exploit this entropy difference to generate work.

An extreme exploitation of the dynamical splitting of the energy levels owing to the external driving field results in the two-level engine. **Supplemental Figure 1** presents a schematic operation of the engine. Two excitations from the hot bath are required to generate a gain in the upper manifold. A clever choice of coupling to the bath is required to generate this gain, and a model of such an engine has been worked out (57, 58). The maximum efficiency of such an engine becomes \( \eta \leq 1/2 \leq \eta_c \) as two-pump steps are required for one output step.

### 2.6. Power Storage
An integral part of an engine is a device that allows one to store the power and retrieve it on demand, a flywheel. A model based on the tricycle of such a device just disconnects the power bath and stores the energy in the power oscillator, \( \omega_w \hat{c} \hat{c}^\dagger \). Once there is positive gain generated by the hot and cold baths, energy will flow to this oscillator. A model based on the three-level amplifier used this approach (21, 34, 59). The drawback of these studies is that they use a local description of the master equation. When the flywheel oscillator has a large amplitude, it will modify the system-bath coupling in analogy to the dressed state picture (9).

Other studies also considered coupling to a cavity mode (54, 56, 60). The issue to be addressed is the amount of energy that can be stored and then extracted. Because the storage mode is entangled with the rest of the engine, this issue is delicate (60). The amount of possible work to be extracted is defined by an entropy-preserving unitary transformation to a passive state (61).

### 3. CONTINUOUS REFRIGERATORS
In a nutshell, refrigerators are just inverted heat engines. They employ power to pump heat from a cold to a hot bath. As with heat engines, the first and second laws of thermodynamics impose the same restrictions on the refrigerator’s performance. The distinction between the static and dynamical viewpoints also applies. The key elements in a refrigerator are entropy extraction and disposal. This entropy disposal problem is enhanced at low temperatures. The unique feature of refrigerators is therefore the third law of thermodynamics.
3.1. The Third Law of Thermodynamics

Two independent formulations of the third law of thermodynamics have been presented, both originally stated by Nernst (62–64). The first is a purely static (equilibrium) one, also known as the Nernst heat theorem, phrased as follows: The entropy of any pure substance in thermodynamic equilibrium approaches zero as the temperature approaches zero. The second formulation is known as the unattainability principle: It is impossible by any procedure, no matter how idealized, to reduce any assembly to absolute zero temperature in a finite number of operations (64, 65).

There is an ongoing debate on the relations between the two formulations and their relation to the second law, regarding which, and if at all, one of these formulations implies the other (4, 66–68). Quantum considerations can illuminate these issues. The tricycle model is the template used to analyze refrigerator performance as $T_c \to 0$.

We first analyze the implications of the Nernst heat theorem and its relations to the second law of thermodynamics. At steady state, the second law implies that the total entropy production is non-negative (see Equation 8):

$$\frac{d}{dt}\Delta S_t = -\frac{J_c}{T_c} - \frac{J_h}{T_h} - \frac{J_w}{T_w} \geq 0.$$  

This behavior can be quantified by a scaling exponent; as $T_c \to 0$, the cold current should scale with temperature as

$$J_c \propto T_c^{1+\alpha_c},$$

with an exponent $\alpha$.

When the cold bath approaches the absolute zero temperature, it is necessary to eliminate the entropy production divergence at the cold side; when $T_c \to 0$, the entropy production scales as

$$\Delta S_c \sim -T_c^{\alpha_c}, \quad \alpha_c \geq 0.$$  

(36)

For the case in which $\alpha = 0$, the fulfillment of the second law depends on the entropy production of the other baths, $-J_h/T_h - J_w/T_w > 0$, which should compensate for the negative entropy production of the cold bath. The first formulation of the third law modifies this restriction. Instead of $\alpha \geq 0$, the third law imposes $\alpha > 0$, guaranteeing that at absolute zero the entropy production at the cold bath is zero: $\Delta S_c = 0$. The Nernst heat theorem then leads to the scaling condition of the heat current with temperature (11):

$$J_c \sim T_c^{\alpha + 1} \quad \text{and} \quad \alpha > 0.$$  

(37)

We now examine the unattainability principle. Quantum mechanics enables a dynamical interpretation of the third law, modifying the definition to the following: No refrigerator can cool a system to absolute zero temperature at finite time. This form of the third law is more restrictive, imposing limitations on the system-bath interaction and the cold bath properties when $T_c \to 0$ (35). To quantify the unattainability principle, one finds that the rate of temperature decrease of the cooling process should vanish according to the characteristic exponent $\zeta$:

$$\frac{dT_c(t)}{dt} = -\zeta T_c^\zeta, \quad T_c \to 0,$$

(38)

where $\epsilon$ is a positive constant. Solving Equation 38, one obtains

$$T_c(t) = T_c(0)e^{-\epsilon t}, \quad \text{for} \quad \zeta < 1,$$

$$T_c(t) = T_c(0)e^{-\epsilon t}, \quad \text{for} \quad \zeta = 1,$$

$$\frac{1}{T_c(t)} \geq T_c(0)e^{-\epsilon t} + \epsilon t, \quad \text{for} \quad \zeta > 1.$$  

(39)
From Equation 39, it is apparent that the cold bath can be cooled to zero temperature at finite
time for $\zeta < 1$. The third law therefore requires $\zeta \geq 1$. The two third-law scaling exponents can
be related by accounting for the heat capacity $c_V(T_c)$ of the cold bath:

$$J_c(T_c(t)) = -\epsilon_V(T_c(t)) \frac{dT_c(t)}{dt}, \quad (40)$$

where $\epsilon_V(T_c)$ is determined by the behavior of the degrees of freedom of the cold bath at low
temperatures at which $\epsilon_V \sim T_c^\eta$ when $T_c \to 0$. Therefore, the scaling exponents are related,
$\zeta = 1 + \alpha - \eta$ (31).

The third-law scaling relations given in Equations 37 and 40 can be used as an independent
check for quantum refrigerator models (69). Violation of these relations points to flaws in the
quantum model of the device, typically in the derivation of the master equation.

3.2. The Quantum Power–Driven Refrigerator

Laser cooling is a crucial technology for realizing quantum devices. When the temperature is
decreased, degrees of freedom freeze out, and systems reveal their quantum character. Inspired
by the mechanism of solid state lasers and their analogy with Carnot engines (6), investigators
realized that inverting the operation of the laser at the proper conditions will lead to refrigeration
(70–73). A few years later, a different approach for laser cooling was initiated based on the Doppler
shift (74, 75). In this scheme, translational degrees of freedom of atoms or ions were cooled by
laser light detuned to the red of the atomic transition. Unfortunately, the link to thermodynamics
was forgotten. A flurry of activity then followed with the realization that the actual temperatures
achieved were below the Doppler limit, $k_B T_{\text{Doppler}} = h\gamma/2$, where $\gamma$ is the natural line width
(76–78).

Elaborate quantum theories were devised to unravel the discrepancy. The recoil limit is based
on the assumption that the photon momentum is in quasi-equilibrium with the momentum of the
atoms, $p = h\kappa$. As a result, the temperature is related to the average kinetic energy:

$$k_B T_{\text{recoil}} = \frac{h^2 k^2}{2M}, \quad (41)$$

where $h\kappa$ is the photon momentum and $M$ the atomic mass of the particle being cooled. For sodium, the Doppler limit is $T_{\text{Doppler}} = 235 \mu K$ and the recoil limit $T_{\text{recoil}} = 2.39 \mu K$ (79).

Thermodynamically, these limits do not bind; the only hard limit is absolute zero, $T_c = 0$.
To approach this limit, one has to optimize the cooling power to match the cold bath temper-
ature. Unoptimized refrigerators become restricted by a minimum temperature above absolute
zero.

A reexamination of the elementary three-level model stresses this point. By examining the
heat engine model of Figure 1, one finds that if the power direction is reversed, a refrigerator is
generated, provided the gain is negative: $G = p_2 - p_1 < 0$. Assuming quasi-equilibrium conditions,
this leads to

$$\frac{\omega_1}{\omega_6} = \frac{\omega_10}{\omega_20} \leq \frac{T_c}{T_B}, \quad (42)$$

which translates to a minimum temperature of $T_c(\text{min}) \geq (\omega_1/\omega_6)T_B$. We can consider typical
values for laser cooling of sodium based on the D line of 589.592 nm, which translates to $\omega_6 = 508.838 \times 10^{13}$ Hz. If the translational cold bath is coupled to the hyperfine structure splitting, then
one finds that $F_2 \to F_1$ of the ground state $3S_1/2$, meaning $\omega_6 = 1.7716 \times 10^{10}$ Hz. The cooling
ratio becomes $\omega_1/\omega_6 = 3.48 \times 10^{-6}$, and the minimum temperature, assuming room temperature
of the hot bath, is $T_c(\text{min}) \sim 1.510^{-3}$ K. With the use of a magnetic field, the hyperfine levels $F_1$
split in three, leading to further splittings in the megahertz limit, increasing the cooling ratio by three orders of magnitude. When the cooling is in progress, one can reduce the hyperfine splitting by changing the magnetic field such that it follows the cold bath temperature. In principle, any temperature above \( T_c = 0 \) is reachable.

### 3.3. Dynamical Refrigerator Models

A quantum dynamical framework of the refrigerator is based on the tricycle template. The work bath is replaced by a time-dependent driving term, and the cooling current can be calculated analytically. One can convert the dynamical equations of the heat engine (Equation 23) to those of a refrigerator by inverting the sign of all heat currents. Consequently, one finds that the gain \( G_1 \leq 0 \) (Equation 24). For example, when \( \epsilon > \kappa \), the cooling current becomes

\[
J_c = \frac{1}{2} \left( \frac{\hbar \omega_c}{\Gamma_c + \Gamma_b} (N_c^- - N_c^+) + \frac{\hbar \omega_c}{\Gamma_c + \Gamma_b} (N_c^+ - N_c^-) \right). \tag{43}
\]

The phenomenon of the refrigerator splitting into two parts is also found here. In a similar fashion, the three-level amplifier can be reversed to become a refrigerator (9). The heat currents are identical to Equation 23 with the two-level-system definition of \( N_{i/b} \).

In a refrigerator, the object of optimization is the cooling power \( J_c \) compared to the output power \( P \) in an engine. The efficiency is defined by the coefficient of performance (COP), the ratio between \( J_c \) and the input power \( P \):

\[
\text{COP} = \frac{J_c}{P} \leq \text{COP}_c \leq \text{COP}_o, \tag{44}
\]

where \( \text{COP}_o = \omega_c / \nu \) and \( \text{COP}_c = T_c / (T_h - T_c) \). All power-driven refrigerators are restricted by the Otto COP\(_o\). In Section 3.5, we analyze the performance of power-driven refrigerators at low temperatures.

### 3.4. The Quantum Absorption Refrigerator

An absorption chiller is a refrigerator that employs a heat source to replace mechanical work for driving a heat pump (80). The first device, developed in 1850 by the Carré brothers, became the first useful refrigerator. In 1926, Einstein & Szilárd (81) invented an absorption refrigerator with no moving parts. This invention is an inspiration for miniaturizing the device to solve the growing problem of heat generated in integrated circuits. Even more challenging proposal is to miniaturize to a few-level quantum system. Such a device could be incorporated into a quantum circuit. The first quantum version was based on the three-level refrigerator (12) driven by thermal noise. A more recent model of an autonomous quantum absorption refrigerator with no external intervention based on three qubits (36) has renewed interest in such devices. A setup of optomechanical refrigerators powered by incoherent thermal light was introduced in Reference 82. The study showed that cooling increases while the photon number increases up to the point that the fluctuation of the radiation pressure becomes dominant and heats the mechanical mode. Studies of cooling based on electron tunneling have also been considered. The working medium comprises quantum dots (83, 84) or a normal metal insulator superconductor junction (85). Heat is removed from the reservoir by electron transport induced by thermal or shot noise.

#### 3.4.1. The three-level absorption refrigerator

In the three-level absorption refrigerator, the power source has a finite temperature \( T_w \). The coefficient of performance becomes \( \text{COP} = J_c / J_w \).
From the second and first laws of thermodynamics (Equations 7 and 8) using steady-state conditions, one obtains

$$\text{COP} \leq \frac{(T_w - T_b)T_c}{(T_b - T_c)T_w},$$

(45)

and $T_w > T_b > T_c$ (80). Under the assumption that the three baths are uncorrelated, the three-level state is diagonal in the energy representation and determined by the heat conductivities (12, 86). From these values, one can evaluate the heat currents. In this refrigerator, no internal coherence is required for operation.

### 3.4.2. Tricycle absorption refrigerator.

The template to understand the absorption refrigerator is the tricycle model, a refrigerator connected to three reservoirs with the Hamiltonians given in Equations 5 and 6. A thermodynamical consistent description requires that one first diagonalizes the Hamiltonian and then obtains the generalized master equations for each reservoir. The final step is to obtain the steady-state heat currents $J_c$, $J_h$, and $J_w$. The nonlinearity in Equation 6 hampers this task.

One remedy to this issue is to replace the harmonic oscillators in the model by qubits (37). The other approach is to consider the high-temperature limit of the power reservoir or a noise-driven refrigerator (35, 87). This three-qubit refrigerator model was introduced as the ultimate miniaturization model (36, 88, 89). The free Hamiltonian of the device has the form

$$\hat{H}_F = \hbar \omega_h \hat{\sigma}_h^z + \hbar \omega_c \hat{\sigma}_c^z + \hbar \nu \hat{\sigma}_w^z,$$

(46)

and the interaction Hamiltonian is

$$\hat{H}_I = \hbar \epsilon \left( \hat{\sigma}_h^z \otimes \hat{\sigma}_c^z \otimes \hat{\sigma}_w^z + \hat{\sigma}_h^z \otimes \hat{\sigma}_c^z \otimes \hat{\sigma}_w^z \right),$$

(47)

where $\hat{\sigma}$ are two-level operators. The diagonal energy levels are described in the Supplemental Appendix.

Of major importance is the distinction between the complete nonlocal description of the tricycle and a local description. In the local case, the dissipative terms $L_{hc}$ are set to equilibrate only the local qubit. In the nonlocal case, the local qubit is mixed with the other qubits owing to the nonlinear coupling. As a result, in the nonlocal approach it is impossible to reach the Carnot coefficient of performance, COP, regardless of how small the internal coupling $\epsilon$ is. This can be observed in Figure 11, which shows the coefficient of performance versus the cooling current $J_c$ for different coupling strengths, $\epsilon$. Conversely, COP, is reachable in the local model. Interestingly, a universal limit for the coefficient of performance at the maximum cooling power was found applicable for all absorption refrigerator modes of $\text{COP} \leq \frac{(d/d + 1)}{\text{COP}^w}$, where $d$ is the dimensionality of the phonon cold bath (86).

### 3.4.3. Noise-driven refrigerator.

An ideal power source generates zero entropy: $\Delta S_w = -J_w/T_w$. A pure mechanical external field achieves this goal. Another possibility is a thermal source; when $T_w \to \infty$, it also generates zero entropy. An unusual power source is noise. Gaussian white noise also carries with it zero entropy production. Analysis demonstrates that the performance of refrigerators driven by all these power sources is very similar when approaching the absolute zero temperature (87).

Employing the tricycle as a template, we examine the option of employing noise as a power source. The simplest option is a Gaussian white noise source. As a result, the interaction nonlinear term in Equation 6 is replaced with

$$\hat{H}_{int} = f(t)(\hat{a} \hat{b} + \hat{a}^\dagger \hat{b}^\dagger) = f(t)\hat{X},$$

(48)
Figure 11
The coefficient of performance, COP/COP∗, of the three-qubit refrigerator versus the normalized cold current Jc/J∗ for different values of the internal coupling constant, ϵ. The coefficient of performance at maximum power is found to be bound by 3/4COP∗, independent of ϵ. Figure courtesy of L.A. Correa and J.P. Palao.

where f(t) is the noise field; \( \hat{X} = (\hat{a}^{\dagger}\hat{b} + \hat{a}\hat{b}^{\dagger}) \) is the generator of a swap operation between the two oscillators and is part of a set of SU(2) operators, \( \hat{Y} = i(\hat{a}^{\dagger}\hat{b} - \hat{a}\hat{b}^{\dagger}) \), \( \hat{Z} = (\hat{a}^{\dagger}\hat{a} - \hat{b}^{\dagger}\hat{b}) \), and the Casimir \( \hat{N} = (\hat{a}^{\dagger}\hat{a} + \hat{b}^{\dagger}\hat{b}) \).

A Gaussian source of white noise is characterized by zero mean \( \langle f(t) \rangle = 0 \) and delta time correlation \( \langle f(t) f(t') \rangle = 2\eta \delta(t - t') \). The Heisenberg equation for a time-independent operator \( \hat{O} \) is reduced to

\[
\frac{d}{dt} \hat{O} = i[\hat{H}_s, \hat{O}] + L_{n,c}^{(a)}(\hat{O}) + L_{h,c}^{(a)}(\hat{O}),
\]

where \( \hat{H}_s = \hbar \omega_{hc}\hat{a}^{\dagger}\hat{a} + \hbar \omega_{hc}\hat{b}^{\dagger}\hat{b} \). The noise dissipator for Gaussian noise is \( L_n^{(a)}(\hat{O}) = -\eta[\hat{X}, [\hat{X}, \hat{O}]] \) (46). The bath relaxation generators \( L_{h,c}^{(a)} \) are the standard local energy relaxation terms (see the Supplemental Appendix) (90).

The equations of motion including the dissipative part are closed to the SU(2) set of operators. To derive the cooling current \( J_c = \langle L_c(h\omega_c\hat{b}^{\dagger}\hat{b}) \rangle \), we solve for stationary solutions of \( \hat{N} \) and \( \hat{Z} \). The cooling current becomes

\[
J_c = \hbar \omega_c \frac{2\eta \Gamma}{2\eta + \Gamma}(N_c - N_h),
\]

where the effective heat conductance is \( \Gamma = (\Gamma_{c} + \Gamma_{h})/(\Gamma_{c} + \Gamma_{h}) \). Cooling occurs for \( N_c > N_h \Rightarrow \omega_c/T_c > \omega_h/T_h \). The coefficient of performance for the absorption chiller is defined by the relation \( \text{COP} = J_c/J_n \); with the help of Equation 50, we obtain the Otto COP∗ (23) (see Equation 44).

We now show the equivalence of the noise-driven refrigerator with the high-temperature limit of the work bath \( T_{\text{h}} \). Based on the weak coupling limit, the dissipative generator of the power
The Poisson noise generates an effective Hamiltonian comprising
\[ L'_u(\hat{O}) = \Gamma_u(N_w + 1) \left( \hat{a}^\dagger \hat{b} \hat{O}^\dagger \hat{a} - \frac{1}{2} (\hat{a}^\dagger \hat{a} \hat{b} \hat{b}^\dagger, \hat{O}) \right) + \Gamma_u N_w \left( \hat{a} \hat{b} \hat{O}^\dagger \hat{a}^\dagger \hat{b} \hat{b}^\dagger - \frac{1}{2} (\hat{a}^\dagger \hat{a} \hat{b} \hat{b}^\dagger, \hat{O}) \right), \]

where \( N_w = (\exp((\hbar \omega_0)/(kT_b)) - 1)^{-1} \). At finite temperature, \( L'_u(\hat{O}) \) does not lead to a close set of equations. But in the limit of \( T_w \to \infty \), it becomes equivalent to the Gaussian noise generator:
\[ L'_u(\hat{O}) = -\eta/2(\hat{X}, [\hat{X}, \hat{O}]) + [\hat{Y}, [\hat{Y}, \hat{O}]], \]

where \( \eta = \Gamma_u N_w \). This noise generator leads to the same current \( J_r \) and coefficient of performance as in Equations 44 and 50. We conclude that Gaussian noise represents the singular bath limit equivalent to \( T_w \to \infty \).

One can employ Poisson white noise as a power source. This noise is typically generated by a sequence of independent random pulses with exponential interarrival times (91, 92). These impulses drive the coupling between the oscillators in contact with the hot and cold baths, leading to
\[ \frac{d\hat{O}}{dt} = (i/\hbar)[\hat{H}, \hat{O}] + (i/\hbar)[\hat{H}', \hat{O}] + L'_u(\hat{O}). \]

The unitary part is generated with the addition of the Hamiltonian \( \hat{H}' = \hbar \tilde{X} \) and with the interaction
\[ \epsilon = -\frac{\lambda}{2} \int d\xi P(\xi) (2\xi/\hbar - \sin(2\xi/\hbar)). \]

This term can cause a direct heat leak from the hot to the cold bath. The noise generator \( L'_u(\hat{\rho}) \) can be reduced to the form \( L'_u(\hat{O}) = -\eta(\hat{X}, [\hat{X}, \hat{O}]) \), with a modified noise parameter:
\[ \eta = \frac{\lambda}{4} \left( 1 - \int d\xi P(\xi) \cos(2\xi/\hbar) \right). \]

The Poisson noise generates an effective Hamiltonian comprising \( \hat{H} \) and \( \hat{H}' \), modifying the energy levels of the working medium. One must incorporate this new Hamiltonian structure into the derivation of the master equation; otherwise, the second law will be violated. The first step is to rewrite the system Hamiltonian in its dressed form (93). A new set of bosonic operators is defined
\[ \hat{A}_1 = \hat{a} \cos(\theta) + \hat{b} \sin(\theta), \]
\[ \hat{A}_1 = \hat{b} \cos(\theta) - \hat{a} \sin(\theta). \]

The dressed Hamiltonian is given by
\[ \hat{H}_d = \hbar \Omega_+ \hat{A}_1^\dagger \hat{A}_1 + \hbar \Omega_- \hat{A}_2^\dagger \hat{A}_2, \]

where \( \Omega_{\pm} = (\omega_0 + \omega_c)/2 \pm \sqrt{((\omega_0 - \omega_c)/2)^2 + \epsilon^2} \) and \( \Omega_{\pm} = (\omega_0 - \Omega_+)/\Omega_- - \Omega_- \). Equation 57 imposes the restriction \( \Omega_+ > 0 \), which can be translated to \( \omega_0, \omega_c > \epsilon^2 \). The master equation in the Heisenberg representation becomes
\[ \frac{d\hat{O}}{dt} = (i/\hbar)[\hat{H}, \hat{O}] + L'_u(\hat{O}) + L'_u(\hat{O}) + L'_u(\hat{O}). \]
where the details can be found in Reference 35. The noise generator becomes

$$\mathcal{L}^f_\sigma(\hat{\mathbf{O}}) = -\eta [\hat{W}, [\hat{W}, \hat{\mathbf{O}}]]$$

where $\hat{W} = \sin(2\theta)\hat{Z} + \cos(2\theta)\hat{X}$. Again, the SU(2) algebra is employed to define the operators $\hat{X} = (\hat{A}_1^+ \hat{A}_1 + \hat{A}_1 \hat{A}_1^+), \hat{Y} = i(\hat{A}_1^+ \hat{A}_2 - \hat{A}_2 \hat{A}_1^+), \text{and} \hat{Z} = (\hat{A}_1^+ \hat{A}_2 - \hat{A}_2 \hat{A}_1^+).$ The total number of excitations is accounted for by the operator $\hat{N} = (\hat{A}_1^+ \hat{A}_1 + \hat{A}_2^+ \hat{A}_2)$. Once the set of linear equations is solved, one can extract the exact expressions for the heat currents: $J_b = \langle \mathcal{L}^f_\sigma(\hat{H}_b) \rangle, J_c = \langle \mathcal{L}^f_\sigma(\hat{H}_c) \rangle,$ and $J_n = \langle \mathcal{L}^f_\sigma(\hat{H}_n) \rangle$.

The distribution of impulse determines the performance of the refrigerator. For a normal distribution of impulses in Equation 52, $P(\xi) = (1/\sqrt{2\pi}\sigma^2)e^{-\frac{(\xi - \xi_0)^2}{2\sigma^2}}$. The energy shift is controlled by

$$\epsilon = -\frac{\lambda}{2} \left(2\xi_0/h - e^{-\frac{2\lambda^2}{\hbar^2}} \sin(2\xi_0/h)\right).$$

(60)

The effective noise strength becomes

$$\eta = \frac{\lambda}{4} \left(1 - e^{-\frac{2\lambda^2}{\hbar^2}} \cos(2\xi_0/h)\right).$$

(61)

In the limit of $\sigma \to 0$, one finds that $P(\xi) = \delta(\xi - \xi_0)$. Another possibility is an exponential distribution: $P(\xi) = (1/\xi_0)e^{-\frac{\lambda}{\xi_0}}$. Then

$$\epsilon = -\frac{\lambda}{4(1 + (\xi_0/h)^2)} \left(\frac{\xi_0}{h}\right)^3$$

(62)

and

$$\eta = \frac{\lambda}{4} \left(\frac{\xi_0}{h}\right)^2 \frac{1 + 4(\xi_0/h)^2}{1 + 4(\xi_0/h)^2}.$$  

(63)

The Poissonian noise plays two opposing roles. On the one hand, it increases the cooling current $J_c$, by increasing $\eta$. On the other hand, it decreases $\epsilon$ (becomes more negative) and by that decreases $J_i$. Both parameters $\eta$ and $\epsilon$ depend linearly on $\lambda$, which can be interpreted as the rate of photon absorption in the system that enhances the cooling process. Supplemental Figure 2 shows that $J_i$ increases with $\lambda$ until a point at which $\epsilon$ dominates and $J_c$ decreases.

The coefficient of performance for the Poisson-driven refrigerator is restricted by the Otto and Carnot coefficients of performance:

$$\text{COP} = \frac{\Omega_+ - \Omega_-}{\Omega_+} \leq \frac{\omega_b}{\omega_b - \omega_c} \leq \frac{T_c}{T_b - T_c}.$$  

(64)

### 3.5. Refrigerators Operating as $T_c \to 0$ and the Third Law of Thermodynamics

The performance of all types of refrigerators at low temperature has universal properties. In power-driven refrigerators, the lower split manifold is dominant, leading to the cold current

$$J_i \approx h\omega_c \frac{2\epsilon^2\tilde{\Gamma}}{4\epsilon^2 + \Gamma_i \Gamma_c} \cdot G,$$

where the gain $G = \Delta N^* - N^*$ and $\tilde{\Gamma} = (\Gamma_i + \Gamma_c)/\Gamma_c$. The three-level absorption refrigerator, the cold current becomes

$$J_c = h\omega_c \frac{\Gamma_i \Gamma_c \Gamma_w}{\Delta} \cdot G,$$

(66)
where $\Delta$ is a combination of kinetic constants and $G = e^{-\frac{h\omega_\epsilon}{k_B T}}e^{-\frac{h\omega_\epsilon}{k_B T_f}} - e^{-\frac{h\omega_\epsilon}{k_B T_f}}$ (86).

In the Gaussian noise-driven refrigerator, Equation 50 becomes

$$\mathcal{J}_c = \frac{h\omega_c}{2\eta + \Gamma} \cdot G,$$

(67)

where $G = N_c - N_b$.

In the Poisson-driven refrigerator, we obtain

$$\mathcal{J}_c \approx \frac{h\Omega}{2\eta + \Gamma} \cdot G,$$

(68)

with $G = (N_c^{-} - N_b^{-})$ and $\Omega^{-} \approx \omega_b - \epsilon^2/(\omega_b - \omega_c)$. All expressions for the cooling current $\mathcal{J}_c$ are a product of an energy quant $h\omega_c$, the effective heat conductance, and the gain $G$.

To approach the absolute zero temperature, we require a further optimization. Optimizing the gain $G$ is obtained when $\omega_b \rightarrow \infty$, which leads to $G \sim e^{-\frac{h\omega_b}{k_B T}}$. In addition, the driving amplitude should be balanced with the heat conductivity. One can also express the cooling current in terms of the relaxation rate to the bath via the relation $\gamma(\omega)e^{-\frac{h\omega}{k_B T}} = \Gamma(\omega)N(\omega)$. The universal optimized cooling current as $T_c \rightarrow 0$ becomes

$$\mathcal{J}_c = h\omega_c \cdot \gamma_c \cdot e^{-\frac{h\omega_c}{k_B T_c}}.$$

(69)

This form is correct both in the weak coupling limit and in the low density limit. One can interpret the heat current $\mathcal{J}_c$ as the quant of energy $h\omega_c$ extracted from the cold bath at the rate $\gamma_c$, multiplied by a Boltzmann factor. Further optimization with respect to $\omega_c$ is dominated by the exponential Boltzmann factor (optimizing the function $x^{\alpha}e^{-x/\beta}$ leads to $x^{\alpha} \propto b$). As a result, one finds that $\omega_c \propto T_c$, obtaining $\mathcal{J}_c \propto \omega_c^{\beta} \cdot \gamma_c(\omega_c)$. The linear relation between the optimal frequency $\omega_c$ and $T_c$ allows one to translate the temperature scaling relations to the low-frequency scaling relations of $\gamma_c(\omega) \sim \omega^\alpha$ and $\gamma_c(\omega) \sim \omega^\beta$ when $\omega \rightarrow 0$.

To fulfill the Nernst heat theorem (Equation 37), one restricts the scaling of the relaxation rate to $\gamma_c(\omega) \sim \omega^\alpha$ and $\alpha > 0$. The fulfillment of the unattainability principle (Equation 40) depends on the ratio between the relaxation rate and the heat capacity $\gamma_c/\epsilon V \sim \omega^{\xi - 1}$, where $\xi > 1$.

We now examine the low-frequency properties of the relaxation rate $\gamma_c$ and the heat capacity $\epsilon V$. For three-dimensional ideal degenerate Bose gases, $\epsilon V$ scales as $T_c^{1/2}$. For degenerate Fermi gases, $\epsilon V$ scales as $T_c$. In both cases, the fraction of the gas that can be cooled decreases with temperature. Based on a collision model, when cooling occurs owing to inelastic scattering, we have found the scaling exponent $\xi = 3/2$ for both degenerate Bose and Fermi gases.

The most studied generic bath is the harmonic bath. It includes the electromagnetic field (a photon bath), a macroscopic piece of solid (a phonon bath), or Bogoliubov excitations in a Bose-Einstein condensate. The standard form of the bath’s Hamiltonian is

$$\hat{\mathbf{H}}_{\text{int}} = (\hat{\mathbf{b}} + \hat{\mathbf{b}}^\dagger) \left( \sum_k (g(k)\hat{a}(k) + \bar{g}(k)\hat{a}^\dagger(k)) \right), \quad \hat{\mathbf{H}}_{\beta} = \sum_k \omega(k)\hat{a}^\dagger(k)\hat{a}(k).$$

(70)

where $\hat{a}(k)$ and $\hat{a}^\dagger(k)$ are the annihilation and creation operators for a mode $k$. For this model, the weak coupling limit procedure leads to the GKS-L generator with the cold bath relaxation rate given by (35)

$$\gamma_c = \gamma_c(\omega_c) = \pi \sum_k |g(k)|^2 \delta(\omega(k) - \omega_c) \left[ 1 - e^{-\frac{h\omega(k)}{k_B T_c}} \right]^{-1}. \quad (71)$$
For the bosonic field in $d$-dimensional space, with the linear low-frequency dispersion law ($\alpha(k) \sim |k|$), one obtains the following scaling properties for the cooling rate at low frequencies: $\gamma \sim \alpha^2 \alpha^{-1}$, where $\alpha^2$ represents the scaling of the form factor $|g(\omega)|^2$, and $\alpha^{-1}$ is the scaling of the density of modes. For $\omega \sim T_\gamma$, the final current scaling becomes $J_{\text{eq}} \sim T_\gamma^{d+\kappa}$ or $\alpha = d + \kappa - 1$.

At low temperatures, the heat capacity of the bosonic systems scales as follows: $c_V(T) \sim T_d$, which produces the scaling $\zeta = \kappa$. This means that $\kappa \geq 1$ to fulfill the third law. To rationalize the scaling, $c_V$ is a volume property and $\gamma$ is a surface property, so $\omega_\gamma \gamma$ scales the same as $c_V$. The scaling of the form factor $\kappa$ is related to the speed at which the excitation can be carried away from the surface.

The exponent $\kappa = 1$ when $\omega \to 0$ is typical in systems such as electromagnetic fields or acoustic phonons that have a linear dispersion law, $\omega(k) = v|k|$. In these cases, the form factor becomes $g(k) \sim |k|/\sqrt{\omega(k)}$; therefore, $|g(\omega)|^2 \sim |k|$. The condition $\kappa \geq 1$ excludes exotic dispersion laws, $\omega(k) \sim |k|^{\delta}$ with $\delta < 1$, which produce infinite group velocity forbidden by relativity theory. Moreover, the popular choice of Ohmic coupling is excluded for systems in dimension $d > 1$ (31).

We note that challenges to the third law have been published (94). Our viewpoint is that these discrepancies are the result of flaws in the reduction of the spin bath to a harmonic spectral density formulation.

4. SUMMARY

Thermodynamics represents physical reality with an amazingly small number of variables. In equilibrium, the energy operator $\hat{H}$ is sufficient to reconstruct the state of the system, from which all other observables can be deduced. Dynamical systems out of equilibrium require more variables. Quantum thermodynamics advocates that only a few additional variables are required to describe a quantum device: a heat engine or a refrigerator. A description based on the Heisenberg equation of motion lends itself to this viewpoint. A sufficient condition for the set of operators to be closed to the Hamiltonian is that they form a Lie algebra. In addition, the Hamiltonian needs to be a linear combination of operators from the same algebra (95). Finally, the operators have to be closed to the dissipative part $\hat{L}_{\text{eff}}$. Most of the solvable models in this review are based on the SU(2) Lie algebra with four operators. We chose them to represent the energy $\hat{H}$, the identity $\hat{1}$, and two additional operators, $\hat{X}$ and $\hat{Y}$. These operators can associate with coherence because $[\hat{H}, \hat{X}] \neq 0$, $[\hat{H}, \hat{Y}] \neq 0$. Generically, in these models, the power is proportional to the coherence $P \propto \langle \hat{Y} \rangle$. We can speculate that the steady-state operation of a working engine or refrigerator is minimally characterized by the SU(2) algebra of three noncommuting operators. This is also the case in reciprocating engines (96–98).

The Hamiltonian can be decomposed to noncommuting local operators that couple to the baths. The nonlocal structure influences the derivation of the master equation. A local approach based on equilibrating $\dot{\hat{H}}$, leads to equations of motion that can violate the second law of thermodynamics (9, 31, 35, 69). A thermodynamically consistent derivation requires a prediagonalization of the system Hamiltonian. This Hamiltonian defines the system-bath weak coupling approximation leading to the GKS-L completely positive generator.

Quantum mechanics introduces dynamics into thermodynamics. As a result, the laws of thermodynamics have to be reformulated, replacing heat with heat currents and work with power. The first law at steady state requires that the sum of these currents is zero. For the devices considered, the entropy production is exclusively generated in the baths and the sum of all contributions is positive. All quantum devices should be consistent with the first and second laws of thermodynamics. Apparent violations point to erroneous derivations of the dynamical equations of motion.
The tricycle template is a universal description for continuous quantum heat engines and refrigerators. This model incorporates basic thermodynamic ideas within a quantum formalism. The tricycle combines three energy currents from three sources by a nonlinear interaction. A nonlinear construction is the minimum requirement for all heat engines or refrigerators. This universality means that the same structure can describe a wide range of quantum devices. The performance characteristics are given by the choice of working medium and reservoirs. In the derivation, the reservoirs are characterized by their temperature and correlation functions. The working medium filters out the channels of heat transfer and power.

The trade-off between power and efficiency is a universal characteristic of the quantum devices. The output power or the cooling current exhibits a turnover with any control parameter (99). For example, when the coupling parameter to the external field is increased first, the power or cooling current increases with it, but then beyond a critical parameter, because of the splitting of the energy levels, a decrease in performance appears. Another turnover phenomenon is observed when the coupling parameter to the baths is increased. Initially increasing the coupling will allow more heat to flow to the device. Above a critical value, dissipation will degrade the coherence and with it the performance. Optimal power and optimal cooling currents are obtained in a balanced operational point far from reversible conditions of maximum efficiency.

The quantum version of the third law originates from the existence of a ground state. One can think of the third law as a limit to the distillation of a pure ground state in a subsystem disentangling it from the environment. We can generalize and apply the third law to any pure state of the subsystem, as such a state is accessible from the ground state by a unitary transformation. Unitary transformations are isentropic and therefore allowed by the Nernst heat theorem at $T = 0$. The quantum dilemma of cooling to a pure state stems from the requirement of a system-bath interaction to induce a change of the entropy of the system. However, such an interaction generates system-bath entanglement so that the system cannot be in a pure state. Approaching the absolute zero temperature is a delicate maneuver. The system-bath coupling has to be reduced while the cooling takes place, eventually vanishing when $T \to 0$. As a result, the means to cool are exhausted before the target is reached (100, 101). The universal behavior of all quantum refrigerators as $T \to 0$ emphasizes this point.

How do quantum phenomena such as coherence and entanglement influence performance? This issue can be separated into global effects, which include the reservoirs, and internal quantum effects within the device. The description of heat transport into the device by the GKS-L master equation is the thermodynamic equivalent of an isothermal partition. In this description, the system and bath are not entangled (31). The issue of internal quantum effects is more subtle. Internal coherence is necessary for operation in quantum-driven devices. For quantum autonomous absorption refrigerators, it has been recently claimed that, although present, coherence is not a determinant in engine operation (86). Other papers claim the opposite (50, 84, 102). Entanglement has been related to the extraction of additional work (60, 103, 104). These issues require further study.

For completeness, here we mention what was excluded from the review above: reciprocating heat engines and refrigerators. Significant insight can be gained from the analysis of such devices. The template is the quantum Otto four-stroke cycle in which the unitary adiabatic branches are separated from the heat transport (22–24, 33, 98, 105–110). The analysis of reciprocating branches is simplified owing to this separation. The disadvantage is that the timing of the branches is determined externally. A review of reciprocating engines and refrigerators will be treated in a separate publication.

Finally, we can conclude from this review that, when analyzed according to the laws of quantum mechanics, quantum devices are consistent with a dynamical interpretation of the laws of thermodynamics.
DISCLOSURE STATEMENT
The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS
This review was supported by the Israel Science Foundation. The authors thank Tova Feldmann, Eitan Geva, Jose P. Palao, Jeff Gordon, Lajos Diosi, Peter Salamon, Gershon Kuritzky, and Robert Alicki for sharing their wisdom. Part of this work was supported by the COST Action MP1209 “Thermodynamics in the quantum regime.”

LITERATURE CITED

www.annualreviews.org • *Quantum Heat Engines* 391
Contents

A Journey Through Chemical Dynamics
William H. Miller ................................................................. 1

Chemistry of Atmospheric Nucleation: On the Recent Advances on
Precursor Characterization and Atmospheric Cluster Composition
in Connection with Atmospheric New Particle Formation
M. Kulmala, T. Petäjä, M. Ehn, J. Thornton, M. Sipilä, D.R. Worsnop,
and V.-M. Kerminen ............................................................. 21

Multidimensional Time-Resolved Spectroscopy of Vibrational
Coherence in Biopolymers
Tiago Buckup and Marcus Motzkus .......................................... 39

Phase Separation in Bulk Heterojunctions of Semiconducting
Polymers and Fullerenes for Photovoltaics
Neil D. Treat and Michael L. Chabinyc .................................... 59

Nitrogen-Vacancy Centers in Diamond: Nanoscale Sensors for
Physics and Biology
Romana Schirhagl, Kevin Chang, Michael Loretz, and Christian L. Degen .......... 83

Superresolution Localization Methods
Alexander R. Small and Raghuveer Parthasarathy ........................................... 107

The Structure and Dynamics of Molecular Excitons
Christopher J. Bardeen ........................................................... 127

Advanced Potential Energy Surfaces for Condensed Phase Simulation
Omar Demerdash, Eng-Hui Yap, and Teresa Head-Gordon ......................... 149

Ion Mobility Analysis of Molecular Dynamics
Thomas Wytenbach, Nicholas A. Pierson, David E. Clemmer,
and Michael T. Bowers .......................................................... 175

State-to-State Spectroscopy and Dynamics of Ions and Neutrals by
Photoionization and Photoelectron Methods
Cheuk-Yiu Ng ........................................................................... 197

Imaging Fluorescence Fluctuation Spectroscopy: New Tools for
Quantitative Bioimaging
Nirmalya Bag and Thorsten Wobland .............................................. 225
Elucidation of Intermediates and Mechanisms in Heterogeneous Catalysis Using Infrared Spectroscopy
Aditya Savara and Eric Weitz ....................................................... 249

Physicochemical Mechanism of Light-Driven DNA Repair by (6-4) Photolyases
Shirin Faraji and Andreas Dreuw .................................................... 275

Advances in the Determination of Nucleic Acid Conformational Ensembles
Loïc Salmon, Shan Yang, and Hashim M. Al-Hashimi .......................... 293

The Role of Ligands in Determining the Exciton Relaxation Dynamics in Semiconductor Quantum Dots
Mark D. Peterson, Laura C. Cass, Rachel D. Harris, Kedy Edme, Kimberly Sung, and Emily A. Weiss .................................................. 317

Laboratory-Frame Photoelectron Angular Distributions in Anion Photodetachment: Insight into Electronic Structure and Intermolecular Interactions
Andrei Sanov .................................................................................. 341

Quantum Heat Engines and Refrigerators: Continuous Devices
Ronnie Kosloff and Amikam Levy .................................................... 365

Approaches to Single-Nanoparticle Catalysis
Justin B. Sambur and Peng Chen ...................................................... 395

Ultrafast Carrier Dynamics in Nanostructures for Solar Fuels
Jason B. Baxter, Christiaan Richter, and Charles A. Schmuttenmaer ....... 423

Nucleation in Polymers and Soft Matter
Xiaofei Xu, Christina L. Ting, Isamu Kusaka, and Zhen-Gang Wang ........ 449

H- and J-Aggregate Behavior in Polymeric Semiconductors
Frank C. Spano and Carlos Silva ...................................................... 477

Cold State-Selected Molecular Collisions and Reactions
Benjamin K. Stuhl, Matthew T. Hummon, and Jun Ye ......................... 501

Band Excitation in Scanning Probe Microscopy: Recognition and Functional Imaging

Dynamical Outcomes of Quenching: Reflections on a Conical Intersection
Julia H. Lehman and Marsha I. Lester ................................................. 537

Binuclear Recombination in Organic Photovoltaics
Girish Lakhwani, Akshay Rao, and Richard H. Friend .......................... 557

vi Contents
Mapping Atomic Motions with Ultrabright Electrons: The Chemists’
Gedanken Experiment Enters the Lab Frame
R.J. Dwayne Miller .............................................................. 583

Optical Spectroscopy Using Gas-Phase Femtosecond
Laser Filamentation
Johan Odhner and Robert Levis ........................................... 605

Indexes
Cumulative Index of Contributing Authors, Volumes 61–65 .............. 629
Cumulative Index of Article Titles, Volumes 61–65 ........................ 632

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