Energy conservation and reciprocity of random rough surface scattering

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The principle of reciprocity and energy conservation is a well-established concept in electromagnetic scattering and diffraction dealing with random and deterministic rough surfaces.\(^1,2\) It has been shown that existing rough surface...
scattering models, including the Kirchhoff scattering and perturbation models, satisfy the reciprocal relationship. Recently Okayama and Ogura presented experimental results which showed the nonreciprocal properties of light scattering from very rough surfaces. They have found that as irregular surfaces of glass plates become rougher, the angular behavior of intensity scattering tends to violate further the reciprocal relationship.

The purpose of this Letter is to explain this anomalous behavior using a theoretical scattering model. We choose a random surface scattering model based on the Kirchhoff approximation for the verification of experimental results in Ref. 5. In what follows, we briefly discuss the theoretical behavior of bistatic scattering intensity of the Kirchhoff scattering model in terms of single- and multiple-scattering processes. Theoretical scattering behavior is then compared to the experimental results, and a possible explanation for nonreciprocal responses of the experimental observation is suggested.

The Kirchhoff scattering model for a random rough surface scattering is one of the most extensively used models due mainly to its mathematical tractability. The Kirchhoff scattering model is based on the high-frequency approximation and single-scattering theory. The single-scattering assumption inevitably results in the unfulfillment of energy conservation particularly when a surface becomes rougher. Several attempts have been made to improve the energy characteristics of scattering models by incorporating the effects of multiple scattering into the single Kirchhoff scattering theory.

To investigate the reciprocal relation regarding surface scattering, it is essential to use an energy-conserving surface scatter model which correctly includes the multiple-scattering effects. In this paper, we use the multiple-scattering Kirchhoff model developed by Fung and Eom. It is shown in Ref. 8 that the process of multiple scattering can be accounted for in the form of an infinite series which can be summed up in a closed form. It is also shown that the multiple-scattered bistatic scattering coefficients can fulfill energy conservation of a rough surface. For ease of reference, we plot the energy conservation character of both single and multiple scattering in Fig. 1. Sums of scattered intensities are shown vs incidence angles. Note that the energy characteristic curves corresponding to single and multiple scattering become poorer as surface roughnesses increase, while multiple scattering curves remain satisfactory.

According to the Kirchhoff scattering model in Ref. 8, the polarized scattered intensity \( \mathbf{S}_s \) is related to the incident intensity \( \mathbf{S}_i \) by the phase matrix \( \mathbf{M} \):

\[
\mathbf{S}_s = \mathbf{M} \mathbf{S}_i,
\]

where \( \mathbf{S}_s \) and \( \mathbf{S}_i \) are \((4 \times 1)\) column vectors representing 4-Stokes parameters. Matrix \( \mathbf{M} \) denotes a \((4 \times 4)\) Mueller matrix of 4-Stokes vector representation. Hence the unpolarized scattered intensity is seen to be \((M_{uy} + M_{yh} + M_{hu} + M_{hh})\) when incident intensity is also unpolarized. \( M_{uv}, M_{uv}, M_{hv}, \) and \( M_{hh} \) denote the first four elements in \( \mathbf{M} \) corresponding to vertical and horizontal polarizations. Note that the contents of Mueller matrix are bistatic scattering coefficients. This means that when single- (or multiple-) scattered phase matrices are used in \( \mathbf{M} \), the scattered intensity \( \mathbf{S}_s \) corresponds to all scattered rays resulting from single (or multiple) scattering. Note that single-scattered phase matrices of the Kirchhoff surface scattering model satisfy the law of reciprocity.

To check the reciprocity property of multiple-scattering phase matrices, we compute \( \mathbf{S}_s \) using the multiple-scattered phase matrices in Eq. (1). The computational results are shown in Table I for multiple scattering. The results for single scattering are not shown since they are shown to satisfy the reciprocal relation in the Kirchhoff approximation. It shows that the law of reciprocity is clearly violated after inclusion of multiple scattering. A qualitative agreement is shown to exist between experimental data in Ref. 5 and the theory (Table I). Both theory and experimental results indicate that the rougher the surface is the more the curve deviates from the reciprocal case (a perfect plane case). This is because the process of multiple scattering becomes dominant as the surface roughness becomes larger, so that the interchange of transmitter and receiver can no longer be viewed as a simple reciprocal process.

<table>
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<th>( \gamma )</th>
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Table 1. Reciprocity of Multiple Kirchhoff Scatter Theory

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To summarize: the laws of energy conservation and reciprocity of rough surfaces are discussed using the Kirchhoff surface scattering model. Theoretical results verify the experimental observation that the law of reciprocity is unfulfilled in a very rough surface scattering problem. Some reasons for failure of the reciprocity law have been considered, and the effect of multiple scattering is one of them.

References

Magnification equations for a two-lens system

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There are some useful expressions for the magnification of a two-lens system which the author has been unable to find in the literature. The trick is in specifying distances relative to the focal points of the lenses. In the optical system shown in Fig. 1 the first lens has a front focal length $f_1$ and a rear focal length $f'_1$; the second lens has focal lengths $f_2$ and $f'_2$. The distance from the rear focal point of the first lens to the front focal point of the second is $q$, the distance from the front focal point of the first lens to the axial object point is $s$, and the distance from the near focal point of the second lens to the axial image point is $s'$. All distances are directed, with the sign convention that those from left to right are positive and conversely. Using Newton’s relations, the system magnification is easily found to be

$$m = m_0 \frac{1}{1 = \frac{s}{q} \frac{f_2}{f_1'}}$$

![Fig. 1. Two-lens system. $F_1$ and $F'_1$ are the front and rear focal points of the first lens, $F_2$ and $F'_2$ of the second, $O$ is the axial object position, and $O'$ is the axial image position.](image)

Ion-assisted deposition of magnesium fluoride films on substrates at ambient temperatures

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This Letter reports on the optical properties of magnesium fluoride films deposited by evaporation in a vacuum while under the bombardment of ions. One of the most significant advances in optical thin-film technology has been introduction of ion-assisted deposition (IAD). When a growing film is bombarded with low-energy ions at room temperature, the packing density of the film can be increased to the point at which water penetration by capillary action is virtually eliminated due to the ion-induced modification of the film microstructure. The resulting films have stable and reproducible refractive indices that are close to that of the bulk material. Other ion-induced effects in IAD films are increased adhesion, reduced stress, modification of the crystal structure, and, in the case of a compound, stoichiometric changes. The last is not desirable for low optical absorbance and is a consequence of preferential sputtering, usually of the lighter elements. In the case of dielectric oxides such as SiO₂,