Longitudinal and transverse displacements of a bounded microwave beam at total internal reflection*

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Longitudinal and transverse shifts of an 8 cm parallel bounded beam of 34.2 GHz (8.77 mm) microwaves totally reflected from a paraffin prism have been investigated. The 45°-45°-90° prism is 18 cm high by 25 cm on the sides and the index of refraction is 1.491. Longitudinal shifts as large as 3 cm have been measured in a single reflection near the critical angle for a beam linearly polarized in the plane of incidence. The shift for perpendicular polarization is approximately half this value. The results are in general agreement with the classical theory for the Goos-Hänchen effect. An incident beam polarized at 45° to the incidence plane produces both parallel and perpendicular polarization shifts with values similar to the above. The shifts for both polarizations are reduced but are still distinctly separate if either a second prism or a metallic reflector is brought into the evanescent wave at millimeter distances from the interface. These results are in accordance with stationary phase calculations for two interfaces. It is found that a small (6 mm) transverse shift results if the prism is illuminated with circularly polarized microwaves.

I. INTRODUCTION

A bounded beam of light that enters a prism and is totally reflected at the prism-vacuum interface is known to be displaced longitudinally from the position of geometrical reflection. The displacement is small, usually on the order of the illuminating wavelength, and it is larger for light polarized within the plane of incidence than perpendicular to it. This phenomenon, commonly known as the Goos-Hänchen shift, has been the object of considerable theoretical and experimental studies over the past few years, not only because of its importance in research on the fundamental properties of light but also because of its relation and application to the expanding fields of optical imaging and integrated optics.

Theoretical treatments of the effect have stressed the important role played by the inhomogeneous, evanescent wave associated with total reflection that appears on the vacuum side of the interface. Accurate determinations of the shift have been made on the assumption that energy from the beam flows into the evanescent wave, travels along the surface for a certain distance and then passes back into the reflected beam. Other treatments, such as the stationary phase theory, implicitly take the surface wave into account through Fresnel’s equations. Still other theories explain the shift in terms of a time delay of scattering process.

In the usual beam-shift experiments light is linearly polarized either parallel or perpendicular to the plane of incidence, but there have been some interesting variations using circularly polarized and unpolarized light. With circular polarization the beam is shifted transversely, out of the plane of incidence. The direction depends on the sense of polarization, and the magnitude is smaller than that for the longitudinal shift. The transverse shift has been given as evidence for the noncollinearity of velocity and momentum of a spinning photon.

Using unpolarized light two distinct longitudinal shifts occur, the larger corresponding to parallel polarization, and the smaller, to perpendicular polarization, with no intermediate values. Some authors have been able to explain this experiment on the basis of existing electromagnetic theory, while others have cited it as evidence that the photon has a small but finite mass.

There are many interesting questions to be investigated and resolved with total reflection phenomena, but the experiments using visible light have not always been easy to perform. Since the shift is so small, special experimental techniques to amplify it have generally been required. This involves allowing the beam to be reflected many times before emerging from the prism, and then delicate measurement techniques are employed.

Experiments carried out at microwave frequencies, using wavelengths of about 1 cm, are more amenable to analysis and interpretation and probably easier than those done in the optical region where the wavelength is much larger for light polarized within the plane of incidence, larger than perpendicular to it. This phenomenon, commonly known as the Goos-Hänchen shift, has been the object of considerable theoretical and experimental studies over the past few years, known as the Goos-Hänchen shift, delay of scattering process. Such as the stationary phase theory, implicitly take the energy from the beam flows into the evanescent wave, travels along the surface for a certain distance and then passes back into the reflected beam. These results are in general agreement with the classical theory for the Goos-Hänchen effect. An incident beam polarized at 45° to the incidence plane produces both parallel and perpendicular polarization shifts with values similar to the above. The shifts for both polarizations are reduced but are still distinctly separate if either a second prism or a metallic reflector is brought into the evanescent wave at millimeter distances from the interface. These results are in accordance with stationary phase calculations for two interfaces. It is found that a small (6 mm) transverse shift results if the prism is illuminated with circularly polarized microwaves.

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smaller. For example, only a single reflection is required to measure the beam shift accurately. Also, studies of the evanescent wave, which extends on the order of a wavelength from the prism face into air, are much easier to accomplish for total reflection of microwaves than for light waves. \(^{13}\)

This paper is a report of studies of the total reflection of a bounded beam of microwaves from a paraffin prism. \(^{14}\) The longitudinal shift with parallel, perpendicular, and intermediate polarization states was investigated as well as the transverse shift with circular polarization. Also the effect on the shift of placing a metallic reflector or second paraffin prism into the evanescent wave at millimeter distances from the interface was studied.

II. EXPERIMENTAL APPARATUS AND PROCEDURE

A diagram of the experimental arrangement is shown in Fig. 1. Microwaves with a free space wavelength of \(\lambda = 0.877 \) cm were produced by a klystron operating at a frequency of 34.2 MHz. The beam was fed by waveguide through a directional coupler and an attenuator to a flexible waveguide that was in turn attached to a 3.5 in. \(\times 3.5\) in. square transmitting horn containing a phase correction lens. The receiving horn was identical to this and the beam was terminated in a square-law crystal detector. The signal was fed into an amplifier and the \(y\) axis of a pen recorder.

The prism was made of paraffin, the least absorbing most commonly available material. The transmission through paraffin slabs of various thicknesses was measured and an absorption coefficient of \(\kappa = 3.35 \times 10^{-4}\) was obtained. An interferometric technique was used to determine the index of refraction. The microwave beam was split into two parts, one passing through a slab of paraffin and the other, through a waveguide containing a phase shifter. By adjusting the phase \(\phi\) for maximum and minimum intensities for various slab thicknesses and using the phase change formula

\[
\Delta \phi = 2\pi (n - 1) t/\lambda ,
\]

where \(t\) is the paraffin thickness, \(n\) is the index of refraction, and \(\lambda\) is the free-space wavelength, an index of refraction of \(n = 1.491 \pm 0.002\) was obtained.

The prism was cast in a cube, then cut diagonally and milled into two \(45^\circ - 45^\circ - 90^\circ\) prisms, each 18 cm high and 25 cm on the sides. One of these was used in the reflection experiments, and the other, as a dielectric to be placed near the reflecting interface. Precision ball bearing mounts were made for the horns that allowed them to be moved about an axis perpendicular to the plane of incidence, for varying the angle of incidence, and about an axis through their center, to vary the polarization. The prism and horns were then mounted on an optical bench (a modified lathe) that allowed the receiving horn to be moved back and forth or up and down parallel to the prism face. The motion was fed through a potentiometer to the \(x\) axis of the recorder. This arrangement was used for the linear polarization experiments.

For the circular polarization experiments the square horns were replaced with 3-in. -diam round horns containing phase correcting lenses. The feed to the rectangular waveguide and the change to circular polarization was accomplished by attachment to each horn of a rectangular-to-round transducer waveguide section and a quarter-wave phase retarder. The retarders were straight Cu tubes either squeezed in the middle or filled with a short section of Cu wire to achieve the necessary retardation for circular polarization.

In general the shift was measured by first fixing the angle of incidence and exit, which were identical for both horns, and the state of polarization. A metallic reflector (an aluminum plate) was then placed over the hypotenuse of the prism and the signal recorded by scanning the detector over the prism face. The reflector was then removed and the scan repeated. By doing this it was assumed that with the plate in position the evanescent wave could not form and that the wave would thus be reflected without being shifted. This is the same technique used in the optical experiments where a layer of metal is evaporated onto half the totally reflecting side of the glass prism and the beam displacement of the uncoated half is measured relative to the coated one.

III. RESULTS AND DISCUSSION

A. Longitudinal shift \(p\) and \(s\) polarization

Measurements of the longitudinal shift were made for both \(p\) and \(s\) polarizations for angles of incidence \(\theta\) in the vicinity of the critical angle. The critical angle is given by

\[
\sin \theta_c = 1/n
\]

and is \(\theta_c = 42.1^\circ\) for the paraffin prism. The maximum shift for both polarizations was found near the critical angle, as expected.

The shift can be defined as either the parallel translation \(D\) of the beam or as the translation \(X\) of the beam along the reflecting surface (Fig. 1). We have used the latter designation in this work. From the geometry of the experimental arrangement it follows that
where $Re$ denotes "the real part of" $D_v(y)$ is the parabolic-cylinder function of order $v$ and $y$ is the perpendicular coordinate. Values of $r$ and $\gamma$ are obtained from Fresnel's equations. 15 The stationary phase theory is useful in the present work because it can be easily extended to the case of additional interfaces, such as would be encountered, if absorbers or dielectrics were placed near the reflecting surface. However, it suffers the usual deficiency common to most treatments in that at exactly the critical angle, it diverges.

For comparison with experimental values close to the critical angle, therefore, we have utilized the bounded beam theory of Horowitz and Tamir. 16 The shift, as given by Eq. (39) of their paper, is

$$X = \frac{1}{2} d \left(2^{1/2}(1 + \tan \theta)\right),$$  

(3)

where $d$ is the beam shift measured along the short side of the prism. The shift is that distance between the centers of the curves at half-maximum. Generally the curves were approximately Gaussian in shape so that the center found this way coincided with the peak.

Typical values obtained were $d = 2.4$ cm for $p$ polarization and $d = 1.0$ cm for $s$ polarization at $\theta = \theta_e$. From Eq. (3) the shifts $X$ are, respectively, $X = 3.3$ cm and $1.4$ cm. Measured values of $d$ from other runs at the critical angle differed no more than a few mm from the above values.

The results for the value of the shift in the vicinity of the critical angle are shown in Fig. 2. In the region of total reflection $\theta > \theta_e$, a comparison was first made with the stationary phase theory. 6 In this theory the reflected field is represented as a spectrum of plane waves

$$E = \int A(S) e^{iK(Sx+Cy)} \gamma e^{i\alpha} dS,$$  

(4)

where $A(S)$ is the wave amplitude, $S = \sin \theta$, $\gamma = \cos \theta$, $K = 2\pi n/\lambda$ is the wave number within the prism, $r$ is the reflection amplitude, and $\alpha$ is the reflection phase; $x$ is the beam coordinate parallel to the reflecting surface and $y$ is the perpendicular coordinate. Values of $r$ and $\alpha$ are obtained from Fresnel’s equations. 15 The stationary-phase condition is that the phase $\psi$ of the integral be maximized by the relation

$$\frac{\partial \psi}{\partial S} = 0.$$  

(5)

In this case

$$\psi = K(Sx+Cy) + \alpha,$$  

(6)

from which we obtain

$$\frac{\partial}{\partial S} (Sx+Cy) = \frac{1}{K} \frac{\partial \alpha}{\partial S} = 0$$  

(7)

and

$$\frac{\partial}{\partial S} (Sx+Cy) = x - y \tan \theta$$  

(8)

$$\alpha = x - x_0$$  

(9)

$$X = x_0,$$  

(10)

where $x_0$ corresponds to the position of geometrical reflection. Thus the beam shift is

$$X = \frac{1}{K} \frac{\partial \alpha}{\partial S}.$$  

(11)

This result is identical to that obtained by Artmann. 6 The physical argument is that a spectrum of plane waves has components incident at different angles, and each undergoes a different phase change. The maximization of the phase, which implicitly contains the beam shift, means that the reflected field intensity is maximized when all the plane-wave components are superimposed. Using Eq. (11) and Fresnel’s equations for a dielectric-air interface, the beam shifts for parallel and perpendicular polarization are, respectively,

$$X_p = \frac{\lambda \tan \theta}{\pi (u^2 S^2 - C^2)(u^2 S^2 - 1)^{1/2}};$$  

(12)

$$X_s = \frac{\lambda \tan \theta}{\pi (u^2 S^2 - 1)^{1/2}}.$$  

(13)

These relations are similar to other equations for this effect derived by different methods. Almost identical results are obtained with Eqs. (12) and (13) as from those using energy conservation arguments. 5 One sees from Fig. 2 that the agreement with the experimental values at angles greater than $\theta \approx 43^\circ$ is quite good. This is remarkable considering that the theory is an approximation that does not take into account the dimensions of the incident bounded beam. The stationary phase theory is useful in the present work because it can be easily extended to the case of additional interfaces, such as would be encountered, if absorbers or dielectrics were placed near the reflecting surface. However, it suffers the usual deficiency common to most treatments in that at exactly the critical angle, it diverges.

FIG. 2. Variation of shift with incidence angle showing comparison of measured shift with the classical stationary phase and bounded beam theories.
with

\[ m = \begin{cases} n^2 & \text{p polarization,} \\ 1 & \text{s polarization.} \end{cases} \]

In a separate measurement made with a waveguide probe across the face of the transmitting horn, the field distribution for p polarization was found to consist of a central maximum lobe and two subsidiary lobes on either side of it, while for s polarization there was only a single central lobe. When the receiving horn is moved past the transmitter, these variations are smoothed out, so that one records only a single smooth approximately Gaussian distribution. The net effect is that the curves for p are inherently broader than for s. Also the recorded signal appears broader at the base but more peaked than the transmitted beam because it is proportional to the convolution of this distribution as one horn scans past the other one. For purposes of comparison with the theory of Horowitz and Tamir, the beam width was measured at 1/e of the maximum intensity for the transmitting horn field distribution. For preliminary calculations we took an average value for both polarizations to be 8 cm.

Using values of \( \lambda \) and \( \theta_e \) given previously, \( X \) was calculated from Eq. (14) for a beam half-width, \( w = 4 \) cm (\( w/\lambda = 4.58 \)), and the results, for angles \( \theta \) in the vicinity of \( \theta_e \) are given in Fig. 2. At \( \theta_e \), we found that the shift for \( p \) was actually less than that for \( s \), with \( X_p = 0.37 \) cm and \( X_s = 0.71 \) cm, but the agreement with experiment was better for angles \( \theta > \theta_e \) and at \( \theta < 39^\circ \). Also included in Fig. 2 are the calculated \( X \) values assuming a larger beam halfwidth (\( w/\lambda = 10 \)) that appeared to give the best agreement with experiment. Applying the second-order correction of Ref. 16 did not significantly improve the results.

The cusp-like behavior of the calculated value of the shift at \( \theta = \theta_e \) is probably due to the approximations that have been made in deriving Eq. (14). In particular, the theory is valid for large values of \( w/\lambda \) (\( w/\lambda \gg 1 \)), while in our case, \( w/\lambda \) is small.

An exact numerical treatment for the shift has been carried out incorporating Fresnel's equations by Carniglia and McGuirk. The preliminary results have yielded smooth curves that match the experimental data quite well for reasonable values of \( w/\lambda \) (\( w/\lambda = 5 \) and 6). These studies are presently being completed and shall be published separately.

Future theoretical treatments may require including effects that have been neglected up to now. One is that the distance between transmitting and receiving horns should be taken into account and the other is that we may be operating in a region close to the near field of the transmitter, while far-field conditions have been assumed.

**B. Variation of the shift with incident beam polarization**

When the incident beam polarization was changed to an intermediate position and the receiver set for either p or s polarization, two distinct shifts were found, one corresponding to p and the other to s. The results, for the beam incident at \( \theta_i \) and at azimuth angles ranging from \( \psi_i = 0^\circ \) to \( \psi_i = 90^\circ \), are shown in Fig. 3. The amplitudes are maximum for s and p, respectively, at \( \psi_i = 0^\circ \) and \( \psi_i = 90^\circ \) because these are simply the linear polarization cases repeated, where the entire energy of the beam goes into that particular polarization state. For the other angles, the energy of the beam appears to divide into either of the two polarization components, with there being no intermediate values. Thus the prism had the effect, in total reflection, of acting as a polarization filter. This is the same phenomenon that was observed earlier in the optical region.

When on the other hand, the incident azimuth value was set halfway between p and s at \( \psi_i = 45^\circ \) and the receiver polarization was varied, the results shown in Fig. 4 were obtained. In this case a mixture of p and s signals was recorded, the amount of each depending on the receiver orientation angle. Since the curves for receiver azimuth angles other than \( \psi_r = 0^\circ \) or \( \psi_r = 90^\circ \) are a superposition of separate p and s signals, they actually appear as single peaks because of the inability of our apparatus to resolve the two polarization states. A simple calculation to determine the beam shape was made, assuming a superposition of fields for two Gaussian distributions shifted with respect to each other (i.e., p and s). For \( \psi_i = 45^\circ \) the halfwidth of the sum curve was found to be greater than for either p or s individually. This indeed is the case experimentally and is most clearly seen in Fig. 4(b). The analysis further showed that the p and s peaks will not be clearly resolved because of the small relative value of the shift compared to the halfwidths of these two distributions.

It can be noted from Fig. 3 that some of the curves are slightly asymmetric; i.e., the peaks do not always correspond to the centers of the curves. Similar behavior can also be seen in some of the subsequent figures. This could be attributed in part to an inherent asymmetry, as has been pointed out in Refs. 4 and 18, but the effect is generally small. The most likely reason is that reflections have caused the peak position to be uncertain. This explanation is reinforced by the fact that reproducibility was more consistent for measurements of the centers of the curves than the peaks.
C. Effect of placing a reflector or a second paraffin prism into the evanescent wave

When the beam was polarized on either \( p \) or \( s \) and incident at the critical angle, it was found that the shift was changed when either a reflector or a second paraffin prism was introduced into the evanescent wave.

Figure 5 shows the effect of the metallic reflector for \( s \) polarization. The values \( a = 0 \) and \( a = \infty \) correspond, respectively, to the no shift condition and the maximum shift, as before, but with the reflector set parallel to the surface at \( a = 5 \) mm, the shift was reduced from 10 to 4 mm.

When a second paraffin prism was introduced into the beam the results shown in Fig. 6 were obtained. Not only did we find lower values for both polarizations but the intensities were also reduced. This latter condition could be expected since the narrower the air gap becomes the more the evanescent wave is influenced by the second prism, particularly for distances here which are on the order of the exponential decay length of this wave. For these distances a portion of the wave tunnels across the air gap and is transmitted into the second prism, and thus in this sense the second prism may be regarded as an "absorber."

A theoretical evaluation of the shift considering the paraffin dielectric was again made with the stationary phase theory, but using Fresnel's equations for two interfaces rather than one and where we have, in effect, the same dielectric separated by an air gap. To use this theory in a region where it is valid closest to the critical angle and yet to still obtain a nondivergent result, we assumed an incident angle of \( \theta = 43^\circ \). The results of the calculation are given in Fig. 7 (see Appendix). It is seen that the shifts become smaller as the air gap is decreased and that the amount of reduction is almost independent of the state of polarization. This means that even with the shifts being reduced the shift for \( p \) will always be larger than \( s \) for any dielectric distance (except zero). As can be seen from Fig. 7 the theory is in general agreement with the results found experimentally.

D. Effect of the second prism on the shift for \( \psi_i = 45^\circ \)

A further study was made to see how the shift was affected when the second prism was moved into the evanescent wave, but with the incident azimuth angle set on \( \psi_i = 45^\circ \). With the detector set first on \( p \) and then on \( s \), the results shown in Fig. 8 were obtained. As the air gap spacing was reduced, the intensities became smaller and the shift was decreased, just as had been found earlier for \( \psi_i = 0^\circ \) and \( \psi_i = 90^\circ \). Furthermore, the \( p \) and \( s \) signals remained distinctly separate, and the magnitude...
of each shift was in general agreement with the stationary phase theory. Thus what had been observed earlier with the polarization filtering effect for arbitrary beam polarization appeared to hold whether a dielectric was present or not.

This result is interesting because of its relation to the optical beam-shift experiment conducted with unpolarized light. It appears to support the views of Chiu and Quinn and also Costa de Beauregard and Imbert who contended that the splitting into definite polarization states is fully accounted for within existing electromagnetic theory. It does not appear to support a different interpretation of this experiment proposed by de Broglie and Vigier, who suggested that the splitting of the reflected shifted beam into two distinct components could be accounted for by the assumption of a small but finite photon mass. It was predicted that if absorbers were placed into the evanescent wave in total reflection, so that not all of the energy would be reflected, the finite mass effect would be manifested by the beam not splitting into two polarization components, but into a single intermediate value or sets of values depending on the incident beam polarization.

The implications of this experiment are sufficiently important, however, to justify repeating it, but perhaps using a slightly different technique. It would be interesting, for example, to do the experiment with a detector having less pronounced polarization characteristics (assuming that p and s intensities could be resolved), thus making it more nearly equivalent to what would be detected optically by photographic means.

**E. Transverse shift with circular polarization**

The final case that was investigated was the transverse beam shift with circular polarization. The experiment was done by setting the receiver and transmitting horns to the critical angle of incidence, adjusting the retarders until both horns were in the same sense of circular polarization, and then scanning vertically (i.e., transversely). The sense of polarization of both horns was then reversed and the vertical scan was repeated. In this manner it was unnecessary to determine an absolute sense of rotation for either one of the polarizations, and the measured transverse shift was thus the sum of the separate shifts above and below the plane of incidence.

Figure 9 shows the results of a typical measurement, which indicates a transverse shift of 6 mm, or 3 mm to either side of the incidence plane (the shifts were always symmetric about the incidence plane). This is roughly an order of magnitude less than the maximum shift in the longitudinal direction for p polarization but is about the same ratio that Imbert found in his work with circularly polarized light with glass prisms. Imbert's equation for the transverse shift out of the plane of incidence, is

\[ L = \frac{\lambda}{\pi n} \sin \theta_s \cos \theta_s. \]

(16)

Applying this to the present case, a shift of \( L = 3.7 \) mm
to either side of the incidence plane was obtained.

Thus there is not only a transverse shift in the microwave region, but its value is of about the order of magnitude that would be expected using classical theoretical arguments.

Since the receiving horn had to be moved longitudinally from the position of the geometrically reflected beam to maximize the signal, there existed, at the same time, both a transverse and a longitudinal shift. This does not contradict the statement of Costa de Beauregard and Imbert\textsuperscript{10} that these shifts cannot simultaneously exist. Their result applies to a shift that is amplified by many reflections and to the impossibility of multiplying both the transverse and longitudinal shifts simultaneously. This restriction does not apply here since there is only a single reflection.

III. SUMMARY

We have performed an experiment to measure the longitudinal and transverse shifts of a bounded beam of microwaves totally reflected from a paraffin prism. We investigated the longitudinal displacements for beams linearly polarized parallel, perpendicular, and at arbitrary angles to the plane of incidence, both with and without a second prism or reflector being placed in the evanescent wave near the reflecting interface. The dependence of the shift as a function of incidence angle was also studied. Using Fresnel’s equations and the method of stationary phase, equations were developed for the longitudinal shift, both in the presence of a second dielectric, and without it. The transverse shift was studied with circularly polarized microwaves.

For an illuminating wavelength of approximately 9 mm, maximum longitudinal shifts of over 3 cm were measured for beams polarized parallel to the plane of incidence and values about half this were obtained for perpendicular polarization. The shifts decreased either in the presence of a reflector or second dielectric or when the incidence angle was changed beyond the critical value for total reflection. The beam displacement for arbitrary linear polarization divided into either the parallel or perpendicular polarized state whether a second dielectric was present or not. A small transverse shift (6 mm) was detected in the presence of circular polarization.

All these results seem to be in substantial agreement with predictions of classical electromagnetic theory.

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APPENDIX

For the special case of two dielectrics of the same type, separated by an air gap, Fresnel’s equations take the form\textsuperscript{15}

\[
R_0 = R(1 - \gamma)/(1 - R^2\gamma)
\]

where \( R \) is Fresnel’s reflection coefficient at the dielectric-vacuum boundary, and \( \gamma \) and \( \beta \) are the reflection amplitude and phase, respectively, for two interfaces. Also

\[
\gamma = e^{-2\alpha S},
\]

where \( \alpha \) is the air gap separation and \( \rho = (2\pi/\lambda)(n_0^2S^2 - 1)^{1/2} \).

One finds the following simple relation between the phase \( \beta \) for two interfaces, \( \alpha \) for the single interface, and the gap spacing exponential \( \gamma \):

\[
\tan \beta = [(1 + \gamma)/(1 - \gamma)] \tan \alpha.
\]

Defining the shift with the second dielectric present as \( X = (1/K)(\theta \beta /\alpha S) \) and that without it as \( X_{\text{max}} = -(1/K) \times (\theta \alpha /\alpha S) \), we obtain the following ratio:

\[
X /

X_{\text{max}} = \frac{(1 - \gamma^2)(1 + A)^2 - 4\gamma QhAF(1 - A)}{(1 - \gamma^2)(1 + A)^2 + 4(1 + \gamma)^2 A},
\]

where

\[
A = \frac{Q}{n^2 S^2 - 1}]^{1/2} + 2\pi/\lambda;
\]

\[
F = \begin{cases}
\begin{array}{c}
\text{p polarization}, \\
\text{s polarization}, \\
1
\end{array}
\end{cases}
\]

This is seen to reduce to \( X/X_{\text{max}} = 1 \) when \( a = \infty \) (second dielectric absent) and to \( X/X_{\text{max}} = 0 \) when \( a = 0 \).

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Correlation of isotope shifts with $|\psi(0)|^2$ for actinide configurations

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We have used both Hartree-Fock functions and the pseudorelativistic HX functions of Cowan to examine the relationship between observed isotope shifts and s-electron densities at the nucleus for Cm, Am, Pu, and U. There are significant changes in relative electron densities when the relativistic correction is included which greatly improve the correlation between observed isotope shifts and electron density. The improvement is especially dramatic for those configurations which fit most poorly with HF values of $|\psi(0)|^2$. The slope $S = \Delta IS/\Delta(2\sigma|\psi(0)|^2)$ per unit effective mass difference is found to be nearly constant for the actinides considered. This allows prediction of isotope shifts for $^{135}\text{Cf}$-$^{258}\text{Cf}$ which have not been measured. Isotope shifts are also predicted for Cm, Am, Pu, and U configurations which have not yet been identified.

INTRODUCTION

The isotope shift (IS) of a given atomic energy level consists of a mass shift and a nuclear volume shift. Since the mass shift is proportional to $1/A^2$, it is usually neglected for elements with $Z > 80$. Consequently, one can expect actinide ISs to arise entirely from the volume effect, i.e., the change in the overlap of the nuclear and electronic charge distributions from one isotope to another. The volume shift can be written

$$ IS = |\psi(0)|^2 \pi \left(\frac{a_0^2}{Z}\right) C = FC, \quad (1) $$

where $F$ depends only on the electron density at the nucleus, and all nuclear effects are contained in the IS constant $C$.\(^1\) Since in the Slater-Condon theory all levels of a configuration have the same electron radial wave functions, the IS is expected to be constant for all levels

See, for example, the review by H. K. V. Lotsch, Optik 32, 116 (1970); 32, 193 (1970); 32, 229 (1971); 32, 563 (1970), and references therein.


T. Tamir, Optik 36, 209 (1972); 37, 204 (1973); 38, 269 (1973). Both this reference and Ref. 2 and 3 above contain excellent descriptions of the physical phenomena involved. See also H. Kogelnik, in Integrated Optics, edited by T. Tamir, Topics in Applied Physics, Vol. 7 (Springer, New York, Heidelberg, Berlin, 1979), pp. 15–16.


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