INTRODUCTION TO THE HIGH INTENSITY PHYSICS
OF ATOMS AND FREE ELECTRONS

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ABSTRACT

The new and rapidly growing field of high intensity atomic physics is reviewed from an experimental perspective.

1. INTRODUCTION

These lectures introduce the subject of high intensity atomic physics, the study of electrons and atoms in intense laser fields. This subject has a simple and fascinating experimental basis: all neutral atoms are observed to be unstable in laser intensities above a few times $10^{13}$ W/cm$^2$. Even helium, with a binding energy of 24.6 eV, cannot survive intensities above a few times $10^{14}$ W/cm$^2$. In this range we find that the electromagnetic field can no longer be treated as a weak perturbation of the atomic Hamiltonian. The quantum mechanics describing ionization and scattering must be approached in a new way, in order to accommodate processes involving dozens or even hundreds of photons absorbed or scattered in a single step. This theoretical challenge has attracted widespread attention over the past few years, and forms the subject of several other lectures in this volume. This paper will focus on the new phenomena that have been observed in this regime. These include:

1. The scattering of free electrons by light;
2. Bragg scattering of electrons from a periodic light lattice;
3. Above-threshold, or excess photon, ionization;
4. Narrow resonances in ATI photoelectron spectra obtained with sub-picosecond pulses;
5. Asymmetries in photoelectron angular distributions with elliptically polarized light.

The first half of this paper deals with free electrons in intense laser fields. The second considers atomic photoionization. There is a vast and rapidly growing literature in both areas, and the reader interested in pursuing these subjects is advised to begin with some review articles and conference proceedings (Eberly 1967, Chio 1984, Cooke 1987, Smith 1988, Eberly 1988).

Stated in the usual (though mixed!) units, intensities in excess of $10^{13}$ W/cm$^2$ seem enormous. However, such high intensities are readily available these days. A typical high intensity experiment utilizes a Nd:YAG laser, which is a common solid state laser used in physics and industry, operating at a near infrared wavelength of 1064 nm (photon energy of 1.165 eV). The laser is usually mode-
locked, which in practical terms means that it produces pulses approximately 50 to 100 picoseconds in duration. In such a configuration, it is straightforward to amplify the pulses to 0.1 joules of energy, or a peak power of about 1 GW. In order to produce a peak intensity of $10^{14}$ W/cm², one need only focus this laser light to a beam diameter of about 30 μm. These parameters, 1064 nm, 100 psec duration, and $10^{14}$ W/cm², form a "standard example," which will be referred to several times in this paper.

Other choices for high intensity lasers include excimer lasers operating in the near ultraviolet (see I. McIntyre’s article in this volume) and dye lasers, which only deliver a few mJ of energy, but correspondingly shorter pulse durations. Future laser development is likely to concentrate on increasing the peak intensity by shortening the laser pulse at fixed energy, rather than increasing the energy. Most of these new generation lasers rely on a technique borrowed from pulsed radar, called chipped amplification. We don’t have the space to discuss this here, but the reader may turn to recent review articles by G. Mourou (1988).

One way to characterize high intensity in the realm of quantum mechanics is to consider the photon density of the laser beam. A frequent statement in quantum mechanics textbooks is that when the photon density exceeds one per cubic wavelength, the quantum description in terms of photons is no longer important, and the field can be considered classical. (For example, see J.J. Sakurai, 1967, p.35) In these terms, our lasers are intense indeed. Our standard example above corresponds to a density of

$$\rho = \frac{n^3}{4\pi c} = 2\times10^{10} \text{ photons per cubic wavelength} \quad (1)$$

Evidently, one photon per cubic wavelength is far below the threshold for high intensity behavior. However, this example suggests, for example, that the laser beam will behave more like a classical wave than a collection of photons, and so we should consider classical properties such as the electric field strength. For linear polarization, the peak electric field is related to the laser intensity in our example by:

$$E_0 = \left[ \frac{8\pi I}{c} \right]^{1/2} = 2.7\times10^8 \text{V/cm for } 10^{14} \text{ W/cm}^2 \quad (2)$$

again, an enormous number when stated in laboratory units. The peak magnetic field is approximately one million gauss! Since the electric field is oscillating, it imparts a wiggling motion to any charged particle in its path. The time averaged kinetic energy of wiggling is known as the "ponderomotive energy" of the classical particle, and its value is

$$U_p = \frac{e^2 E_0^2}{4m_0 c^2} \quad (3)$$

where m is the mass of the classical particle. In the non-relativistic limit, the ponderomotive energy is linear in laser intensity (quadratic in field strength), and inversely proportional to the square of the laser frequency. For our example above, the ponderomotive energy of a free electron is 10 eV. In other words, the classical wiggle energy exceeds the light quantum $\hbar v$ by a factor of 10. It is this feature, the dominance of classical over quantum energy scales, that lead to new phenomena in laser-electron scattering at these intensities. This is also true for laser-atom interactions; but before discussing these, we first turn our attention to the free electron in an intense laser field.

2. FREE ELECTRONS IN HIGH INTENSITY LIGHT

The first discussions of the interaction of charged particles with oscillating fields can be found in the early works of William Thomson (later Lord Kelvin; 1845). These papers, published when Thomson was only 21 years old, predate both the discovery of the electron and Maxwell’s equations! The more modern approach states that at ordinary intensities, the interaction of electrons with light is confined to elastic (Compton) scattering, described by the Klein-Nishina formula (1929; see also the derivation in any standard textbook on relativistic quantum mechanics, such as Bjorken and Drell, 1964):

Here $\sigma_0$ is the "classical electron cross section", and $\alpha$ is the angular frequency of oscillation of energy and momentum. The perturbation theory, where it can be followed (or preceded) by emission or absorption.

At optical frequencies the electron, so the Klein-Nishina formula was first derived in 1929. Integrating over the invariants

$$\sigma_{\text{Thomson}} = \frac{8\pi n^2}{3} = 0.67\times10^{-24} \text{ cm}^2$$

Now, it is a curious fact that the number of scatterings suffering a ħ/\hbar \approx 0.04. Even if a scattering event is only enough to change its energy is overwhelmed by ordinary Thomson scattering, essentially at the same time, increases the scattering probability; this stimulated scattering effect is electromagnetic field. Many high intensity classical equations of motion, and so we continue with the picture as needed.

2.1 The Time-Averaged Lorentz Force

In a laser focus, the wigging is by two effects. First, since the laser trajectory is not simply one-dimensional, the electron actually describes a trajectory that further are effects due to the spatial. These effects when taken together, understand the results of real experiment.

The problem can be simplified, assume that the spatial and temporal wiggle wiggle of the electron. In the adiabatic approximation seems justifiable.

We now compute the time-averaged force in general, but we will now focus on the momentum for the electron will be calculated. The magnetic fields at point A are taken to be

**E.**

(Note that we employ Gaussian units, the electron describes a figure...
approximately 50 to 100 picoseconds to amplify the pulses to 0.1 joules of peak intensity of $10^{14}$ W/cm$^2$, one picosecond. These parameters, 1064nm, 100 picoseconds, will be referred to several times in the text.

In attempting to operate in the near ultraviolet (see table 1), we may deliver a few mJ of energy, but that is likely to concentrate on increasing the energy, rather than increasing the energy.

The energy would be delivered from pulsed radar, called chirped pulses. In this case, the reader may turn to recent review articles in [3].

The goal of quantum mechanics is to consider the photon in a quantum mechanical context. A common description in terms of photons is inadequate. For example, see J.J. Sakurai, 1967, and references therein. The example above corresponds to a laser beam of the form

$$\mathbf{B}(x)=\mathbf{B}_0 \cos \omega t \hat{\mathbf{z}}$$

threshold for high intensity behavior. A laser beam will behave more like a classical electromagnetic field, and we must consider classical properties such as the magnetic field strength to be related to the laser intensity in the form

$$B_0 = \frac{\rho I_0}{2\pi c}$$

(2)

The peak magnetic field is approximated by $B_0$. This field imparts a wiggling motion to any charge moving along it. This wiggling is known as the "ponderomotive force".

$$\mathbf{F} = \mathbf{E} \times \mathbf{B}$$

(3)

In the classical limit, the ponderomotive energy is approximately proportional to the square of the frequency. A major difference is that the energy of a free electron is 10 eV. In contrast, the ponderomotive force is only 10 eV. This is the field, that leads to new phenomena in laser-atom interactions; but before we discuss this, we need to introduce an intense laser field.

The ponderomotive force is defined as the force on a charged particle in an electromagnetic field. It is given by

$$F = q \mathbf{E} + q \mathbf{v} \times \mathbf{B}$$

where $q$ is the charge of the particle, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. In the case of an intense laser field, the ponderomotive energy can become comparable to the kinetic energy of the electron.

With oscillating fields can be found in textbooks on quantum mechanics. One of the earliest papers, published when Thomson was a student, was by Ritz and Max Planck in 1900. However, the interaction of electrons with light is still a topic of active research, and the Klein-Nishina formula (1929; or see the references, such as Bjorken and Drell, 1965). The Klein-Nishina formula is easily calculated using covariant perturbation theory, where it can be described as the absorption of a photon from the laser field, followed (or preceded) by emission of a different photon into a formerly unoccupied mode.

At optical frequencies the energy of a photon is negligible compared to the rest energy of the electron, so the Klein-Nishina formula simplifies to Thomson scattering, first described by J.J. Thomson in 1898. Integrating over angles, one obtains the total Thomson cross section

$$\sigma_{\text{Thomson}} = \frac{8\pi r_0^2}{3} = 0.67 \times 10^{-29} \text{ cm}^2.$$  

(4)

Here $r_0$ is the "classical electron radius" $e^2/mc^2$, $\omega$ is the angular frequency of the scattered radiation, and $\omega'$ is the angular frequency of the scattered photon, as required by the simultaneous conservation of energy and momentum. The Klein-Nishina formula is easily calculated using covariant perturbation theory, where it can be described as the absorption of a photon from the laser field, followed (or preceded) by emission of a different photon into a formerly unoccupied mode.

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(4)

Now, it is a curious fact that for our sample experiment described above, the effects of Thomson scattering are utterly negligible. The photon fluence $\rho$ is approximately $6 \times 10^{22}$ photons/cm$^2$, so that the number of scatterings suffered by an electron in the middle of the laser focus is $\rho \sigma_{\text{Thomson}} = 0.04$. Even if a scattering event occurs, the momentum imparted to the electron, about 1 eV/c, is only enough to change its energy by 0.1% for a 1 eV electron.

Thomson scattering is overwhelmed at high intensities by stimulated scattering. This is similar to ordinary Thomson scattering, but the scattered photon enters an occupied laser mode, which increases the scattering probability by $n$, the number of photons in the mode. In the classical limit, this stimulated scattering effect is the wiggling motion of the electron in the oscillating classical electromagnetic field. Many high intensity phenomena are most easily understood in terms of the classical equations of motion, and so we now introduce these, returning from time to time to the quantum picture as needed.

2.1 The Time-Averaged Lorentz Force

In a laser focus, the wiggling motion of a free electron due to the oscillating fields is complicated by two effects. First, since the laser beam contains magnetic as well as electric fields, the electron trajectory is not simply one-dimensional harmonic motion in response to the electric driving force. The electron actually describes a "figure-8" trajectory in the $\hat{\mathbf{e}} \cdot \hat{\mathbf{k}}$ plane. Complicating this still further are effects due to the spatial and temporal inhomogeneities inherent in a pulsed laser focus. These effects taken together, cause the electron to drift and accelerate. If we are ever to understand the results of real experiments, we must be able to analyze this complicated motion.

The problem can be simplified considerably through the "adiabatic" approximation, in which we assume that the spatial and temporal inhomogeneities of the focus are small over the course of a single wiggle of the electron. In the standard example, the wiggle amplitude is only 1.4 nm, so the adiabatic approximation seems justified.

We now compute the time-averaged Lorentz force in our standard example. This is a messy business in general, but we will make liberal use of the adiabatic approximation. The 30 nm laser focus is depicted schematically in figure 1. The laser is assumed polarized along $\hat{x}$, and two initial positions for the electron will be considered, labeled A and B in the figure. The electric and magnetic fields at point A are taken to be

$$E_{A} = A \hat{\mathbf{x}} \cos \omega t$$

$$B_{A} = A \hat{\mathbf{y}} \cos \omega t$$

(5)

(Note that we employ Gaussian units.) Neglecting for the moment all spatial gradients in the intensity, the electron describes a figure-8 motion, with a displacement

$$\delta x = -\frac{eE_{A}}{m\omega^2} \cos \omega t$$

(6a)

along $\hat{x}$ (amplitude 1.4 nm) and...
\[ \delta z = \frac{c}{\omega} \frac{e^2 E_A^2}{8m^2 \omega^2 c^3} \sin 2\omega t \]  

(6b)

along \( \hat{z} \) (amplitude 15 fm). Obviously, the motion along \( \hat{z} \) is far less important than motion along \( \hat{x} \). The orbital motion of the electron is not periodic, since the field intensity varies over the wiggle amplitude. Since the electromagnetic fields at A are mostly varying in the \( \hat{x} \) direction, the Lorentz force may be written

\[ \dot{\mathbf{x}} = \dot{\mathbf{x}} \left[ \frac{eE_A}{m} \cos \omega t + \frac{\partial}{\partial x} \frac{eE_A}{m} \delta \cos \omega t \right] \]

\[ + \frac{\dot{\mathbf{x}}}{2} \frac{e^2 E_A^2}{4m^2 \omega^2 c^3} \cos 2\omega t \cos \omega t \]

(7)

\[ \frac{\dot{\mathbf{x}}}{2} \frac{e^2 E_A^2}{4m^2 \omega^2 c^3} \cos 2\omega t \cos \omega t \]

Figure 1. Cross section of a laser focus. Lines are contours of constant intensity.

In this expression, the first two terms describe the anharmonic force due to the electric field gradient, the third term is the induced electric field due to the curl in the inhomogeneous magnetic field, and the last terms are the velocity-dependent magnetic forces. The time-averaged Lorentz force is considerably simpler, since all terms above average to zero over one cycle of the field, except for the second term:

\[ <\dot{\mathbf{x}}> = -\frac{\dot{\mathbf{x}}}{2} \frac{e^2 E_A^2}{4m^2 \omega^2} \cdot \frac{\omega}{\omega} \]

(8)

Thus there is a time-averaged force proportional to the intensity of the laser focus (quadratic in \( E \)) and directed out of the focus.

The same procedure may be applied to an electron at position B, where the field gradients are along \( \hat{y} \), perpendicular to \( \hat{z} \). In that case, the Lorentz force may be written

\[ \dot{\mathbf{x}} = \dot{\mathbf{x}} E_B \cos \omega t - \frac{e^2 E_A}{m\omega c} \sin \omega t (\hat{x} \times B) \]

(9)

We have neglected the motion along \( \hat{z} \), which is an unnecessary complication. The magnetic field at B is

\[ B_B = \hat{y} \]

so the time-averaged force is

\[ <\dot{\mathbf{x}}> = \dot{\mathbf{x}} \frac{e^2 E_A^2}{4m^2 \omega^2 c^3} \]

again, out of the focus, and with the same period. Since the envelope function for the field is

2.2 The Ponderomotive Potential

The general case is most easily understood for an electromagnetic field polarized in the \( \hat{x} \) direction.

\[ \mathbf{A} \]

(More precisely, in a laser focus the field can be considered to fairly loosely focus in space in time. This is the "easiness" of the problem.)

The classical Hamiltonian of an electron in a strong, time, and space-dependent electric field is

\[ H = \frac{\mathbf{p}^2}{2m} - \mathbf{E} \cdot \mathbf{A} \]

Taking advantage of the adiabaticity of the laser pulse period. Since the envelope function for the electric field is constant, the time-averaged, or drift portion, of the electric field potential is

\[ <H> = \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{E} \cdot \mathbf{A}}{2} \]

The first term is the energy of the electron in the electric field, the second term is the energy of the electron in the magnetic field, and hence acts as an effective potential in the steady-state case, i.e. when the laser is on.

An electron in an inhomogeneous laser field is affected by a force equal to \(-\nabla U(\mathbf{r})\). The expressions above are sufficient for the discussion of the general topic. In our standard example, the ponderomotive potential is not constant in \( \mathbf{r} \) so that an electron with kinetic energy \( E \) is scattered from one focus to another during a time interval \( \delta t \) is scattered by an amount

\[ \delta \mathbf{r} = \mathbf{A} \frac{\mathbf{p}}{m} \frac{\delta t}{\omega} \]

An electron overtaken by a light pulse is scattered from one focal point to another due to kinetic energy by accelerating in the electric field. The energy acquired by the electron does work on the electron just as the work done by a vector field is non-conservative. The scattering is known as "surfing".

2.3 Elastic and inelastic scattering

An experiment demonstrating "surfing" has been performed by employing a pulsed beam of electrons circularly polarized 100 picoseconds long.
less important than motion along $\hat{z}$. If field intensity varies over the wiggle lying in the $\hat{x}$ direction, the Lorentz force due to the electric field gradient, inhomogeneous magnetic field, and time-averaged Lorentz force is constant cycle of the field, except for the

\[ \mathbf{B}_R \approx \frac{\gamma}{c} \mathbf{E}_R \cos \theta - \frac{\gamma}{c} \frac{\partial}{\partial y} \mathbf{E}_R \sin \theta, \]

so the time-averaged force is

\[ \langle \mathbf{m} \mathbf{\dot{x}} \rangle = -\frac{\gamma}{c} \frac{\partial}{\partial y} \frac{e^2 \mathbf{E}_R^2}{4mc^2}. \]

again, out of the focus, and with the same dependence on the intensity gradient.

2.2 The Ponderomotive Potential

The general case is most easily handled using Hamiltonian mechanics. The vector potential for an electromagnetic field polarized along $\hat{e}$ and propagating along $\hat{z}$ may be written

\[ \mathbf{A} = A_0(x,t) \hat{e} \sin(\kappa z - \omega t) \]

(More precisely, in a laser focus $\hat{e}$ is also a function of position; we neglect this, restricting consideration to fairly loose foci, f/10 or greater.) The function $A_0(x,t)$ is assumed to be slowly varying in space in time. This is the "envelope function" that describes the laser pulse in the intense field. The classical Hamiltonian of an electron in the electromagnetic field may be derived from the field free Hamiltonian using the "minimal coupling" prescription $p \rightarrow p - (e/c)A$:

\[ H = \frac{\left[ p - \frac{e}{c} A(x,t) \right]^2}{2m} + \frac{e^2 A_0^2(x,t)}{4mc^2} = T + U_p(x,t) \]

Taking advantage of the adiabatic approximation, we now average the Hamiltonian over one laser period. Since the envelope function $A_0(x,t)$ is essentially fixed over this interval, the time-averaged Hamiltonian is

\[ \langle H \rangle = \frac{p^2}{2m} + \frac{e^2 A_0^2(x,t)}{4mc^2} = T + U_p(x,t) \]

The first term on the right hand side is the "drift" kinetic energy. Note that the canonical momentum $p$ is not equal to the actual mechanical momentum, which is oscillating with the field, but is only the time-averaged, or drift portion. The second term on the right hand side is the average energy due to wiggling. It appears in the Hamiltonian as a positive-definite scalar function of position and time, and hence acts as an effective potential energy, known as the ponderomotive potential. In the steady-state case, i.e. when the laser intensity is approximately constant, the potential is conservative. An electron in an inhomogeneous region of the beam therefore experiences a time-averaged force equal to $-\nabla U(t)$. The expressions given above for the time-averaged Lorentz force are equal to this.

In our standard example, the ponderomotive potential exceeds 10 eV in the center of the laser focus, so that an electron with kinetic energy of a few eV will actually scatter away from the laser focus.

In a real experiment, the scattering of the electron by the light is not perfectly elastic, because the laser intensity, and thus $U_p$, is not constant in time. The incremental energy change of the electron during a time interval $\delta t$ is

\[ \delta E = \frac{\partial U_p}{\partial t} \delta t \]

An electron overtaken by a light pulse gains ponderomotive potential energy, which can be converted to kinetic energy by accelerating in the ponderomotive potential gradients. In this way, the light does work on the electron just as a water wave can do work on a surfer, and this form of inelastic scattering is known as "surfing".

2.3 Elastic and inelastic scattering of electrons by the Ponderomotive potential

An experiment demonstrating the ideas in the preceding section was recently performed. It employed a pulsed beam of electrons and an intense focused Nd:YAG laser (Bucksbaum, 1987) A circularly polarized 100 psec laser pulse with a peak ponderomotive potential of 8 eV, was focused
between the electron source and the detector, so that the ponderomotive potential could block the electron pulse. (Note that the ponderomotive potential does not depend on the direction of polarization; use of circularly polarized light had some technical advantages in the experiment.) A pulsed point source of monoenergetic electrons at several discrete energies below 5 eV was provided by photoionizing xenon using 532 nm 100 psec light pulses. The relative timing could be varied to show that the scattering was due to the presence of the light only. (See figure 2)

![Figure 2. Scattering of electrons by light. 0.54 eV electrons were directed through the focus of a 100 psec laser pulse with peak ponderomotive potential of 8 eV. Those arriving early or late were undeflected. Electrons passing through the leading and trailing edges were accelerated or decelerated, respectively, due to the phenomenon of "surfing", described in the text. Electrons encountering the peak of the pulse were scattered away from the detector. (From Buck sem, 1987).](image)

Electron "surfing", that is, inelastic scattering, occurred on the rising or falling temporal edge of the 100 picosecond laser pulse. Figure 2 shows how the .54 eV electron peak was shifted to higher energy when the electrons gained ponderomotive potential energy by passing through the rising leading edge of the laser pulse. Conversely, when the electrons arrived late in the pulse, they decelerated due to the spatial gradients, and then remained at lower energy as the laser pulse passed by.

2.4 Kapitza-Dirac Effect

We have constructed two mental pictures of electron scattering in a laser field. In the quantum picture, the electron scatters photons, primarily by stimulated Compton effect. In the classical picture, valid at high intensities, the electron quivers back and forth under the influence of a periodic electric driving force in the laser beam. The correspondence between these two ideas is most easy to see in a situation where there are only two modes present, so that stimulated scattering involves transfer of photons between one mode and the other. For example, in a standing wave, the k-vectors of the two modes are equal but opposite. Each photon exchanged between the two modes results in a momentum transfer of 2nk to the electron.

The exchange of photons between the two laser modes in a standing wave is easy to calculate in second order perturbation theory. The steady state rate $\Gamma$ is given by

$$\Gamma = \frac{\Delta \sigma}{\Delta \nu}$$

$$U_p = \frac{\Gamma}{\hbar^2}$$

where $n$ is the number of photons in the laser beam, and $\Delta \sigma/\Delta \nu$ is the detection channel. This has a simple interpretation: $\Delta \nu$ is the laser bandwidth. This means that the scattering of particles from a standing wave is indicated. Actually, our rate is larger than the one assumed that the light in the standing wave.

The K-D effect has also been observed for the standing wave. If a standing wave scatters from a periodic lattice, the laser intensity and momentum leads to the standing wave. The wavelength $\lambda$, the optical wavelength.

In order to investigate this process, we need to know the laser intensity. The standing wave effect, and the power and intensity of the standing wave, were studied in (1968), (1971), (1973), (1976).

Equation (16) encounters some important issues. It is valid so long as $\Gamma$ does not exceed a critical value $(\approx 2.8 \times 10^{14} \text{Hz})$ of 100 psec pulses.

$$\Gamma = 5 \times 10^{14}$$

This imposes an upper limit of about five times the experiment.

At higher intensities, the relevant effect is inelastic scattering. This suggests the substitution

$$\Gamma = \frac{U_p}{\hbar} = 160$$

Thus we arrive at an important difference in the K-D effect; for high enough intensities, the standing wave; for higher intensities, the standing wave; for higher intensities, the standing wave; for higher intensities, the standing wave.

For still higher intensities, the standing wave becomes the dominant mode. The standing wave is observed, and as we shall see, the standing wave scattering at this intensity.

Our high intensity standing wave experiment.

The pulses were 100 psec long, and the frequency standing wave during the interaction. The scattered electrons came from ATI in...
electron momentum could block the (depending on the direction of polarizations in the experiment.) A pulsed energy below 5 eV was provided by the relative timing could be varied to (see figure 2)

![Focus](image)

![Det](image)

electrons were directed by the ponderomotive potential in the detector. Electrons passing through or accelerated, respectively, in the text. Electrons away from the detector.

The rising or falling temporal edge of the electron peak was shifted to higher energy by passing through the linear lead- edge late in the pulse, they decelerated by the laser pulse passed by.

![Laser Field](image)

In the quantum electrodynamic effect. In the classical picture, the electron under the influence of a periodic field, these two ideas is most easy to see that stimulated scattering involves in a standing wave, the k-vectors between the two modes results in no standing wave is easy to calculate in these.

\[
\Gamma = n \sigma \frac{\partial \sigma_{\text{gap}}}{\partial \Omega} = \left( \frac{e^2}{\hbar \nu^2} \frac{1}{\Delta \nu} \right) \left( \frac{1}{\hbar \nu} \right) \left( \frac{e^4}{m_0^2 c^4} \right) = \frac{U_p^2}{\hbar^2 \Delta \nu},
\]

(16)

where \( n \) is the number of photons per mode in the standing wave, \( \rho \) is the photon flux from either laser beam, and \( \partial \sigma_{\text{gap}}/\partial \Omega \) is the differential Thomson cross section, evaluated in the backscattering direction. This has a simple interpretation in terms of the ponderomotive potential, as shown, where \( \Delta \nu \) is the laser bandwidth. This rate was first calculated by Kapitza and Dirac (1933), and the scattering of particles from a standing optical wave is generally called the Kapitza-Dirac effect. Actually, our rate is larger than the original Kapitza-Dirac formula by a factor of two, because they assumed that the light in the standing wave was unpolarized.

The K-D effect has also been called the wave-particle dual of Bragg scattering, since a particle beam scatters from a periodic lattice of light, exchanging integer multiples of momentum \( nG = 2\pi k \), where \( G \) is the reciprocal lattice vector of an optical standing wave. Simultaneous conservation of energy and momentum leads to a Bragg condition \( nG = 2\pi k \). Equation (16) encounters some serious problems at this intensity. First of all, it should only be valid so long as \( \Gamma \) does not exceed the bandwidth of the light. For counterpropagating Nd:YAG laser pulses (\( \nu = 2.8 \times 10^{14} \text{Hz} \)) of 100 psec duration (\( \Delta \nu = 5 \text{GHz} \)), equation (16) yields

\[
\Gamma = 5.2 \left( \frac{1}{1 \text{ MW/cm}^2} \right)^2 \text{ MHz.}
\]

(17)

This imposes an upper limit of \( I < 30 \text{ MW/cm}^2 \) in our example, far below any intensity used in our experiment.

At higher intensities, the relevant coherence time is no longer \( 1/\Delta \nu \), but the time since the last scattering. This suggests the substitution \( \Delta \nu \rightarrow \Gamma \), leading to

\[
\Gamma = \frac{U_p}{\hbar} = 160 \left( \frac{1}{1 \text{ GW/cm}^2} \right) \text{ GHz (I>30MW/cm}^2).\)

(18)

Thus we arrive at an important difference between the high and low intensity regimes: for low intensities, the stimulated scattering rate depends on the product of the two laser intensities in the standing wave; for higher intensities, the rate is linear in intensity, and therefore linear in the ponderomotive potential.

For still higher intensities, the K-D effect becomes more complicated. The electron mean free path approaches a single wavelength for \( I = 10 \text{ GW/cm}^2 \), so that the Bragg condition no longer holds; at \( 10^{13} \text{ W/cm}^2 \), \( \Gamma \) exceeds the laser frequency! This is the relevant regime for electron scattering following ATI, and as we shall now show, even equation (18) does not predict the main features of the scattering at this intensity.

Our high intensity standing waves were made by colliding two focused 1064 nm laser pulses. The pulses were 100 psec long, and approximately fourier-transform limited, to insure a uniform stationary standing wave during the \( \approx 10-20 \) psec transit of the electrons through the focus. The scattered electrons came from ATI in xenon or krypton in the standing wave.
The detector used to observe electrons under these conditions is shown schematically in figure 3a.

Figure 3 (a). Experimental geometry. Photoelectrons produced in a standing wave scatter out of the focus, and are detected on a screen. (b)-(f): Electron angular distributions, for electrons that reach the detector with 9 to 12 eV of energy. (b). Typical distribution from xenon AT1 in a single 1064 nm laser beam. The peak laser intensity is approximately \(8 \times 10^{13} \text{W/cm}^2\) (c). K-D effect in a linearly polarized standing wave. (d). K-D effect in a circularly polarized standing wave, with opposite helicities in the two beams. (e). K-D effect is absent in a circularly polarized standing wave with equal helicities in the beams. (f). reduced contrast standing wave, made from beams of unequal intensities. (From reference 15.)

It was an image-intensified detection screen subtending 0.08 sr (66° opening angle), 5 cm from the focus. Retarding grids at the screen could select electrons at specified energies.

Figure 3b is a histogram of the photoelectron angular distribution data from a single focused laser beam (no standing wave). Most of the photoelectrons are emitted along the polarization direction, as usual for linear polarization. The elongation in azimuthal (θ) angles is due to ponderomotive scattering from intensity gradients as the electrons leave the focus (Freeman, 1986).

When a second beam of equal intensity is introduced, it also splits to form two peaks symmetrically about the K-D effect at high intensity. The beams have a separation of order of 1000 fL of momentum.

Circularly polarized standing wave beams show a single peak, but with opposite helicities (hence the photocurrents are summed), whereas unpolarized laser beams show no helicity (figure 3d); however, if the laser beam is blocked, the helicity is not observed (figure 3e). Finally, if the laser beam is blocked, we observe only one peak, which is the standing wave.

There have been several attempts to explain these phenomena (Rytov, 1985; Gush, 1971). Since the predictions of the classical polarization theory are reliable, they predict that the above equations hold for any circularly polarized laser beam:

\[
P_\epsilon = \frac{\mathcal{E}_\perp}{\cos \theta}
\]

where \(A(x,t)\) is the time-dependent amplitude of the propagating plane waves along \(\hat{z}\):

\[
A(x,t) = A(t) \cos \theta
\]

For a single traveling plane wave (as discussed in the text below); but for a standing wave, we have

\[
A(x,t) = A(t) \cos \theta_x \cos \theta_y
\]

However, it is convenient to make a few changes to equation (13) in place of equation (14) of the initial solution (Chan, 1971). This takes the form of

\[
\frac{\partial P_\epsilon}{\partial \zeta} = \frac{\mathcal{E}_\perp}{\cos \theta} \cos \theta_x \cos \theta_y
\]

Figure 4. The calculated trajectory of an AT1 electron drifting out of the standing wave, showing one cycle of bounded motion in a potential trough, followed by a transition from bound to unbound motion along \(\hat{z}\). (From reference 15.)
When a second beam of equal intensity collides with the first in the focus, the angular distribution splits to form two peaks symmetric with respect to the polarization plane (figure 3c). This is the K-D effect at high intensity. The electrons at the peaks of the distribution have absorbed on the order of 1000 keV of momentum.

Figure 5. Polar angle distributions for electrons of different energies in the same standing wave. All distributions are plotted vs. polar angle (top scale). Bottom scale shows the momenta transfer for each energy. Solid lines are data, collected in a 2 keV window bracketing the energy shown. Dashed lines are results of Monte Carlo electron trajectories. (From reference 15.)

Circularly polarized standing waves are shown in figures 3d and 3e. When the laser beams have opposite helicity (hence the photons have equal angular momentum), the K-D scattering is quite pronounced (figure 3d); however, for equal helicity, hence the opposite angular momentum, all stimulated backscattering is forbidden by angular momentum conservation, and no K-D scattering is observed (figure 3e). Finally, figure 3f shows how the momentum transfer is reduced for a lower contrast standing wave.

There have been several attempts to deal with the theory for high intensities (Bartell, 1967; Coutsiakas, 1985; Gush, 1971). Since perturbation theory is unwieldy in this regime nonperturbative methods that treat the light as a time-varying potential are more attractive (Chan, 1984). In the nonrelativistic theory, Schroedinger's equation is given by

$$\frac{\{p - \frac{e}{c} A(x,t)\}^2}{2m_e} \psi(x,t) = \frac{i}{2} \frac{\partial \psi(x,t)}{\partial t}$$  \hspace{1cm} (18A)

where \( A(x,t) \) is the time-dependent vector potential for a standing wave produced by two counterpropagating plane waves along \( \hat{z} \):

$$A(x,t) = A_0(\hat{e}_1 \cos(kz+\omega t) + \hat{e}_2 \cos(kz-\omega t))$$  \hspace{1cm} (19)

For a single traveling plane wave, the solutions to this equation are known (see Volkov states, below); but for a standing wave, the solutions are much more complicated.

It is convenient to make a further, classical approximation: we employ the classical Hamiltonian in equation (13) in place of equation (18A), and reintroduce the time-averaged ponderomotive potential (Chan, 1971). This takes the form \( U_p(z) = U_1 + U_2 + 2U_1U_2 \hat{\ell}_1 \hat{\ell}_2 \cos 2kz \). Here \((U_1, U_2)\)
is the ponderomotive potential for the traveling wave in the (+z, -z) direction. Equation (18) must be replaced by a *position-dependent* scattering rate

$$\Gamma_{1 \rightarrow 2}(z) = \frac{1}{2nk} \frac{\partial U_p}{\partial z} = \frac{2}{\hbar} \frac{1}{2} \frac{1}{\hbar} e_1 e_2 | \sin 2kz |,$$

where 1 → 2 means that this is the *net* scattering rate of photons from beam 1 to beam 2. This rate can be positive or negative.

It so happens that this is also the Hamiltonian for a simple pendulum of mass m and length 1/(2k) under a uniform force 4kU1U2. (See any standard mechanics textbook, for example, Slongerfeld, 1952.) Low energy electrons are localized in a sinusoidal potential well centered on an electric field node of the standing wave, where they oscillate. For example, the oscillation period for a 1 eV electron in a peak potential of 10 eV (corresponding to \( \approx 10^{14} \) W/cm^2 for Nd:YAG radiation) is 0.3 psec.

The electron distributions in this experiment were compared to calculated electron trajectories employing this classical approximation, with a time-varying spatially periodic ponderomotive potential appropriate for a standing wave formed by two perfectly overlapped 100 psec pulses meeting in a TEM00 gaussian focus. Electrons were launched in the simulation according to a very simple model of ATI, in which the highly nonlinear intensity dependence is approximated by a single threshold intensity of approximately \( 3 \times 10^{13} \) W/cm^2 (Freeman, 1986). Electrons were initially directed along the laser polarization, in accordance with figure 3b; but they immediately began to oscillate in the k direction under the influence of the ponderomotive spatial grating. This is shown for a single trajectory in figure 4. Histograms for Monte Carlo generated electron distributions at several energies are shown in figure 5. Results are quite consistent with the data in figure 5, and lend strong support to the ponderomotive hypothesis.

2.5 Do electrons really wiggle?

Given the great success of classical models in describing the Kapitza-Dirac effect and ponderomotive scattering of free electrons, there is a tendency to overstate the case for wiggling electrons. As we learned in the Kapitza-Dirac experiment, the stimulated scattering rate at very high intensities is approximately \( U_p/\hbar \). Even when this equals the laser frequency, as, for example \( 10^{13} \) W/cm^2 in Nd:YAG, the scattering rate is still not great enough to think of the electron as smoothly oscillating under the influence of the classical driving force; it only exchanges momentum with the field about once per optical cycle. We observe that on time average, the electron motion is the same as that of an oscillating charge with mass \( m_e \); but one should keep in mind that this is only a model.

2.6 Volkov States: the Quantum Mechanics of Electrons in Intense Light

A more appropriate description of electrons in intense fields must include quantum mechanics. We would like, however, to avoid perturbation theory. Perhaps the simplest way to do this is to treat the laser field as a time-varying potential, and write Schroedinger’s equation for a free spinless electron in the presence of this potential. In the \( \mathbf{A} \cdot \mathbf{p} \) gauge, we then have

$$\left[ \frac{m_e}{2} \partial^2 }{\partial t^2} - \frac{e}{\hbar} A(x, t) \right] \psi(x, t) = \frac{i}{\hbar} \frac{\partial \psi(x, t)}{\partial t}$$

where \( A(x, t) \) is the classical (non-quantized) vector potential. In the electric dipole approximation,

$$A(x, t) = A_0 \sin \omega t$$

The solutions, called Volkov states, are the quantum-mechanical analogs of free wiggling electrons (Volkov, 1935). They are plane waves, plane waves, with an oscillating phase, modulated by the electromagnetic vector potential:

$$\psi_{\text{Volkov}}(x, t) = \exp \left[ \frac{i}{\hbar} \mathbf{p} \cdot \mathbf{x} - \frac{i}{\hbar} \frac{\mathbf{p}^2}{2m_e} t \right]$$

The solutions are simultaneous eigenstates of the mechanical variables, and are periodic energy fluctuations caused by the ponderomotive potential of a classical particle in the sum. The change due to the ponderomotive potential is \( \approx 2\omega \); and the last is the restoring force in the direction of the driving field.

A classical wiggling electron exciton state, however, is a superposition of the states

$$\psi_{\text{Volkov}}(x, t) = e^{-i \omega n t} \psi_n(x)$$

Here \( \psi_n \)'s are cylindrical Bessel functions in the Volkov phase.

Volkov states can also be calculated to contain a fully relativistic calculation (Berestetskii 1982, section 40).

3. Electron scattering in really intense light

The laser in our “standard experiment” is supersedible in the laboratory by a four-wave mixing technique, first developed for high intensities in the 1960s. The process occur when laser rods and other intensity \( 10^{19} \) W/cm^2. In chirped pulse amplification, short pulses are passing through a delay line, with a delay line and a short pulse is amplified to a very short pulse. For example, a 100 mJ, 100 fs, after recompression, the output pulse may exceed \( 10^{19} \) W/cm^2.

3.1 The Relativistic Ponderomotive Effect

What should we expect from this calculation by considering the free electron? If we are, however, we must use relativistic quantum mechanics, and a polarized light field whose electric field is

$$\mathbf{E} = \mathbf{E_0} \cos \omega t$$

An electron that is at rest on time average, this simple non-relativistic solution has
(20)

From beam 1 to beam 2. This rate

to calculated electron trajectories
sion, and the pendulum of mass \(m_0\) and length \(l\) pre-rectangular potential well centered on an
exposition, for example, Som-ter, 1992). For example, the oscillation period for
lent \(10^{-14}\) W/cm\(^2\) for Nd:YAG radiation)

Schrödinger equation. However, for wiggling elec-
tron with period of the ponderomotive potential; the next represents periodic fluctuations with this value
at frequency \(2\omega\) and the last is oscillation at \(\omega\) (the \(\tilde{c}\cdot p\) term), due to the component of the

First, the Kapitza-Dirac effect and ponderomotive the case for wiggling electrons. Scattering rate at very high intensities is
key, as, for example, \(10^{15}\) W/cm\(^2\) in electron as smoothly oscillating
field of the laser, the momentum with the field about electron motion is the same as that of growth.

3. Electron scattering in really intense light

The laser in our "standard example" was state-of-the-art a few years ago, but it will soon be superseded in the laboratory by a new class of lasers that employ "chirped pulse amplification." This technique, first developed for high intensity radar, overcomes serious materials damage problems that occur when laser rods and other optical materials are subjected to intensities above about
\(10^{15}\) W/cm\(^2\). In chirped pulse amplification, a very short laser pulse with low power is stretched by passing it through a positive dispersive medium. This might be a long optical fiber, or successive refraction from diffraction gratings. Stretching the pulse distributes the energy over a longer time, and lowers the intensity accordingly. Moreover, since the light is spectrally smeared or chirped, the short pulse can be reconstructed by transmission through a negatively dispersive medium. The stretched pulse is amplified to just below the damage limit in the amplifying medium, and then recompressed. For example, a 100 fs pulse laser pulse can be stretched to 100 ps, and amplified to
100 mJ. After recompression, the laser power can approach one terawatt, and focused intensities may exceed \(10^{19}\) W/cm\(^2\).

3.1 The Relativistic Ponderomotive Energy

What should we expect from laser intensities of this magnitude? We begin to answer this question by considering the free electron in a classical field, just as for the lower intensity case. Now, however, we must use relativistic mechanics. The simplest case by far is a free electron in a circularly polarized light field whose electric polarization vector varies as

\[
E = \frac{E_0}{\sqrt{2}}(\hat{x}\cos \omega t + \hat{y}\sin \omega t).
\]  

An electron that is at rest on time average, executes circular motion in response to this field. The simple non-relativistic solution has speed
\[ 1v1 = \frac{1}{\sqrt{2}} \frac{eE_0}{m0} \]  

about a circle with radius 
\[ r = \frac{1}{\sqrt{2}} \frac{eE_0}{m0c^2}. \]  

The kinetic energy is therefore 
\[ T = \frac{1}{2} mv^2 = \frac{e^2 E_0^2}{4mc^2} = U_{P,N.R.}, \]  

which we recognize immediately as the ponderomotive potential, labeled “N.R.” to show that this is the non-relativistic prediction. As the intensity increases, however, this simple formula breaks down, since the velocity cannot exceed c, nor the radius exceed \( \lambda/2\pi \). Instead, we have 
\[ eE = \frac{dp}{dt} = \beta \gamma mc \]  

where the usual definitions hold 
\[ \gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = v/c. \]  

The relativistic expression for the speed of the particle in its circular orbit is 
\[ \frac{v}{c} = \left[ 1 + \frac{mc^2}{2U_{P,N.R.}} \right]^{-1/2} < 1, \]  

and its kinetic energy, which we call the relativistic ponderomotive energy, is 
\[ U_{P,Rel} = mc^2 \left[ \sqrt{1 + \frac{2U_{P,N.R.}}{mc^2}} - 1 \right]. \]  

This reduces to \( U_{P,N.R.} \) in the non-relativistic limit. At ultra-relativistic intensities, this simplifies to 
\[ U_{P,Rel} \rightarrow \sqrt{2U_{P,N.R.}mc^2} = \frac{E_0 \lambda}{\sqrt{2} \pi}. \]  

which is equal to the amount of work done by the optical field on an ultra-relativistic particle traveling in the direction of the field for time \( 1/\omega_0 \). Note that in the relativistic limit, the ponderomotive potential is linear (rather than quadratic) in the field strength, linear (not quadratic) in the charge, and independent of the particle mass! Some of these features are shown in figure 6.

### 3.2 Relativistic Mass Increase

The relativistic ponderomotive potential can be interpreted as a relativistic mass increase in the electron. The mass in the laser beam is increased by 
\[ m \rightarrow m' = m + U_{P,Rel}/c^2 = m \sqrt{1 + 2U_{P,N.R.}/mc^2} \]  

### 3.3 Nonlinear Compton Scattering

As figure 6 shows, deviations from the non-relativistic ponderomotive potential are not very pronounced for laser intensities below \( 10^{29} \text{W/cm}^2 \) (for \( \lambda = 1.06 \mu\text{m} \)). At lower intensities, however, some relativistic effects are evident. An example is nonlinear Compton scattering. In terms of classical electromagnetism, this is higher multipole radiation due to deviations from simple harmonic motion of a charged particle in a laser field. The deviations are caused by relativistic or magnetic effects. In the quantum picture, the nonlinear Compton effect is a higher order scattering process involving the absorption of two or more photons, followed by emission of a single photon at the sum frequency. However, as we have already seen, stimulated scattering processes are so dominant at high intensities that they must be taken into account. The simple Feynman Diagram rules suggested by this particle scattering description are not expected to work very well.
(26)

(27)

(28)

(29)

(30)

(31)

(32)

(33)

(34)

Figure 6. The ponderomotive potential as a function of laser intensity, for two different lasers: Nd:YAG (1.06 μm), and CO₂ (10.6 μm). The solid line is for electrons, and the dashed line is for protons. At high intensities, the potential becomes independent of mass.

Light generation at the second harmonic is forbidden in the electric dipole approximation if the scattering medium possesses inversion symmetry, as an unpolarized electron certainly does (Shen, 1984; Yariv, 1975). However, the magnetic fields of the laser break the inversion symmetry of the system. The electrons execute a figure-8 motion (see section 2.1) which causes them to radiate at the second harmonic. The classical calculation has been done by Vachaspati (1962).

Quantum calculations for second and higher harmonics were an active area of research over twenty years ago, and a review by Eberly on this subject in 1969 is still a good summary of the theory (Eberly, 1969). One approach, used by Nikishov and Ritus (1964) among others, was to consider light scattering from a relativistic Volkov state. They showed that the probability for an s-photon process goes as \( \delta^{-1} \), where \( \delta \) is a dimensionless ratio

\[
\delta = \left( \frac{eA}{mc} \right)^2 = \frac{2U_{P,N.R.}}{mc^2} \tag{35}
\]

At quite low intensities, where the perturbative result for ordinary spontaneous Compton scattering is valid, this scaling law provides an easy way to estimate the harmonic scattering cross-section \( \sigma_s \):

\[
\sigma_s = \sigma_{Compton} \delta^{-1}. \tag{36}
\]

Deviations from this simple scaling law occur as \( \delta \) approaches 1. The calculation becomes quite complicated in this range. For the ultra-high intensity case of \( \delta \approx 1 \) the problem simplifies once again, becoming similar to the problem of scattering in crossed static electric and magnetic fields. A good discussion of this appears in Berestetskii (1982, section 101).

The Compton shift, that is, the photon energy shift due to the recoil of the electron, is quite small at optical frequencies. However, the shift is not quite the same for nonlinear Compton scattering, due to the additional momentum of the extra photons, and also due to the "relativistic" intensity-dependent increase in the electron's effective mass. The modified formula for the shift is (Brown 1965, Nikishov and Ritus 1964, and Berestetskii 1982)

\[
\omega' = \frac{\omega}{1 + (\pi \alpha / m' c^2) (1 - \cos \theta)}, \tag{37}
\]

where \( \theta \) is the scattering angle, and \( m' \) is the renormalized mass defined above. An additional doppler shift contribution will be discussed below.
3.4 Observation of Nonlinear Compton Scattering

In the first section of these lectures we pointed out that the amount of spontaneous Compton or Thomson scattering in these laser experiments is totally negligible. Nonlinear scattering is orders of magnitude smaller than this. It therefore should be considered an experimental tour de force that this effect has been observed in the laboratory, in an experiment carried out at \( \delta = 8 \times 10^{-5} \).

The experiment was performed at the University of Wyoming by T.J. Englert and E.A. Rinehart (1983), using a Nd:Glass laser. The laser beam was focused into a vacuum chamber, where it intersected an electron beam. Scattered photons were optically filtered to reject background light. The major problem in an experiment of this type is that harmonic photons are generated everywhere, not just by the free electrons. The infrared laser power is well over 10 GW, and scattered light from the windows and walls of the vacuum chamber generally produces so much second harmonic, that a green glow is sometimes visible to the unaided eye. To reject this, the experimenters used a fast electron beam, so that the nonlinear Compton scattered photons would be doppler shifted. They were thus able to reduce background light to less than one photon in 600 laser shots. There signal was only about one photon every 30 laser shots, but was clearly visible in the data.

3.5 Relativistic Doppler Shifts

Englert and Rinehart used a fast external electron beam to doppler shift the scattered radiation. But at higher intensities, doppler shifts are actually unavoidable, no matter what the energy of the electron. That is because when a laser pulse overtakes an electron, it imparts some forward momentum. Classically, this is an ExB drift. A simple way to calculate the drift velocity is to think of the electron as "dressed" by photons in the laser beam. The time-averaged number of photons the electron absorbs is \( U_{P,NR}/\hbar \), and the electron must take on the energy and momenta of those photons. If the electron is at rest before the beam overtakes it, its total energy in the beam is \( mc^2 + U_{P,NR} \), while its total momentum is \( U_{P,NR}/c \). In other words, using standard 4-vector notation, the electron 4-momentum \( p_\mu \) is \((mc^2,0,0,0)\) out of the beam, and \( p_\mu = (mc^2+U_{P,NR},0,0,U_{P,NR}/c) \) in the beam. Therefore, its velocity is

\[
\nu_{\text{def}} = \frac{U_{P,NR}}{mc^2 + U_{P,NR}}.
\]

Note that the "rest mass" of the electron in the beam, i.e. its invariant 4-momentum, is

\[
(p_\mu p^\mu)^{1/2} = mc^2 \sqrt{1 + \frac{2U_{P,NR}}{mc^2}} = mc^2 + U_{P,Rel},
\]

just as in our previous calculation of the relativistic ponderomotive potential. In fact, that calculation was just a special case where the electron is at rest (on time average) in the beam. This calculation shows that for electrons at rest before the beam arrives, a substantial part of the total energy supplied to the electron by the laser field goes into drift rather than the ponderomotive potential.

3.6 Spin and intense light.

All our calculations have assumed that the electron has no spin. Indeed, spin-dependent effects are generally small, and none yet been observed in intense laser interactions with free electrons. However, the magnetic fields in these laser beams are quite enormous. In our standard example, the magnetic field is approximately one million gauss. We would like to estimate whether magnetic effects are significant in these experiments.

We start with the Bargmann-Michel-Telegdi, or BMT, equation for the precession of a classical spin \( s \) with magnetic moment \( \mu = s(\hbar/2mc) \) in slowly varying magnetic and electric fields (Bargmann 1959; A clear derivation may be found in Jackson 1975):

\[
\frac{ds}{dt} = \frac{c}{mc} \left[ \frac{s}{2} \left( \frac{g}{\gamma} + 1 \right) B - \left( \frac{g}{2} - 1 \right) \frac{\gamma}{\gamma^1} \left[ B \cdot \frac{\gamma}{\gamma^1} \right] \right]
\]

For the present, we will assume \( g = 1 \).

The first term is the precession of \( s \) in the motional magnetic field proportional "Thomas factor", a relativistic effect (Thompson, 1927).

We consider a circularly polarized light and magnetic fields given by

\[
E(z=0,0) = E_0
\]

\[
B(z=0,1) = B_0
\]

The motional term for these fields is

\[
\omega_{\text{motional}} = \frac{g}{2} \frac{E_0}{B_0}
\]

and the precession frequency is

\[
\omega_{\text{precession}} = \frac{g}{2} \frac{E_0}{B_0}
\]

This is a kind of spin-orbit interaction, like a circulating current on a circle, i.e. along \( \hat{z} \). The magnetic field is in the polarization plane. This precession approaches relativistic limits, however, with a significant amount (=1 radian) during the transit.

The first term of the non-relativistic limit predicts that the magnetic field of the laser. Since the laser is quite complicated, a great simple way to look at the magnetic field is to consider it as a magnetic resonance problem: when the magnetic field is constant, the effective magnetic field is constant.

In the frame rotating at \( \Omega \), all vectors are constant. The electron spin equation is

\[
\frac{ds}{dt} = \frac{c}{mc} \left( \frac{g}{2} - 1 \right) \frac{\gamma}{\gamma^1} \left[ B \cdot \frac{\gamma}{\gamma^1} \right]
\]

The precession frequency in this case is

\[
\omega_{\text{precession}} = \frac{g}{2} \frac{E_0}{B_0}
\]
The amount of spontaneous Compton or \textit{e}-\textit{g} scattering is orders of magnitude larger than the experimental \textit{tour de force} that this \textit{e}-\textit{g} scattering could be measured in the experiments led to rejection of background light. The photons are generated everywhere, not just in the focal point. The scattered light from the focal point is so much second harmonic light, that a Doppler shift of the scattered radiation is not expected to be visible in the data.

However, a Doppler shift of the scattered radiation, no matter what the energy of the beam, is expected to show a shift of some forward momenta. The drift velocity is to think of the net momentum of photons the energy and momenta of those photons. The energy in the beam is $mc^2 + U_{P,N.R.}$, and the standard 4-vector notation, the electron momentum is $\frac{eE_0}{c}$ in the beam.

\begin{equation}
\text{(38)}
\end{equation}

Doppler shift is given by

\begin{equation}
\text{(39)}
\end{equation}

the potential. In fact, that calculation (average) in the beam. This calculation is the antiall part of the total energy supplied by the ponderomotive potential.

In the beam, indeed, \textit{e}-\textit{g} dependent effects are observed. In our standard example, the electron like to estimate whether magnetic field influence the precession of a classical electron in a magnetic and electric fields (Bargmann, 1959).

\begin{equation}
Y_{\frac{1}{2}}(\vec{B} \times \vec{v})
\end{equation}

For the present, we will assume $g=2$ and take the non-relativistic limit (first order in $v/c=\beta$):

\begin{equation}
\frac{ds}{dt} = \frac{e}{mc} s_{\text{rot}} \left[ B \vec{v} \times \vec{E} \right] \tag{41}
\end{equation}

The first term is the precession of the spin in the magnetic field. The second term is the precession in the motional magnetic field produced by the electric field in the lab frame. The factor 1/2 is the famous "Thomas factor", a relativistic kinematic effect first observed in the spin-orbit interaction in atoms (Thomas, 1927).

We consider a circularly polarized plane wave laser beam traveling along $z$, with electric and magnetic fields given by

\begin{align}
E_z(\mathbf{r},t) &= \frac{1}{\sqrt{2}} E_0 [\hat{x} \cos \omega t + \hat{y} \sin \omega t] \\
B_z(\mathbf{r},t) &= \frac{1}{\sqrt{2}} E_0 [\hat{x} \sin \omega t + \hat{y} \cos \omega t] \tag{42}
\end{align}

The motional term for these fields is

\begin{equation}
\frac{eE_0}{mc^2} \frac{eE_0}{4mc} \tag{43}
\end{equation}

and the precession frequency is

\begin{equation}
\Omega_{\text{motional}} = \frac{U_{P,N.R.}}{mc^2} = \frac{3}{2} \tag{44}
\end{equation}

This is a kind of spin-orbit interaction. The electron driven around in a circle by the circularly polarized light, is like a circulating current. This produces a magnetic moment normal to the plane of the circle, i.e. along $z$. The magnetic moment interacts with the spin, producing a precession in the polarization plane. This precession is quite slow compared to the laser frequency, until the intensity approaches relativistic limits; however, in our standard example, the electron precesses a significant amount (=1 radian) during the laser pulse.

The first term of the non-relativistic BMT equation describes the precession of the spin due to the magnetic field of the laser. Since the field is rotating at the laser frequency, the solution appears quite complicated. A great simplification is possible, however, by borrowing a trick from nuclear magnetic resonance problems: we transform to a rotating coordinate system, where the magnetic field is constant. The effective magnetic field in the laboratory reference frame, including the spin-orbit contribution, is

\begin{align}
B_x &= -\frac{E_0}{\sqrt{2}} \sin \omega t \\
B_y &= \frac{E_0}{\sqrt{2}} \cos \omega t \tag{45} \\
B_z &= \frac{eE_0^2}{4mc}
\end{align}

In the frame rotating at $\Omega$, all vectors $\vec{v}$ take on a fictitious time dependence $\Omega \times \vec{v}$. For $\Omega = \omega$, the electron spin equation is

\begin{equation}
\frac{d\mathbf{s}_{\text{rot}}}{dt} = \frac{e}{mc} s_{\text{rot}} \times (B + \frac{mc}{e} \vec{a}) \tag{46}
\end{equation}

The precession frequency in this frame is
The conclusion is that in the lab frame, the magnetic precession term and the spin-orbit term in the BMT equation nearly cancel. Therefore, spin effects on free electrons in currently accessible laser fields are unimportant.

3.7 Pure QED Effects; The Ultimate Intensity

No introduction to this subject is complete without at least mentioning the effect of really intense light on the quantum vacuum. The quantum vacuum is a sea of electromagnetic field fluctuations, which are present even at zero temperature in vacuum. In this sea are virtual electron-positron pairs which can interact with light. (To be accurate, vacuum field fluctuations can exist even in a classical theory; however, the existence of virtual matter-antimatter pairs in the vacuum is a quantum phenomenon. See Boyer 1980.)

At low intensities the quantum vacuum is difficult to detect. It usually appears as a very slight energy correction (e.g. the Lamb shift). At high intensities, however, new phenomena should occur. One that might be observed using next-generation lasers, is the scattering of light by light. This is Compton scattering of laser photons from virtual $e^+e^-$ pairs. For example, if two powerful laser pulses with the same frequency $\omega$ collide head-on, they can in principle scatter two back-to-back photons into a different direction, also at frequency $\omega$. In perturbation theory, the matrix element for this is fourth order in the photon field. The calculation is straightforward (see, for example, Berestetskii 1982 section 127), and leads to the dishearteningly small total cross-section (for $\hbar\omega<mc^2$)

$$\sigma \approx 0.031\alpha^2 r_0^2 \left( \frac{\hbar\omega}{mc^2} \right)^6 \equiv 10^{-66} \text{ for } \lambda = 1.06 \mu\text{m}. \quad (48)$$

Of course, this calculation does not include non-perturbative effects. For example, the electron and positron in the calculation are not Volkov states, and their masses have not been corrected for the field (see above).

The most glamorous, if least accessible, process involving the quantum vacuum is surely the direct production of matter out of light. In a sense, the threshold for this process represents an ultimate intensity, since above this limit, radiation is converted into matter. Unfortunately, the intensity required to observe this effect seems beyond our grasp at present. A very simple calculation illustrates this.

Virtual pairs in the vacuum exist for a very short time, as governed by the Heisenberg uncertainty principle:

$$\tau_{\text{pair}} \approx \frac{\hbar}{2mc^2} \approx 7 \times 10^{-22} \text{ sec} \quad (49)$$

During this time the particle separation can be no greater than

$$\approx \frac{\hbar}{mc} = \lambda_c, \quad (50)$$

the Compton wavelength. The threshold for pair production occurs when the work done by the fields on the virtual pair exceeds the rest mass.

$$eE_{\text{critical}}\lambda_c > 2mc^2$$

$$E_{\text{critical}} = \frac{2mc^2}{e\lambda_c}$$

$$I_{\text{critical}} = 9.7 \times 10^{18} \text{ W/cm}^2$$

There have been proposals to pair-produce light with high energy gamma rays.

If matter is present in the laser beam: electrons can lose ponderomotive forces and bremsstrahlung radiation can produce plasma as a catalyst for converting laser radiation to plasma.

Laser-plasma interactions at high intensities can seed plasma formation, the ultimate goal in this line of research.

4. ATOMS IN HIGH INTENSITY LASERS

4.1 Above-Threshold Ionization

Above-threshold ionization, or ATI, occurs at laser intensities above $10^{13}$ to $10^{14}$ W/cm$^2$. The number of photons necessary to overcome an electron's kinetic energy, or its ponderomotive, forces, and ponderomotive terms.

We will now review recent ATI experiments, angular distributions as a function of time, and learn about the nature of the high intensity limits.

The active experimental interest in ATI of Tompkins at the University of Chicago and earlier work of Agostini (1975) on xenon ATI experiments in the $10^{13}$ W/cm$^2$ range is $E_o = 12.13$ eV. The absorption in a Nd:YAG laser ($h\nu = 1.65$ eV) with a xenon.

The regular series of peaks are an extension to n photons of Einstein's photon concept.

This neat agreement obscures the fact that the new experiments involve laser intensities of this magnitude. In the next section, we will drop the "N.R." in the subscript.

Since a focused laser beam contains ponderomotive potential, the spectrum of photoelectron energies will shift because the atomic levels are energy shifted. The situation is simple in experiments, the ionization potential is equal to $U_p$. In that case, the final value it would have had in the absence of the ponderomotive potential effectively higher.
in term and the spin-orbit term in the

electrons in currently accessible laser

mentioning the effect of really intense

electromagnetic field fluctuations, 

sea are virtual electron-positron pairs

fluctuations can exist even in a classical

pairs in the vacuum is a quantum

It usually appears as a very slight

however, new phenomena should occur.

scattering of light by light. This is

For example, if two powerful laser

in principle scatter two back-to-back

scattering theory, the matrix element for

straightforward (see, for example,

small total cross-section (for

equal).

(48)

Effects. For example, the electron and

masses have not been corrected for the

the quantum vacuum is surely the

field for this process represents an ul-

timate limit for the intensity of such

A very simple calculation illus-

tered by the Heisenberg uncertainty

(49)

(50)

presents when the work done by the fields

\[ E_{\text{critical}} = \frac{2mc^2}{\epsilon_0 c^2} = \frac{2me^3}{\epsilon_0} = 2.7 \times 10^{16} \text{V/cm} \]

\[ I_{\text{critical}} = 9.7 \times 10^{29} \text{W/cm}^2 \] (51)

There have been proposals to produce pairs from the vacuum by colliding high intensity laser

pulses with high energy gamma rays at a particle accelerator (See Reiss 1971).

If matter is present in the laser focus, pairs can be produced by a different, lower-intensity pro-

cess: electrons can lose ponderomotive or drift energy in violent collisions with ions. The resulting

bremsstrahlung radiation can produce pairs if the collisions are violent enough. We can think of the

plasma as a catalyst for converting laser energy into matter.

Laser-plasma interactions at high intensity are beyond the scope of these lectures. However, the

seed for plasma formation, the multiphoton ionization of isolated atoms, is a fascinating field in its

own right. We now turn to this subject.

4. ATOMS IN HIGH INTENSITY LIGHT

4.1 Above-Threshold Ionization

Above-threshold ionization, or ATI, occurs in atomic photoinization experiments with peak laser

intensities above $10^{13}$ to $10^{14}$ W/cm$^2$. In ATI, atomic electrons absorb far more than the minimum

number of photons necessary to overcome their binding energy. The extra photons go into the

electron's kinetic energy, or its ponderomotive energy. There is a close connection between ATI and

ponderomotive forces, and ponderomotive effects usually affect the electron spectra in ATI experi-

ments.

We will review recent ATI experiments on rare gases. By studying the photoelectron spectra and

angular distributions as a function of laser intensity, pulse width, and polarization, much has been

learned about the nature of the high intensity light-matter interaction.

The active experimental interest in ATI can be traced back to two experiments, the "Garton and

Tompkins" of this field. These are: the dissertation work of Knut (1983) at FOM institute, who used

time-of-flight electron spectrometer to observe electrons from multiphoton ionization of xenon gas;

and earlier work of Agostini (1979). Figure 7 shows similar spectra of photoelectrons observed in

xenon ATI experiments in the $10^{13}$ W/cm$^2$ range (McIlrath 1987): The ionization potential of neutral

xenon is $E_0 = 12.13$ eV. The above-threshold photoinization involves up to 20 photons from a

Nd:YAG laser ($h

v=1.16$ eV) with a pulsewidth of approximately 100 psec.

The regular series of peaks are separated by single photon energies, according to the natural

extension to n photons of Einstein's photoelectric formula:

\[ E_{\text{electron}} = nhv - E_0 \] (52)

This neat agreement obscures the fact that the atoms and the electrons have enormous energy shifts

in fields of this magnitude. In the case of the free electron, the shift is the ponderomotive potential

$U_p$. (Since all of the work in this section involves intensities where $U_p$ is much less than $mc^2$, we

will drop the "N.R." in the subscript.)

Since a focused laser beam contains a continuous intensity distribution, we might expect the pon-

deromotive accelerations of electrons produced throughout the focal volume to produce a continuous

spectrum of photoelectron energies in an ATI experiment. Evidently, this does not happen. We

think this is because the atomic levels undergo A.C. Stark shifts that nearly match the free electron

energy shifts. The situation is shown schematically in figure 8, for 2.0 eV photons. In most ATI

experiments, the ionization potential shifts with respect to the ground state by an amount nearly

equal to $U_p$. In that case, the final state acceleration merely replenishes the electron energy to the

value it would have had in the absence of any shifts. In other words, the Stark effect and the pon-

deromotive potential effectively hide each other.
measured at low intensities, since it
directly if very short laser pulses are

![Figure 9. Photoelectron spectra for six-photon resonance of 616nm pulse.](image)

For pulses shorter than 1 psec, only
the electron's energy. The ponderomotive
which shows spectra obtained by full
sub-picosecond 616nm pulses. These
electrons with different energies, in a
laser pulse, each of these electrons
ponderomotive potential, so that all
however, the electrons appear slower
ion, the greater the energy that was

The fine structure peaks are the
bound states are in resonance with
Stark shifts predicts that these peaks
positions by an amount roughly equal
to zero intensity. (See figure 8)

The success of this model, and
confirmations that the ionization potential

### 4.3 Calculating ATI

ATI is easy to identify experimentally
signature. The theory is much more
theory is still valid at these intensities
impractical at best, and possibly it
have been tried. Most rely on a single
Yet, sub-picosecond experiments show
resonances that are left out of these

---

This cancellation of the Stark shift not only requires that the Rydberg states undergo the same self-energy correction as a free electron, but also that the ground state shift is very small. Otherwise, we should expect to see an additional energy shift in the electron peaks, corresponding to the work done on the ground state neutral atom as the laser turns on prior to ionization. Perturbation theory calculations show that the ground state shift should indeed be quite small (Pan 1987). It can be
measured at low intensities, since it is related to the optical index of refraction of the gas.

### 4.2 ATI With Sub-Picosecond Pulses

The hypothesis that A.C. Stark shifts are equal to the ponderomotive potential can be tested directly if very short laser pulses are used, as shown in figure 9 (From Freeman 1987).

![Figure 9. Photoelectron spectrum in xenon for 7 photon ionization by sub-picosecond 616 nm pulses. The peaks line up well with the expected positions of six-photon resonant enhancements, provided that the resonant intermediate states shift by the full ponderomotive potential of a free electron in the laser field, as shown in figure 8.](image)

For pulses shorter than 1 psec, ponderomotive forces act for too short a time to significantly alter the electron’s energy. The ponderomotive potential is no longer conservative, as we saw in figure 2. The nonconservative effects were quite small for 100 pico-second pulses. Compare that to figure 9, which shows spectra obtained by R.R. Freeman, H.M. Milchberg, and collaborators (1987), using sub-picosecond 616nm pulses. These show that each ATI peak really consists of a "fine structure" of electrons with different energies, ionized when the laser intensity was at different values. In a long laser pulse, each of these electrons regains nearly all of its energy deficit by accelerating out of the ponderomotive potential, so that all electrons pile up under an ATI peak. For very short pulses, however, the electrons appear slower, near the energy where they were "born". The slower the electron, the greater the energy that went into the binding potential.

The fine structure peaks are thought to be enhancements in ionization that occur when excited bound states are in resonance with an intermediate number of photons. The hypothesis about A.C. Stark shifts predicts that these peaks should appear at energies displaced from the Einstein equation positions by an amount roughly equal to the detuning of the state from intermediate resonance at zero intensity. (See figure 8)

The success of this model, and the appearance of ATI peaks for long laser pulses, are the major confirmations that the ionization potential shift is equal to the ponderomotive potential.

### 4.3 Calculating ATI

ATI is easy to identify experimentally: the absorption of extra photons during ionization is its signature. The theory is much more difficult, however, and remains controversial. If perturbation theory is still valid at these intensities, it must be used in very high order; this renders the approach impractical at best, and possibly intractable altogether. Several approaches using scattering theory have been tried. Most rely on a simplified model of the atomic potential and the scattering states. Yet, sub-picosecond experiments show clear evidence for dominance by intermediate bound state resonances that are left out of these treatments.
Several lectures on the theory appear in this volume (see Reiss, Pan, and Lambropoulos). We will concentrate on describing some of the experiments that have attempted to make contact with various theoretical predictions, particularly those of scattering theories. These experiments have explored the role of laser polarization in ATI.

4.4 Circular Polarization

S-matrix theories of high order multiphoton ionization were originally put forward by Keldysh, over 20 years ago (Keldysh 1965). There has been renewed interest in them since 1986, when ATI was first observed for circularly polarized light (figure 10, Bucksbaum 1986). Until then, all experiments had used linear polarization.

![ATI spectrum for circularly polarized 1064 nm light in xenon](image)

Figure 10. Top: ATI spectrum for circularly polarized 1064 nm light in xenon (from Bucksbaum 1986). Bottom: A computer simulation of the experiment, using ionization rates from the Keldysh theory of H. Reiss (1980), integrated over the measured temporal and spatial profile of the laser pulse.

Comparing figures 10 and 7 it is immediately obvious that circular polarization distorts the ATI spectrum significantly. Low energy electrons are suppressed much more for circular polarization than for linear polarization at the same intensity. One may understand this result in a qualitative way by considering the role of angular momentum in the ionization process. An atom that absorbs n photons from the circularly polarized laser field also absorbs n/2 units of angular momentum. These must be carried off as orbital angular momentum. The final state continuum electron wave function must be a high angular momentum state. Such states are expelled from the region around the atom by a repulsive centrifugal potential barrier. Electrons near threshold have such low energy, that this repulsion effectively eliminates any overlap of the outgoing electron wave with the atom, nearly totally suppressing any ionization into these channels.

Shortly after this effect was published, H. Reiss pointed out that certain Keldysh-type theories (Reiss 1980, Faisal, 1973) could accurately reproduce the circular polarized ATI spectra, with laser intensity as the only adjustable parameter. (Reiss, 1987) These theories simplify the ionization calculation by assuming that the final states may be approximated by Volkov states, i.e., free electrons in the laser field. These approximations, which have come to be known as Keldysh-Faisal-Reiss, or KFR models, seem to work well for circular polarization, but achieve much less success for linear polarization. There the final state wave functions have low angular momentum, and therefore penetrate the atomic potential where the differences between Volkov states and the atomic continuum are most important. High angular momentum states only sample regions of space where the Keldysh approximation is good, i.e., regions where the ion potential is weak.

4.5 ATI in Elliptically Polarized Light

The KFR model makes specific predictions about ATI electron spectra. For circular polarizations, the final states have angular momenta in the plane of polarization; thus the final states are a superposition of Volkov states produced by elliptically polarized light, which ranges from parallel to perpendicular, resulting in the time-varying electric field.

![Comparison of ATI spectra for Xenon and Krypton](image)

Figure 11. Comparison of ATI spectra for Xenon and Krypton, showing the effect of elliptical polarization on the ATI process. (h=+0.82, and data obtained from Baskansky, 1988.)

The KFR model makes specific predictions about ATI electron spectra. For circular polarizations, the final states have angular momenta in the plane of polarization; thus the final states are a superposition of Volkov states produced by elliptically polarized light, which ranges from parallel to perpendicular, resulting in the time-varying electric field.

Our data show that, although the KFR model makes specific predictions, the final states do not respect the requirement of circular polarization. The light is also circularly polarized, with respect to rotations by π. The light is elliptical, in order to satisfy spatial isotropy.

4.6 Asymmetric Angular Distribution

The form and magnitude of the angular distribution depend on the atomic structure and the orientation of the polarization axis. These effects are not yet fully understood, but some progress has been made in understanding the role of atomic structure in determining the angular distribution. Atomic structure effects appear to be strongest at small angles relative to the polarization axis. However, these effects are still not fully understood, and the role of atomic structure in determining the angular distribution remains an open question.
4.5 ATI in Elliptically Polarized Light

The KFR model makes specific predictions about shape, intensity, and angular distributions of ATI electron spectra. For circular polarization, the angular distribution is trivial, i.e., isotropic in the azimuthal (polarization) plane. Since linear polarization experiments involve low angular momentum states, where the KFR model fails, we have begun to investigate the properties of ATI for elliptical polarization, where the final states have high angular momentum, and Keldysh theories make definite predictions about angular distributions as well as spectra (Bashkansky 1988). Our aim was to discover whether the distributions of the ATI electrons followed the simple patterns that are characteristic of Volkov states produced by elliptically polarized light. These four-fold symmetric patterns in the polarization plane, which range from butterfly-shapes to clover leaves to figure-eights, are the result of wave-mechanical interference in the outgoing electron wave, as it is driven back and forth by the time-varying electric field.

Our data show that, although the electron distributions have many features in common with KFR predictions, the final states do not resemble free Volkov electrons in one crucial respect: the distributions do not display the required 4-fold symmetry. The distributions are only symmetric with respect to rotations by π. The light-atom system must be invariant with respect to this rotation in order to satisfy spatial isotropy.

4.6 Asymmetric Angular Distributions

The form and magnitude of the asymmetries, shown in figures 11 and 12, provide important clues about their origin.

![Table illustrating ATI distributions](image)

Figure 11. Comparison between data obtained with positive helicity light (h=+0.82), and data obtained under the same conditions, but with the helicity reversed (h=-0.82). The laser pulse was 0.10 to 0.12 nsec. Xe 1064nm: I_{peak}=4×10^{13} W/cm^2; P_{3/2} and P_{3/2} final states were not resolved, so numbers indicate photons absorbed for the P_{3/2} final state. Kr 1064nm: I_{peak}=4×10^{13} W/cm^2; P_{3/2} final states only. Xe 532nm: I_{peak}=1×10^{13} W/cm^2; primed numbers designate photons absorbed to the final P_{3/2} state; unprimed numbers designate the P_{3/2} state. Kr 532nm: I_{peak}=1.5×10^{13} W/cm^2; primes mean the same as for Xe 532nm. Helium 532nm: I_{peak}=1×10^{14} W/cm^2. (From Bashkansky, 1988.)

They depend on the helicity h of the light, reversing with the sign of h as required to maintain parity invariance. 4-fold symmetry is restored for linear polarization (h = 0). There is also a strong intensity dependence, as seen in figure 12.

Atomic structure effects appear to play some role in at least one instance (six photon 532nm ionization of xenon to the P_{3/2} final state ion in figure 11), where the sense of the asymmetry is reversed relative to the other data. However, these asymmetries appear in all atoms at both wavelengths used.
The interaction between the electron and the field is obtained by multiplying the ordinary complications of quantum mechanics; momentum \( \mathbf{p} \) and \( z \)-component momentum \( \mathbf{m} \) must always be written in the form: 

\[
\mathbf{u}_n \mathbf{S}_n \mathbf{a}_n \mathbf{f}_n \mathbf{g}_n \mathbf{h}_n \mathbf{i}_n \mathbf{j}_n \mathbf{k}_n \mathbf{l}_n \mathbf{m}_n \mathbf{n}_n \mathbf{o}_n \mathbf{p}_n \mathbf{q}_n \mathbf{r}_n \mathbf{s}_n \mathbf{t}_n \mathbf{u}_n \mathbf{v}_n \mathbf{w}_n \mathbf{x}_n \mathbf{y}_n \mathbf{z}_n
\]

Here \( E \) is the electron energy, and the spherical electron wave, \( \delta \) vanish. The function \( j_n(\mathbf{p}/\mathbf{r}) \), \( \mathbf{p} \) is the electron momentum, phase shift is not well defined, arithmic r-dependent phase affect symetries through interference. One

\[ a_0 \]

where the \( a_0 \)'s are linear combinations. The configuration set by the size of the polarization. In the case of linear polarization, but opposite \( a_0 \) contribute equally, but no angular shift.

No satisfactory method of estimating has been elaborated to date. Other approaches describe some of them.

5. SUMMARY

Before ending, we should remark the direction of nuclear electrons. The simplest experiments have been made of the order of nuclear polarization. (See, for example, L'Huillier, 1980). Harmonic generation in the plane of the gas also present. The connection between the experimental investigation at this moment.

Finally, I must acknowledge my collaborators, particularly

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The interaction between the outgoing electron and the ion potential leads to a complex phase \( e^{i\delta} \) multiplying the ordinary complex azimuthal dependence of spherical waves with definite angular momentum \( l \) and \( z \)-component \( m_z \). For potentials that fall off faster than \( 1/r \), the asymptotic wave can always be written in the form: (See, for example, Bethe and Salpeter 1957)

\[
\psi_{nl}(r,\theta,\phi) = e^{i\delta(l)} R_{nl}(r) P_{nl}(\theta)e^{im\phi}.
\]

(54)

Here \( E \) is the electron energy, and \( P_{nm}(\theta) \) is an associated Legendre polynomial. For a free outgoing spherical electron wave, \( \delta(l) \) vanishes, and \( R_{nl}(r) \) is (except for a normalization) a spherical Bessel function \( j_l(pr/R) \). (\( p \) is the electron radial momentum.) For a Coulomb potential, the asymptotic phase shift is not well defined, because it continues to increase as \( \log(pr/R) \); however, this logarithmic \( r \)-dependent phase affects all partial waves equally, and therefore cannot give rise to asymmetries through interference. On the other hand, there is also an \( l \)-dependent phase shift, given by

\[
\delta_l = \arg \Gamma(l+1+\frac{\ln p}{\mu a_0})
\]

(55)

where \( a_0 \) is the Bohr radius. Since this phase factor depends on \( l \) rather than \( m_z \), it cannot affect azimuthal distributions either, unless the final state is a superposition of partial waves with different \( l \) and \( m_z \). However, this is precisely the case for ATI using elliptically polarized light.

We should therefore expect asymmetries of the form

\[
\sum_n c_n \cos(2\pi n + \delta_n)
\]

(56)

where the \( \delta_n \)'s are linear combinations of the \( \delta_l \)'s in equation (55). The scale of the angular distortion is set by the size of the phase shifts, which are on the order of radians for these ATI experiments. In the case of linear polarization, the asymmetries vanish, since partial waves with the same \( l \) but opposite \( m_z \) contribute equally, and interfere to form wave functions with an overall phase shift, but no angular shift.

No satisfactory method of incorporating these phase shifts into KFR theory has been demonstrated to date. Other approaches have also been suggested, and other lectures in this volume describe some of them.

5. SUMMARY

Before ending, we should mention some of the ATI experiments that do not involve detecting electrons. The simplest experiments involve collecting ions following ionization. Many studies have been made of the order of nonlinearity, i.e., the nonlinear power dependence of ATI using ion collection. (See, for example, L’Huillier 1983a,b, and, Lompre 1985.) In addition, recent studies of harmonic generation in atomic gases have shown extremely high harmonics at intensities where ATI is also present. The connection between these two phenomena is an area of active theoretical and experimental investigation at this time.

Finally, I must acknowledge the important contributions to some of the work reviewed here made by my collaborators, particularly D.W. Schumacher and M. Bashkansky.

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