A Peculiar Motor of Ampère

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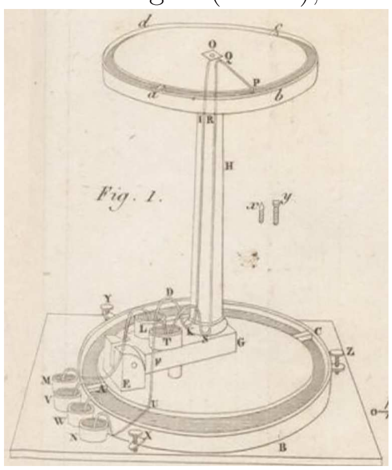
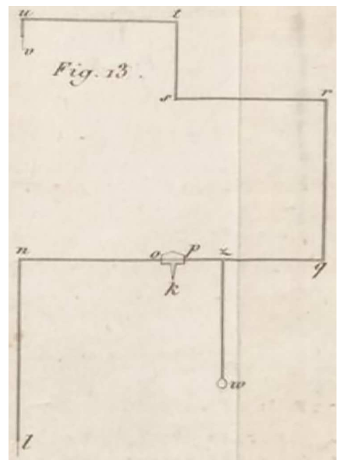
After Faraday's first demonstration of an electromagnetic motor in 1821 [1, 2, 3], Ampère soon demonstrated variants of such motors in [4, 5], one of which is described in [6].[1,2]

Here, I consider a kind of electric motor studied by Ampère in 1822 [5], based on the apparatus shown in his Fig. 1 (below), and the peculiar electrode shown in his Fig. 13.





My reconstruction of this motor is sketched on the next page.

Two wooden disks each have a circular trough that includes one radial partition. Each trough is filled with mercury. The two disks are mounted coaxially, one above the other. Leads from a battery are inserted into the mercury in the two troughs, such that an electric current can flow in the mercury (once a conducting armature links the two troughs), counterclockwise in the lower through as seen from above, but clockwise in the upper trough.

The armature of the motor, shown in red, can pivot about a vertical axis through point P in the sketch on the next page. The lower, bent end of the armature enters the lower trough, while the upper end of the armature is bent downwards so as to enter the mercury in the upper trough. This defines an intricate path for the electric current I, which flows counterclockwise from the positive electrode of the battery connected to the lower trough, into the lower end of the armature, up and across the lower trough, then upwards, and across the top of the upper trough, downwards into the upper trough, and finally clockwise towards negative electrode of the battery connected to the upper trough.

A sense of the motion can be gotten from consideration of the system when initially at rest. According to the Biot-Savart/Lorentz law,[3] the force $d\mathbf{F}$ on a current element $I\, d\mathbf{l}$ in a

[1]Refs. [4, 5] were reprinted in [7].

[2]Numerous electrostatc motors were exhibited in the 1700's, as recounted in [8], including an "electric fly"; see Figs. 10-11, pp. 24-25. This type of device, which is driven by electrostatic-field emission of electrons from the tips of the electrically charged rotor, was described by Ampère as a *moulinet électrique* (electric windmill); see p. 421 of [5], p. 103 of [7], p. 113 of [9], and p. 289 of [11].

[3]The force laws of Biot-Savart and Ampère have been reviewed by the author in Appendices 14.11-12 of [12]. Note that Ampère stated the "Biot-Savart" law in the first form of our eq. (3).

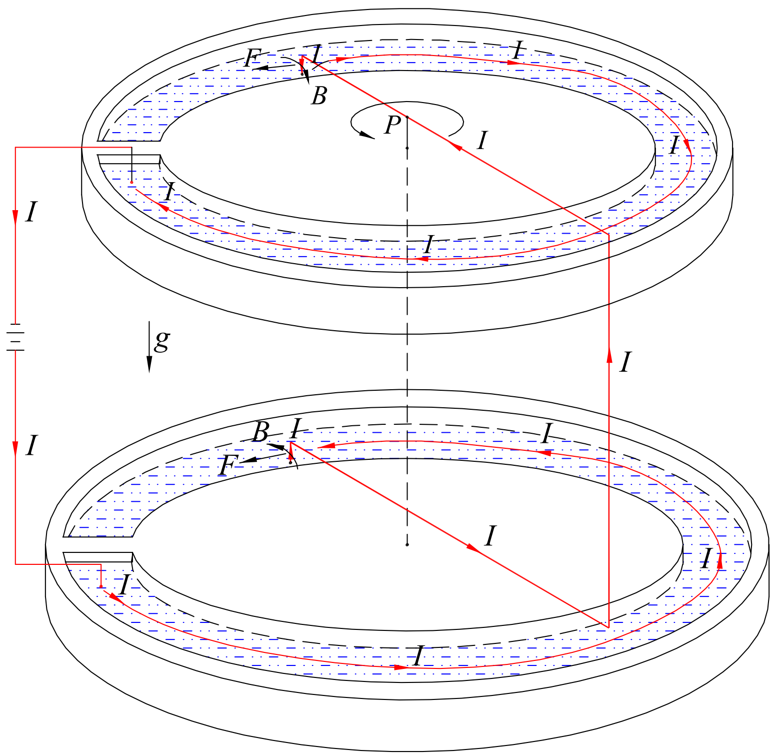
magnetic field **B** is

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}, \tag{1}$$

where the magnetic field can be computed as

$$\mathbf{B} = \frac{\mu_0}{4\pi} \oint \frac{I\, d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}\,, \tag{2}$$

with **r** being the distance vector from the current element to the observation point.[4]



The magnetic field due to the current in the mercury near the bent, lower end of the armature (in the lower trough) points radially outwards, as in the sketch above, and hence the $I\, d\mathbf{l} \times \mathbf{B}$ force on the upward current in that end of the armature points in the counterclockwise direction, as seen from above.

Meanwhile, the magnetic field due to the current in the mercury leaving the bent end of the armature in the upper trough points radially inwards, so the $I\, d\mathbf{l} \times \mathbf{B}$ force on the downward current in that bent end also points in the counterclockwise direction, as seen from above.

Therefore, the armature rotates about the vertical in the counterclockwise sense, making at most one full turn before hitting the wooden partition. Thus, the action of the system is briefly that of an electric motor, which was still a very new phenomenon in 1822,

[4] For two current elements $I_1\, d\mathbf{l}_1$ and $I_2\, d\mathbf{l}_2$, the force on element 1 due to element 2 is

$$d^2\mathbf{F}_{\text{on1}} = \frac{\mu_0}{4\pi} I_1 I_2 \frac{(d\mathbf{l}_1 \cdot \hat{\mathbf{r}})\, d\mathbf{l}_2 - (d\mathbf{l}_1 \cdot d\mathbf{l}_2)\, \hat{\mathbf{r}}}{r^2} = I_1\, d\mathbf{l} \times \frac{\mu_0}{4\pi} \frac{I_2\, d\mathbf{l}_2 \times \hat{\mathbf{r}}}{r^2} \neq -d^2\mathbf{F}_{\text{on2}} \qquad \text{(Biot-Savart)}, \tag{3}$$

where $\mathbf{r} = \mathbf{l}_1 - \mathbf{l}_2$ is the distance from a current element $I_2\, d\mathbf{l}_2$ at $\mathbf{r}_2 = \mathbf{l}_2$ to element $I_1\, d\mathbf{l}_1$ at $\mathbf{r}_1 = \mathbf{l}_1$.

Ampère did not favor the Biot-Savart law, and considered that the action of the motor was successfully predicted by his force law,

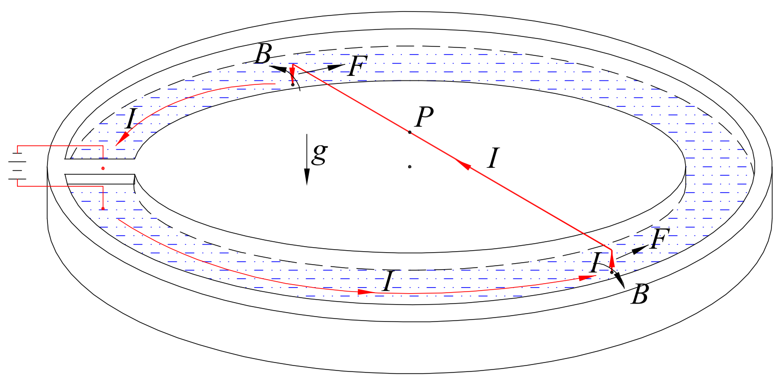
$$d^2\mathbf{F}_{\text{on }1} = \frac{\mu_0}{4\pi} I_1 I_2 [3(\hat{\mathbf{r}} \cdot d\mathbf{l}_1)(\hat{\mathbf{r}} \cdot d\mathbf{l}_2) - 2\, d\mathbf{l}_1 \cdot d\mathbf{l}_2] \frac{\hat{\mathbf{r}}}{r^2} = -d^2\mathbf{F}_{\text{on }2}. \qquad \text{(Ampère)}, \tag{4}$$

where $\mathbf{r} = \mathbf{l}_1 - \mathbf{l}_2$ is the distance from a current element $I_2\, d\mathbf{l}_2$ at $\mathbf{r}_2 = \mathbf{l}_2$ to element $I_1\, d\mathbf{l}_1$ at $\mathbf{r}_1 = \mathbf{l}_1$.[5]

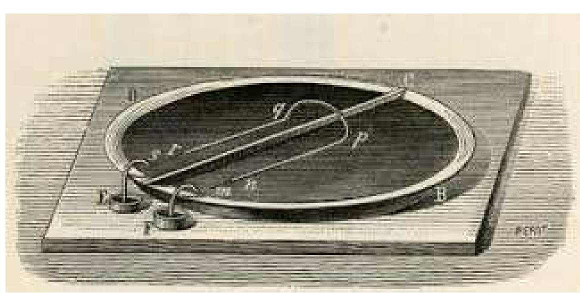
For element 1 as, say the bent, lower end of the armature sketched above, and element 2 in the counterclockwise current in the mercury near that end, $\hat{\mathbf{r}} \cdot d\mathbf{l}_1$ and $\hat{\mathbf{r}} \cdot d\mathbf{l}_2$ are both positive while $d\mathbf{l}_1 \cdot d\mathbf{l}_2 = 0$, so the force on the upward current element 1 is in the $\hat{\mathbf{r}}$ direction (from element 2 to element 1), which has a horizontal component in the counterclockwise direction as seen from above.

Similarly, with element 1 as the bend end of the armature in the upper trough, and element 2 in the counterclockwise current leaving that end in the upper trough, $\hat{\mathbf{r}} \cdot d\mathbf{l}_1$ and $\hat{\mathbf{r}} \cdot d\mathbf{l}_2$ are both negative while $d\mathbf{l}_1 \cdot d\mathbf{l}_2 = 0$, so the force on the upward current element 1 is in the $\hat{\mathbf{r}}$ direction (from element 2 to element 1), which has a horizontal component in the counterclockwise direction as seen from above.

If instead the experiment had been as sketched below, using only a single trough, the force on both bents ends of the armature (in red) would be in the same direction, and the motion of the armature would be a translation rather than a rotation.



This possibility was explored by Ampère and de la Rive in 1822 [16], as illustrated in their Fig. 12 (below, see also pp. 327 and 331 of [11]), which was the first demonstration of a "railgun".[6] Starting from rest, the motion of the armature is predicted to be away from the battery by the force laws of both Biot-Savart and Ampère, as observed in the experiment.



[5]Ampère's insistence that the force law for steady currents obeys Newton's 3$^{\text{rd}}$ law earned him the sobriquet of the "Newton of electricity" by Maxwell in Art. 528 of [13].

[6]Discussion by the author of railguns is in [14].

Ampère was chiefly interested in the experimental confirmation of his (static) force law (4), and downplayed any time dependence exhibited in his experiments, including an early hint of electromagnetic induction in 1822 [15, 16].[7,8,9]

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See also p. 417 of the English translation in [10].

[7] See also letters 485 and 490 from Ampère to Faraday and de la Rive in 1833, pp. 763 and 773 of [17].

[8] For historical commentary, see, for example, [18]-[21] and also pp. 60-67 of [22].

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