A Misuse of Relativity in Circuit Analysis

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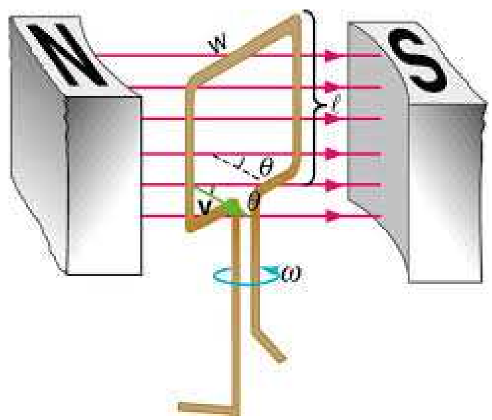
1 Problem

In sec. 3.3 of [1] the author has discussed how many associations of Faraday's law and special relativity are somewhat misguided. This problem concerns one such case, given in sec. 14.4 of [2].

Consider a dynamo in which a rectangle of wire of size $l \times w$ is rotated with constant angular velocity ω in a uniform magnetic field $\mathbf{B}$ that is perpendicular to the axis of rotation, as sketched below. For simplicity, suppose the rectangle has electrical resistance R which is large compare to that in the rest of the circuit.



What is the electric field $\mathbf{E}$ in the various segments of the rotating rectangle when angle $\theta = 90°$, neglecting the small effect of the fields induced by the current that flows in this dynamo.

Give analyses in the lab frame, in the rotating frame of the wire rectangle, and in an inertial frame that has velocity $u = \omega\, w/2$ perpendicular to $\mathbf{B}$.

2 Solution

2.1 Analysis in the Lab Frame

Here, we apply Faraday's flux rule for moving circuits, that the *electromotive force* ($\mathcal{EMF}$) induced in a loop can be computed by,[1]

$$\mathcal{EMF} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_{\rm loop} \mathbf{B}\cdot d\mathbf{Area}. \tag{1}$$

The magnetic flux Φ through the loop when $\theta = \omega t$ is given by,

$$\Phi = Blw\cos\theta, \tag{2}$$

so the induced $\mathcal{EMF}$ is, according to eq. (1),

$$\mathcal{E} = Blw\omega\sin\theta = 2Blv\sin\theta, \tag{3}$$

where $v = \omega w/2$ is the velocity of a vertical segment of the rotating loop. W hen $\theta = 90°$, the $\mathcal{EMF}$ has value $\mathcal{E} = 2Blv$, and the current I in the circuit at this time follows from Ohm's law as,

$$I = \frac{\mathcal{E}}{R} = \frac{2Blv}{R}\,, \tag{4}$$

neglecting the effect of the self inductance of the rectangular loop.

Note that our use of Ohm's law here is in the rotating frame of the loop, and that we have tacitly assumed that all quantities in eq. (4) have the same values in these two frames. We will review the validity of this assumption in sec. 2.2 below.

As we wish to deduce the electric field inside the wire loop, it is useful to note that there exists an alternative expression for the induced $\mathcal{EMF}$ of eq. (1),

$$\mathcal{EMF} = \oint (\mathbf{v}\times\mathbf{B} + \mathbf{E})\cdot d\mathbf{l} = \mathcal{EMF}_{\rm motional} + \mathcal{EMF}_{\rm fixed\ \ loop}, \tag{5}$$

where the motional $\mathcal{EMF}$ is defined by,

$$\mathcal{EMF}_{\rm motional} = \oint_{\rm loop} \mathbf{v}\times\mathbf{B}\cdot d\mathbf{l}, \tag{6}$$

[1]Faraday's law (1) was never stated by Faraday. The earliest statement of this law is by Maxwell in a letter to W. Thomson, Nov. 13, 1854, p. 703 of [3]: *The electromotive force along any line is measured by the number of lines of pol*$^{\rm n}$ *(i.e.*, of the magnetic field) *wh(*ich) *that line cuts in unit of time. Hence the electromotive force round a given circuit depends on the decrease of the number of lines wh: pass thro it in unit of time, that is, on the decrease of the whole pol*$^{\rm n}$ *of any surface bounded by the circuit.*

In Art. 530 of his *Treatise* [4], Maxwell considered electromagnetic induction in four different configurations, and then stated in Art. 531:
The whole of these phenomena may be summed up in one law. When the number of lines of magnetic induction which pass through the secondary circuit in the positive direction is altered, an electromotive force acts round the circuit, which is measured by the rate of decrease of the magnetic induction through the circuit.

However, $\mathcal{EMF}$ of eq. (1) is not a force in the Newtonian sense. As remarked by Maxwell at the end of Art. 598 of [4]: *The electromotive force at a point, or on a particle, must be carefully distinguished from the electromotive force along an arc or a curve, the latter quantity being the line integral of the former.*

in which $\mathbf{v}$ is the velocity (in the inertial lab frame of the calculation) of an element $d\mathbf{l}$ of the loop (which is a line, but which may or may not be inside a conductor), and,[2]

$$\begin{aligned} \mathcal{EMF}_{\rm fixed\ loop} &= \oint_{\rm loop} \mathbf{E} \cdot d\mathbf{l} = \int_{\rm loop} \boldsymbol{\nabla} \times \mathbf{E} \cdot d\mathbf{Area} = -\int_{\rm loop} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{Area} \\ &= -\frac{\partial}{\partial t} \int_{\rm loop\ at\ time\ \mathit{t}} \mathbf{B} \cdot d\mathbf{Area} = -\frac{\partial \Phi}{\partial t}\,. \end{aligned} \tag{7}$$

In the present example, the motional $\mathcal{EMF}$ when $\theta = 90^\circ$ is,

$$\mathcal{E}_{\rm motional}(\theta = 90^\circ) = \oint_{\rm loop} \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} = 2Blv = \mathcal{E}, \tag{8}$$

such that,

$$\mathcal{EMF}_{\rm fixed\ loop} = \oint_{\rm loop} \mathbf{E} \cdot d\mathbf{l} = 0. \tag{9}$$

To deduce the field $\mathbf{E}$ in the various wire segments, we now consider the generalized version of Ohm's law in the law frame for a moving conductor,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{10}$$

where $\mathbf{J}$ is the current density in the wire of cross sectional area A, $\sigma = 2(l + w)/AR$ is the electrical conductivity of the wire loop of length $2(l + w)$ and $\mathbf{v}$ is the lab-frame velocity of the point under consideration.

We consider only the quasistatic behavior of the current in the loop, for which we approximate the current as entirely along the wire. That is, we approximate the component of current density transverse to the wire as zero.

For a horizontal wire segment of length w, where $\mathbf{v} \times \mathbf{B}$ is perpendicular to the wire, $\mathbf{J}_\perp = 0$ implies the existence of a transverse electric field,

$$\mathbf{E}_\perp = -\mathbf{v} \times \mathbf{B}, \qquad E_\perp(\theta = 90^\circ) = Bv \qquad \text{(horizontal segment)}, \tag{11}$$

while the axial field is related by, recalling eq. (4),

$$\mathbf{E}_\parallel = \frac{\mathbf{J}_\parallel}{\sigma} \qquad E_\parallel = \frac{I}{\sigma A} = \frac{2Blv}{R} \frac{R}{2(l + w)} = \frac{l}{l + w} Bv \qquad \text{(horizontal segment)}. \tag{12}$$

[2] In this author's view, eqs. (5)-(7) are implicit in Arts. 598-599 of Maxwell's *Treatise* [4], although the discussion there is not very clear. Maxwell did not explicitly identify the electric field there, but invoked potentials ψ and $\mathbf{A}$ which he related elsewhere in the *Treatise* to the electromagnetic fields according to $\mathbf{E} = -\boldsymbol{\nabla}\Psi - \partial\mathbf{A}/\partial t$ and $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$. Then, for a closed, fixed loop, $\oint \boldsymbol{\nabla}\Psi \cdot d\mathbf{l} = 0$ and $\oint \mathbf{E} \cdot d\mathbf{l} = -\oint \partial\mathbf{A}/\partial t \cdot d\mathbf{l}$, such that eq. (5) can also be written as $\mathcal{EMF} = \oint (\mathbf{v} \times \mathbf{B} - \partial\mathbf{A}/\partial t) \cdot d\mathbf{l}$, whose integrand was given by Maxwell as eq. (B) of Art. 598.

However, the equivalence of our eqs. (1) and (5) was not recognized for many years. The first clear statement of this equivalence may be in sec. 86 of the text of Abraham (1904) [5], which credits Hertz (1890) [6] for inspiration on this. An early verbal statement of this in the American literature was by Steinmetz (1908), pp. 1352-53 of [7], with a more mathematical version given by Bewley (1929) in Appendix I of [8]. Textbook discussions in English include that by Becker, pp. 139-142 of [9], by Sommerfeld, pp. 286-288 of [10], by Panofsky and Phillips, pp. 160-163 of [11], and by Zangwill, sec. 14.4 of [12].

Similarly, for a vertical wire segment of length l, where $\mathbf{v} \times \mathbf{B}$ is parallel to the wire, $\mathbf{J}_\perp = 0$ implies that,

$$\mathbf{E}_\perp = 0 \qquad \text{(vertical segment)}, \tag{13}$$

while the axial field is related by,

$$\mathbf{E}_\parallel = \frac{\mathbf{J}_\parallel}{\sigma} - \mathbf{v} \times \mathbf{B}, \quad E_\parallel(\theta = 90^\circ) = \frac{2Blv}{R}\frac{R}{2(l+w)} - Bv = -\frac{w}{l+w}Bv \quad \text{(vertical segment)}. \tag{14}$$

Then, for $\theta = 90^\circ$,

$$\oint_{\text{loop}} \mathbf{E} \cdot d\mathbf{l} = \oint_{\text{loop}} E_\parallel \, dl = 2w\frac{l}{l+w}Bv - 2l\frac{w}{l+w}Bv = 0, \tag{15}$$

as expected from eq. (9).[3]

2.2 Analysis in the Rotating Frame

The preceding analysis, nominally in the lab frame, used the assumption that Ohm's law has the same form in the lab frame as in the rotating frame of the rectangular loop. To what extent is electrodynamics the same in the lab frame and in a rotating frame?

This question is not trivial, in that physics a rotating (accelerated) frame requires some of the insights of general relativity, or at least of general covariance, in examples like the present where "gravity" is negligible.

A review by the author of this topic is given in [13], with a summary (which we follow here) in sec. 2.2.5. In particular, eq. (66) of [13] affirms that Ohm's law has the same form in the lab frame and in the rotating frame, although the general relations of electrodynamics in the two frames are rather complex.

We also note from eq. (44) of [13] that the transformations of the lab-frame electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ into those in the rotating frame, $\mathbf{E}'$ and $\mathbf{B}'$, are, for points in the lab frame with rotational velocity $v \ll c$,

$$\mathbf{B}' = \mathbf{B}, \qquad \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{E} = \mathbf{E}' - \mathbf{v} \times \mathbf{B}'. \tag{16}$$

For a horizontal wire segment of length $w' = w$, where $\mathbf{v} \times \mathbf{B}$ is perpendicular to the wire,

$$\mathbf{E}'_\perp = \mathbf{E}_\perp + \mathbf{v} \times \mathbf{B} = 0 \qquad \text{(horizontal segment)}, \tag{17}$$

recalling eq. (11), while the axial field is related by, recalling eq. (12),

$$\mathbf{E}'_\parallel = \mathbf{E}_\parallel = \frac{\mathbf{J}_\parallel}{\sigma} \qquad E_\parallel = \frac{l}{l+w}Bv \qquad \text{(horizontal segment)}. \tag{18}$$

[3]We have only deduced the electric field in the wire for the special case that $\theta = 90^\circ$. As angle θ varies with time, the electric field in the wire does also. At any particular time, the electric field within the wire is shaped by an appropriate distribution of electric charge on the surface of the wire. The time-dependent electric field is associated with a time dependent surface-charge distribution, which in turn is associated with time-dependent currents in the wire that are largely transverse to its axis. We have neglected the small effect of these time-dependent currents on the electromagnetic fields.

Similarly, for a vertical wire segment of length $l' = l$, where $\mathbf{v} \times \mathbf{B}$ is parallel to the wire,

$$\mathbf{E}'_\perp = \mathbf{E}_\perp = 0 \qquad \text{(vertical segment)}, \tag{19}$$

recalling eq. (13), while the axial field is related by,

$$\mathbf{E}'_\parallel = \mathbf{E}_\parallel + \mathbf{v} \times \mathbf{B} = \frac{\mathbf{J}_\parallel}{\sigma}, \quad E_\parallel = \frac{l}{l+w} Bv, \quad \text{(vertical segment)}. \tag{20}$$

recalling eq. (14). Then,

$$\oint_{\text{loop}} \mathbf{E}' \cdot d\mathbf{l}' = \oint_{\text{loop}} \mathbf{E}' \cdot d\mathbf{l} = \oint_{\text{loop}} E_\parallel \, dl = 2(l+w) \frac{l}{l+w} Bv = 2Blv. \tag{21}$$

This rotating-frame result hold for any angle θ, and is equal to the lab-frame motional $\mathcal{EMF}$ for $\theta = 90^\circ$, eq. (8).

All this seems reasonable, except that eq. (17) is somewhat odd, that the transverse electric field inside a horizontal wire segment is zero in the rotating frame but nonzero in the lab frame. In the lab frame, we argued (footnote 3 above) that there exists a surface density of electric charge on the horizontal wire. In the rotating frame, there also exists a "fictitious" charge density whose effect is to counter that of the "true" surface charge, bringing $\mathbf{E}'_\perp$ to zero inside the horizontal wire. That is, electrodynamics is not as simple in a rotating frame as in an inertial frame.

2.3 Analysis in an Inertial Frame with $\mathbf{u} \perp \mathbf{B}$

We also consider an inertial frame with velocity $\mathbf{u} \perp \mathbf{B}$ and $u = \omega w/2 \ll c$, with emphasize on the moment when $\theta = 90^\circ$ in the (inertial) lab frame. Then, the plane of the rotating loop is momentarily perpendicular to $\mathbf{u}$, which is out of the page in the figure on p. 1.

The electromagnetic fields inside the wire loop are then, according to observers in the moving inertial frame (the $''$ frame),

$$\mathbf{E}'' = \mathbf{E} + \mathbf{u} \times \mathbf{B}, \qquad \mathbf{B}'' = \mathbf{B} - \frac{\mathbf{u}}{c^2} \times \mathbf{E}, \tag{22}$$

where we neglect quantities of order u^2/c^2, such that $\gamma = 1/\sqrt{1 - u^2/c^2} \approx 1$.

Since the lab-frame electric fields are all of order u, the magnetic field in the $''$ frame differs by that in the lab frame by terms of order $u^2/c2$, which we neglect. That is, $\mathbf{B}'' \approx \mathbf{B}$.

For a horizontal wire segment of length $w'' = w$, where $\mathbf{v} \times \mathbf{B}$ and $\mathbf{u} \times \mathbf{B}$ are both perpendicular to the wire,

$$\mathbf{E}''_\perp = \mathbf{E}_\perp + \mathbf{u} \times \mathbf{B} = (\mathbf{u} - \mathbf{v}) \times \mathbf{B} \qquad \text{(horizontal segment)}, \tag{23}$$

which varies with position along the wire, recalling eq. (11). The axial field is related by,

$$\mathbf{E}'_\parallel = \mathbf{E}_\parallel = \frac{\mathbf{J}_\parallel}{\sigma} \qquad E_\parallel = \frac{l}{l+w} Bv \qquad \text{(horizontal segment)}, \tag{24}$$

recalling eq. (12).

Similarly, for a vertical wire segment of length $l'' = l$, where $\mathbf{v} \times \mathbf{B}$ and $\mathbf{u} \times \mathbf{B}$ are both parallel to the wire,

$$\mathbf{E}'_\perp = \mathbf{E}_\perp = 0 \qquad \text{(vertical segment)}, \tag{25}$$

recalling eq. (13). For the axial field, we must distinguish between the two vertical segments, with velocities $\pm\mathbf{u}$ in the lab frame. The segment with velocity $\mathbf{v} = \mathbf{u}$ has axial field in the $''$ frame given by,

$$\mathbf{E}''_\parallel = \mathbf{E}_\parallel + \mathbf{u} \times \mathbf{B} = \frac{\mathbf{J}_\parallel}{\sigma}, \qquad E_\parallel = \frac{w}{l+w} Bv, \qquad \text{(vertical segment with } \mathbf{v} = \mathbf{u}\text{)}, \tag{26}$$

recalling eq. (14), while the other vertical segment has axial field,

$$\mathbf{E}''_\parallel = \mathbf{E}_\parallel + \mathbf{u} \times \mathbf{B} = \frac{\mathbf{J}_\parallel}{\sigma} - 2\mathbf{v} \times \mathbf{B}, \quad E''_\parallel = -\frac{l+2w}{l+w} Bv \quad \text{(vertical segment with } \mathbf{v} = -\mathbf{u}\text{)}. \tag{27}$$

Then (for angle $\theta = 90°$),

$$\oint_{\text{loop}} \mathbf{E}'' \cdot d\mathbf{l}'' = \oint_{\text{loop}} \mathbf{E}'' \cdot d\mathbf{l} = \oint_{\text{loop}} E''_\parallel \, dl - 2w \frac{l}{l+w} Bv + l \frac{l}{l+w} Bv = l \frac{l+2w}{l+w} Bv = 0. \tag{28}$$

This is to be expected, as eq. (7) for $\mathcal{EMF}_{\text{fixed loop}}$ holds in the $''$ frame (substituting $\mathbf{E}'' \cdot d\mathbf{l}''$ for $\mathbf{E} \cdot d\mathbf{l}$), and noting that $\partial\Phi''/\partial t'' = 0$ when $\theta = 90°$. That is,

$$\mathcal{EMF}''_{\text{fixed loop}}(\theta = 90°) = \oint_{\text{loop}} \mathbf{E}'' \cdot d\mathbf{l}'' = 0. \tag{29}$$

We can also evaluate the motional $\mathcal{EMF}$ (6) in the $''$ frame,

$$\mathcal{EMF}''_{\text{motional}}(\theta = 90°) = \oint_{\text{loop}} \mathbf{v}'' \times \mathbf{B}'' \cdot d\mathbf{l}'' = \oint_{\text{loop}} \mathbf{v}'' \times \mathbf{B} \cdot d\mathbf{l} = 2vBl, \tag{30}$$

since the integrand is nonzero only on the vertical wire segment with velocity $\mathbf{v} = -\mathbf{u}$ in the lab frame, *i.e.*, where $\mathbf{v}'' = -2\mathbf{u}$.

According to eq. (5), as applied to the $''$ frame, the total $\mathcal{EMF}$ in that frame is,

$$\mathcal{EMF}''(\theta = 90°) = \mathcal{EMF}''(\theta = 90°)_{\text{fixed loop}} + \mathcal{EMF}''(\theta = 90°)_{\text{motional}} = 2vBl. \tag{31}$$

This is equal to the $\mathcal{EMF}$ in the lab frame. However, if the $\mathcal{EMF}$ is measured by a voltmeter at rest in the lab frame, one should not say its value is $2vBl$ because that is the result according to an observer in the moving $''$ frame.

Another issue is that the motional $\mathcal{EMF}$ in the $''$ is entirely due to the vertical segment with velocity $\mathbf{v} = -\mathbf{u}$ in the lab frame, while in the lab frame it is equally due to both vertical segments. Thus, while it is often said that the motional $\mathcal{EMF}$ identifies the "seat of the $\mathcal{EMF}$", this identification is actually frame dependent.

2.4 Carter's Analysis

After the lengthy preliminaries about, we come to consideration of sec. 14.4 of [2].

The argument there is very peculiar. It begins with consideration of the fields in our $''$ frame of sec. 2.3 above in the absence of the wire loop, but given the uniform magnetic field $\mathbf{B}$ (and zero electric field in the lab frame). As per our eq. (22), there would be a nonzero electric field in the $''$ frame, perpendicular to both $\mathbf{u}$ and $\mathbf{B}$. Then, it is claimed that in case the wire loop is present, *the conductor short-circuits this electric field, so far as the frame of reference* ($''$) *is concerned.* ($E''_\parallel$) *is compelled to be zero; this brings into being in* (the lab frame)*, an electric field* ($E_\parallel$) *where* ($0 = E_\parallel - uB$).

Here, Carter was discussing the wire segment which, according to our conventions, is vertical and with velocity $\mathbf{v} = \mathbf{u}$ in the lab frame when our $\theta = 90^\circ$.[4] We found the field in this segment to be given by our eq. (26) in the $''$ frame, and by eq. (14) in the lab frame. Thus, Carter's claim about "short-circuits" is bogus.

Carter's discussion continued with the implication that the total $\mathcal{EMF}$ is twice that along the wire segment he considered, *i.e.*, $2Blv$ in our notations. That is, he arrived at the correct value by a spurious "relativistic" argument.

2.5 Carter's Rule

Carter also implied in his sec. 14.4 that his "relativistic" argument "proved" an interesting "safe working rule", given on p. 170 of [2], regarding Faraday's generalized flux rule (1):
The equation $\mathcal{E} = -d\Phi/dt$ always gives the induced e.m.f. correctly, provided the flux-linkage is evaluated for a circuit so chosen that at no point are particles of the material moving across it.[5]

I believe this statement may well be correct, but that it should also include the proviso *..., and at no time is there a discontinuous change in the linked flux.*

However, I am unaware of any valid "proof" of this statement.[6]

References

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[2] G.W. Carter, *The Electromagnetic Field in its Engineering Aspects*, 2nd ed. (Longmans, 1967), http://kirkmcd.princeton.edu/examples/EM/carter_67_ch8.pdf http://kirkmcd.princeton.edu/examples/EM/carter_67_ch14.pdf

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[4] In Carter's analysis the axis of rotation is horizontal, so his horizontal is our vertical, and *vice versa*.

[5] That is, a valid path through the interior of a material must be at rest with respect to that material.

[6] Additional discussion by the author of Carter's rule is given in sects. 2.3-4 of [1].

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[11] W.K.H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, 2nd ed., sec. 9-3 (Addison-Wesley, 1962), http://kirkmcd.princeton.edu/examples/EM/panofsky-phillips.pdf

[12] A. Zangwill, *Modern Electrodynamics* (Cambridge U. Press, 2012),
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