Wheel Rolling on a Merry-Go-Round

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1 Problem

In Sec. 19.4 of Vol. 1 of his *Lectures on Physics* [1], Feynman used the example of a man who walks radially inwards on a merry-go-round that is driven with constant angular velocity $\mathbf{\Omega}$ (about its vertical axis) to illustrate the Coriolis force.

Consider instead a wheel in the form of a uniform disk of mass m, radius a and thickness d that rolls without slipping on the horizontal platform of the merry-go-round, when launched from initial distance r_0 from the center of the merry-go-round with initial inward velocity v_0.

What is the minimum value of v_0 such that the wheel can reach the center of the merry-go-round?

What is the minimum value of the thickness d such that the wheel does not fall sideways as it rolls?

Consider also variants in which the merry-go-round, with moment of inertia I_{MGR} about its vertical axis, is not driven, but rotates without friction, as well as the case that the wheel has an internal energy source that can drive it with constant inward velocity v_0 in the rotating frame of the merry-go-round.

2 Solution

We begin by working in the rotating frame of the merry-go-round, where the equation of motion of the wheel can be written as

$$m\frac{d\mathbf{v}}{dt} = m\mathbf{g} + \mathbf{N} + \mathbf{F}_{\rm friction} + m\mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}) + 2m\mathbf{v} \times \mathbf{\Omega} + m\mathbf{r} \times \dot{\mathbf{\Omega}}, \tag{1}$$

where $\mathbf{r}$ and $\mathbf{v}$ are the position and velocity of the center of mass of the wheel in the rotating frame, the first term on the right is the force of gravity, the second term is the normal force $\mathbf{N} = -m\mathbf{g}$ of the platform of the merry-go-round on the wheel, the third term is the force of friction of the platform on the wheel, the fourth term is the is the centrifugal force, the fifth term is the Coriolis force, and the sixth term would be nonzero if the angular velocity $\mathbf{\Omega}$ were not constant. The radial component of the equation of motion includes the centrifugal force,

$$m\frac{dv_r}{dt} = F_{{\rm friction},r} + m\Omega^2 r, \tag{2}$$

and the tangential component includes the Coriolis force,

$$m\frac{dv_\theta}{dt} = F_{{\rm friction},\theta} - 2m\Omega v_r. \tag{3}$$

We assume that the static friction $F_{\text{friction},\theta}$ is always large enough to counteract the Coriolis force, such that $v_\theta = 0$ always and the motion of the wheel is purely radial. Then,

$$F_{\text{friction},\theta} = 2m\Omega v_r. \tag{4}$$

Torque Equation about the Center of the Wheel

In addition, we must consider the torque equation, which we take about the center of the wheel. Frist, we consider the torque is due to the radial friction force, $F_{\text{friction},r}$, that acts at the point of contact of the wheel with the platform of the merry-go-round, at distance a below the center of the wheel) For rolling without slipping, the wheel has angular velocity $\omega = v_r/a$ about its (horizontal) symmetry axis, with and moment of inertia $I_w = kma^2$ about this axis, where $k = 1/2$ for a uniform wheel. The relevant torque equation is then

$$I_w \frac{d\omega}{dt} = kma^2 \frac{1}{a}\frac{dv_r}{dt} = -aF_{\text{friction},r}. \tag{5}$$

The minus sign in eq. (5) occurs because the convention that $\omega = v_r/a$ implies that the torque due to a positive friction force $F_{\text{friction},r}$ would reduce the angular momentum of the rolling wheel. Then,

$$F_{\text{friction},r} = -km\frac{dv_r}{dt}\,. \tag{6}$$

Using this in eq. (2), we find

$$(1+k)\frac{dv_r}{dt} = \Omega^2 r, \qquad \frac{dv_r}{dt} = \frac{d^2r}{dt^2} = \frac{\Omega^2}{1+k}r, \tag{7}$$

so the radial motion of the center of the wheel has the form

$$r = A\cosh\frac{\Omega t}{\sqrt{1+k}} + B\sinh\frac{\Omega t}{\sqrt{1+k}}\,. \tag{8}$$

Will the Wheel Tip Over?

We must also consider the torque associated with by the Coriolis force (of magnitude $2m\Omega\,|v_r|$, which acts at the center of the wheel), which tends to tip the wheel over sideways. In the limiting case, the wheel remains in contact with the platform of the merry-go-round only at the point on its line of contact at distance $d/2$ from the radial line along which the wheel rolls. It is convenient to consider the torque about this point, where Coriolis torque has magnitude $2m\Omega\,|v_r|\,a$, which is opposed by the torque due to gravity, $mgd/2$. The wheel will tip over unless

$$2m\Omega\,|v_r|\,a < \frac{mgd}{2}\,, \qquad d > \frac{4\Omega a\,|v_r|}{gd}\,. \tag{9}$$

For v_r directed inwards, the magnitude of the velocity is greatest at the initial time when $|v_r| = v_0$. That is, the wheel will tip over unless its thickness d obeys

$$d > \frac{4\Omega a v_0}{gd}\,. \tag{10}$$

2.1 Merry-Go-Round Driven with Constant Angular Velocity

We now consider the case that the merry-go-round is driven (by a motor) at constant angular velocity Ω in the lab frame.

2.1.1 The Wheel Rolls Freely

We first consider the case that the wheel rolls freely from initial position r_0 with initial (inward) velocity $v_r(0) = -v_0$, for positive r_0 and v_0. Then, $A = r_0$ and $B\,\Omega/\sqrt{1+k} = -v_0$,

$$r = r_0 \cosh \frac{\Omega t}{\sqrt{1+k}} - v_0 \frac{\sqrt{1+k}}{\Omega} \sinh \frac{\Omega t}{\sqrt{1+k}}\,, \tag{11}$$

$$-v_0 \cosh \frac{\Omega t}{\sqrt{1+k}} .v_r = \frac{dr}{dt} = \frac{r_0 \Omega}{\sqrt{1+k}} \sinh \frac{\Omega t}{\sqrt{1+k}} \tag{12}$$

For large enough Ω, the wheel does not reach the center of the merry-go-round at $r = 0$. Supposing that the wheel does reach the center, at time $t_{\rm final}$, we have that

$$r_{\rm final} = 0 = r_0 \cosh \frac{\Omega t_{\rm final}}{\sqrt{1+k}} - v_0 \frac{\sqrt{1+k}}{\Omega} \sinh \frac{\Omega t_{\rm final}}{\sqrt{1+k}}\,. \tag{13}$$

$$\tanh \frac{\Omega t_{\rm final}}{\sqrt{1+k}} = \frac{r_0}{v_0} \frac{\Omega}{\sqrt{1+k}}\,. \tag{14}$$

Since $\tanh \Omega t_{\rm final}/\sqrt{1+k} \le 1$, the wheel can reach the center only if $v_0 \ge r_0 \Omega/\sqrt{1+k}$, and for the limiting case of equality, the wheel takes infinite time to reach the origin.

Power Absorbed by the Merry-Go-Round Drive Motor

As the wheel moves inwards, it kinetic energy decreases, and this loss of energy must be absorbed by the Merry-Go-Round drive motor.

The drive motor is in the (inertial) lab frame of the system, so we should consider the changes of energy that the system induces on the merry-go-round in this frame.

In the lab frame, the only forces on the merry-go-round associated with the motion of the wheel are the normal force and the frictional force of the wheel on the merry-go-round.

The normal force is perpendicular to the velocity of the point of contact the wheel with the merry-go-round, so does not contribute to the power delivered to the merry-go-round.

The frictional force on the wheel is the same in the lab frame and in the rotating frame of the merry-go-round. This follows from eqs. (4), (6), (7), (11) and (12) as

$$F_{{\rm friction},r} = -km \frac{dv_r}{dt} = -\frac{k}{1+k} m\Omega^2 r, \qquad F_{{\rm friction},\theta} = 2m\Omega v_r \tag{15}$$

The force on the platform is the negative of eq. (15), and the power ($\mathbf{F} \cdot \mathbf{v}$) absorbed by the platform is, recalling that $v_\theta = \Omega r$,

$$P_{\rm absorbed} = -F_{{\rm friction},r} v_r - F_{{\rm friction},\theta} v_\theta = \frac{k}{1+k} m\Omega^2\, r v_r - 2m\Omega^2\, r v_r = -\frac{(2+k)m\Omega^2}{1+k} r v_r$$

$$= \frac{(2+k)m\Omega^2}{1+k} \left[r_0 v_0 \left(2\cosh^2 \frac{\Omega t}{\sqrt{1+k}} - 1 \right) - \left(r_0^2 \frac{\Omega}{\sqrt{1+k}} + v_0^2 \frac{\sqrt{1+k}}{\Omega} \right) \cosh \frac{2\Omega t}{\sqrt{1+k}} \right]. \tag{16}$$

As a check, the power absorbed by the merry-go-round motor should be $-dT/dt$ where T is the kinetic energy of the wheel in the lab frame. We recall that the kinetic energy of a rigid body is the sum of the kinetic energy associated with the motion of its center of mass plus the kinetic energy of the motion relative to the center of mass. For the wheel of radius a and thickness d,

$$T_{\rm cm\ motion} = \frac{mv^2}{2} = \frac{m(v_r^2 + v_\theta^2)}{2} = \frac{m(v_r^2 + \Omega^2 r^2)}{2}, \tag{17}$$

and

$$T_{\rm rel\ to\ cm} = \frac{I_w (v_r/a)^2}{2} + \frac{I'_w \Omega^2}{2} \tag{18}$$

where the moment of inertia about the symmetry axis of the wheel is

$$I_w = kma^2, \tag{19}$$

and the moment of inertia about an axis perpendicular to the symmetry axis and which contains the center of the wheel is

$$I'_w = \frac{kma^2}{2} + \frac{md^2}{12}, \tag{20}$$

with $k = 1/2$ for a solid cylinder and $k = 1$, $d = 0$ for a hoop.

When the merry-go-ground is driven with constant angular velocity Ω, we have, recalling eq. (7), that

$$\frac{dT}{dt} = mv_r \frac{dv_r}{dt} + m\Omega^2 r v_r + kma^2 \frac{v_r}{a^2}\frac{dv_r}{dt} = \frac{2+k}{1+k} m\Omega^2 r v_r, \tag{21}$$

which is the negative of eq. (16), as expected.

2.1.2 The Wheel is Driven with Constant Inward Velocity

We now suppose that the wheel has some kind of internal energy source and mechanism that can drive the wheel (inward) with constant speed v_0 in the rotating frame of the platform. Then, the radial position of the center of the wheel is given by

$$r = r_0 - v_0 t. \tag{22}$$

In the lab frame, the kinetic energy of the wheel is given by eqs. (17)-(18), but now with constant radial velocity $v_r = v_0$, The power absorbed by the motor of the merry-go-round to keep the angular velocity Ω constant is now

$$P_{\rm absorbed} = -\frac{dT}{dt} = -m\Omega^2 r v_r = -m\Omega^2 r v_0. \tag{23}$$

In the rotating frame of the platform, the force equation (2) tells us that the radial force of friction of the platform on the wheel is now given by

$$F_{{\rm friction},r} = m\Omega^2 r, \tag{24}$$

and points inward.

The energy source inside the rolling wheel must deliver some power if the wheel is to roll with constant speed while the centrifugal force pushes outward on the wheel (in the rotating frame). The wheel motor is in the rest frame of the center of the wheel, we must consider the work done on the wheel in this frame (the $'$ frame).

The origin of the $'$ frame is accelerated with respect to the lab frame, so the equation of motion (1) includes an additional term (coordinate force) $-m\ddot{\mathbf{r}}_{\rm cm}$ where the acceleration of the center of the wheel in lab frame is given by

$$\ddot{\mathbf{r}}_{\rm cm} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\boldsymbol{\theta}} = -r\Omega^2\,\hat{\mathbf{r}} - 2v_0\Omega\,\hat{\boldsymbol{\theta}}. \tag{25}$$

These terms serve to cancel the centrifugal and Coriolis forces in the $'$ frame (where the center of the wheel is always at rest). The frictional forces do no work in the $'$ frame, but the inward radial friction exerts a torque of magnitude $\tau = aF_{{\rm friction},r} = m\Omega^2 ar$ on the wheel, which must be countered by the motor inside the wheel. The frictional torque tends to decrease the magnitude of the angular velocity ω of the wheel about its symmetry axis, so the countertorque provided by the wheel motor tends to increase the angular velocity, and delivers positive power to the wheel, given by

$$P_{\rm wheel\ motor} = \tau\omega = m\Omega^2 ar(v_0/a) = m\Omega^2 rv_0. \tag{26}$$

This is to be the negative of the power (23) absorbed by the motor that drives the merry-go-round. Thus, power flows from the energy source inside the wheel, through the wheel and into the platform where it ends up absorbed by the merry-go-round motor.

2.2 The Merry-Go-Round Rotates Freely

This problem could be extended by consideration of cases where the merry-go-round is not driven by a motor, but rotates freely. Then the total angular momentum of the system in the lab frame is conserved, and the motion is more intricate than that considered above.

For the time being I don't have the energy to pursue to details of this case further.

Thanks to Mario Pinheiro for e-discussions of this problem.

References

[1] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. 1, Sec. 19-4 (Addison-Wesley, 1964), https://www.feynmanlectures.caltech.edu/I_19.html