Concepts of Mass in General Relativity

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A recent paper [1] discussed three concepts of mass, and the Introduction can lead the reader to suppose that the discussion applies to general relativity.[1] But, the discussion in Ref. [1] is actually limited to Newtonian gravity, where it is generally considered that the three types of mass, "inertial", "active" and "passive", can be taken as equal.

In general relativity one does not speak of forces, but only of positions, velocities and accelerations (in some frame other than the rest frame of the body itself) of a body with (positive) rest mass m. Hence, the concept of "inertial" mass is little used in general relativity.[2] The rest mass of the body also corresponds to the "active" mass of Bondi, which is the source of its gravitational field. The "passive" mass of Bondi is that which an external gravitational field acts upon, but this is not well defined in general, although it equals the (positive) "active" mass for low velocities.

For example, consider a body with (positive) rest mass m moving with velocity close to c in the field of another mass M at rest, with distance $\mathbf{r} = \mathbf{r}_m - \mathbf{r}_M$ (large compared to the Schwarzschild radius $2GM/c^2$, where G is Newton's constant and c is the speed of light) between the two masses. Then, mass m experiences transverse acceleration $\approx 2m\,\mathbf{g} \approx -2mMG\,\hat{\mathbf{r}}/r^2$ when the velocity of m is perpendicular to the vector $\mathbf{r}$ (in so-called isotropic coordinates with source mass M at rest and centered on the origin). In contrast, when the velocity is parallel to $\mathbf{r}$ and $v > c/\sqrt{3}$ the acceleration is away from mass M (antigravity).[3] In the limit as $v \to c$, the acceleration is $-2m\,\mathbf{g}$ for $\mathbf{v} \parallel \mathbf{r}$.

A photon has zero rest mass, but one can define an effective mass for a photon of energy E as $m_{\rm eff} = E/c^2$. One might expect this effective mass to serve as the "active" mass that produces the gravitational field of the photon, which lies only in the plane perpendicular to the motion of the photon (which plane contains the photon). However, it seems that the field far from the photon does not depend on $m_{\rm eff}$ but rather is related to the system that emitted the photon [5]. The effect of an external gravitational field on the photon is the same as that on a massive particle with $v \approx c$, which depends on the direction of motion of the photon with respect the the vector $\mathbf{r}$ from the source of the external field. That is, neither "active" nor "passive" mass are well defined for a photon.

[1] This possible confusion is exacerbated by the title of the 1957 paper of Bondi [2] that first introduced three, rather than two concepts of mass.

[2] June 30, 2026. In special relativity it is sometimes convenient to speak of the "relativistic mass" $\gamma m = m/\sqrt{1 - v^2/c^2}$ of an object with rest mass m and velocity $\mathbf{v}$, where c is the speed of light. Newton's second law is $\mathbf{F} = (d/dt)(\gamma m\mathbf{v})$ so the "relativistic mass" can be regarded as the "inertial" mass. However, this concept is not relevant in general relativity.

[3] This result was first deduced by Hilbert in 1916 [3], but is seldom discussed in textbooks. For a review, see [4].

References

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