Do Past Experiments Exclude Light Majorana Neutrinos?

http://physics.princeton.edu/~mcdonald/examples/majorana.pdf

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1896: H. Becquerel discovered that uranium salts can activate photographic film through black paper, as if penetrating rays are emitted.

1897 M. Curie found similar behavior for thorium, and coined the term "radioactivity."

1898: M. and P. Curie discovered radioactive elements polonium and radium.

1899: E. Rutherford showed that radioactive materials have an exponential decay, and that there are 2 types of radioactivity, alpha (not very penetrating) and beta (more penetrating).

1901: Becquerel showed that beta rays are electrons.

1914: J. Chadwick gave first clear evidence that the energy spectrum of electrons in beta decay is continuous, which implies apparent energy nonconservation.

Spin-½ Particles and Antiparticles



1928: Dirac formulated a relativistic quantum theory of spin- $\frac{1}{2}$ particles, including negative-energy states that he first interpreted as "holes," with "electron holes" having positive charge, so perhaps protons.

P.A.M. Dirac, Proc. Roy. Soc. A 117, 610 (1928) P.A.M. Dirac, Proc. Roy. Soc. A 126, 360 (1930)

Only in 1931 did he identify "electron holes" as the antiparticles of electrons, now called positrons. *P.A.M. Dirac, Proc. Roy. Soc. A* 133, 60 (1931)



1929: Weyl noted that massless spin- $\frac{1}{2}$ states have only 2 independent components in Dirac's theory, in which case the "superfluous" negative-energy states are absent. The remaining 2 components are left- and righthanded. Even for Dirac states with mass, the notion of left- and righthandedness may be useful, said Weyl.

Since mass couples to gravity, Weyl speculated that Dirac states with mass and electric charge may provide a connection between electromagnetism and gravity. In pursuit of this, he introduced the term gauge invariance.

H. Weyl, Proc. Nat. Acad. Sci. 15, 323 (1929)



Neutrinos



1930: Pauli notes that if a new particle is produced in beta decay, this would restore conservation of energy, also Fermi statistics if the particle has spin $\frac{1}{2}$.

This is the first solution to a problem in particle physics by invention of a new particle.

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the "wrong" statistics of the N and 6 Li nuclei and the continuous β -spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $\frac{1}{2}$ and obey the exclusion principle The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous β -spectrum would then become understandable by the assumption that in β -decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant. For the moment, however, I do not dare to publish anything on this idea So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December.



W. Pauli



Neutrons, Neutrinos and Intermediate Bosons

1932: Chadwick "discovered" the neutron, a partner of the proton in nuclei.

J. Chadwick, Proc. Roy. Soc. A 136, 692 (1932)



1932: Heisenberg invents isospin as a symmetry relating the proton and neutron, as a doublet analogous to spin-up and -down.

W. Heisenberg, Z. Phys. 78, 156 (1932)



1934: Fermi named Pauli's new particle the neutrino, and supposed that its mass is much less than that of the electron, perhaps zero. He gave a theory of the (short range) weak interaction of spin- $\frac{1}{2}$ particles as like the electromagnetic interaction, but with a "contact interaction" instead of photon exchange. Fermi. Z. Phys. 88, 161 (1934)



1935: Yukawa conjectured that the short-range interactions, strong and weak, might both be due to exchanges of heavy bosonic quanta. The W[±] bosons are the heavy quanta of the weak interaction, as foreseen by Yukawa (although the strong interaction is short range due to "color confinement" of the massless, colored gluons that mediate this interaction).

H. Yukawa, Proc. Phys.-Math. Soc. Japan, 17, 45 (1935)

Majorana



1937: Majorana gave a "symmetric theory of electrons and positrons," in which there might be no distinction between particles and antiparticles.

He noted that this aspect apparently doesn't apply to charged particles like electrons and positrons, but might apply to neutrinos.

E. Majorana, Nuovo Cimento 14, 171 (1937)

An English translation, by L. Miani, of Majorana's 1937 paper appears on pp. 218-233 of Ettore Majorana Scientific Papers (Springer, 2006).

Rumor: Majorana's paper was not written by him, but by Fermi.

F. Wilczek, Nature Physics 5, 214 (2009)

1941: Pauli commented on Majorana's paper as implying that we should consider states of the form $\frac{\psi+\psi^C}{\sqrt{2}}$, where ψ^C is the charge conjugate (antiparticle) of ψ .

Eqs. (99-100) of W. Pauli, Rev. Mod. Phys. 13, 203 (1941)





Double-Beta Decay



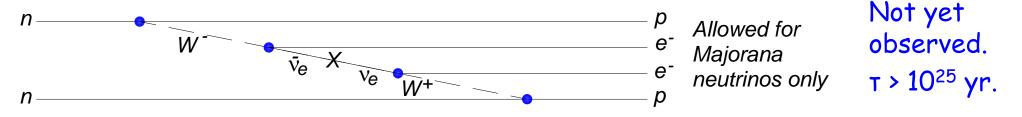
1939: Furry argued that a process which could occur for Majorana neutrinos, but not Dirac neutrinos, is "neutrinoless" double-beta decay,

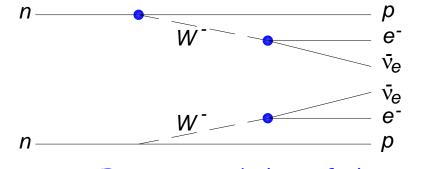
$$(A,Z) \rightarrow (A,Z+2)e^-e^-$$

in contrast to 2-neutrino double-beta decay,

$$(A,Z) \rightarrow (A,Z+2)e^-e^-\bar{v}_e\bar{v}_e$$

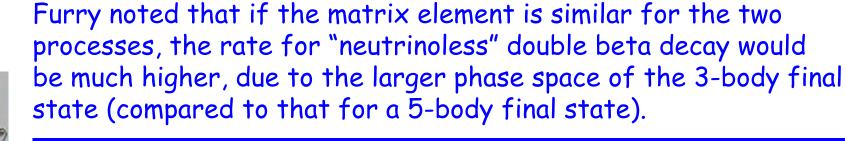
which is allowed for Dirac neutrinos. W.H. Furry, Phys. Rev. 56, 1184 (1939)





Allowed for Dirac or Majorana neutrinos

Several examples observed. $\tau \approx 10^{21} \text{ yr.}$





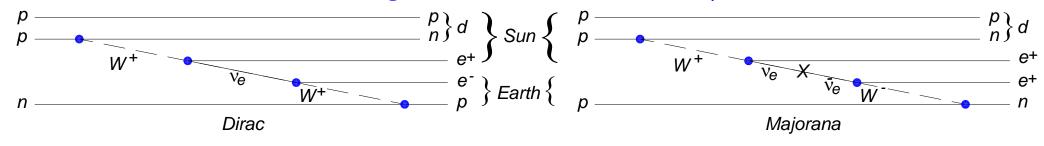
Pontecorvo



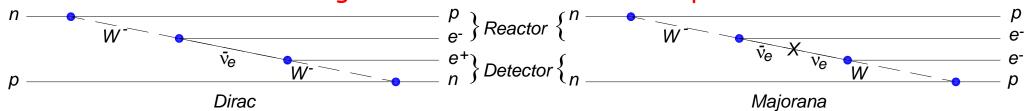
1946: Pontecorvo argued that reactor-neutrino experiments, and solar-neutrino experiments might include reactions possible only via Majorana neutrinos.

B. Pontecorvo, Chalk River PD-205 (1946)

Possible diagrams in solar-neutrino experiments



Possible diagrams in reactor-neutrino experiments



The diagrams on the right, for Majorana neutrinos, have the same form at that for "neutrinoless" double-beta decay, except that the virtual neutrino lives longer.

However, present conventional wisdom is that Majorana neutrinos could not contribute to these "ordinary" reactions.

Davis



1955: Following a suggestion of Pontecorvo, Davis searched for the reaction $v + Cl^{37} \rightarrow Ar^{37} + e^-$

with a detector placed near a nuclear reactor.

He obtained no signal, but remarked that the detector mass (4 tons) was too small for a signal to have been observed, even if the nominal antineutrinos from a reactor were actually neutrinos as per Majorana.

R. Davis Jr, Phys. Rev. 97, 766 (1952)

This version of Davis' experiment has never been repeated.

Davis switched his efforts to the detection of solar neutrinos, deep underground and far from any nuclear reactor, with now-famous results.



Cowan and Reines





1953: Cowan and Reines noted that a better way to detect reactor antineutrinos (produced via the beta decay $n \to p \ e^- \ \bar{v}_e$) is via the inverse-beta-decay process \bar{v}_e p $\rightarrow n e^+$, using a liquid-scintillator detector that first observes the positron, and

then the delayed capture of the thermalized neutron on a nucleus, with

F. Reines and C.L. Cowan Jr, Phys. Rev. **90**, 492 (1953)

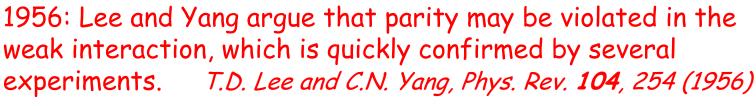
They reported marginal evidence for detection of antineutrinos in 1953, and then more compelling evidence in 1956.

> F. Reines and C.L. Cowan Jr, Phys. Rev. 92, 830 (1953) C.L. Cowan Jr et al., Science 124, 103 (1956)



subsequent emission of γ -rays.

The V-A Theory of the Weak Interaction



1958: Feynman and Gell-Mann reformulate Fermi's vector theory of the weak interaction as V-A, vector - axial vector, which is maximally parity violating.

Only the lefthanded components of spin- $\frac{1}{2}$ particles, and the righthanded components of spin- $\frac{1}{2}$ antiparticles, participate in the weak interaction.

R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)

1n 1967, S. Weinberg and A. Salam recast the V-A theory as a gauge theory, in which the heavy spin-1 quanta, the W^{\pm} and \angle ⁰ bosons, get their mass via the Higgs mechanism (1964).

> P. Higgs, Phys. Rev. Lett. 13, 508 (1964) 5. Weinberg, Phys. Rev. Lett. 19, 1264 (1967)

The W and Z bosons were first observed directly in 1983, and the Higgs (spin-0) boson was observed in 2012.





Chirality and Helicity

The concepts of right- and lefthanded spin- $\frac{1}{2}$ particles mentioned by Weyl in 1929 were formalized in 1957 as chirality states. For a general spin-1/2 4-spinor,

 ψ , its right- and lefthanded components are defined by

$$\psi_R = \frac{1+\gamma^5}{2}\psi$$
, $\psi_L = \frac{1-\gamma^5}{2}\psi$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

S. Watanabe, Phys. Rev. 106, 1306 (1957), K.M. Case, Phys. Rev. 107, 307 (1957)

Helicity states (originally called spirality) are defined by the component of the spin along the direction of motion (so ill-defined for a particle at rest).

I use + and - to indicate helicity states; the positive-helicity state ψ_+ has spin parallel to its momentum p, while the negative helicity state ψ_- has spin antiparallel to its momentum.

An important factoid is that for relativistic states, with $E\gg m$, the chirality and helicity states are approximately the same.

Since the mass of neutrinos is known to be less than 1 eV, neutrino chirality and helicity states are essentially the same in most experiments. An exception is for cosmic-microwave-background neutrinos, that are to be studied in the PTOLEMY

experiment. http://physics.princeton.edu/~mcdonald/examples/neutrinos/tully_060815_ktm.pptx

Chirality and Helicity Antiparticles

Pauli introduced the concept of charge conjugation in 1936, which operation takes a particle to its antiparticle (to within an overall ± sign).

W. Pauli, Ann. Inst. H. Poincaré 6, 106 (1936); Rev. Mod. Phys. 13, 203 (1941)

I write $\psi^{\it C}$ as the antiparticle of ψ .

For spin- $\frac{1}{2}$ particles, I write the spinors of particles, with spacetime dependence $e^{-ipx} = e^{-i(Et-p\cdot x)}$ in case of plane waves, as u, while the symbol for antiparticle spinors, with spacetime dependence e^{ipx} , is v.

For a pair of particle and antiparticle spinors $u=v^{\mathcal{C}}=i\,\gamma^2v^*$, $v=u^{\mathcal{C}}=i\,\gamma^2u^*$, helicity antiparticles are simply related, $u_+ = v_+^{\mathcal{C}} = (v_+)^{\mathcal{C}} = (u^{\mathcal{C}})_+$.

However, since γ^5 anticommutes with γ^2 ,

$$(u_R)^C = i \gamma^2 \frac{1+\gamma^5}{2} u^* = \frac{1-\gamma^5}{2} i \gamma^2 u^* = \frac{1-\gamma^5}{2} v \equiv v_R$$



$$(u^C)_R = \frac{1+\gamma^5}{2}i \gamma^2 u^* = \frac{1+\gamma^5}{2}v \equiv v_L.$$



What is a Majorana State in the V-A Theory?

Majorana neutrinos are in some sense not distinguishable from (the same as) Majorana antineutrinos, and the two are combined into a single state.

But, in the V-A theory, only lefthanded neutrinos, and righthanded antineutrinos participate in the weak interaction.

So it does not make sense (to me) to suppose that a lefthanded neutrino u_L is indistinguishable from (to be combined with) $(u_L)^C = v_L$.

Rather, we should consider that a lefthanded neutrino, u_L ,

is to be combined with $(u^C)_L = v_R$.

K.M. Case, Phys. Rev. 107, 307 (1957)

In other words, the proper way to form a (normalized) lefthanded Majorana neutrino state is to first form a Majorana state for a particle/antiparticle pair $\{u,v\}$, and then take the lefthanded component of this,

$$\psi_L = \left(\frac{u+u^C}{\sqrt{2}}\right)_L = \left(\frac{u+v}{\sqrt{2}}\right)_L = \frac{u_L+v_R}{\sqrt{2}}.$$

A consequence of this is $(\psi_L)^C = \left(\frac{u_L + v_R}{\sqrt{2}}\right)^C = \frac{v_L + u_R}{\sqrt{2}} = \psi_R$,



and that $\{\psi_L,\psi_R\}$ obey the coupled equations $i\gamma^\mu\partial_\mu\,\psi_{L,R}=m\psi_{R,L}$.

Another View

Many books and papers seem to use $\psi'_L = \frac{u_L + (u_L)^C}{\sqrt{2}} = \frac{u_L + v_L}{\sqrt{2}} = (\psi'_L)^C.$ $\psi'_R = \frac{u_R + (u_R)^C}{\sqrt{2}} = \frac{u_L + v_R}{\sqrt{2}} = (\psi'_R)^C.$

Since the lefthanded antineutrino v_L does not participate in the V-A weak interaction, there is no physical difference in single-neutrino interactions of a Dirac lefthanded neutrino u_L or the above Majorana state ψ'_L , except for the normalization factor $1/\sqrt{2}$.

If Majorana states normalized with the factor $1/\sqrt{2}$, single-neutrino interaction rates would be down by $\frac{1}{2}$ compared to those for Dirac neutrinos .

Can fix this by multiplying the electroweak coupling constant g by $\sqrt[4]{2}$.

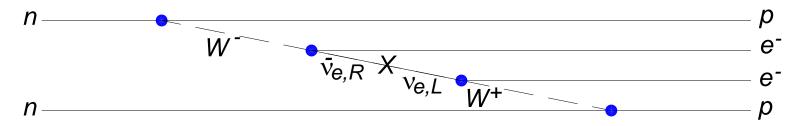
But, then should also multiply g by $\sqrt[4]{2}$ to keep the Weinberg angle $\theta_W = tan^{-1}g'/g$ the same, which would increase the predicted decay width of the Z^0 by $\sqrt{2}$, in disagreement with experiment.



KT McDonald

Majorana Mass Term

"Neutrinoless" double-beta decay would not occur either, in that the righthanded (antineutrino) Majorana state ψ'_R produced at the "first" vertex is distinct from the lefthanded (neutrino) Majorana state ψ'_L needed at the "second" vertex.



This issue is fixed by the so-called Majorana mass term in the Lagrangian, that provides a coupling between the ψ'_L and the ψ'_R , of strength m_v/E_v.

This coupling flips chirality, but with very small amplitude in most experiments.

An exception is cosmic-microwave-background neutrinos, whose energy is \approx 1/4000 eV. These are being searched for in the PTOLEMY experiment.

"Conventional Wisdom"



Where Does the "Conventional Wisdom" Come From?

As far as I call tell, the "conventional wisdom" is not really "derived" anywhere, but is stated as "easy to see by inspection" in an influential paper by Li and Wilczek.

L.F. Li and F. Wilczek, Phys. Rev. D 25, 143 (1982).

From p. 144:

From the fact that X_e , is a Majorana field $X_e^C = X_e$, this means that X_e can produce either e^- or e+, but with different chiralities. Since only the mass term can flip the chirality, in the zero-mass limit, where chirality is the same as the helicity, these two processes involving different chiralities will not interfere with each other and the Majorana field is equivalent to the Dirac field.

The Feynman rules for calculation with Majorana neutrinos of course can be read off from the above. It is then easy to see "by inspection" that even in nuclear decays like $H^3 \rightarrow He^3 + \bar{v} + e^+$, where the \bar{v} is very soft, there will be no detectable difference between Dirac and Majorana neutrinos. Differences only arise the neutrino and antineutrino of opposite chiralities can interfere.



Why Do People Want to Believe the "Conventional Wisdom"

A very interesting idea emerged in the mid 1970's that a possible explanation for the tiny masses of the observed neutrinos ν is that they are Majorana states which are partners with very heavy neutrinos states X, whose mass is at the grand-unification scale.

H. Fritzsch, M. Gell-Mann and P. Minkowski, Phys. Lett. B 59, 256 (1975)

Further, the mass matrix for these neutrinos might have off-diagonal terms of order of the mass of the Higgs boson, with the implication that

$$m_{\nu} \approx \frac{m_{Higgs}^2}{m_{\nu}} \approx \frac{(100 \text{ GeV})^2}{10^{15} \text{ GeV}} = 10^{-11} \text{ GeV} = 0.01 \text{ eV}.$$

This is the famous "see-saw" mechanism.

Clearly, we would like to believe that it is true, so most people find it convenient to accept without much question that it is impossible to determine whether neutrinos are Dirac or Majorana states via existing experiments, except for "neutrinoless" double-beta decay (which experiments are not yet sensitive enough to decide the issue).



Implications of a Majorana Neutrino Chirality State $\psi_L = \frac{u_L + v_R}{\sqrt{2}}$.

If the lefthanded chirality neutrinos that participate in the V-A weak interaction have the form $\psi_L=\frac{u_L+v_R}{\sqrt{2}}$, then many existing experiments exclude this.

A good place to start is the charged-pion decay, $\pi^+ \to \mu_R^+ v$, $\pi^- \to \mu_L^- v$, where the muon is almost at rest in the pion frame, $KE_\mu \approx 4~MeV$.

Then, a lefthanded chirality μ_L^- (or righthanded chirality μ_R^+) has essentially equal probabilities to be either positive or negative helicity.

The pion has spin zero, a lefthanded neutrino has almost pure negative helicity, and a righthanded antineutrino has almost pure positive helicity.

Hence, a neutrino can only appear in the final state together with a negative-helicity muon (and an antineutrino can appear only with a positive helicity muon).

$$\nu_{\mu}$$
 $\bar{\nu}_{\mu}$ $\bar{\nu}_{\mu}$ $\bar{\nu}_{\mu}$ $\bar{\nu}_{\mu}$

There would be essentially equal rates for the two decay modes

$$\pi^+
ightarrow \mu_R^+ \, v$$
, $\pi^+
ightarrow \mu_R^+ \, ar v$, and for the two modes $\pi^-
ightarrow \mu_L^- \, ar v$, $\pi^-
ightarrow \mu_L^- \, ar v$.



Only the first of each pair is allowed for Dirac neutrinos.

Charged-Pion Decay, II

The amplitudes for each decay mode with Majorana neutrinos have a factor of $1/\sqrt{2}$, so the rates for each mode are $\frac{1}{2}$ that for Dirac neutrinos. The total rate for $\pi^{\pm} \to \mu^{\pm} v$ is the same for Dirac or Majorana neutrinos.

However, for Majorana neutrinos, the final state muons have both helicities with essentially equal probability, such that the decay does not appear to be parity violating, even though the decay follows the V-A theory in use of only lefthanded particles and righthanded antiparticles.

Experiments on the decay of muons produced by the decay of pions would not find polarized muons.

This is contradicted by famous results on parity violation from 1957.

R.L. Garwin. L.M. Lederman and M. Weinrich, Phys. Rev. 105, 1415 (1957)

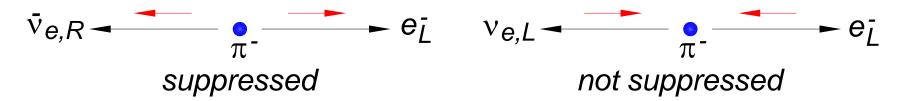
This one result is enough to exclude that muon neutrinos are Majorana states, in my view. However, many more examples of this exclusion can be given.

Charged-Pion Decay, III

In case of Dirac neutrinos, the decays $\pi^- \to e_L^- \, \bar{v}_{e,R}$ and $\pi^- \to \mu_L^- \, \bar{v}_{\mu,R}$ the righthanded antineutrino has positive helicity, and then so must the e_L^- and the μ_L^- to have spin zero in the final state.

This is probable for the nonrelativistic muon, but improbable for the relativistic electron.

For Majorana neutrinos, the $\bar{v}_{e,R}$ could also be a $v_{e,L}$ with negative helicity, so the mode $\pi^- \to e_L^- v_{e,L}$ would not be suppressed, and the branching fraction for pions to electrons or muons would be similar, whereas data show it to be of order $\left(m_e/m_u\right)^2$.



This excludes that electron neutrinos are Majorana states.

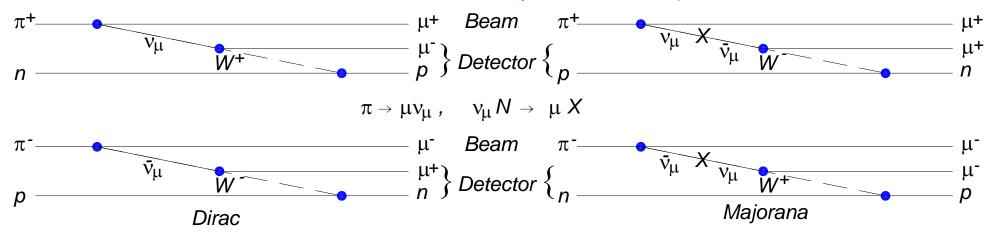


Interaction of Muon Neutrinos from Charged-Pion Decay

In case of Dirac neutrinos, $\bar{v}_{\mu.R}$ are produced in the decay $\pi^- o \mu_L^- \, \bar{v}_{\mu.R}$, while $v_{\mu,L}$ are produced from $\pi^+ \to \mu_R^+ v_{\mu,L}$.

If these neutrinos interact with nuclear targets (with equal numbers of protons and neutrons, as for carbon), then the cross section from muon neutrinos from π^+ is found to be twice that for muon antineutrinos from π^- .

However, for Majorana neutrinos, both π^- and π^+ decays would produce equal numbers of muon neutrinos and antineutrinos, so the cross sections for interactions of neutrinos from these 2 decay would be equal.



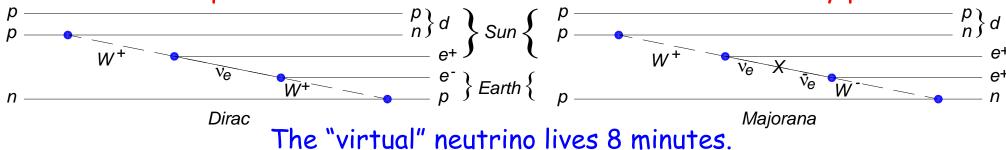
Note that the combination of neutrino production and interaction is a



"neutrinoless" double-beta decay. So "conventional wisdom" should accept that the diagrams on the right exist.

Solar-Neutrino Experiments

Solar-neutrino experiments are also "neutrinoless double-beta decay process.



The nuclear reactions that produce the neutrinos can be either Fermi ($e_R^+ v$ with spin 0) or Gamow-Teller ($e_R^+ v$ with spin 1).

Over most of the energy spectrum, the positrons are relativistic, so Fermi transitions are suppressed for the solar neutrinos, and presumably the transitions are Gamow-Teller.

For Majorana states, the antineutrinos would be suppressed in Gamow-Teller transitions, but favored in Fermi transitions. If Fermi transitions are probable, then the rate of solar Majorana antineutrinos would be roughly the same as for the observed solar neutrinos.

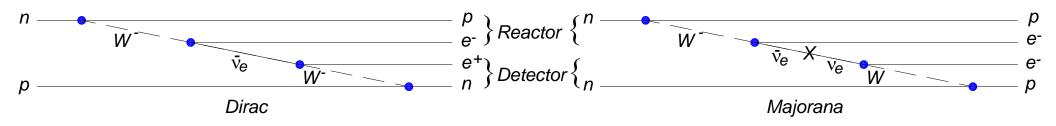
However, no solar antineutrinos have been observed to date.

M. Agostini et al. Phys. Rev. D 92, 031101 (1015)



Reactor-Neutrino Experiments

Reactor-neutrino experiments are also "neutrinoless" double-beta decay process.



The $e_L^- \bar{v}_R$ final state is suppressed for Fermi transitions, but not for Gamow-Teller.

For Majorana neutrinos, a $e_L^- v_L$ final state could occur without suppression in Fermi transitions. However, present reactor-neutrino experiments are not designed to observe the reaction $v_e \ n \to p \ e^-$, which has only a "prompt"-energy signal.

But also, in case of Majorana neutrinos, the production of reactor antineutrinos is only $\frac{1}{2}$ that for Dirac neutrinos (due to the factor $1/\sqrt{2}$ in the Majorana state), so the observed rates of the inverse beta-decay \bar{v}_e p $\rightarrow n$ e^+ would be only $\frac{1}{2}$ of standard expectations. This is excluded by 20 σ in the Daya Bay experiment.

F.P. An et al. Phys. Rev. Lett. 116, 061801 (2016)



Summary

There is an ambiguity in the meaning of Majorana neutrinos in the V-A theory, where only lefthanded chirality states participate. This arises because

$$(u_L)^C = \frac{1+\gamma^5}{2}v = v_L,$$
 $(u^C)_L = \frac{1-\gamma^5}{2}v = v_R.$

for a particle spinor \bar{u} and its antiparticle spinor $v=u^{c}$.

The Majorana state of u and v is $\psi_L = \frac{u+v}{\sqrt{2}}$, which is its own antiparticle.

My lefthanded chirality Majorana state is $\psi_L = \left(\frac{u+v}{\sqrt{2}}\right)_L = \frac{u_L + v_R}{\sqrt{2}} = \frac{u_L + (u^C)_L}{\sqrt{2}}$.

Many present experiments exclude that observed neutrinos are this state.

Others use
$$\psi'_{L} = \frac{u_{L} + (u_{L})^{C}}{\sqrt{2}} = \frac{u_{L} + v_{L}}{\sqrt{2}} = (\psi'_{L})^{C}$$
.

The v_L component of this state does not participate in the weak interaction, so the state ψ'_L cannot be distinguished from the Dirac state u_L , if the electroweak coupling constant g is multiplied by $\sqrt[4]{2}$ to compensate for the $1/\sqrt{2}$ normalization.

But then, must also multiply g' by $\sqrt[4]{2}$ to keep $\theta_W = tan^{-1}g'/g$ the same,





Bonus Puzzler

The spin-0, charged pion decays according to $\pi^+ \to \mu^+ v$, which takes place in the Standard Model via a spin-1 intermediate vector boson W^+ .

Does this mean that angular momentum is not conserved in the Standard Model?



Solution?

As Peter Higgs remarked in his Nobel Lecture,

"... in this model the Goldstone massless (spin-0) mode became the longitudinal polarization of a massive spin-1 photon, just as Anderson had suggested."

P.W. Higgs, Rev. Mod. Phys. 86, 851 (2014)

That is, in the Higgs' mechanism, the $S_z=0$ state of a W^+ .boson is more or less still a spin-0 "particle."

Likewise, Weinberg in his Nobel Lecture stated that: "The missing Goldstone bosons appear instead as helicity-zero states of the vector particles, which thereby acquire a mass."

S. Weinberg, Rev. Mod. Phys. **52**, 515 (1980)

A similar view is given in N. Nakanishi, Mod. Phys. Lett. A 17, 89 (2002).

