Past Experiments
Exclude Light Majorana Neutrino States

Unnati Akhouri
Miranda House, University of Delhi, New Delhi, 110021, India

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

(August 11, 2016; updated March 3, 2021)

1 Problem

In 1937, Majorana [1] remarked that a possible variant Dirac 4-spinor states [2] of electrically neutral, spin-1/2 particles is that they are their own antiparticles, and suggested that neutrinos may be a realization of this in Nature. Discuss how light neutrinos in the Glashow-Weinberg-Salam Standard Model of the electroweak interaction [3, 4, 5] are excluded by existing data from being Majorana states.

2 Solution

A key to the solution is the distinction between Majorana neutrino states (which are their own antiparticles), and Majorana-mass terms (which can apply even to "Dirac" neutrino states).[1] We argue that Majorana states are excluded by present experimental data, but it remains that the (Dirac) neutrino states could be associated with Majorana-mass terms, which permit nonzero rates of neutrinoless double-beta decay, and also the "see-saw" mechanism that could explain the low masses of observed neutrinos in a grand-unified context.

2.1 Majorana States of the Form $(\nu_L + \bar{\nu}_L)/\sqrt{2}$ Are Excluded by Data

To show specific disagreements between the concept of Majorana states of the known, light neutrinos and experimental data, interpreted in the Glashow-Weinberg-Salam model, we need to recall some details about spin-1/2 particle/antiparticle states.[2]

2.1.1 Dirac Spinors

Dirac 4-spinors ψ for a spin-1/2 particle (or antiparticle) of mass m obey Dirac's equation,

$$i\gamma^\mu \partial_\mu \psi = m\psi, \tag{1}$$

(in units with $c = 1 = \hbar$) where the 4×4 matrices γ^μ, $\mu = 0, 1, 2, 3$ have the anticommutation relations,

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} I_4 \qquad \left[\Rightarrow \quad (\gamma^0)^2 = I_4, \quad (\gamma^i)^2 = -I_4\right], \tag{2}$$

[1] This distinction is often ignored, but is emphasized, for example, in sec. 5 of [6].
[2] A more extensive set of notes on this topic by one of the authors is at [7].

the diagonal matrix $\eta^{\mu\nu}$ has diagonal elements $1, -1, -1, -1$, and I_4 is the unit 4×4 matrix.[3] Plane-wave-spinor states with energy $E > 0$, 3-momentum $\mathbf{p}$, 4-momentum $p = p_\mu = (E, \mathbf{p})$ are considered to be particles if their spacetime dependence is $e^{-ipx} = e^{-ip_\mu x^\mu} = e^{i(Et-\mathbf{p}\cdot\mathbf{x})}$, and are called antiparticles if their spacetime dependence is e^{ipx}. The antiparticle states are the "negative-energy" particle states interpreted by Dirac as "holes", *i.e.*, the absence of a particle [9, 10],[4] and later interpreted by Stueckelberg [11, 12] and Feynman [13, 14] as positive-energy particle moving backwards in time. Particle spinors will sometimes be symbolized by u, and antiparticle spinors by v.

2.1.2 Electric-Charge Conjugation

Pauli introduced the concept of electric-charge conjugation of Dirac spinors in 1936 [15, 16, 17],[5] such that, in the Dirac representation,

$$\psi^{(C)} = i\gamma^2\psi^* \tag{4}$$

is the charge-conjugate state to spinor ψ, with the same energy, momentum and spin components as ψ, but if ψ is a particle plane-wave state with spacetime wavefunction e^{-ipx}, then $\psi^{(C)}$ is an antiparticle state with spacetime wavefunction e^{ipx}, and *vice versa.* If state ψ has electric charge q, then $\psi^{(C)}$ has electric charge $-q$.

2.1.3 Majorana States

Pauli noted in eqs. (99)-(100) of [16] that any Dirac 4-spinor ψ can be expressed as the sum of two Majorana states, ψ_1 and ψ_2, which are composed of ψ and its charge-conjugate (antiparticle) state $\psi^{(C)}$ [15, 17],

$$\psi = \frac{\psi_1 + i\psi_2}{\sqrt{2}} \tag{5}$$

$$\psi_1 = \frac{\psi + \psi^{(C)}}{\sqrt{2}}\ , \qquad \psi_2 = \frac{\psi - \psi^{(C)}}{\sqrt{2}\, i}\ , \tag{6}$$

$$\psi^{(C)} = i\gamma^2\psi^*, \qquad \psi_{1,2}^{(C)} = \psi_{1,2}. \tag{7}$$

For the Majorana theory to be more than the identity (5), in view of relations (6)-(7), it must be that the Dirac state ψ is replaced by either Majorana state ψ_1 or ψ_2. Here, we suppose

[3] A particular representation of the γ-matrices was given by Dirac [2], $\gamma^0 = \alpha_4 = \rho_3$ of pp. 614-615 of [2], while $\gamma^j = \alpha_j = \rho_1\sigma_j$,

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \qquad \gamma^j\gamma^k = -i\epsilon^{jkl}\begin{pmatrix} \sigma_l & 0 \\ 0 & \sigma_l \end{pmatrix}, \tag{3}$$

where I_2 is the 2×2 unit matrix, and $\sigma_i,\ i = 1, 2, 3$ are the Pauli spin matrices [8]. Some other representations of the γ^μ are summarized in Appendix A of [7].

[4] Dirac interpreted electron "hole" states as protons in [9], and as what are now called positrons in [10]. The later paper also introduced the Dirac magnetic monopole.

[5] In gauge theories with other kinds of charge than electric charge, following the spirit of Yang and Mills [18] we should consider other charge-conjugation operators within the Glashow-Weinberg-Salam model, as in sec. 2.2 below.

that, in the Majorana theory, a neutrino (or antineutrino) state ψ in the Dirac theory is instead the Majorana state $\psi_1 = (\psi + \psi^{(C)})/\sqrt{2}$.

2.1.4 V−A Theory and Majorana Chirality States

The Glashow-Weinberg-Salam model builds on the V−A theory [19, 20, 21], that only lefthanded-chirality states participate in the weak interaction.[6]

Already in 1929, Weyl [23] had commented that the right- and lefthanded chirality states $\psi_{R,L}$ of Dirac 4-spinors ψ might play an important role in physics.[7] But, because these states are not invariant under space inversion (parity), they were initially not considered to be physically relevant.[8] These states are the eigenstates of the chirality projection operator,

$$P_{R,L} = \frac{I_4 \pm \gamma^5}{2}, \qquad \psi_{R,L} = \frac{I_4 \pm \gamma^5}{2}\psi, \tag{8}$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which anticommutes with the γ^μ: $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$ for $\mu = 0, 1, 2, 3$.[9]

The convention is that (neutrino) particle chirality states $\nu_{R,L}$ are eigenstates of the chirality projection operator $P_{R,L}$, but (neutrino) antiparticle chirality states $\bar{\nu}_{R,L}$ are eigenstates of $P_{L,R}$,

$$\nu_{R,L} = \frac{I_4 \pm \gamma^5}{2}\nu, \qquad \bar{\nu}_{R,L} = \frac{I_4 \mp \gamma^5}{2}\bar{\nu}. \tag{10}$$

In this convention, $\bar{\nu}_{L,R}$ is the antiparticle of $\nu_{L,R}$,

$$(\nu_{R,L})^{(C)} = i\gamma^2\nu^*_{R,L} = i\gamma^2\frac{I_4 \pm \gamma^5}{2}\nu^\star = \frac{I_4 \mp \gamma^5}{2}i\gamma^2\nu^\star = \frac{I_4 \mp \gamma^5}{2}\bar{\nu} = \bar{\nu}_{R,L}, \tag{11}$$

noting that in the Dirac representation,[10] $\gamma^{5\star} = \gamma^5$.

Then, the simplest Majorana chirality states ψ_1 of eq. (6) have the forms,

$$\psi_L = \frac{\nu_L + \nu_L^{(C)}}{\sqrt{2}} = \frac{\nu_L + \overline{\nu}_L}{\sqrt{2}}, \qquad \psi_R = \frac{\nu_R + \nu_R^{(C)}}{\sqrt{2}} = \frac{\nu_R + \overline{\nu}_R}{\sqrt{2}}. \tag{13}$$

---

[6] Righthanded-chirality neutrinos have no interactions in the Standard Model (although they interact gravitationally), and can be called sterile neutrinos [22].

[7] The term chirality as applied to spinors first appeared in [24].

[8] See, for example, p. 226 of [25].

[9] In any representation, $(\gamma^5)^2 = I_4$. In the Dirac representation,

$$\gamma^5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}. \tag{9}$$

[10] Because γ^5 anticommutes with the γ^μ, the chirality states (8) do not obey the Dirac equation (1), but rather the coupled Dirac-like equations, $i\gamma^\mu\partial_\mu\psi_{R,L} = m\psi_{L,R}$, noting that,

$$i\gamma^\mu\partial_\mu\psi_{R,L} = i\gamma^\mu\partial_\mu\frac{I_4 \pm \gamma^5}{2}\psi_{R,L} = \frac{I_4 \mp \gamma^5}{2}i\gamma^\mu\partial_\mu\psi_{R,L} = \frac{I_4 \mp \gamma^5}{2}m\psi_{L,R} = m\psi_{L,R}. \tag{12}$$

A more general form of a Majorana state is,

$$\psi = a\psi_L + b\psi_R = \frac{a\nu_L + a\overline{\nu}_L + b\nu_R + b\overline{\nu}_R}{\sqrt{2}}\,, \tag{14}$$

where the real numbers a and b obey $|a|^2 + |b|^2 = 1$.

2.1.5 $W\pm$ and Z^0 Decay Rates

A key feature is the $1/\sqrt{2}$ in the Majorana states (13), which factor seems to have been neglected in works like [26] that are often cited as showing how processes which could occur with Dirac neutrinos cannot distinguish these from Majorana neutrinos. Instead, if the weak coupling strength is the same in a theory with Dirac neutrinos and in one with Majorana neutrinos, than the amplitude for a reaction with a single neutrino would be $1/\sqrt{2}$ smaller in the Majorana theory.

If the electroweak coupling constants g (associated with the $W^{\pm}$ bosons) and g' (associated with the Z^0 boson) of the Weinberg-Salam theory [4, 5] were the same for Majorana neutrinos as for Dirac neutrinos, then the rates for various single-neutrino reactions for Majorana neutrinos would be 1/2 that for Dirac neutrinos. This is excluded by many experiments. For example, in the theory with Dirac neutrinos, the decay width of the $W^{\pm}$ gauge bosons is roughly 1/3 due to decays to leptons plus neutrinos, and 2/3 due to nonleptonic decays, which prediction agrees with experiment to 2% accuracy (see sec. 10.4.4 of [27]). For Majorana neutrinos the leptonic width would be only 1/2 those for Dirac neutrinos, and the W decay width would be reduced by 16%, which is excluded by 8 standard deviations. Similarly, for Dirac neutrinos the width of the Z^0 gauge boson is predicted to be 1/5 due to $\nu\bar{\nu}$ decays and 4/5 due to other decays, which prediction agrees with data to 0.1%. For Majorana neutrinos, the rate $Z^0 \to \nu\bar{\nu}$ would be 1/4 that for Dirac neutrinos, and the width of the Z would be reduced by 15%, which is excluded by 150 standard deviations.

However, it could be that the electroweak coupling constants g and g' are different in a theory with Majorana neutrinos than one with Dirac neutrinos. This possibility is compatible with Fermi's theory of the weak interaction [61, 62], as was the best view at the time of Majorana, but is inconsistent with experimental data interpreted in the Glashow-Weinberg-Salam model of the electroweak interaction [3, 4, 5]. For example, if g were increased by $\sqrt{2}$ to keep the leptonic decays rates of the $W^{\pm}$ the same for Majorana neutrinos as for Dirac neutrinos, then the nonleptonic rates would be doubled, in conflict with experiment. Further, the ratio of the electroweak coupling constants is constrained by observation of the masses of the weak vector bosons,

$$\frac{m_W}{m_Z} = \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{1}{\sqrt{1 + (g'/g)^2}}\,, \tag{15}$$

where θ_W is the Weinberg angle [4]. Hence, if g is altered in a theory with Majorana neutrinos compared to that with Dirac neutrinos, g' must be altered by the same factor. However, then the rates for the decay of Z^0 to states other than $\nu\bar{\nu}$ would be the changed by the square of that factor, in severe disagreement with experiment.

In sum, the factor of $1/\sqrt{2}$ in a Majorana neutrino state of neutrino and antineutrino implies incompatibilities with experimental data, whether the electroweak coupling constants are the same or different in a theory with Majorana, rather than Dirac neutrinos. Past experiments exclude that the observed light neutrinos are Majorana states of the form (13).

This argument also applies to the general form (14), since such a state always has 50% probability of containing the noninteracting (sterile) neutrinos ν_R and $\overline{\nu}_L$.

2.2 Majorana States of the Form $(\nu_L + \bar{\nu}_R)/\sqrt{2}$ Are Excluded by Data

There is an ambiguity as to what is meant by a Majorana neutrino state in a V−A theory of the weak interaction [19, 20], where the lefthanded coupling is only to the lefthanded neutrino $\nu_L = (1-\gamma^5)\nu/2$ and the righthanded antineutrino $\bar{\nu}_R = (1-\gamma^5)\bar{\nu}/2$. It could be that we should consider a lefthanded Majorana state to be the combination,[11]

$$\psi_L = \frac{1-\gamma^5}{2}\frac{\nu+\overline{\nu}}{\sqrt{2}} = \frac{\nu_L + \bar{\nu}_R}{\sqrt{2}}\,, \tag{16}$$

rather than eq. (13) as in sec. 2.1.4. Then, there would be no coupling to the other Majorana neutrino state, $\psi_R = (\nu_R + \bar{\nu}_L)/\sqrt{2}$.

Indeed, in the Glashow-Weinberg-Salam model, the $\bar{\nu}_R$ is the antiparticle of the ν_L (and the $\bar{\nu}_L$ is the antiparticle of the ν_R). That is, if we introduce the charge-conjugation operator $C_W = \gamma^5 C$ (in the Dirac representation) of the Standard model, then $\bar{\nu}_{R,L} = C_W \nu_{L,R}$,

However, many experiments rule out the possibility that a Majorana state (16) occurs in Nature.

2.2.1 Charged-Pion Decay

In all neutrino experiments to date, except the PTOLEMY search for cosmic microwave background neutrinos [29], the neutrinos are ultrarelativistic ($E_\nu \gg m_\nu$) such that their chirality states are essentially helicity states as well, $\nu_L \approx \nu_-$ and $\tilde{\nu}_R \approx \tilde{\nu}_+$. In the $V-A$ theory with Dirac neutrinos, the ν_L is paired with a righthanded-chirality charge anti lepton $\tilde{l}^+_R$, while the $\tilde{\nu}_R$ is paired with l^+_L, in a production reactions such as $\pi^+ \to \tilde{l}^+_R \nu_L$ and $\pi^- \to l^+_L \tilde{\nu}_R$. In the decay of a spinless charged pion, the lepton and neutrino must have the same helicity. For (relativistic) neutrinos, this restricts the possibilities to $\pi^+ \to \tilde{l}^+_{R,-}\nu_{L,-}$ and $\pi^- \to l^+_{L,+}\tilde{\nu}_{R,+}$, where $\tilde{l}^+_{R,-}$ is a righthanded-chirality, negative-helicity antilepton, *etc.* If the (anti)leptons were relativistic, as for electrons/positrons in pion decay, their right(left)handed-chirality states would be essentially positive(negative) helicity, such that charged-pion decay electrons/positrons is suppressed. In contrast, muons from charged-pion decay have only 4-MeV kinetic energy in the frame of the pion, such that a muon chirality state is roughly an equal mixture of positive and negative helicity states, which permits decay to the "wrong helicity" muons with high probability.

For Majorana neutrinos, charged-pion decay could also proceed via $\pi^+ \to \tilde{l}^+_R \tilde{\nu}_R \approx \tilde{l}^+_{R,+}\tilde{\nu}_{R,+}$ and $\pi^- \to l^+_L \nu_L \approx l^+_{L,-}\nu_{L,-}$, which violate lepton number conservation. Charge-pion decay to electrons/positrons would not be suppressed in these "right helicity" decays,

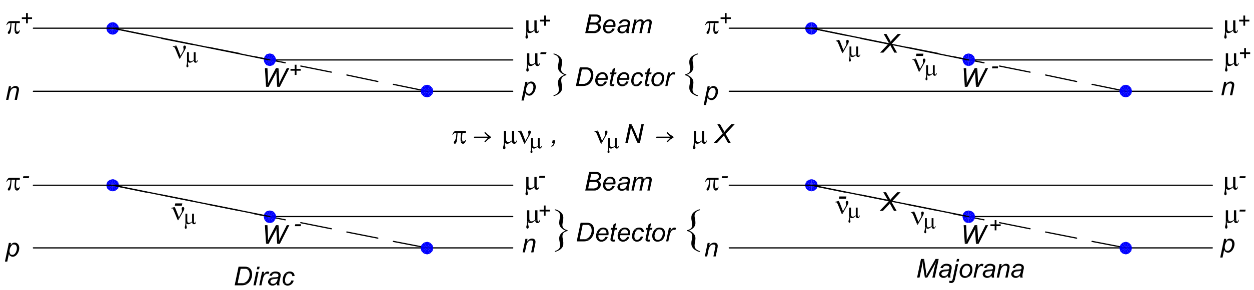
[11] The form (16) was considered in [28], shortly before the V−A theory was devised.

and the decay rates to electrons and muons would be roughly equal, rather than in the observed ratio of $(m_e/m^\mu)^2$ [21].

For Majorana neutrinos, the charge-pion decay $\pi \to \mu\nu$ would have roughly equal numbers of muons with positive and negative helicity, and hence would exhibit little parity violation, in contrast to experiments [30, 31] that observe "maximal" parity violation.

2.2.2 Interactions of Neutrinos from Charged-Pion Decay

In case of Dirac neutrinos, $\pi^+(\pi^-)$-decay leads primarily to $\nu_\mu(\tilde{\nu}_\mu)$, whereas for Majorana neutrinos roughly equal numbers of ν_μ, $\tilde{\nu}_\mu$ and $\tilde{\nu}_e$ ($\tilde{\nu}_\mu$, ν_μ and ν_e) would be produced. The Majorana-neutrino hypothesis conflicts with evidence from "neutrinoless" double-beta-decay experiments with neutrino beams from charged-pion decay (the first beta decay), where in particular charged-current (anti)neutrino-nucleon scattering reactions (the second, inverse beta decay) would have equal cross sections for nominal $\nu_\mu N$ and $\tilde{\nu}_\mu N$ initial states if the neutrinos were Majorana rather than Dirac. Rather, data from a large number of experiments, reviewed in Fig. 49.1 of [32], show that the observed ratio of these cross sections is 2:1, in agreement with models based on Dirac neutrinos.



2.2.3 Leptonic Beta Decay

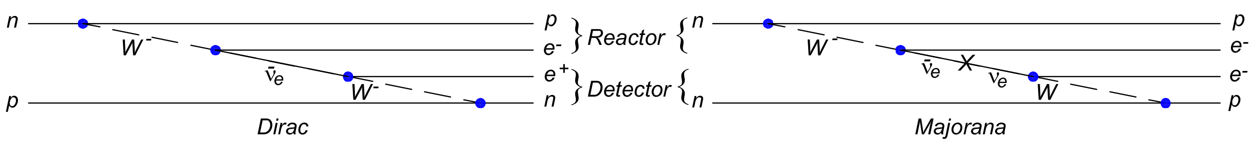
Leptonic beta-decay, such as $\mu^- \to e^-\nu\nu$, can be well-calculated in the Standard $V - A$ theory, where the reaction reads $\mu^- \to e^-\tilde{\nu}_{eR}\nu_{\mu L}$, with results that agree with experiment to within a few percent (as noted already in [19]). If the neutrinos were Majorana states (16), one might infer that the muon lifetime would be 4 times longer than if they are Dirac states, noting that the electron is relativistic in muon decay, such that only one of ν or $\tilde{\nu}$ couples at each vertex in the decay diagram, leading to a factor of 1/2 in the decay amplitude compared to that for Dirac neutrinos. However, as pointed out in the last line of [33], in case of Majorana neutrinos there exists a second, crossed diagram, in which the "second" neutrino comes from the muon vertex rather then the electron vertex, while the "first" neutrino comes from the electron vertex. These two diagrams have equal amplitudes, each 1/2 that of the single diagram in case of Dirac neutrinos, and hence the lifetime of the muon (and Fermi's coupling constant G) is the same for either Dirac or Majorana neutrinos. For additional discussion of this issue, including possible distinguishing effects in the decay angular distribution, see [34, 35].

2.2.4 Helicity of the Neutrino

The neutrino was determined to have negative helicity in an *extrémement ingénieuse* experiment [36] on the electron-capture process ${}_{63}\mathrm{Eu}^{152} + e^- \rightarrow {}_{62}\mathrm{Sm}^{152} + \nu$, in which the neutrino energy is 840 keV and the captured electron was bound by about 100 keV. The lefthanded chirality electron had nearly equal probability to have either positive or negative helicity, such that Majorana neutrinos and antineutrinos would have roughly equal probability of being produced, and the net helicity of the final state neutrino would be near zero. The latter was excluded by about four standard deviations in the experiment, which translates into a similar exclusion of Majorana neutrinos.

2.2.5 Neutrino Experiments at Nuclear Reactors

Turning to nuclear-reactor experiments, in the Standard Model of beta-decay (with Dirac neutrinos) only antineutrinos are produced in reactions of the form $n \rightarrow p\,e^-\tilde{\nu}_e$ (while only neutrinos are produced in solar fusion reactions). Hence, most reactor-based neutrino experiments are designed to observed the inverse beta-decay reaction $\tilde{\nu}_e\, p \rightarrow e^+ n$ [39], where the neutron can be detected as a delayed coincidence after thermalization by capture on nucleus, as first proposed by Cowan and Reines [40, 41]. However, the earliest proposal to detect neutrinos produced in nuclear reactors was due to Pontecorvo [42], who supposed that neutrinos and antineutrinos were not distinct (*i.e.*, were Majorana states), such that reactor neutrinos could lead to reactions such as $\mathrm{Cl}^{37}(\nu, e^-)\mathrm{A}^{37}$, which require a neutrino rather than an antineutrino.

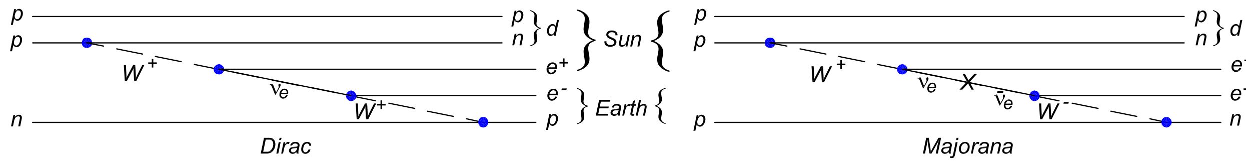


Davis' Reactor-Neutrino Experiment

Pontecorvo's suggestion was implemented by Davis [43, 44] in a 4-m^3 detector close to the Brookhaven Lab nuclear reactor, but only 6 m underground, with a result consistent with the rate of interactions expected from cosmic-ray electron neutrinos. Nonetheless, this result has been considered as evidence against Majorana neutrinos [45]. Davis' reactor-experiment search for Majorana neutrinos has never been repeated, although he went on to make important measurements of solar neutrinos (not antineutrinos) via the chlorine reaction in a 400-m^3 detector deep underground [46].

Solar-Neutrino Experiments

Solar-neutrino experiments other than Davis' have searched for a inverse beta-decay events, $\tilde{\nu}_e\, p \rightarrow e^+ n$, producing limits [47, 48, 49] that the solar-antineutrino flux is small compared to that of solar neutrinos, which appears to exclude that solar neutrinos are Majorana states, although a quantitative assessment of the exclusion remains to be made. Note that solar neutrino experiments are "neutrinoless" double-beta-decay processes in which the "virtual" neutrino lives for about 8 minutes.



Recent Reactor-Neutrino Experiments

Following the success of Cowan and Reines' reactor-antineutrino experiment [50, 51] (which also is a type of "neutrinoless" double-beta-decay process with a "virtual" neutrino in the intermediate state but no neutrino in the final state), and the experimental evidence that the electron and muon are associated with different neutrinos [52], Pontecorvo suggested [22] the possibility of oscillations between the two (or more) neutrino types. Initial experiments used a single detector at a single distance from a nuclear reactor, and compared the rate of detected antineutrino interactions (inverse beta-decay) with expectations based on the neutrino flux as inferred from the reactor power. The first experiment of this type to obtain a reasonably significant result was [53] which reported (1981) a ratio 0.955 ± 0.035 (stat.) ± 0.110 (syst.) of the observed to expected rates of inverse beta-decay reactions. It is now known that for a detector at the optimal distance from the reactor the observed rate would be 0.91 of that expected in the Standard Model assuming no oscillations [54]. A recent result [55], taking into account the existence of neutrino oscillations, found a ratio 0.946 ± 0.022 of the observed rate of inverse beta-decay reactions compared to that of a particular model of the beta-decays in the nuclear reactor complex.

If the neutrinos produced by the nuclear reactor were lefthanded Majorana states (16), for a given flux of such neutrinos the rate of inverse beta-decay $\tilde{\nu}_R\, p \to e^+ n$ would be only 1/2 that from Dirac neutrinos, as lefthanded Majorana neutrinos have only a 50% probability of being a righthanded antineutrino. Hence, reactor experiments that study inverse beta-decay have excluded that the (low-mass) neutrinos produced in nuclear beta decays are Majorana states, by 5 standard deviations in 1981 [53] and more recently by 20 standard deviations [55].

2.3 Comments

Despite lack of experimental evidence for a Majorana character to neutrinos, enthusiasm for this remains high in view of the "see-saw" mechanism [57, 58, 59, 60] that provides a possible explanation for low-mass neutrinos together with partners of mass at the grand-unification scale.

However, the Majorana character required for the see-saw mechanism is not that neutrinos be Majorana states (which are their own antiparticles), but that they are associated with "Majorana-mass" terms, which can exist even for "Dirac" neutrinos.

Furthermore, a nonzero rate for neutrinoless-double-beta decay does not require Majorana neutrino states, but could occur if "Dirac" neutrinos have Majorana-mass terms.[12,13]

Quasiparticles labeled Majorana fermions have been reported in condensed-matter experiments [69, 70]. These nonpropagating "Majorana zero modes" have only one spin state, with a participating electron that is shared between two surfaces of the sample. Such states have been described [71] as "spin-zero half-fermions," that have only electromagnetic interactions, and are rather different entities than the weakly interacting Majorana-neutrino chirality states considered here.

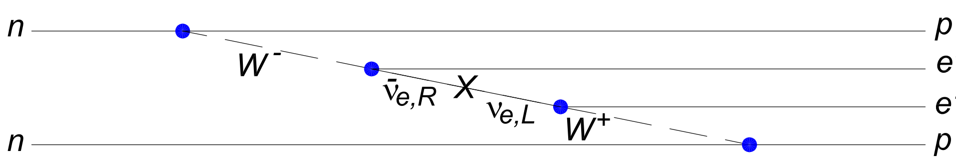
The authors thank Frank Calaprice, Duncan Haldane and Robert Shrock for discussions of this topic.

References

[1] E. Majorana, *Teoria simmetrica dell'elettrone e del positrone*, Nuovo Cim. **14**, 171 (1937), http://kirkmcd.princeton.edu/examples/EP/majorana_nc_14_171_37.pdf.

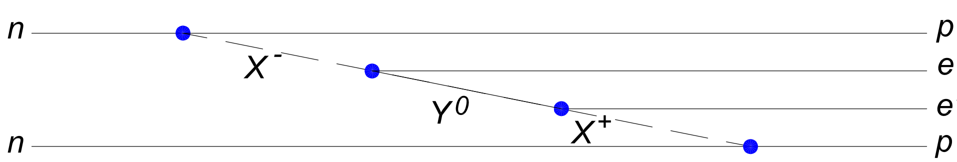
[12]In Majorana's time the theory of the weak interaction was that due to Fermi [61, 62]. An early application of Majorana's vision to Fermi's theory was by Furry [63], who pointed out that Majorana neutrinos permit the phenomenon of "neutrinoless" double-beta decay, $A(Z) \to A'(Z+2)e^-e^-$, of a nucleus $A(Z)$, in addition to the 2-neutrino double-beta decay $A(Z) \to A'(Z+2)e^-e^-\nu\tilde{\nu}$ studied by Goeppert-Mayer [64]. In the Fermi theory, one expects the matrix elements for the two forms of double-beta decay to be similar, but the 5-body final state of 2-neutrino double-beta decay has much smaller phase volume than does the 3-body final state of neutrinoless double-beta decay, so Furry predicted that the latter has much higher rate (shorter lifetime). Present experimental limits [65, 66] on the lifetime for neutrinoless double-beta decay are several orders of magnitude longer than that of observed 2-neutrino double-beta decays.

A diagram for "neutrinoless" double-beta decay appears below, where the X on the virtual-neutrino line indicates that transition from an antineutrino to neutrino.



While this could be due to the antineutrino being part of a Majorana state, it is also accommodated by the existence of a Majorana-mass term. See, for example, [67, 68].

[13]It could also be that there exist $X^\pm$ bosons and Y^0 Majorana fermions that obey a non-gauge theory, in which neutrinoless double-beta decay is possible.



It could be that the Y^0 fermions have low mass as per a see-saw mechanism. But, the $X^\pm$ bosons would have to be heavy, as decays like $n \to pX^- \to ne^-Y^0$ seem not to be observed. Decays like $\pi^\pm \to X^\pm \to \mu^\pm Y^0$ would be forbidden due to violation of conservation of angular momentum, unless the $X^\pm$ bosons had spin 0.

In this case, the rate for the neutrinoless double-beta decay would be heavily suppressed.

Similarly, if the $X^\pm$ bosons were relatively light while the Y^0 fermions were heavy, the decay rate would also be heavily suppressed. If so, neutrinoless double-beta decay is unlikely to be observed soon.

English translation by L. Maiani, *Ettore Majorana Scientific Papers* (Springer-Verlag, 2006), p. 218, http://kirkmcd.princeton.edu/examples/EP/majorana_nc_14_171_37_english.pdf.

[2] P.A.M. Dirac, *The Quantum Theory of the Electron*, Proc. Roy. Soc. London A **117**, 610 (1928), http://kirkmcd.princeton.edu/examples/QED/dirac_prsla_117_610_28.pdf.

[3] S.L. Glashow, *Partial-Symmetries of Weak Interactions*, Nucl. Phys. B **22**, 579 (1961), http://kirkmcd.princeton.edu/examples/EP/glashow_np_22_579_61.pdf

[4] S. Weinberg, *A Model of Leptons*, Phys. Rev. Lett. **19**, 1264 (1967), http://kirkmcd.princeton.edu/examples/EP/weinberg_prl_19_1264_67.pdf

[5] A. Salam, *Weak and electromagnetic interactions*, in *Elementary Particle Theory*, N. Svartholm, ed. (Wiley, 1968), p. 367, http://kirkmcd.princeton.edu/examples/EP/salam_ns_367_68.pdf

[6] R.D. Klauber, *The Seesaw Mechanism*, http://www.quantumfieldtheory.info/TheSeesawMechanism.pdf

[7] K.T. McDonald, *Notes on the Dirac Equation* (Mar. 3, 2021), http://kirkmcd.princeton.edu/examples/dirac.pdf

[8] W. Pauli Jr, *Zur Quantenmechanik des magnetischen Elektrons*, Z. Phys **43**, 601 (1927), http://physics.princeton.edu/~mcdonald/examples/QM/pauli_zp_43_601_27.pdf http://physics.princeton.edu/~mcdonald/examples/QM/pauli_zp_43_601_27_english.pdf

[9] P.A.M. Dirac, *A Theory of Electrons and Protons*, Proc. Roy. Soc. London A **126**, 360 (1930), http://kirkmcd.princeton.edu/examples/QED/dirac_prsla_126_360_30.pdf

[10] P.A.M. Dirac, *Quantised Singularities in the Electromagnetic Field*, Proc. Roy. Soc. London A **133**, 60 (1931), http://kirkmcd.princeton.edu/examples/QED/dirac_prsla_133_60_31.pdf

[11] E.C.G. Stueckelberg, *Remarque à propos de la création de paires de particules en théorie de relativité*, Helv. Phys. Acta **14**, 588 (1941), http://kirkmcd.princeton.edu/examples/QED/stueckelberg_hpa_14_588_41.pdf

[12] E.C.G. Stueckelberg, *La mécanique du point matériel en théorie de relativité et en théorie des quanta*, Helv. Phys. Acta **15**, 23 (1942), http://kirkmcd.princeton.edu/examples/QED/stueckelberg_hpa_15_23_42.pdf

[13] R.P. Feynman, *A Relativistic Cut-Off for Classical Electrodynamics*, Phys. Rev. **74**, 939 (1948), http://kirkmcd.princeton.edu/examples/QED/feynman_pr_74_939_48.pdf

[14] R.P. Feynman, *The Theory of Positrons*, Phys. Rev. **76**, 749 (1949), http://kirkmcd.princeton.edu/examples/QED/feynman_pr_76_749_49.pdf

[15] The operation of charge conjugation for Dirac spinors and the symbol C (but not the term charge conjugation) were introduced on p. 130 of W. Pauli, *Contributions mathématique à la théorie des matrices de Dirac*, Ann. Inst. H. Poincaré **6**, 109 (1936), http://kirkmcd.princeton.edu/examples/QED/pauli_aihp_6_109_36.pdf.

[16] W. Pauli, *Relativistic Field Theories of Elementary Particles*, Rev. Mod. Phys. **13**, 203 (1941), http://kirkmcd.princeton.edu/examples/QED/pauli_rmp_13_203_41.pdf

[17] H.A. Kramers, *The use of charge conjugated wavefunctions in the hole theory of the electron*, Proc. Kön. Neer. Akad. Wet. **40**, 814 (1937), http://kirkmcd.princeton.edu/examples/QED/kramers_pknaw_40_814_37.pdf

[18] C.N. Yang and R.L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Phys. Rev. **96**, 191 (1954), http://kirkmcd.princeton.edu/examples/EP/yang_pr_96_191_54.pdf

[19] R.P. Feynman and M. Gell-Mann, *Theory of the Fermi Interaction*, Phys. Rev. **109**, 193 (1958), http://kirkmcd.princeton.edu/examples/EP/feynman_pr_109_193_58.pdf.

[20] E.C.G. Sudarshan and R.K. Marshak, *Chirality Invariance and the Universal Fermi Interaction*, Phys. Rev. **109**, 1860 (1958), http://kirkmcd.princeton.edu/examples/EP/sudarshan_pr_109_1860_58.pdf.

[21] That the weak interaction might be V and A was inferred earlier from the small ratio $\Gamma_{\pi\to e\nu}/\Gamma_{\pi\to\mu\nu}$ in M. Ruderman and R. Finkelstein, *Note on the Decay of the π-Meson*, Phys. Rev. **76**, 1458 (1949), http://kirkmcd.princeton.edu/examples/EP/ruderman_pr_76_1458_49.pdf.

[22] The notion of a sterile neutrino seems to have been introduced on p. 986 of B. Pontecorvo, *Neutrino Experiments and the Problem of Conservation of Leptonic Charge*, Sov. Phys. JETP **26**, 984 (1968), http://kirkmcd.princeton.edu/examples/neutrinos/pontecorvo_spjetp_26_984_68.pdf.

[23] H. Weyl, *Gravitation and the Electron*, Proc. Nat. Acad. Sci. **15**, 323 (1929), http://kirkmcd.princeton.edu/examples/GR/weyl_pnas_15_323_29.pdf
Elektron und Gravitation. I, Z. Phys. **56**, 330 (1929), http://kirkmcd.princeton.edu/examples/EP/weyl_zp_56_330_29.pdf

[24] S. Watanabe, *Chirality of K Particle*, Phys. Rev. **106**, 1306 (1957), http://kirkmcd.princeton.edu/examples/EP/watanabe_pr_106_1306_57.pdf.

[25] W. Pauli, *Die allgemeinen Prinzipien der Wellenmechanik*, *Handbuch der Physik* **24**, 83 (1933), http://kirkmcd.princeton.edu/examples/QM/pauli_hp_24_83_33.pdf.

[26] L.F. Li and F. Wilczek, *Physical processes involving Majorana neutrinos*, Phys. Rev. D **25**, 143 (1982), http://kirkmcd.princeton.edu/examples/neutrinos/li_prd_25_143_82.pdf.

[27] J. Erler and A. Freitas, *Electroweak model and constraints on new physics*, Particle Data Group (2015), http://pdg.lbl.gov/2016/reviews/rpp2016-rev-standard-model.pdf

[28] K.M. Case, *Reformulation of the Majorana Theory of the Neutrino*, Phys. Rev. **107**, 307 (1957), http://kirkmcd.princeton.edu/examples/neutrinos/case_pr_107_307_57.pdf.

[29] A.J. Long, C. Lunardini and E. Sabancilar, *Detecting non-relativistic cosmic neutrinos by capture on tritium: phenomenology and physics potential*, J. Cosmol. Astro. Phys. **8**, 38 (2014), http://kirkmcd.princeton.edu/examples/neutrinos/long_jcap08_038_14.pdf.

[30] R.L. Garwin, L.M. Lederman and M. Weinrich, *Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*, Phys. Rev. **105**, 1415 (1957),
http://kirkmcd.princeton.edu/examples/EP/garwin_pr_105_1415_57.pdf.

[31] J.I. Friedman and V.L. Telegdi, *Nuclear Emulsion Evidence for Parity Nonconservation in the Decay Chain* $\pi^+ \to \mu^+ \to e^+$, Phys. Rev. **105**, 1681 (1957),
http://kirkmcd.princeton.edu/examples/EP/friedman_pr_105_1681_57.pdf.

[32] G.P. Zeller, *Neutrino Cross Section Measurements*, Particle Data Group (2015),
http://pdg.lbl.gov/2015/reviews/rpp2015-rev-nu-cross-sections.pdf.

[33] R.E. Shrock, *Pure Leptonic Decays with Massive Neutrinos and Arbitrary Lorentz Structure*, Phys. Lett. B **112**, 382 (1982),
http://kirkmcd.princeton.edu/examples/neutrinos/shrock_pl_112b_382_82.pdf.

[34] M. Doi *et al.*, *Majorana Neutrinos and* μ*-Decay*, Prog. Theor. Phys. **67**, 281 (1982),
http://kirkmcd.princeton.edu/examples/neutrinos/doi_ptp_67_281_82.pdf.

[35] M. Doi, T. Kotani and E.T. Akasugi, *Double Beta Decay and Majorana Neutrino*, Prog. Theor. Phys. Suppl. **83**, 1 (1985), Fig. 11.2,
http://kirkmcd.princeton.edu/examples/neutrinos/doi_ptps_83_1_85.pdf.

[36] M. Goldhaber, L. Grodzins and A.W. Sunyar, *Helicity of Neutrinos*, Phys. Rev. **109**, 1015 (1958), http://kirkmcd.princeton.edu/examples/neutrinos/goldhaber_pr_109_1015_58.pdf.
This experiment is deserving of the superlative bestowed by Poincaré [37] on Sommerfeld's analysis [38] of diffraction by a straight edge.

[37] H. Poincaré, *Sur la Polarisation par Diffraction*, Ann. Math. **20**, 313 (1897),
http://kirkmcd.princeton.edu/examples/EM/poincare_am_20_313_97.pdf.

[38] A. Sommerfeld, *Mathematische Theorie der Diffraction*, Math. Ann. **47**, 317 (1896),
http://kirkmcd.princeton.edu/examples/EM/sommerfeld_ma_47_317_96.pdf.

[39] H. Bethe and R. Peierls, *The "Neutrino"*, Nature **133**, 532 (1934),
http://kirkmcd.princeton.edu/examples/neutrinos/bethe_nature_133_532_34.pdf.

[40] F. Reines and C.L. Cowan, Jr, *A Proposed Experiment to Detect the Free Neutrino*, Phys. Rev. **90**, 492 (1953),
http://kirkmcd.princeton.edu/examples/neutrinos/reines_pr_90_493_52.pdf.

[41] C.L. Cowan, Jr *et al.*, *Large Liquid Scintillation Detectors*, Phys. Rev. **90**, 493 (1953),
http://kirkmcd.princeton.edu/examples/neutrinos/reines_pr_90_493_53.pdf.

[42] B. Pontecorvo, *Inverse* β *Process*, Chalk River report PD-205 (1946), in *B. Pontecorvo, Selected Scientific Works* (Societa Italiana di Fisica, Bologna, 1997), p. 21,
http://kirkmcd.princeton.edu/examples/neutrinos/pontecorvo_pd-205_46.pdf.

[43] R. Davis, Jr, *Attempt to Detect the Antineutrinos from a Nuclear Reactor by the $Cl^{37}(\nu, e^-)A^{37}$ Reaction*, Phys. Rev. **97**, 766 (1955), http://kirkmcd.princeton.edu/examples/neutrinos/davis_pr_97_766_55.pdf.

[44] R. Davis, Jr, *An Attempt to Observe the Capture of Reactor Neutrinos in Chlorine-37*, in *Radioiostopes in Scientific Research*, Vol. 1, R.C. Extermann ed. (Pergamon, 1958), p. 728, http://kirkmcd.princeton.edu/examples/neutrinos/davis_rsr_728_58.pdf.

[45] See, for example, p. 4 of Y. Ne'eman, *The Weak Interaction: Past Answers, Present Questions*, Ann. N.Y. Acad. Sci. **294**, 3 (1977), http://kirkmcd.princeton.edu/examples/EP/neeman_anyas_294_3_77.pdf.

[46] R. Davis, Jr, D.S. Harmer and K.C. Hoffmann, *Search for Neutrinos from the Sun*, Phys. Rev. Lett. **20**, 1205 (1968), http://kirkmcd.princeton.edu/examples/neutrinos/davis_prl_20_1205_68.pdf.

[47] Y. Gando *et al.*, *Search for $\bar{\nu}_e$ from the Sun at Super-Kamiokande-I*, Phys. Rev. Lett. **90**, 171302 (2003), http://kirkmcd.princeton.edu/examples/neutrinos/gando_prl_90_171302_03.pdf.

[48] B. Aharmin *et al.*, *Electron antineutrino search at the Sudbury Neutrino Observatory*, Phys. Rev. D **70**, 093014 (2004), http://kirkmcd.princeton.edu/examples/neutrinos/aharmin_prd_70_093014_04.pdf.

[49] A. Ianni *et al.*, *High significance measurement of the terrestrial neutrino flux with the Borexino detector*, J. Phys. Conf. Ser. **718**, 062025 (2016), http://kirkmcd.princeton.edu/examples/neutrinos/ianni_jpcs_718_062025_16.pdf.

[50] C.L. Cowan, Jr *et al.*, *Detection of the Free Neutrino: a Confirmation*, Science **124**, 103 (1956), http://kirkmcd.princeton.edu/examples/neutrinos/cowan_science_124_103_56.pdf.

[51] F. Reines *et al.*, *Detection of the Free Neutrino*, Phys. Rev. **117**, 159 (1960), http://kirkmcd.princeton.edu/examples/neutrinos/reines_pr_117_159_60.pdf.

[52] G. Danby *et al.*, *Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos*, Phys. Rev. Lett. **9**, 36 (1962), http://kirkmcd.princeton.edu/examples/neutrinos/danby_prl_9_36_62.pdf.

[53] H. Kwon *et al.*, *Search for neutrino oscillations at a fission reactor*, Phys. Rev. D **24**, 1097 (1981), http://kirkmcd.princeton.edu/examples/neutrinos/kwon_prd_24_1097_81.pdf.

[54] F.P. An *et al.*, *Observation of Electron-Antineutrino Disappearance at Daya Bay*, Phys. Rev. Lett. **108**, 171803 (2012), http://kirkmcd.princeton.edu/examples/neutrinos/an_prl_108_171803_12.pdf.

[55] F.P. An *et al.*, *Measurement of the Reactor Antineutrino Flux and Spectrum at Daya Bay*, Phys. Rev. Lett. **116**, 061801 (2016), http://kirkmcd.princeton.edu/examples/neutrinos/an_prl_116_061801_16.pdf.

[56] In a model of neutrinoless double-beta decay via the exchange of a virtual, lefthanded Majorana neutrino, the spin-1/2 propagator is combined with two factors of $1 - \gamma^5$, leading to a factor in the matrix element of $(1 - \gamma^5)(\not{q} - m_\nu)(1 - \gamma^5) = -2m_\nu(1 - \gamma^5)$. Hence, the rate includes a factor of $(m_\nu/E_\nu)^2$, where $m_\nu = \sum_i U_{ei}^2 m_i$ in a 3-neutrino scenario with neutrino mass states m_i and MNS mixing matrix $U_{\alpha i}$, and E_ν is a characteristic energy of the virtual neutrino. This factor is small, which would heavily suppress the rate of neutrinoless double-beta decay compared to the early estimate of Furry [63] if Majorana neutrinos existed.

[57] H. Fritzsch, M. Gell-Mann and P. Minkowski, *Vectorlike Weak Currents and New Elementary Fermions*, Phys. Lett. B **59**, 256 (1975),
http://kirkmcd.princeton.edu/examples/EP/fritzsch_pl_59b_256_75.pdf.

[58] H. Fritzsch and P. Minkowski, *Vectorlike Weak Currents, Massive Neutrinos, and Neutrino Beam Oscillations*, Phys. Lett. B **62**, 72 (1976),
http://kirkmcd.princeton.edu/examples/EP/fritzsch_pl_62b_72_76.pdf.

[59] T.P. Cheng, *Hierarchy of lepton masses in a vectorlike theory with Majorana particles*, Phys. Rev. D **14**, 1367 (1976),
http://kirkmcd.princeton.edu/examples/EP/cheng_prd_14_1367_76.pdf.

[60] P. Minkowski, *$\mu \to e\gamma$ at a Rate of One out of 10^9 Muon Decays?*, Phys. Lett. B **67**, 421 (1977), http://kirkmcd.princeton.edu/examples/EP/minkowski_pl_67b_421_77.pdf.

[61] E. Fermi, *Tentativo di una Teoria dei Raggi β*, Nuovo Cim. **11**, 1 (1934),
http://kirkmcd.princeton.edu/examples/neutrinos/fermi_nc_11_1_34.pdf.

[62] E. Fermi, *Versuch einer Theorie der β-Strahlen. I*, Z. Phys. **88**, 161 (1934),
http://kirkmcd.princeton.edu/examples/neutrinos/fermi_zp_88_161_34.pdf.
English translation by F.L. Wilson, Am. J. Phys. **36**, 1150 (1968),
http://kirkmcd.princeton.edu/examples/neutrinos/fermi_zp_88_161_34_english.pdf.

[63] W.H. Furry, *On Transition Probabilities in Double Beta-Disintegration*, Phys. Rev. **56**, 1184 (1939), http://kirkmcd.princeton.edu/examples/EP/furry_pr_56_1184_39.pdf.

[64] M. Goeppert-Mayer, *Double Beta-Disintegration*, Phys. Rev. **48**, 512 (1935),
http://kirkmcd.princeton.edu/examples/EP/goeppert-mayer_pr_48_512_35.pdf.

[65] EXO-200 Collaboration, *Search for Majorana neutrinos with the first two years of EXO-200 data*, Nature **510**, 229 (2014),
http://kirkmcd.princeton.edu/examples/neutrinos/exo-200_nature_510_229_14.pdf.

[66] K. Asakura *et al.*, *Search for double-beta decay of ^{136}Xe to excited states of ^{136}Ba with the KamLAND-Zen experiment*, Nucl. Phys. A **946**, 171 (2016),
http://kirkmcd.princeton.edu/examples/neutrinos/asakura_np_a946_171_16.pdf.

[67] J. Schechter and J.W.F. Valle, *Neutrinoless double-β decay in SU(2) $\times$ SU(1) theories*, Phys. Rev. D **25**, 2951 (1982),
http://kirkmcd.princeton.edu/examples/neutrinos/schechter_prd_25_2951_82.pdf.

[68] S.P. Rosen, *Double Beta Decay* (Oct. 1, 1992), https://arxiv.org/abs/hep-ph/9210202

[69] S. Nadj-Perge *et al.*, *Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor*, Science **346**, 602 (2014),
http://kirkmcd.princeton.edu/examples/CM/nadj-perge_science_346_602_14.pdf.

[70] A. Banerjee *et al.*, *Proximate Kitaev quantum spin liquid behaviour in a honeycomb magnet*, Nature Mat. **15**, 733 (2016),
http://kirkmcd.princeton.edu/examples/CM/banerjee_nm_15_733_16.pdf.

[71] F.D.M. Haldane, private communication.