Kirchhoff's Circuit Node Law

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One of Kirchhoff's famous circuit laws [1, 2] is that the sum of the currents into (or out of) a node/junction in an electrical circuit is zero. This holds for DC circuits because $\boldsymbol{\nabla} \cdot \mathbf{J} = 0$ where $\mathbf{J}$ is the electric-current density. But in general, the divergence of the third Maxwell equation tells us that $\boldsymbol{\nabla} \cdot (\mathbf{J} + \epsilon_0\, \partial \mathbf{E}/\partial t) = 0$. At a circuit node between copper conductors, we have that $\mathbf{J} = \sigma \mathbf{E}$ where the electrical conductivity of copper is $\sigma \approx 6 \times 10^6$ in SI units (Siemens/meter). In an AC circuit with period T, we estimate that $\partial \mathbf{E}/\partial t \approx \pi \mathbf{E}/T$, such that $\boldsymbol{\nabla} \cdot [\mathbf{J}(1 + \pi\epsilon_0/\sigma T)] = 0$. For, say, $T = 10^{-2}$ s, $\pi\epsilon_0/\sigma T \approx 4\pi^2 \times 10^{-7}/(6 \times 10^6 \cdot 10^{-2}) \approx 7 \times 10^{-11}$. This implies that $\boldsymbol{\nabla} \cdot \mathbf{J} = 0$ is a good approximation for such AC circuits, and the correction to Kirchhoff's node law is negligible in practice for typical AC circuits.

For copper circuits at frequencies of 100 GHz the correction to Kirchhoff's circuit node law would be roughly 10 %, while small skin depth at such high frequencies greatly increases resistive losses, both of which effect limit the use of higher frequencies. In nerve cells, the conductivity is $\sigma \approx 1$ S/m while $T \approx 0.1$ s, so the correction to Kirchhoff's circuit node law is of order 4×10^{-5}, which is still rather negligible here.

References

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