Ampère's Hairpin Spaceship

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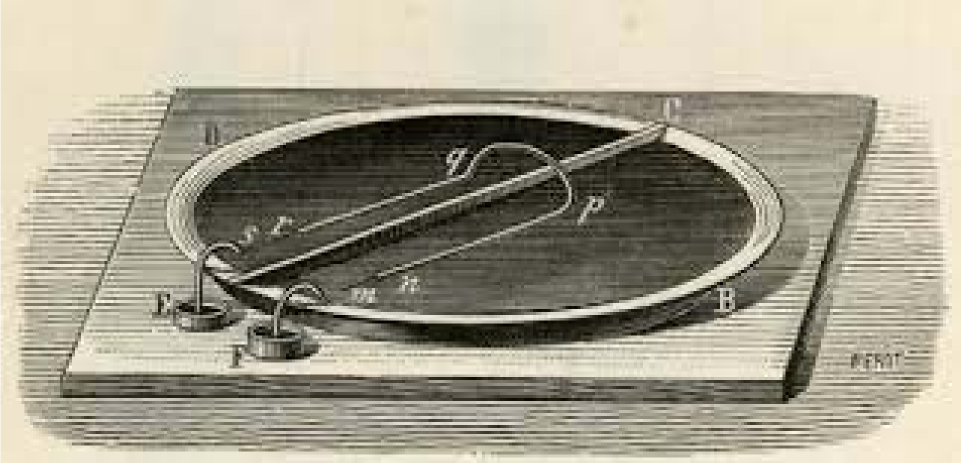
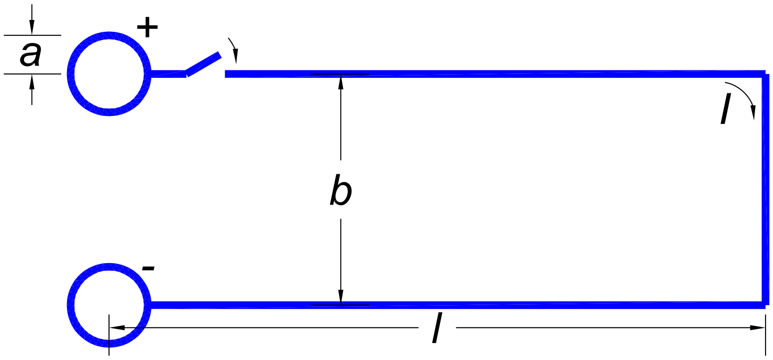
1 Problem

In 1822, Ampère and de La Rive [1] conducted an experiment, illustrated below, in which a conducting "hairpin" *npqr* floated on two troughs of mercury that were connected to the two terminals of a Voltaic pile (battery). The floating hairpin was observed to move away from the end of the circuit where the battery was located (towards point C in the figure).



Consider the variant shown below in which a U-shaped wire of radius r_0, length l, width b and electrical resistance R is connected to a capacitor made from two conducting spheres of radii a with initial charges $\pm Q_0$. Discuss the motion of the system (assumed to be a rigid body) after the switch is closed and the capacitor discharges.



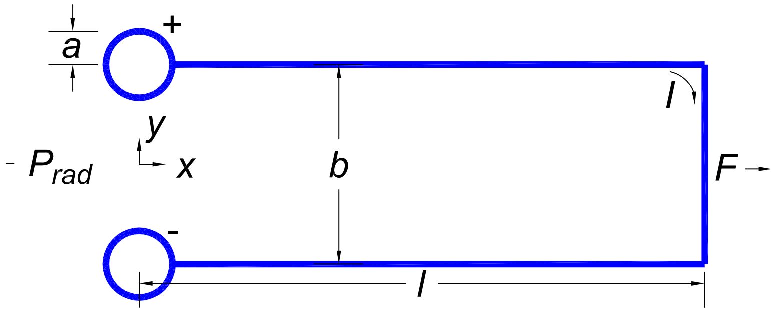
2 Solution

Ampère's hairpin experiment, considered as the first railgun is discussed in [2].

Taking the switch to be closed at time $t = 0$, an electric current $I(t > 0)$ flows between the + and the − spheres, given approximately by,

$$I(t) = \frac{Q_0}{RC}\, e^{-t/RC} \equiv \frac{Q(t)}{RC}\,, \tag{1}$$

where $C \approx a/2$ is the capacitance of the system, and we neglect the transient rise of the current from zero to $\approx Q_0/RC$ over time $\approx L/R$, where $L \approx (4l/c^2)\ln(b/r)$ is the self inductance.



The capacitance of a conducting sphere of radius a is just a (in Gaussian units), so the capacitance of two spheres in series is $a/2$ in the first approximation. See sec. 2(a) of [3] for the next approximation, and [4]) for a discussion of higher approximations.

To estimate the self inductance, we take the spheres be centered at $(x, y, z) = (0, \pm b/2, 0)$, and the crosspiece of the hairpin to be at $(l, -b/2 < y < b/2, 0)$, with $l \gg b \gg a \gg r$. Then, the magnetic field in the plane $z = 0$, between the x-segments and away from their ends, is approximately that associated with infinite-length wires,

$$B_z(b \ll x \ll l - b, -b/2 + r_0 < y < b/2 - r_0, 0) \approx -\frac{2I}{c(b/2 + y)} - \frac{2I}{c(b/2 - y)}\,. \tag{2}$$

The self inductance of the circuit is related to the magnetic flux Φ through it according to $\Phi = cLI$. Supposing the field (2) holds for $0 < x < l$, the magnetic flux is,

$$\Phi \approx 2\int_{r_0}^{b} \frac{2I}{cy'}\, l\, dy' = \frac{4lI}{c} \ln\frac{b}{r_0}, \tag{3}$$

and the self inductance L is approximately (see Art. 685 of [5] for a slightly better approximation),

$$L = \frac{\Phi}{cI} \approx \frac{4l}{c^2} \ln\frac{b}{r_0}\,. \tag{4}$$

2.1 Self Force on the Hairpin

The magnetic field due to the current I exerts a Lorentz force on itself, with an x-component on the crosspiece (and no net force in the y- and z-directions).[1]

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[1] That is, the Lorentz force does not obey Newton's third law (of action and reaction), which this problem was chosen to illustrate.

At the crosspiece, where $x = l$, the z-component of the magnetic field is approximately one half of that given by eq. (2) because the x-currents only for $x < l$. That is,

$$B_z(l, -b/2 + r_0 < y < b/2 - r_0, 0) \approx -\frac{I}{c(b/2 + y)} - \frac{I}{c(b/2 - y)} . \tag{5}$$

The Biot-Savart-Lorentz force on the crosspiece is then approximately,

$$F_x^{\rm c} = -\int_{-b/2+r_0}^{b/2-r_0} \frac{I B_z(x_{\rm c}, y, 0)}{c}\, dy \approx \frac{2I^2}{c^2} \ln\frac{b - r_0}{r_0} \approx \frac{2I^2}{c^2} \ln\frac{b}{r_0} , \tag{6}$$

supposing that the wire radius r_0 is small compared to the separation b.[2]

In addition, the changing magnetic field associated with the changing current leads to an electric field that act on the charges on the capacitor. As we are mainly interest in the x-component of the force, QE_x, we compute $E_x = -\partial A_x/\partial ct$ via the vector potential (ignoring effects of retardation),

$$A_x(0, b/2, 0) \approx \int_a^l \frac{I\, dx}{cx} - \int_a^l \frac{I\, dx}{c\sqrt{x^2 + b^2}} = \frac{I}{c} \ln\frac{l}{a} - \frac{I}{c} \ln\frac{l + \sqrt{l^2 + b^2}}{a + \sqrt{a^2 + b^2}} \approx \frac{I}{c} \ln\frac{b}{a} , \tag{7}$$

for $a \ll b \ll l$. The vector potential at the spheres is mainly due to the current near the spheres, and so is largely independent of the length l of the hairpin.

The force on the positively charge sphere is then, recalling eq. (1),

$$F_x^+ = -\frac{Q}{c}\frac{\partial}{\partial t} A_x(0, b/2, 0) \approx \frac{QI}{c^2 RC} \ln\frac{b}{a} = \frac{I^2}{c^2} \ln\frac{b}{a} . \tag{8}$$

The force F_x^- on the negatively charged sphere is the same as that of eq. (8) since $A_x(0, -b/2, 0) = -A_x(0, b/2, 0)$. The total x-force on the system is largely independent of the length l of the hairpin,

$$F_x = F_x^{\rm c} + F_x^+ + F_x^- \approx \frac{I^2}{c^2} \ln\frac{b^3}{a^2\, r_0} . \tag{9}$$

A circuit of mass m takes on final x-velocity,

$$v_f = \frac{1}{m} \int F_x\, dt \approx \frac{2}{mc^2} \ln\frac{b^3}{a^2\, r_0} \int I^2\, dt \approx \frac{1}{mc^2} \frac{Q^2}{RC} \ln\frac{b^3}{a^2\, r_0} = \frac{2U_C}{mc^2} \frac{1}{R} \ln\frac{b^3}{a^2\, r_0} , \tag{10}$$

where $U_C = Q^2/2C$ is the initial electric energy stored in the capacitor. The final momentum of the circuit is,

$$P = mv_f \approx \frac{1}{c^2} \frac{Q^2}{RC} \ln\frac{b^3}{a^2\, r_0} = \frac{2U_C}{c^2} \frac{1}{R} \ln\frac{b^3}{a^2\, r_0} . \tag{11}$$

Note also that the U-shaped wire is electrically neutral, and so is an “isolated” current element in the sense of Ampère. For such elements the force laws of Ampère and Biot-Savart-Grassman-Lorentz are not equivalent, and the Lorentz force law is the correct on to use. See, for example, [2] for further discussion of these force laws.

[2] Note that even if the current I changes sign as the capacitor discharges, the force is always in the $+x$ direction.

Noting that a resistance R in Gaussian units is the resistance in Ohms divided by $3c$,

$$\frac{v_f}{c} = \frac{6}{R[\Omega]} \frac{U_C}{mc^2} \ln \frac{b^3}{a^2\, r_0}\,, \tag{12}$$

"Practical" values of the parameters can be chosen such that v_f is roughly the escape velocity of the Earth (11 km/s $\approx 3 \times 10^{-4} c$).

2.2 Radiation of Momentum

The nonzero momentum acquired by the hairpin as the capacitor discharges is balanced by an equal and opposite increase in the electromagnetic field momentum,[3] whose density is related by,

$$\mathbf{p}_{\rm EM} = \frac{\mathbf{E} \times \mathbf{B}}{4\pi c} = \frac{\mathbf{S}}{c^2} \tag{13}$$

where $\mathbf{S}$ is the Poynting vector [7]. In some cases, the field momentum remains close to the sources of the electromagnetic fields [8, 9], and in quasistatic examples where the field momentum is nonzero the equal and opposite "mechanical" momentum is often characterized as "hidden" [10]. However, in the present example the electric charge and current densities go to zero with time, and hence so do the near electromagnetic fields and any electromagnetic field momentum temporarily stored in the near fields. Rather, the field momentum is radiated away, and the present example is a kind of antenna problem [11, 12].[4]

The attempt below to model the radiation via analytic approximations is, however, not very satisfactory.

To compute the radiation of momentum by the hairpin as it discharges, we make the approximation that its bulk motion can be neglected, although for extreme parameters this is not a good approximation. We follow sec. 66 of [14] in noting that the radiation (far zone) fields can be deduced from the vector potential $\mathbf{A}$ according to,

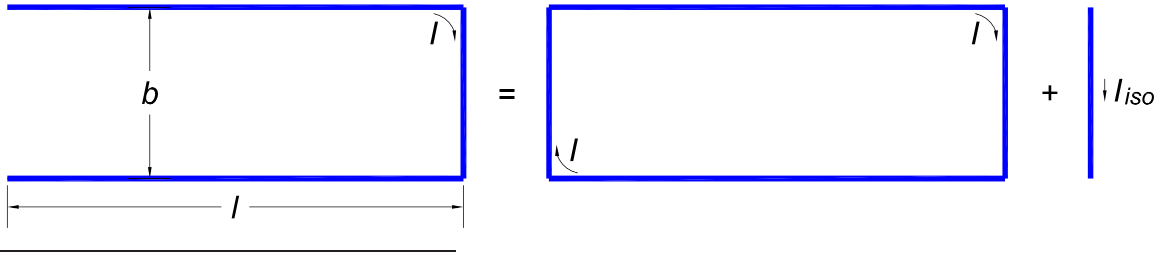
$$c\,\mathbf{B}(\mathbf{r},t) = \left[\dot{\mathbf{A}}\right] \times \hat{\mathbf{r}}, \qquad c\,\mathbf{E}(\mathbf{r},t) = \hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \left[\dot{\mathbf{A}}\right]\right), \tag{14}$$

where,

$$\left[\dot{f}(\mathbf{r},t)\right] = \frac{\partial}{\partial t} f(\mathbf{r}, t - r/c), \tag{15}$$

is the time derivative at the retarded time $t' = t - r/c$.

The vector potential depends on the current in the three segments of the hairpin, which are equivalent to a closed current loop plus an isolated current element that flows from $(0, b/2, 0)$ to $(0, -b/2, 0)$.



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[3] Field momentum of the form (13) was first discussed by J.J. Thomson [6].

[4] For a review of the related topic of rocket propulsion by lasers, see, for example, [13].

The closed loop is a magnetic dipole of moment $\mathbf{m} = -(Ibl/c)\,\hat{\mathbf{z}} = -(Qbl/cRC)\,\hat{\mathbf{z}}$, centered at $(l/2, 0, 0)$, while the isolated current element has a vector potential in the far zone,

$$\mathbf{A}_{\rm iso}(\mathbf{r}, t) = \int \frac{[I]}{cr}\, d\mathbf{l} \approx -\frac{b\,[I]}{cr_{\mathbf{d}}}\,\hat{\mathbf{y}} = -\frac{b\,[Q]}{cr_{\mathbf{d}}\,RC}\,\hat{\mathbf{y}} = -\frac{[\mathbf{d}]}{cr_{\mathbf{d}}\,RC} = \frac{\left[\dot{\mathbf{d}}\right]}{cr_{\mathbf{d}}}\,, \tag{16}$$

where the approximation holds for observers far from the hairpin, $\mathbf{d} = Qb\,\hat{\mathbf{y}}$ is its electric dipole moment. and $r_{\mathbf{d}} = r$ is the distance to the observer from the center $(0, 0, 0)$ of the isolated current element. Altogether, the vector potential is that due to the electric dipole moment $\mathbf{d}$ and the (perpendicular) magnetic dipole moment $\mathbf{m}$. The radiation fields then follow from eq. (71.4) of [14] as,

$$\mathbf{B}_{\rm rad} = \frac{\left[\ddot{\mathbf{d}}\right] \times \hat{\mathbf{r}}_{\mathbf{d}}}{c^2 r_{\mathbf{d}}} + \frac{\hat{\mathbf{r}}_{\mathbf{m}} \times (\hat{\mathbf{r}}_{\mathbf{m}} \times [\ddot{\mathbf{m}}])}{c^2 r_{\mathbf{m}}} = \frac{e^{-[t]/RC}}{c^2 R^2 C^2} \left\{ Qb \frac{\hat{\mathbf{y}} \times \hat{\mathbf{r}}}{r} + \frac{Qbl}{cRC} \frac{\hat{\mathbf{z}} - (\hat{\mathbf{r}}_{\mathbf{m}} \cdot \mathbf{z})\,\hat{\mathbf{r}}_{\mathbf{m}}}{r_{\mathbf{m}}} \right\}, \tag{17}$$

$$\mathbf{E}_{\rm rad} = \frac{\hat{\mathbf{r}}_{\mathbf{d}} \times \left(\hat{\mathbf{r}}_{\mathbf{d}} \times \left[\ddot{\mathbf{d}}\right]\right)}{c^2 r_{\mathbf{d}}} - \frac{[\ddot{\mathbf{m}}] \times \hat{\mathbf{r}}_{\mathbf{m}}}{c^2 r_{\mathbf{m}}} = \frac{e^{-[t]/RC}}{c^2 R^2 C^2} \left\{ Qb \frac{-\hat{\mathbf{y}} + (\hat{\mathbf{r}} \cdot \mathbf{y})\,\hat{\mathbf{r}}}{r} + \frac{Qbl}{cRC} \frac{\hat{\mathbf{z}} \times \hat{\mathbf{r}}_{\mathbf{m}}}{r_{\mathbf{m}}} \right\}, \tag{18}$$

where $\mathbf{r}_{\rm m} = \mathbf{r} - l/2\,\hat{\mathbf{x}}$ is the vector from the center of the magnetic dipole to the observer. We are interested in the radiated momentum, whose density is proportional to $\mathbf{E} \times \mathbf{B}$. However, it is "well known" that the radiation patterns of "simple" dipoles are symmetric and no net momentum is radiated by such dipoles alone. In the present example, radiation of momentum is associated only with the cross terms,

$$\begin{aligned} \mathbf{p}_{\rm rad} &= \frac{\mathbf{E}_{\mathbf{d}} \times \mathbf{B}_{\rm m}}{4\pi c} + \frac{\mathbf{E}_{\mathbf{m}} \times \mathbf{B}_{\rm d}}{4\pi c} \\ &= \frac{Q^2 b^2 l\, e^{-2[t]/RC}}{4\pi c^6 r r_{\rm m} R^5 C^5} \{-\hat{\mathbf{x}} + (\hat{\mathbf{r}}_{\rm m} \cdot \hat{\mathbf{x}})\,\hat{\mathbf{r}} - (\hat{\mathbf{r}}_{\rm m} \cdot \hat{\mathbf{z}})\,\hat{\mathbf{r}}_{\rm m} \times \hat{\mathbf{y}} + (\hat{\mathbf{r}} \cdot \hat{\mathbf{y}})\,\hat{\mathbf{r}} \times \hat{\mathbf{z}} \\ &\qquad - (\hat{\mathbf{r}} \cdot \hat{\mathbf{y}})(\hat{\mathbf{r}}_{\rm m} \cdot \hat{\mathbf{z}})\,\hat{\mathbf{r}} \times \hat{\mathbf{r}}_{\rm m} + (\hat{\mathbf{z}} \cdot \mathbf{r} \times \hat{\mathbf{r}}_{\rm m})\,\hat{\mathbf{y}}\}. \end{aligned} \tag{19}$$

In the limit that $l = 0$, such that $\mathbf{r} = \mathbf{r}_{\rm m}$, we find,

$$\begin{aligned} \mathbf{p}_{\rm rad} &= \frac{Q^2 b^2 l\, e^{-2[t]/RC}}{4\pi c^6 r r_{\rm m} R^5 C^5} \{-\hat{\mathbf{x}} + \cos^2\theta_x\,\hat{\mathbf{x}} + \cos\theta_x \cos\theta_y\,\hat{\mathbf{y}} + \cos\theta_x \cos\theta_z\,\hat{\mathbf{z}} \\ &\qquad - \cos\theta_x \cos\theta_z\,\hat{\mathbf{z}} + \cos^2\theta_z\,\hat{\mathbf{x}} - \cos\theta_x \cos\theta_y\,\hat{\mathbf{y}} + \cos^2\theta_y\,\hat{\mathbf{x}}\} \\ &= 0. \end{aligned} \tag{20}$$

And, to the first approximation, use of $\mathbf{r}_{\rm m} = \mathbf{r} - l/2\,\hat{\mathbf{x}}$ does not lead to a nonzero radiation momentum in the $-\mathbf{x}$ direction. Presumably the various approximation used above are too simplistic to clarify the momentum balance in this example.

References

[1] M. de La Rive fils, *Sur l'Action qu'exerce le globe terrestre sur une portion mobile du circuit voltaïque.* Ann. Chemie Phys. **21**, 24 (1822),
`http://kirkmcd.princeton.edu/examples/EM/ampere_delarive_acp_21_24_22.pdf`

[2] K.T. McDonald, *Capacitor-Driven Railgun: Magnetic Fields Doing Work* (Dec. 28, 2015), `http://kirkmcd.princeton.edu/examples/railgun.pdf`

[3] K.T. McDonald, *Radiation by an AC Voltage Source* (Jan. 5, 2005),
`http://kirkmcd.princeton.edu/examples/acsource.pdf`

[4] J. Lekner, *Capacitance coefficients of two spheres*, J. Electrostat. **11**, 11 (2014), http://kirkmcd.princeton.edu/examples/EM/lekner_je_69_11_11.pdf

[5] J.C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 2 (Clarendon Press, 1873), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v2_73.pdf
Vol. 2, 3rd ed. (Clarendon Press, 1892), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v2_92.pdf

[6] K.T. McDonald, *J.J. Thomson and "Hidden" Momentum* (Apr. 30, 2014), http://kirkmcd.princeton.edu/examples/thomson.pdf

[7] J.H. Poynting, *On the Transfer of Energy in the Electromagnetic Field*, Phil. Trans. Roy. Soc. London **175**, 343 (1884), http://kirkmcd.princeton.edu/examples/EM/poynting_ptrsl_175_343_84.pdf

[8] L. Page and N.I. Adams Jr, *Action and Reaction Between Moving Charges*, Am. J. Phys. **13**, 141 (1945), http://kirkmcd.princeton.edu/examples/EM/page_ajp_13_141_45.pdf

[9] K.T. McDonald, *Onoochin's Paradox* (Jan. 1, 2006), http://kirkmcd.princeton.edu/examples/onoochin.pdf

[10] K.T. McDonald, *On the Definition of "Hidden" Momentum* (July 9, 2012), http://kirkmcd.princeton.edu/examples/hiddendef.pdf

[11] K.T. McDonald, *The Force on an Antenna Array* (May 1, 1979), http://kirkmcd.princeton.edu/examples/antenna_force.pdf

[12] K.T. McDonald, *Tuval's Electromagnetic Spaceship* (Nov. 30, 2015), http://kirkmcd.princeton.edu/examples/tuval.pdf

[13] M.M. Michaelis and A. Forbes, *Laser propulsion: a review*, S. Afr. J. Sci. **102**, 289 (2006), http://kirkmcd.princeton.edu/examples/EM/michaelis_sajs_102_289_06.pdf

[14] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, 4th ed. (Butterworth-Heinemann, 1975), http://kirkmcd.princeton.edu/examples/EM/landau_ctf_71.pdf

[15] P. de Maricourt (Peregrinus), *Epistola ad Sigerum* (1269), http://kirkmcd.princeton.edu/examples/EM/peregrinus_magnet.pdf

[16] W. Gilbert, *De Magnete* (1600), English translation (1893), http://kirkmcd.princeton.edu/examples/EM/gilbert_de_magnete.pdf

[17] R. Descartes, *Principia Philosophiæ* (1644), http://kirkmcd.princeton.edu/examples/EM/descartes_principia.pdf

[18] J. Wallis, *A Summary Account of General Laws of Motion*, Phil. Trans. **3**, 864 (1668), http://kirkmcd.princeton.edu/examples/mechanics/wallis_wren_pt_3_864_68.pdf
C. Wren, *Lex Naturæde Collisione Corporum*, Phil. Trans. **3**, 867 (1668).

[19] C. Hugens, *A Summary Account of the Laws of Motion*, Phil. Trans. **4**, 925 (1669), http://kirkmcd.princeton.edu/examples/mechanics/huyghens_pt_4_925_69.pdf

[20] R. Hooke, *Micrographia: or some Physiological Descriptions of Minute Bodies made by Magnifying Glasses with Observations and Inquiries thereupon* (1667), http://kirkmcd.princeton.edu/examples/EM/hooke_micrographia.pdf

[21] I. Newton, *Philosophiæ Naturalis Principia Mathematica* (1686), http://kirkmcd.princeton.edu/examples/mechanics/newton_principia.pdf

[22] S. Gray, *A Letter containing several Experiments concerning Electricity*, Phil. Trans. **37**, 18 (1731), http://kirkmcd.princeton.edu/examples/EM/gray_pt_37_18_31-32.pdf

[23] P. Dod and Dr. Cookson, *An Account of an extraordinary Effect of Lightning in communicating Magnetism*, Phil. Trans. Roy. Soc. London **39**, 74 (1735), http://kirkmcd.princeton.edu/examples/EM/dod_ptrsl_39_74_35.pdf

[24] J.T. Desaguliers, *Experiments made before the Royal Society, Feb. 2. l737-38*, Phil. Trans. **41**, 193 (1739-1741), http://kirkmcd.princeton.edu/examples/EM/desaguliers_pt_41_193_39-41.pdf

[25] J. Michell, *A Treatise on Artificial Magnets* (Cambridge U. Press, 1750), http://kirkmcd.princeton.edu/examples/EM/michell_magnets.pdf

[26] J. Michell, *On the Means of discovering the Distance, Magnitude, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Distinction Should be found to take place in any of them, and such other Data Should be procured from Observations, as would be farther necessary for that Purpose*, Phil. Trans. Roy. Soc. London **74** , 35 (1783), http://kirkmcd.princeton.edu/examples/GR/michell_ptrsl_74_35_84.pdf

[27] J. Priestly, *The History and Present State of Electricity, with Original Experiments* (1767), http://kirkmcd.princeton.edu/examples/EM/priestly_electricity_67_p731-732.pdf

[28] C.-A. de Coulomb, *Premier Mémoire sur l'Électricité et le Magnétisme*, Mém. Acad. Roy. Sci., 569 (1785), http://kirkmcd.princeton.edu/examples/EM/coulomb_mars_88_569_85.pdf

[29] C.-A. de Coulomb, *Second Mémoire sur l'Électricité et le Magnétisme*, Mém. Acad. Roy. Sci., 578 (1785), http://kirkmcd.princeton.edu/examples/EM/coulomb_mars_88_578_85.pdf

[30] C.-A. de Coulomb, *Collection de Mémoires relatifs a la Physique* (Gauthier-Villars, 1883), http://kirkmcd.princeton.edu/examples/EM/coulomb_memoires.pdf

[31] S. Poisson, *Sur la Distribution de l'Électricité a la Surface des Corps Conducteurs*, Mém. Inst. Imp., 1 (1812), 163 (1813), http://kirkmcd.princeton.edu/examples/EM/poisson_12.pdf

[32] J.C. Örsted, *Experimenta circa Effectum Conflictus Electrici in Acum Magneticam* (pamphlet, Berlin, 1829), http://kirkmcd.princeton.edu/examples/EM/oersted_experimenta.pdf

[33] J.C. Oersted, *Experiments on the Effect of a Current of Electricity on the Magnetic Needle*, Ann. Phil. **16**, 273 (1820),
http://kirkmcd.princeton.edu/examples/EM/oersted_ap_16_273_20.pdf

[34] J.-B. Biot and F. Savart, *Note sur la Magnétisme de la pile de Volta*, Ann. Chem. Phys. **15**, 222 (1820), http://kirkmcd.princeton.edu/examples/EM/biot_acp_15_222_20.pdf
English translation on p. 118 of [128].

[35] A.M. Ampère, *Mémoire sur les effets courans électriques*, Ann. Chem. Phys. **15**, 59 (1820), http://kirkmcd.princeton.edu/examples/EM/ampere_acp_15_59_20.pdf

[36] A.M. Ampère, *Expériences relatives à de nouveaux phénomènes électro-dynamiques*, Ann. Chem. Phys. **20**, 60 (1822),
http://kirkmcd.princeton.edu/examples/EM/ampere_acp_20_60_22.pdf

[37] A.M. Ampère, *Mémoire sur la détermination de la formule qui représente l'action mutuelle de deux portions infiniment petites de conducteurs voltaïques*, Ann. Chem. Phys. **20**, 398, 422 (1822),
http://kirkmcd.princeton.edu/examples/EM/ampere_acp_20_398_22.pdf

[38] F. Savary, *Mémoire sur l'Application du Calcul aus Phénomènes électro-dynamique*, J. Phys. Chem. Hist. Natur. **96**, 1 (1823),
http://kirkmcd.princeton.edu/examples/EM/savary_jpchna_96_1_23.pdf

[39] A.M. Ampère, *Extrait d'un Mémoire sur les Phénomènes électro-dynamiques*, Ann. Chem. Phys. **26**, 134, 246 (1824),
http://kirkmcd.princeton.edu/examples/EM/ampere_acp_26_134_24.pdf

[40] A.M. Ampère, *Description d'un appareil électro-dynamique*, Ann. Chem. Phys. **26**, 390 (1824), http://kirkmcd.princeton.edu/examples/EM/ampere_acp_26_390_24.pdf

[41] J.-B. Biot, *Précis Élémentaire de Physique Expérimentale*, 3rd ed., Vol. 2 (Paris, 1820), pp. 704-774, http://kirkmcd.princeton.edu/examples/EM/biot_precis_24_v2.pdf
English translation on p. 119 of [128].

[42] S. Poisson, *Sur la Théorie du Magnétisme*, Mém. Acad. Roy. Sci., 247, 486 (1824),
http://kirkmcd.princeton.edu/examples/EM/poisson_24.pdf

[43] A.M. Ampère, *Mémoire sur une nouvelle Éxperience électro-dynamique, sur son application à la formule qui représente l'action mutuelle de deux élemens de conducteurs voltaïques, et sur de nouvelle conséquences déduites de cette formule*, Ann. Chem. Phys. **29**, 381 (1825), http://kirkmcd.princeton.edu/examples/EM/ampere_acp_29_381_25.pdf

[44] A.M. Ampère, *Sur la théorie mathématique des phénomènes électro-dynamiques uniquement déduite de l'expérience, dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 et 26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825*, Mém. Acad. Roy. Sci. **8** (1827), http://kirkmcd.princeton.edu/examples/EM/ampere_ars_20-25.pdf

[45] A.M. Ampère, *Théorie mathématique des Phénomènes électro-dynamiques uniquement déduite de l'Expérience*, 2^{nd} ed. (Paris, 1883),
http://kirkmcd.princeton.edu/examples/EM/ampere_theorie_26_83.pdf
English translation in [229].

[46] S. Poisson, *Sur la Théorie du magnétisme en mouvement*, Mém. Acad. Roy. Sci., 441 (1826), http://kirkmcd.princeton.edu/examples/EM/poisson_26.pdf

[47] G. Ohm, *Die galvanishe Kette* (Berlin, 1827),
http://kirkmcd.princeton.edu/examples/EM/ohm_galvanische_kette.pdf
The Galvanic Circuit Mathematically Investigated (Van Nostrand, 1891),
http://kirkmcd.princeton.edu/examples/EM/ohm_galvanic_circuit.pdf

[48] G. Green, *Mathematical Papers* (Macmillan, 1871),
http://kirkmcd.princeton.edu/examples/EM/green_papers.pdf

[49] M. Faraday, *Experimental Researches in Electricity*, Phil. Trans. Roy. Soc. London **122**, 125 (1832), http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_122_163_32.pdf

[50] C.F. Gauss, *Werke*, Vol. 5 (Göttingen, 1867),
http://kirkmcd.princeton.edu/examples/EM/gauss_werke_v5.pdf

[51] M. Faraday, *Experimental Researches in Electricity.—Eleventh Series*, Phil. Trans. Roy. Soc. London **128**, 1 (1838),
http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_128_1_38.pdf

[52] J. MacCullagh, *An Essay toward a dynamical Theory of crystalline Reflexion and Refraction*, Trans. Roy. Irish Acad. **21**, 17 (1839),
http://kirkmcd.princeton.edu/examples/optics/maccullagh_tria_21_17_39.pdf

[53] P.Q.R. (W. Thomson), *On the Uniform Motion of Heat in Homogeneous Solid Bodies, and its Connection with the Mathematical Theory of Electricity*, Cam. Math. J. **3**, 71 (1843), http://kirkmcd.princeton.edu/examples/EM/thomson_cmj_3_71_42.pdf

[54] H. Grassmann, *Neue Theorie der Elektrodynamik*, Ann. d. Phys. **64**, 1 (1845),
http://kirkmcd.princeton.edu/examples/EM/grassmann_ap_64_1_45.pdf
http://kirkmcd.princeton.edu/examples/EM/grassmann_ap_64_1_45_english.pdf

[55] F.E. Neumann, *Allgemeine Gesetze der inducirten elektrischen Ströme*, Abh. König. Akad. Wiss. Berlin **1**, 45 (1845),
http://kirkmcd.princeton.edu/examples/EM/neumann_akawb_1_45.pdf

[56] W. Thomson, *On the Mathematical Theory of Electricity in Equilibrium*, Cam. Dublin Math. J. **1**, 75 (1846), http://kirkmcd.princeton.edu/examples/EM/thomson_cdmj_1_75_46.pdf

[57] M. Faraday, *Experimental Researches in Electricity.—Twentieth Series*, Phil. Trans. Roy. Soc. London **136**, 21 (1846),
http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_136_21_46.pdf

[58] W. Weber *Elektrodynamische Maassbestimmungen*, Abh. König. Sächs. Gesell. Wiss. 209 (1846), http://kirkmcd.princeton.edu/examples/EM/weber_aksgw_209_46.pdf http://kirkmcd.princeton.edu/examples/EM/weber_aksgw_209_46_english.pdf

[59] W. Thomson, *On a Mechanical Representation of Electric, Magnetic, and Galvanic Forces*, Cam. Dublin Math. J. **2**, 61 (1847), http://kirkmcd.princeton.edu/examples/EM/thomson_cdmj_2_61_47.pdf

[60] F.E. Neumann, *Über ein allgemeines Princip der mathematischen Theorie inducirter elektrischer Ströme*, Abh. König. Akad. Wiss. Berlin, 1 (1847), http://kirkmcd.princeton.edu/examples/EM/neumann_akawb_1_47.pdf

[61] W. Thomson, *A Mathematical Theory of Magnetism*, Phil. Trans. Roy. Soc. London **141**, 243, 269 (1851), http://kirkmcd.princeton.edu/examples/EM/thomson_ptrsl_141_243_51.pdf

[62] W. Thomson, *On the Theory of Magnetic Induction in Crystalline and Non-crystalline Substances*, Phil. Mag. **1**, 177 (1851), http://kirkmcd.princeton.edu/examples/EM/thomson_pm_1_177_51.pdf

[63] M. Faraday, *Experimental Researches in Electricity.—Twenty-sixth Series*, Phil. Trans. Roy. Soc. London **141**, 29 (1851), http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_141_21_51.pdf

[64] M. Faraday, *Experimental Researches in Electricity.—Twenty-eighth Series*, Phil. Trans. Roy. Soc. London **142**, 25 (1852), http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_142_25_52.pdf

[65] M. Faraday, *On the Physical Character of the Lines of Magnetic Force*, Phil. Mag. **3**, 401 (1852), http://kirkmcd.princeton.edu/examples/EM/faraday_pm_3_401_52.pdf

[66] M. Faraday, *On some Points of Magnetic Philosophy*, Phil. Mag **9**, 81 (1855), http://kirkmcd.princeton.edu/examples/EM/faraday_pm_9_81_55.pdf

[67] J.C. Maxwell, *On Faraday's Lines of Force*, Trans. Camb. Phil. Soc. **10**, 27 (1956), http://kirkmcd.princeton.edu/examples/EM/maxwell_tcps_10_27_56.pdf

[68] R. Kohlsrausch and W. Weber *Elektrodynamische Maassbestimmungen inbesondere Zurückführung der Stromintensitäts-Messungen auf Mechansishes Maass*, Abh. König. Sächs. Gesell. Wiss. 219 (1856), http://kirkmcd.princeton.edu/examples/EM/weber_aksgw_219_56.pdf

[69] W. Weber and R. Kohlsrausch, *Ueber die Elektricitätsmenge, welche bei galvanischen Strömen durch den Querschnitt der Kette fliesst*, Ann. d. Phys. **99**, 10 (1856), http://kirkmcd.princeton.edu/examples/EM/weber_ap_99_10_56.pdf http://kirkmcd.princeton.edu/examples/EM/weber_ap_99_10_56_english.pdf

[70] J.C. Maxwell, *On Physical Lines of Force*, Phil. Mag. **21**, 161, 281, 388 (1861), http://kirkmcd.princeton.edu/examples/EM/maxwell_pm_21_161_61.pdf

[71] P.G. Tait, *Note on a modification of the apparatus employed for one of Ampère's fundamental experiments in electrodynamics*, Phil. Mag. **21**, 319 (1861), http://kirkmcd.princeton.edu/examples/EM/tait_pm_21_319_61.pdf

[72] J.C. Maxwell, *A Dynamical Theory of the Electromagnetic Field*, Phil. Trans. Roy. Soc. London **155**, 459 (1865), http://kirkmcd.princeton.edu/examples/EM/maxwell_ptrsl_155_459_65.pdf

[73] J. Stefan, *Über die Grundformeln der Elektrodynamik*, Sitzber. Kaiser. Akad. Wiss. Wien **59**, II, 693 (1869), http://kirkmcd.princeton.edu/examples/EM/stefan_skaw_59_2_693_69.pdf

[74] W. de Fonveille, *Thunder and Lightning* (Scribner, 1869), http://kirkmcd.princeton.edu/examples/EM/fonvielle_lightning.pdf

[75] M. Reynard, *Nouvelle Théorie des Actions Électrodynamiques*, Ann. Chem. Phys. **19**, 272 (1870), http://kirkmcd.princeton.edu/examples/EM/reynard_acp_19_272_70.pdf

[76] J.C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1 (Clarendon Press, Oxford, 1873), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v1_73.pdf
Vol. 1, 3rd ed. (Clarendon Press, 1904), http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v1_04.pdf

[77] P.G. Tait, *Note on the Various Possible Expressions for the Force Exerted by an Element of one Linear Conductor on an Element of another*, Proc. Roy. Soc. Edinburgh **8**, 220 (1875), http://kirkmcd.princeton.edu/examples/EM/tait_prse_8_220_75.pdf

[78] E.H. Hall, *On a New Action of the Magnet on Electric Currents*, Am. J. Math. **2**, 287 (1879), http://kirkmcd.princeton.edu/examples/EM/hall_ajm_2_287_79.pdf

[79] J. Stefan, *Über die Abweichungen der Ampère'schen Theorie des Magnetismus von der Theorie der elektromagnetischen Kräfte.*, Sitzber. Kaiser. Akad. Wissensch. Wien **79**, II, 679 (1879), http://kirkmcd.princeton.edu/examples/EM/stefan_skaw_79_2_679_79.pdf

[80] D.J. Korteweg, *Ueber das ponderomotische Elementargesetz*, J. Reine Angew. Math. **90**, 49 (1881), http://kirkmcd.princeton.edu/examples/EM/korteweg_jram_90_49_81.pdf

[81] J.J. Thomson, *On the Electric and Magnetic Effects produced by the Motion of Electrified Bodies*, Phil. Mag. **11**, 229 (1881), http://kirkmcd.princeton.edu/examples/EM/thomson_pm_11_229_81.pdf

[82] G.F. FitzGerald, *On Maxwell's Equations for the Electromagnetic Action of Moving Electricity*, Brit. Assoc. Rep. (1883), http://kirkmcd.princeton.edu/examples/EM/fitzgerald_bar_83.pdf

[83] W. Thomson), *Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light*, (Johns Hopkins 1884); the quotation does not appear in the revised, 1904 edition, http://kirkmcd.princeton.edu/examples/EM/kelvin_baltimore_04.pdf

[84] O. Heaviside, *On the Electromagnetic Effects due to the Motion of Electrification through a Dielectric*, Phil. Mag. **27**, 324 (1889), http://kirkmcd.princeton.edu/examples/EM/heaviside_pm_27_324_89.pdf

[85] J.J. Thomson, *On the Illustration of the Properties of the Electric Field by Means of Tubes of Electrostatic Induction*, Phil. Mag. **31**, 149 (1891), http://kirkmcd.princeton.edu/examples/EM/thomson_pm_31_149_91.pdf

[86] H.A. Lorentz, *La Théorie electromagnétique de Maxwell et son application aux corps mouvants*, Arch. Néer. Sci. Exactes Natur. **25**, 363 (1892), http://kirkmcd.princeton.edu/examples/EM/lorentz_theorie_electromagnetique_92.pdf

[87] J.J. Thomson, *Recent Researches in Electricity and Magnetism* (Clarendon Press, 1893), http://kirkmcd.princeton.edu/examples/EM/thomson_recent_researches_sec_1-16.pdf

[88] *Wilhelm Weber's Werke*, Vol. 3 (Springer, 1893), http://kirkmcd.princeton.edu/examples/EM/weber_werke_v3.pdf

[89] O. Heaviside, *Electrical Papers*, Vol. 1 (Macmillan, 1894), http://kirkmcd.princeton.edu/examples/EM/heaviside_electrical_papers_1.pdf

[90] S.P. Thompson, *Note on a Neglected Experiment of Ampère*, Phil. Mag. **39**, 534 (1895), http://kirkmcd.princeton.edu/examples/EM/thompson_pm_39_534_95.pdf

[91] A. Liénard, *Champ électrique et magnétique produit par une charge électrique contentreé en un point et animée d'un mouvement quelconque*, L'Éclairage Élect. **16**, 5, 53, 106 (1898), http://kirkmcd.princeton.edu/examples/EM/lienard_ee_16_5_98.pdf

[92] E. Wiechert, *Elektrodynamishe Elementargesetze*, Arch. Néerl. **5**, 549 (1900); Ann. Phys. **309**, 667 (1901), http://kirkmcd.princeton.edu/examples/EM/wiechert_ap_309_667_01.pdf

[93] K. Birkeland, *Electromagnetic Gun*, US Patent 754,637 (filed Jan. 2, 1902), http://kirkmcd.princeton.edu/examples/patents/birkeland_us754637_02_gun.pdf

[94] E.F. Northrup, *Some Newly Observed Manifestations of Forces in the Interior of an Electric Conductor*, Phys. Rev. **24**, 474 (1907), http://kirkmcd.princeton.edu/examples/EM/northrup_pr_24_474_07.pdf

[95] C. Hering, *An Imperfection in the Usual Statement of the Fundamental Law of Electromagnetic Induction*, Trans. Am. Inst. Elec. Eng. **27**, 1341 (1908), http://kirkmcd.princeton.edu/examples/EM/hering_taiee_27_1341_08.pdf

[96] E. Whittaker, *A History of the Theories of Aether and Electricity* (Longmans, Green, 1910), p. 89, http://kirkmcd.princeton.edu/examples/EM/whittaker_history.pdf

[97] C.P. Steinmetz, *Mechanical Forces in Magnetic Fields*, Proc. Am. Inst. Elec. Eng. **29**, 1899 (1910), http://kirkmcd.princeton.edu/examples/EM/steinmetz_paiee_29_1899_10.pdf

[98] C. Hering, *The Stretching of a Conductor by Its Current*, J. Franklin Inst. **171**, 73 (1911), http://kirkmcd.princeton.edu/examples/EM/hering_jfi_171_73_11.pdf

[99] C. Hering, *Revision of Some of the Electromagnetic Laws*, J. Franklin Inst. **192**, 599 (1921), http://kirkmcd.princeton.edu/examples/EM/hering_jfi_192_599_21.pdf

[100] C. Hering, *Electrodynamic Forces in Electric Furnaces*, Trans. Am. Electrochem. Soc. **39**, 313 (1921), http://kirkmcd.princeton.edu/examples/EM/hering_taec_39_313_21.pdf

[101] C. Hering, *Electrodynamic Motions in Electric Furnaces*, Trans. Am. Electrochem. Soc. **41**, 303 (1922), http://kirkmcd.princeton.edu/examples/EM/hering_taec_41_303_22.pdf

[102] C. Hering, *Electromagnetic Forces; A Search for More Rational Fundamentals; a Proposed Revision of the Laws*, Trans. Am. Inst. Elec. Eng. **42**, 311 (1923), http://kirkmcd.princeton.edu/examples/EM/hering_taiee_42_311_23.pdf

[103] C. Hering, *Properties of the Single Conductor: New Fundamental Relations*, J. Am. Inst. Elec. Eng. **45**, 31 (1926), http://kirkmcd.princeton.edu/examples/EM/hering_jaiee_45_31_26.pdf

[104] V. Bush, *The Force between Moving Charges*, J. Math. Phys. (MIT) **5**, 129 (1926), http://kirkmcd.princeton.edu/examples/EM/bush_jmp_5_129_26.pdf

[105] W.F. Dunton, *Electromagnetic forces on current-carrying conductors*, J. Sci. Instrum. **4**, 440 (1927), http://kirkmcd.princeton.edu/examples/EM/dunton_jsi_4_440_27.pdf

[106] M. Mason and W. Weaver, *The Electromagnetic Field* (U. Chicago Press, 1929), p. 173, kirkmcd.princeton.edu/examples/EM/mason_emf_29.pdf

[107] F.F. Cleveland, *Magnetic Forces in a Rectangular Circuit*, Phil. Mag. **21**, 416 (1936), http://kirkmcd.princeton.edu/examples/EM/cleveland_pm_21_416_36.pdf

[108] W.F. Dunton, *Validity of Laws of Electrodynamics*, Nature **140**, 245 (1937), http://kirkmcd.princeton.edu/examples/EM/dunton_nature_140_245_37.pdf

[109] A. O'Rahilly, *Electromagnetic Theory* (Longmans, Green, 1938), http://kirkmcd.princeton.edu/examples/EM/orahilly_EM.pdf
See p. 102 for discussion of the equivalence of the force laws of Ampère and of Neumann.

[110] W.F. Dunton, *A Comprehensive Fundamental Electric Formula*, Nature **143**, 601 (1939), http://kirkmcd.princeton.edu/examples/EM/dunton_nature_143_601_39.pdf

[111] S.B.L. Mathur, *Biot-Savart Law and Newton's Third Law of Motion.*, Phil. Mag. **32**, 171 (1941), http://kirkmcd.princeton.edu/examples/EM/mathur_pm_32_171_41.pdf

[112] J.M. Keller, *Newton's Third Law and Electrodynamics*, Am. J. Phys. **10**, 302 (1942), http://kirkmcd.princeton.edu/examples/EM/keller_ajp_10_302_42.pdf

[113] F. Rasetti, *Deflection of mesons in magnetized iron*, Phys. Rev. **66**, 1 (1944), http://kirkmcd.princeton.edu/examples/EM/rasetti_pr_66_1_44.pdf

[114] G.W.O. Howe, *A Problem of Two Electrons and Newton's Third Law*, Wireless Eng. **21**, 105 (1944), http://kirkmcd.princeton.edu/examples/EM/howe_we_21_105_44.pdf

[115] G.W.O. Howe, *The Application of Newton's Third Law to an Electric Circuit*, Wireless Eng. **22**, 521 (1945), http://kirkmcd.princeton.edu/examples/EM/howe_we_22_521_45.pdf

[116] I.A. Robertson, *An Historical Note on a Paradox in Electrodynamics*, Phil. Mag. **36**, 32 (1945), http://kirkmcd.princeton.edu/examples/EM/robertson_pm_36_32_45.pdf

[117] W.A. Tripp, *A Paradox in Electrodynamics*, Phil. Mag. **38**, 149 (1947), http://kirkmcd.princeton.edu/examples/EM/tripp_pm_38_149_47.pdf

[118] G.W.O. Howe, *The Application of Newton's Third Law to an Electric Circuit*, Wireless Eng. **29**, 83 (1952), http://kirkmcd.princeton.edu/examples/EM/howe_we_29_83_52.pdf

[119] A. Sommerfeld, *Electrodynamics* (Academic Press, 1952).

[120] P. Moon and D.E. Spencer, *Interpretation of the Ampère Experiments*, J. Franklin Inst. **257**, 209 (1954), http://kirkmcd.princeton.edu/examples/EM/moon_jfi_257_209_54.pdf

[121] P. Moon and D.E. Spencer, *The Coulomb Force and the Ampère Force*, J. Franklin Inst. **257**, 305 (1954), http://kirkmcd.princeton.edu/examples/EM/moon_jfi_257_305_54.pdf

[122] P. Moon and D.E. Spencer, *A New Electrodynamics*, J. Franklin Inst. **257**, 369 (1954), http://kirkmcd.princeton.edu/examples/EM/moon_jfi_257_369_54.pdf

[123] W.K.H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, 1995; 2nd ed. 1962), http://kirkmcd.princeton.edu/examples/EM/panofsky-phillips.pdf

[124] G. Builder, *Electrodynamics*, Bull. Inst. Phys. **9**, 311 (1958), http://kirkmcd.princeton.edu/examples/EM/builder_bip_9_311_58.pdf

[125] R.A.R. Tricker, *Ampère as a Contemporary Physicist*, Contemp. Phys. **3**, 453 (1963), http://kirkmcd.princeton.edu/examples/EM/tricker_cp_3_453_62.pdf

[126] W.V.G. Rosser, *Electromagnetism as a Second Order Effect III. The Biot-Savart Law*, Contemp. Phys. **3**, 28 (1963), http://kirkmcd.princeton.edu/examples/EM/rosser_cp_3_28_61.pdf

[127] R.C. Lyness, *The Equivalence of Ampère's Electrodynamic Law and that of Biot and Savart*, Contemp. Phys. **4**, 453 (1963), http://kirkmcd.princeton.edu/examples/EM/lyness_cp_4_453_63.pdf

[128] R.A.R. Tricker, *Early Electrodynamics, the First Law of Circulation* (Pergamon, 1965), http://kirkmcd.princeton.edu/examples/EM/tricker_early_em.pdf

[129] P. Graneau, *The Interaction of Current Elements*, Int. J. Elec. **20**, 351 (1966), http://kirkmcd.princeton.edu/examples/EM/graneau_ije_20_351_66.pdf

[130] A.C. English, *Force on a Wire in a Magnetic Field*, Am. J. Phys. **35**, 326 (1967), http://kirkmcd.princeton.edu/examples/EM/english_ajp_35_326_67.pdf

[131] E. Breitenberger, *Magnetic Interactions between Charged Particles*, Am. J. Phys. **36**, 505 (1968), http://kirkmcd.princeton.edu/examples/EM/breitenberger_ajp_36_505_68.pdf

[132] H. Aspden, *The Law of Electrodynamics*, J. Franklin Inst. **287**, 179 (1969), http://kirkmcd.princeton.edu/examples/EM/aspden_jfi_287_179_69.pdf

[133] E.P. Mosca, *Magnetic Forces Doing Work?* Am. J. Phys. **42**, 295 (1974), http://kirkmcd.princeton.edu/examples/EM/mosca_ajp_42_295_74.pdf

[134] J.D. Jackson, *The Nature of Intrinsic Magnetic Dipole Moments*, CERN-77-17 (1977), http://kirkmcd.princeton.edu/examples/EM/jackson_CERN-77-17.pdf

[135] S.C. Rashleigh and R.A. Marshall, *Electromagnetic acceleration of macroparticles to high velocities*, J. Appl. Phys. **49**, 2540 (1978), http://kirkmcd.princeton.edu/examples/EM/rashleigh_jap_49_2540_78.pdf

[136] C.A. Coombes, *Work done on charged particles in magnetic fields*, Am. J. Phys. **47**, 915 (1979), http://kirkmcd.princeton.edu/examples/EM/coombes_ajp_47_915_79.pdf

[137] J.L. Heilbron, *Electricity in the 17th and 18th Centuries* (U. Calif. Press, 1979; Dover, 1999).

[138] W.R. McKinnon, S.P. McAlister and C.M. Hurd, *Origin of the force on a current-carrying wire in a magnetic field*, Am. J. Phys. **49**, 493 (1981), http://kirkmcd.princeton.edu/examples/EM/mckinnon_ajp_49_493_81.pdf

[139] M.O. Peach and L.K. Shirely, *What Ampère could have done: A simple demonstration of the Biot-Savart and magnetic force laws*, Am. J. Phys. **50**, 410 (1982), http://kirkmcd.princeton.edu/examples/EM/peach_ajp_50_410_82.pdf

[140] P. Graneau, *Electromagnetic jet-propulsion in the direction of current flow*, Nature **295**, 311 (1982), http://kirkmcd.princeton.edu/examples/EM/graneau_nature_295_311_82.pdf

[141] P. Graneau, *Application of Ampere's force law to railgun accelerators*, J. Appl. Phys. **53**, 5548 (1982), http://kirkmcd.princeton.edu/examples/EM/graneau_jap_53_6648_82.pdf

[142] P. Graneau, *First Indication of Ampere Tension in Solid Electric Conductors*, Phys. Lett. **97A**, 253 (1983), http://kirkmcd.princeton.edu/examples/EM/graneau_pl_97a_253_83.pdf

[143] A.M. Hillas, *Electromagnetic jet-propulsion: non-lorentzian forces on currents?* Nature **302**, 271 (1983), http://kirkmcd.princeton.edu/examples/EM/hillas_nature_302_271_83.pdf

[144] P. Graneau, *Graneau Replies*, Nature **302**, 271 (1983), http://kirkmcd.princeton.edu/examples/EM/hillas_nature_302_271_83.pdf

[145] P.T. Pappas, *The Original Ampère Force and Biot-Savart and Lorentz Forces*, Nuovo Cim. **76B**, 189 (1983), http://kirkmcd.princeton.edu/examples/EM/pappas_nc_76b_189_83.pdf

[146] P. Graneau, *Compatibility of the Ampère and Lorentz Force Laws with the Virtual-Work Concept*, Nuovo Cim. **78B**, 213 (1983), http://kirkmcd.princeton.edu/examples/EM/graneau_nc_78b_213_83.pdf

[147] L.P. Williams, *What were Ampère's Earliest Discoveries in Electrodynamics?* Isis **74**, 492 (1983), http://kirkmcd.princeton.edu/examples/EM/williams_isis_74_492_83.pdf

[148] V. Namias, *Induced current effects in Faraday's law and introduction to flux compression theories*, Am. J. Phys. **54**, 57 (1986), http://kirkmcd.princeton.edu/examples/EM/namias_ajp_54_57_86.pdf

[149] P. Graneau, *Longitudinal magnet forces?* J. Appl. Phys. **55**, 2598 (1984), http://kirkmcd.princeton.edu/examples/EM/graneau_jap_55_2598_84.pdf

[150] P. Graneau, *Ampere Tension in Electric Conductors*, IEEE Trans. Mag. **20**, 444 (1984), http://kirkmcd.princeton.edu/examples/EM/graneau_ieeetm_20_444_84.pdf

[151] J. Nasilowski, *Ampere Tension in Electric Conductors*, IEEE Trans. Mag. **20**, 2158 (1984), http://kirkmcd.princeton.edu/examples/EM/nasilowksi_ieeetm_20_2158_84.pdf

[152] K.H. Carpenter, *On the Nonexistence of "Ampere Tension" in Electric Conductors*, IEEE Trans. Mag. **20**, 2159 (1984), http://kirkmcd.princeton.edu/examples/EM/carpenter_ieeetm_20_2159_84.pdf

[153] P.G. Moyssides and P.T. Pappas, *Rigorous quantitative test of Biot-Savart-Lorentz forces*, J. Appl. Phys. **59**, 19 (1986), http://kirkmcd.princeton.edu/examples/EM/moyssides_jap_59_19_86.pdf

[154] P. Graneau, *Reply by Peter Graneau*, IEEE Trans. Mag. **20**, 2159 (1984), http://kirkmcd.princeton.edu/examples/EM/carpenter_ieeetm_20_2159_84.pdf

[155] J.G. Ternan, *Equivalence of the Lorentz and Ampère force laws in magnetostatics*, J. Appl. Phys. **57**, 1743 (1985), http://kirkmcd.princeton.edu/examples/EM/ternan_jap_57_1743_85.pdf

[156] D.C. Jolly, *Identity of the Ampere and Biot-Savart Electromagnetic Force Laws*, Phys. Lett. **107A**, 231 (1985), http://kirkmcd.princeton.edu/examples/EM/jolly_pl_107a_231_85.pdf

[157] P. Graneau and P.N. Graneau, *Electrodynamic explosions in Liquids*, Appl. Phys. Lett. **46**, 468 (1985), http://kirkmcd.princeton.edu/examples/EM/graneau_apl_46_468_85.pdf

[158] P. Graneau, *Ampere and Lorentz Forces*, Phys. Lett. **107A**, 235 (1985), http://kirkmcd.princeton.edu/examples/EM/graneau_pl_107a_235_85.pdf

[159] P. Graneau, *Comments on "Equivalence of the Lorentz and Ampère force laws in magnetostatics,"* J. Appl. Phys. **58**, 3638 (1985), http://kirkmcd.princeton.edu/examples/EM/graneau_jap_58_3638_85.pdf

[160] J.G. Ternan, *Reply to "Comments on 'Equivalence of the Lorentz and Ampère force laws in magnetostatics,' "* J. Appl. Phys. **58**, 3639 (1985), http://kirkmcd.princeton.edu/examples/EM/ternan_jap_58_3639_85.pdf

[161] P.T. Pappas and P.G. Moyssides, *On the Fundamental Laws of Electrodynamics*, Phys. Lett. **111A**, 193 (1985), http://kirkmcd.princeton.edu/examples/EM/pappas_pl_111a_193_85.pdf

[162] L.P. Williams, *Why Ampère did not discover electromagnetic induction*, Am. J. Phys. **54**, 306 (1986), http://kirkmcd.princeton.edu/examples/EM/williams_ajp_54_306_86.pdf

[163] J.G. Ternan, *Stresses in Rapidly Heated Wires*, Phys. Lett. **115A**, 230 (1986), http://kirkmcd.princeton.edu/examples/EM/ternan_pl_115a_230_86.pdf

[164] P. Graneau, *The Ampere-Neumann Electrodynamics of Metallic Conductor*, Fortschr. Phys. **34**, 457 (1986), http://kirkmcd.princeton.edu/examples/EM/graneau_fp_34_457_86.pdf

[165] P. Graneau and P.N. Graneau, *The Electromagnetic Impulse Pendulum and Momentum Conservation*, Nuovo Cim. **7D**, 31 (1986), http://kirkmcd.princeton.edu/examples/EM/graneau_nc_7d_31_86.pdf

[166] P. Graneau, *Railgun recoil and relativity*, J. Phys. D **20**, 391 (1987), http://kirkmcd.princeton.edu/examples/EM/graneau_jpd_20_391_87.pdf

[167] P. Graneau, *Wire Explosions*, Phys. Lett. **120A**, 77 (1987), http://kirkmcd.princeton.edu/examples/EM/graneau_pl_120a_77_87.pdf

[168] P. Graneau, *Amperian recoil and the efficiency of railguns*, J. Appl. Phys. **62**, 3006 (1987), http://kirkmcd.princeton.edu/examples/EM/graneau_jap_62_3006_87.pdf

[169] C. Christodoulides, *Equivalence of the Ampere and Biot-Savart force laws in magnetostatics*, J. Phys. A **20**, 2037 (1987), http://kirkmcd.princeton.edu/examples/EM/christodoulides_jpa_20_2037_87.pdf

[170] C. Christodoulides, *Are longitudinal forces predicted in magnetostatics by the Ampère force law in its line-current-element form?* Phys. Lett. **120A**, 120 (1987), http://kirkmcd.princeton.edu/examples/EM/christodoulides_pl_120a_129_87.pdf

[171] J. Nasilowski, *Comment on Ampere's Hairpin Experiment*, IEEE Trans. Mag. **24**, 3260 (1988), http://kirkmcd.princeton.edu/examples/EM/nasilowksi_ieeetm_24_3260_88.pdf

[172] C. Christodoulides, *Comparison of the Ampère and Biot-Savart magnetostatic force laws in the line-element forms*, Am. J. Phys. **56**, 537 (1988), http://kirkmcd.princeton.edu/examples/EM/christodoulides_ajp_56_357_88.pdf

[173] T.A. Weber and D.J. Macomb, *On the equivalence of the laws of Biot-Savart and Ampere*, Am. J. Phys. **57**, 57 (1989), http://kirkmcd.princeton.edu/examples/EM/weber_ajp_57_57_89.pdf

[174] V. Peoglos, *Measurement of the magnetostatic force of a current circuit on a part of itself*, J. Phys. D **21**, 1055 (1988), http://kirkmcd.princeton.edu/examples/EM/peoglos_jpd_21_1055_88.pdf

[175] C.K. Whitney, *Current Elements in Relativistic Field Theory*, Phys. Lett. **128A**, 232 (1988), http://kirkmcd.princeton.edu/examples/EM/whitney_pl_128a_232_88.pdf

[176] C.K. Whitney, *On the Ampere/Biot-Savart discussion*, Am. J. Phys. **56**, 871 (1988), http://kirkmcd.princeton.edu/examples/EM/whitney_ajp_56_871_88.pdf

[177] P. Graneau and P.N. Graneau, *Electromagnetic momentum measurements*, J. Phys. D **21**, 1826 (1988), http://kirkmcd.princeton.edu/examples/EM/graneau_jpd_21_1826_88.pdf

[178] P.G. Moysiddes, *Calculation of the Sixfold Integrals of the Biot-Savart-Lorentz Force Law in a Closed Circuit*, IEEE Trans. Mag. **25**, 4298 (1989), http://kirkmcd.princeton.edu/examples/EM/moyssides_ieeetm_25_4298_89.pdf

[179] P.G. Moysiddes, *Calculation of the Sixfold Integrals of the Ampere Force Law in a Closed Circuit*, IEEE Trans. Mag. **25**, 4307 (1989), http://kirkmcd.princeton.edu/examples/EM/moyssides_ieeetm_25_4307_89.pdf

[180] P.G. Moysiddes, *Experimental Verification of the Biot-Savart-Lorentz and Ampere Force Laws in a Closed Circuit, Revisited*, IEEE Trans. Mag. **25**, 4317 (1989), http://kirkmcd.princeton.edu/examples/EM/moyssides_ieeetm_25_4313_89.pdf

[181] J. Strnad, *Stefan's equations of electrodynamics*, Eur. J. Phys. **10**, 276 (1989), http://kirkmcd.princeton.edu/examples/EM/strnad_ejp_10_276_89.pdf

[182] P.N. Graneau, *Energy Deficiency of the Electromagnetic-Impulse Pendulum with Respect to the Biot-Savart-Lorentz Force Law*, Nuovo Cim. **11D**, 649 (1989), http://kirkmcd.princeton.edu/examples/EM/graneau_nc_11d_649_89.pdf

[183] P.T. Pappas, *The Nonequivalence of the Ampère and Lorentz/Grassmann Force Laws and Longitudinal Contact Interactions*, Phys. Essays **3**, 15 (1990), http://kirkmcd.princeton.edu/examples/EM/pappas_pe_3_15_90.pdf

[184] P.T. Pappas, *On Ampère Electrodynamics and Relativity*, Phys. Essays **3**, 117 (1990), http://kirkmcd.princeton.edu/examples/EM/pappas_pe_3_117_90.pdf

[185] P. Graneau, *Longitudinal Forces in Ampère's Wire-Arc Experiment*, Phys. Lett. **137A**, 87 (1989), http://kirkmcd.princeton.edu/examples/EM/graneau_pl_137a_87_89.pdf

[186] J. Strnad, *On forms of the force law for current elements*, Phys. Lett. **137A**, 11 (1989), http://kirkmcd.princeton.edu/examples/EM/strnad_pl_137a_11_89.pdf

[187] P. Cornille, *On the difference between the Lorentz and Ampère force laws in magnetostatics*, J. Phys. A **22**, 4075 (1989), http://kirkmcd.princeton.edu/examples/EM/cornille_jpa_22_4075_89.pdf

[188] C. Christodoulides, *On the Equivalence of the Ampere and Biot-Savart magnetostatic force laws*, Am. J. Phys. **57**, 680 (1989), http://kirkmcd.princeton.edu/examples/EM/christodoulides_ajp_57_680_89.pdf

[189] J.P. Wesley, *On Peoglos' measurement of the force on a portion of a current loop due to the remainder of the loop*, J. Phys. D **22**, 849 (1989), http://kirkmcd.princeton.edu/examples/EM/wesley_jpd_22_849_89.pdf

[190] M. Rambaut and J.P. Vigier, *The Simultaneous Existence of EM Grassman-Lorentz Forces (Acting on Charged Particles) and Ampère Forces (Acting on Charged Conducting Elements Does Not Contradict Relativity Theory*, Phys. Lett. **A142**, 447 (1989), http://kirkmcd.princeton.edu/examples/EM/rambaut_pl_142a_447_89.pdf

[191] R. Delellis, H. Kelly and A. Marquez, *On the electromotive force in moving conductors*, Am. J. Phys. **58**, 1064 (1990),
`http://kirkmcd.princeton.edu/examples/EM/delellis_ajp_58_1064_90.pdf`

[192] T.E. Phipps and T.E. Phipps Jr, *Observation of Ampere Forces in Mercury*, Phys. Lett. **A146**, 6 (1990), `http://kirkmcd.princeton.edu/examples/EM/phipps_pl_146a_6_90.pdf`

[193] D.J. Griffiths and M.A. Heald, *Time-Dependent Generalizations of the Biot-Savart and Coulomb Laws*, Am. J. Phys. **59**, 111-117 (1991),
`http://kirkmcd.princeton.edu/examples/EM/griffiths_ajp_59_111_91.pdf`

[194] M. Rambaut and J.P. Vigier, *Ampère forces considered as collective non-relativistic limit of the sum of all Lorentz interactions acting on individual current elements: possible consequences for electromagnetic discharge stability and tokamak behaviour*, Phys. Lett. **A148**, 229 (1990),
`http://kirkmcd.princeton.edu/examples/EM/rambaut_pl_148a_229_90.pdf`

[195] J.J. Roche, *A Critical Study of the Vector potential*, in *Physicists Look Back: Studies in the History of Physics*, J.J. Roche ed. (Adam Hilger, 1990), p. 144,
`http://kirkmcd.princeton.edu/examples/EM/roche_144_90.pdf`

[196] M. Rambaut, *Macroscopic non-relativistic Ampère EM interactions between current elements reflect the conducting electron accelerations by the ions electric fields*, Phys. Lett. **A154**, 210 (1991),
`http://kirkmcd.princeton.edu/examples/EM/rambaut_pl_154a_210_91.pdf`

[197] R. Saumont, *Mechanical effects of an electrical current in conductive media. 1. Experimental investigation of the longitudinal Ampère force*, Phys. Lett. **A165**, 307 (1992),
`http://kirkmcd.princeton.edu/examples/EM/saumont_pl_a165_307_92.pdf`

[198] A.K.T. Assis and F.M. Peixoto, *On the Velocity in the Lorentz Force Law*, Phys. Teach. **30**, 480 (1992), `http://kirkmcd.princeton.edu/examples/EM/assis_tpt_30_480_92.pdf`

[199] A.E. Robson and J.D. Sethian, *Railgun recoil, ampere tension, and the laws of electrodynamics*, Am. J. Phys. **60**, 1111 (1992),
`http://kirkmcd.princeton.edu/examples/EM/robson_ajp_60_1111_92.pdf`

[200] A.K.T. Assis, *Weber's Electrodynamics* (Springer, 1994),
`http://kirkmcd.princeton.edu/examples/EM/assis_weber.pdf`

[201] A.K.T. Assis and M. Bueno, *Equivalence Between Ampère and Grassmann's forces*, IEEE Trans. Mag. **32**, 431 (1996),
`http://kirkmcd.princeton.edu/examples/EM/assis_ieeetm_32_431_96.pdf`

[202] J.R. Hofmann, *André-Marie Ampère: Enlightenment and Electrodynamics* Cambridge U. Press, 1995)

[203] A.K.T. Assis and M. Bueno, *Longitudinal Forces in Weber's Eelectrodynamics*, Int. J. ¡pd. Phys. B **9**, 3689 (1995), `http://kirkmcd.princeton.edu/examples/EM/assis_ijmpb_28_3689_95.pdf`

[204] G. Cavalleri, G, Spavieri and G. Spinelli, *The Ampère and Biot-Savart force laws*, Eur. J. Phys. **17**, 205 (1996),
http://kirkmcd.princeton.edu/examples/EM/cavalleri_ejp_17_205_96.pdf

[205] M. Bueno and A.K.T. Assis, *Proof of the identity between Ampère and Grassmann's forces*, Phys. Scr. **56**, 554 (1997),
http://kirkmcd.princeton.edu/examples/EM/bueno_ps_56_554_97.pdf

[206] K.T. McDonald, *The Relation Between Expressions for Time-Dependent Electromagnetic Fields Given by Jefimenko and by Panofsky and Phillips* (Dec. 5, 1996),
http://kirkmcd.princeton.edu/examples/jefimenko.pdf

[207] D.R. Sadedin, *Conservation of Momentum and Recoil in the Railgun*, IEEE Trans. Mag. **33**, 599 (1997), http://kirkmcd.princeton.edu/examples/EM/sadedin_ieeetm_33_599_97.pdf

[208] P. Graneau and N. Graneau, *Railgun Recoil Forces Cannot Be Modeled as Gas Pressure*, IEEE Trans. Mag. **33**, 4570 (1997),
http://kirkmcd.princeton.edu/examples/EM/graneau_ieeetm_33_4570_97.pdf

[209] G. Cavalleri *et al.*, *Experimental proof of standard electrodynamics by measuring the self-force on a part of a current loop*, Phys. Rev. E **58**, 2505 (1998),
http://kirkmcd.princeton.edu/examples/EM/cavallieri_pre_58_2505_98.pdf

[210] P. Graneau and N. Graneau, *Electrodynamic force law controversy*, Phys. Rev. E **63**, 058601 (2001), http://kirkmcd.princeton.edu/examples/EM/graneau_pre_63_058601_01.pdf

[211] O. Darrigol, *Electrodynamics from Ampère to Einstein* (Oxford U. Press, 2000),
http://kirkmcd.princeton.edu/examples/EM/darrigol_em_00.pdf

[212] S. Stringari and R.R. Wilson, *Romagnosi and the discovery of electromagnetism*, Rend. Fis. Acc. Lincei **11**, 115 (2000),
http://kirkmcd.princeton.edu/examples/EM/stringari_rfal_11_115_00.pdf

[213] N. Graneau, T. Phipps Jr, and D. Roscoe, *An experimental confirmation of longitudinal electrodynamic forces*, Eur. Phys. J. D **15**, 81 (2001),
http://kirkmcd.princeton.edu/examples/EM/graneau_epjd_15_87_01.pdf

[214] G. Cavalleri, G, Spavieri and G. Spinelli, *Reply to Electrodynamic force law controversy*, Phys. Rev. E **63**, 058602 (2001),
http://kirkmcd.princeton.edu/examples/EM/cavallieri_pre_63_058602_01.pdf

[215] J.D. Jackson and L.B. Okun, *Historical roots of gauge invariance*, Rev. Mod. Phys. **73**, 663 (2001), http://kirkmcd.princeton.edu/examples/EM/jackson_rmp_73_663_01.pdf

[216] K.T. McDonald, *A Magnetic Linear Accelerator* (Mar. 3 2003),
http://kirkmcd.princeton.edu/examples/lin_accel.pdf

[217] M. Romalis *et al.*, *Princeton U. Ph210 Rail Gun Project* (May 24, 2004),
https://www.princeton.edu/~romalis/PHYS210/railgun/railgun.html

[218] C.J. Carpenter, *Action-reaction forces between current-carrying conductors*, IEE Proc. Sci. Meas. Tech. **153**, 73 (2006), http://kirkmcd.princeton.edu/examples/EM/carpenter_ieepsmt_153_73_06.pdf

[219] V. Courtillot and J.-L. Le Mouël, *The Study of Earth's Magnetism (1269-1950): A Foundation by Peregrinus and Subsequent Development of Geomagnetism and Paleomagnetism*, Rev. Geophys. **45**, RG3008 (2007), http://kirkmcd.princeton.edu/examples/EM/courtillot_rg_45_RG3008_07.pdf

[220] P.J. Cote *et al.*, *On the role of induced fields in railguns*, J. Phys. D **40**, 274 (2007), http://kirkmcd.princeton.edu/examples/EM/cote_jpd_40_274_07.pdf

[221] K.T. McDonald, *Charge Density in a Current-Carrying Wire* (Dec. 20 2010), http://kirkmcd.princeton.edu/examples/wire.pdf

[222] J.A. Redinz, *Forces and work on a wire in a magnetic field*, Am. J. Phys. **79**, 774 (2011), http://kirkmcd.princeton.edu/examples/EM/redinz_ajp_79_774_11.pdf

[223] D.J. Griffiths, *Introduction to Electrodynamics*, 4th ed. (Pearson, 2013), http://kirkmcd.princeton.edu/examples/EM/griffiths_em3.pdf
http://kirkmcd.princeton.edu/examples/EM/griffiths_ex5.3.pdf

[224] S.O. Starr and R.C. Youngquist, *A low voltage "railgun"*, Am. J. Phys. **81**, 38 (2013), http://kirkmcd.princeton.edu/examples/EM/starr_ajp_81_38_13.pdf

[225] T.M. Minteer, *Magnetostatic Stress: Insightful Analysis and Manipulation of Maxwell's Stress Equation for Magnetostatics*, IEEE Trans. Mag. **49**, 5387 (2013), http://kirkmcd.princeton.edu/examples/EM/minteer_ieeetm_49_5387_13.pdf

[226] T.M. Minteer, *Nonequivalence of the magnetostatic potential energy corresponding to the Ampère and Grassmann current element force formulas*, Eur. J. Phys. **34**, 805 (2013), http://kirkmcd.princeton.edu/examples/EM/minteer_ejp_34_805_13.pdf

[227] T.M. Minteer, *A magnetic vector potential corresponding to a centrally conservative current element force*, Eur. J. Phys. **36**, 015012 (2015), http://kirkmcd.princeton.edu/examples/EM/minteer_ejp_36_015012_15.pdf

[228] K.T. McDonald, *Forces on Magnetic Dipoles* (Oct. 26, 2014), http://kirkmcd.princeton.edu/examples/neutron.pdf

[229] A.K.T. Assis and J.P.M.C. Chahib, *Ampère's Electrodynamics* (Aperion, 2015), http://kirkmcd.princeton.edu/examples/EM/assis_ampere_15.pdf

[230] K.T. McDonald, *Birkeland, Darboux and Poincaré: Motion of an Electric Charge in the Field of a Magnetic Pole* (Apr. 15, 2015), http://kirkmcd.princeton.edu/examples/birkeland.pdf

[231] G. Donoso and C.L. Ladera, *The Expected but Well-Hidden EMF in the Jumping Wire Experiment and its Unexpected Role*, Phys. Teach. **53**, 550 (2015), http://kirkmcd.princeton.edu/examples/EM/donoso_pt_53_550_15.pdf