Can Electromagnetic Fields Have a Velocity?

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No.

When Maxwell began to develop his dynamical theory of the electromagnetic field, he initially imagined it to be some kind of medium with mass, which could be in motion.[1] Our view of the electromagnetic field is now more abstract, in which it has densities of energy, momentum and angular momentum, and can support waves, but the electromagnetic field itself does not have a velocity.[2,3,4]

However, we can identify a velocity $\mathbf{v}_{\rm flow}$ of the flow of electromagnetic-field energy density u,

$$u = \frac{E^2 + B^2}{8\pi}\,, \tag{1}$$

in Gaussian units, by recalling the energy-flux vector $\mathbf{S}$ of Poynting [5] (see also [6]),

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B}, \tag{2}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the macroscopic or microscopic electric and magnetic field, and c is the speed of light in vacuum. Then, we define the flow velocity as[5]

$$\mathbf{v}_{\rm flow} = \frac{\mathbf{S}}{u}\,. \tag{3}$$

For a plane electromagnetic wave, such as $\mathbf{E} = E_0\cos(kz-\omega t)\,\hat{\mathbf{x}}$, $\mathbf{B} = E_0\cos(kz-\omega t)\,\hat{\mathbf{y}}$, we find that $\mathbf{v}_{\rm flow} = c\,\hat{\mathbf{z}}$, which is satisfactory.

In general,

$$v_{\rm flow} = |\mathbf{v}_{\rm flow}| = \frac{2c\,|\mathbf{E}\times\mathbf{B}|}{E^2+B^2} \le \frac{2cEB}{E^2+B^2} \le c, \tag{4}$$

[1]Electric and magnetic field lines can be visualized using flax seed and iron filings, which leads many to suppose that field lines (and their possible velocity) are physical. However, they are only a useful mathematical construct [1] (like the electric scalar potential and the magnetic vector potential). While Faraday associated magnetic induction with the "cutting" of magnetic field lines, the field-theoretic view is that induction is due to changing (time-dependent) magnetic flux [2, 3]. One should not speak of "moving flux", as done in [4].

[2]This applies to any field, even the velocity field of hydrodynamics, which field describes the velocity of a fluid. Among physical entities, only mass/energy (including waves thereof) can have velocity. In quantum field theory, the quanta of the field have velocity, but not the field itself.

[3]The electromagnetic field does not rotate, even when the sources involve rotational motion. A famous, controversial issue is whether the magnetic field lines of a cylindrically symmetric magnet rotate if the magnet rotates about its symmetry axis. Since field lines are not physical, this is a "metaphysical" issue. My view is that the field lines do not rotate in the above case, although they do rotate if the field of the magnet is not cylindrically symmetric about the axis of rotation.

[4]That "fields don't move" is stated on p. 105 of the textbook [7].

[5]This definition appears in eq. (26), p. 223 of [8].

noting that $0 \le (E - B)^2 = E^2 + B^2 - 2EB$.

In my view [9], electromagnetic radiation is described by the Poynting vector (2), such that the velocity of electromagnetic radiation is not necessarily c, but is given by eq. (3).

I now give two examples where the energy-flow velocity (3) is less than the speed of light.[6]

1 Electric Charge with Uniform Velocity

Consider an electric charge q that moves along the z axis according to $z = vt$ with constant velocity $\mathbf{v} = v\,\hat{\mathbf{z}}$.

The electromagnetic fields in this case are (see, for example, p. 560 of [10])

$$\mathbf{E}(\mathbf{x}, t) = \frac{q\,\mathbf{r}}{r^3\gamma^2\left(1 - \beta^2\sin^2\theta\right)^{3/2}}\,, \qquad \mathbf{B} = \frac{\mathbf{v}}{c}\times\mathbf{E}, \tag{5}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, $\mathbf{r} = \mathbf{x} - \mathbf{x}_q = \mathbf{x} - (0, 0, vt)$, and θ is the angle between $\mathbf{r}$ and the z-axis. Then, the field-energy density (1) is

$$u = \frac{E^2 + B^2}{8\pi} = \frac{q^2 r^2 (1 + \beta^2\sin^2\theta)}{8\pi r^6\gamma^4\left(1 - \beta^2\sin^2\theta\right)^3}, \tag{6}$$

and the Poynting vector (2) is

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{c}{4\pi}\frac{q^2}{r^6\gamma^4\left(1 - \beta^2\sin^2\theta\right)^3}\frac{r^2}{c}(\mathbf{v} - v\cos\theta\,\hat{\mathbf{r}}) = -\frac{q^2 r^2 v\sin\theta\,\hat{\boldsymbol{\theta}}}{4\pi r^6\gamma^4\left(1 - \beta^2\sin^2\theta\right)^3}\,, \tag{7}$$

where $\hat{\boldsymbol{\theta}}$ is the polar unit vector at the position of the observer in (r, θ, ϕ) spherical coordinates, such that $\mathbf{v} = v\cos\theta\,\hat{\mathbf{r}} - v\sin\theta\,\hat{\boldsymbol{\theta}}$. Finally, the energy-flow velocity (3) is

$$\mathbf{v}_{\rm flow} - \frac{\mathbf{S}}{u} = -\frac{2v\sin\theta}{1 + \beta^2\sin^2\theta}\,\hat{\boldsymbol{\theta}}\,, \qquad \text{with} \qquad \boldsymbol{\nabla}\cdot\mathbf{v}_{\rm flow} = \frac{4v\cos\theta}{r\left(1 + \beta^2\sin^2\theta\right)^2}\,, \tag{8}$$

rather than $\mathbf{v}_{\rm flow} = \mathbf{v}$ as might have been naïvely expected.

For $\theta = 90°$, the observer in is the plane $z = vt$, which is perpendicular to the z-axis and contains the charge. Here, $-\hat{\boldsymbol{\theta}} = \hat{\mathbf{z}}$ and $\mathbf{v}_{\rm flow} = 2\mathbf{v}/(1 + \beta^2) > \mathbf{v}$.

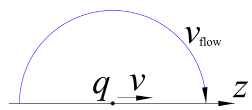
Also, eqs. (7) and (8) imply that no energy flows along the z-axis, although the field energy on the z-axis changes with time. Rather, as the charge approaches some point on the z-axis, the field-energy density increases with time there via flow of energy inward from nearby points off-axis.[7] This behavior is similar to the case of a resistor, where field energy flows into the resistor from transverse directions, rather than along the axis of the resistor.

Field lines of $\mathbf{v}_{\rm flow}$ (and of $\mathbf{S}$) lie on circles centered at the position of the charge. Since $\boldsymbol{\nabla}\cdot\mathbf{v}_{\rm flow}$ is everywhere nonzero, they can begin at any point with $z < vt$ at distance r from

[6] A third example, for the fields in a rectangular waveguide, is given is Secs. 8.4-5 of [10]. The fields do not move, but the field lines move with the phase velocity $v_p > c$, while the energy-flow velocity, eq. (8.53), equals the group velocity $v_g = (c/n)^2/v_p \le c$, where $n = \sqrt{\epsilon\mu}$ is the index of refraction.

[7] As the charge recedes from a point on the z-axis, the field-energy density decreases with time there.

the charge (where $\nabla \cdot \mathbf{v}_{\rm flow} > 0$), and end at any point with $z > vt$ on the circle with radius r and on the same side of the z-axis as the starting point (where $\nabla \cdot \mathbf{v}_{\rm flow} < 0$). Many different field lines all lie on a given circle of radius r, so it is perhaps best to consider only the subset of maximal-length field lines that are semicircles beginning at $(x, y, z) = (0, 0, vt - d)$ and ending at $(0, 0, vt + d)$.[8] One such line is shown below.



The lines of $\mathbf{v}_{\rm flow}$, which are not physical, all have velocity $\mathbf{v}$.

Furthermore, $\mathbf{v}_{\rm flow}$ is not part of a (mechanical) 4-vector, as it is zero in the rest frame of the charge, but is not just $\mathbf{v}$ in a frame where the charge has velocity $\mathbf{v}$. We can compute $\mathbf{v}_{\rm flow}$ in any inertial frame using the transformation of the electromagnetic stress-energy-momentum tensor[9] from the rest frame of the charge, noting that u and $\mathbf{S}$ are components of this tensor.

2 A Battery Connected to a Resistor

Analytic calculations for an example of a battery connected to a resistive wire are possible for a two-dimensional approximation of a current loop by a circular cylinder [11]. See also sec. 2.1.2 of [12]. The conductor is a cylinder of radius a, centered on the z axis, with the "battery" along the line $(r, \phi, z) = (a, \pm\pi, z)$ in cylindrical coordinates, as sketched on the next page.

We suppose that R is the resistance of unit length in z along the cylinder,[10] so that $I = V/R$ is the current per unit length that flows around its circumference due to the battery of voltage V. The magnetic field $\mathbf{B}$ is that of an infinite solenoid,

$$\mathbf{B} = \begin{cases} -\frac{4\pi I}{c}\hat{\mathbf{z}} = -\frac{4\pi V}{cR}\hat{\mathbf{z}} & (r < a), \\ 0 & (r > a). \end{cases} \tag{9}$$

The electric scalar potential Φ is found (see eqs. (5)-(6) of [11] or eq. (13) of [12]) to be

$$\Phi(r, \phi, z) = \frac{V}{\pi} \begin{cases} \tan^{-1} \frac{r \sin\phi}{a + r\cos\phi} = \theta & (r < a), \\ \tan^{-1} \frac{a \sin\phi}{r + a\cos\phi} & (r > a), \end{cases} \tag{10}$$

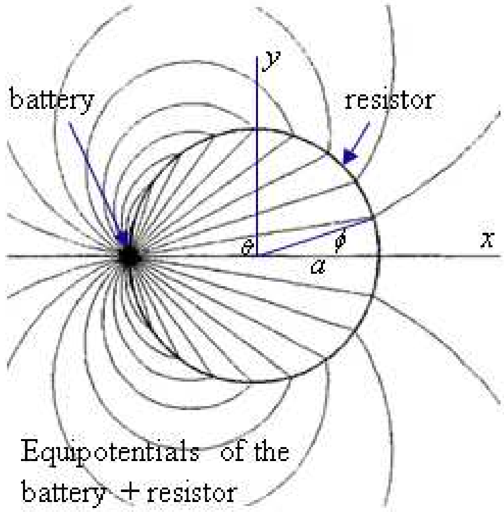
which is continuous at $r = a$, where θ is the azimuthal angle of an equipotential inside the cylinder with respect to the x-axis measured from the location of the battery. In general, $\theta = \phi/2$ inside the cylinder, and for $(r, \phi) = (a, \pi)$ angle θ equals $\pi/2$ and potential $\Phi = V/2$, corresponding to the positive terminal of the battery. The equipotentials are shown in the figure on the next page, adapted from [11].

Inside the circuit the equipotentials are planes that emanate from the battery. Outside the circuit the equipotentials are circular cylinders that pass through the battery.

[8]Thanks to David Griffiths for this insight.

[9]See,for example, Sec. 12.10B of [10].

[10]The dimensions of R are resistance $\times$ length = length/velocity in Gaussian units.

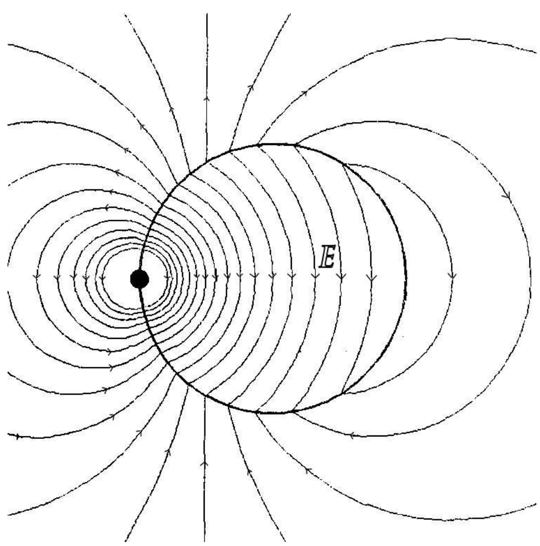


The electric field is found from the scalar potential to have components given by

$$E_r = -\frac{\partial \Phi}{\partial r} = \frac{aV}{\pi} \begin{cases} -\frac{\sin\phi}{r^2+2ar\cos\phi+a^2} & (r < a), \\ \frac{\sin\phi}{r^2+2ar\cos\phi+a^2} & (r > a), \end{cases} \tag{11}$$

$$E_\phi = -\frac{1}{r}\frac{\partial \Phi}{\partial \phi} = -\frac{V}{\pi} \begin{cases} \frac{r+a\cos\phi}{r^2+2ar\cos\phi+a^2} & (r < a), \\ \frac{a}{r}\frac{a+r\cos\phi}{r^2+2ar\cos\phi+a^2} & (r > a). \end{cases} \tag{12}$$

The corresponding electric field lines are shown in the figure below (from [11]).



To evaluate the electromagnetic-energy density momentum (1) and the Poynting vector (2), we first express the electric field for $r < a$ in rectangular coordinates,

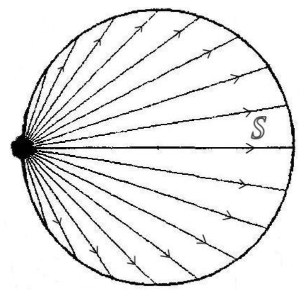
$$\begin{aligned} \mathbf{E}(r < a) &= (E_r\cos\phi - E_\phi\sin\phi)\,\hat{\mathbf{x}} + (E_r\sin\phi + E_\phi\cos\phi)\,\hat{\mathbf{y}} \\ &= \frac{V}{\pi}\frac{r\sin\phi\,\hat{\mathbf{x}} - (a + r\cos\phi)\,\hat{\mathbf{y}}}{r^2 + 2ar\cos\phi + a^2} = E_x\,\hat{\mathbf{x}} + E_y\,\hat{\mathbf{y}}. \end{aligned} \tag{13}$$

Then, the energy density u inside the conducting cylinder is

$$u(r < a) = \frac{V^2}{8\pi^3(r^2 + 2ar\cos\phi + a^2)} + \frac{2\pi V^2}{c^2R^2} = \frac{V^2[c^2R^2 + 16\pi^4(r^2 + 2ar\cos\phi + a^2)]}{8\pi^3c^2R^2(r^2 + 2ar\cos\phi + a^2)}\,, \quad (14)$$

which is consistent in that c^2R^2 has dimensions of length^2.

The Poynting vector, $\mathbf{S} = (c/4\pi)\mathbf{E}\times\mathbf{B}$, is nonzero only inside the circuit, where $\mathbf{S}$ is perpendicular to $\mathbf{E}$ and hence parallel to the equipotentials. That is, the equipotential planes inside the circuit on the upper figure on p, 3 also represent the flow $\mathbf{S}$ of energy from the battery to the resistive cylinder, as shown in the figure below (from [11]).



In detail, the Poynting vector $\mathbf{S}$ inside the cylinder is

$$\mathbf{S}(r < a) = \frac{c}{4\pi}[(E_x\,\hat{\mathbf{x}} + E_y\,\hat{\mathbf{y}}) \times B\hat{\mathbf{z}}] = \frac{V^2}{\pi R}\frac{(a + r\cos\phi)\,\hat{\mathbf{x}} + r\sin\phi\,\hat{\mathbf{y}}}{r^2 + 2ar\cos\phi + a^2}\,, \quad (15)$$

and the energy-flow velocity (3) is

$$\mathbf{v}_{\rm flow}(r < a) = 8\pi^2c^2RV^2\,\frac{(a + r\cos\phi)\,\hat{\mathbf{x}} + r\sin\phi\,\hat{\mathbf{y}}}{c^2R^2 + 16\pi^4(r^2 + 2ar\cos\phi + a^2)}\,. \quad (16)$$

For, say, radius $a = 1$ cm, voltage $V = 1$ volt $= 1/300$ statvolt, and resistance $R = 1\ \Omega$-cm $= 1/9 \times 10^{-11}$ Gaussian units, we have that $(cR)^2 = (3 \times 10^{10} \cdot 1/9 \times 10^{-11})^2 \approx 10^{-3}$ cm^2, which is negligible. Then, the steady-state flow velocity at the center ($r = 0$) of the cylinder is $c^2RV^2/2\pi^2a \approx 10^4$ cm/s, so $a/v \approx 10^{-4}$ s is of the order of the turn-on time of the circuit when the battery is first connected.

This note was inspired by e-discussions with Beth Parks.

References

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See also eq. (8), p. 794 of M. Born and E. Wolf, *Principles of Optics*, 7$^{\text{th}}$ ed. (Cambridge U. Press, 1999). http://kirkmcd.princeton.edu/examples/EM/born_wolf_7ed.pdf
"It (energy-flow velocity) is a useful concept, although, like the phase velocity, it has no directly verifiable physical significance."

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