Is Electrostatic Field Energy "Mechanical"?

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(March 29, 2024)

Despite the great success of Maxwell's "Dynamical Theory of the Electromagnetic Field" [1], Lord Kelvin's view that electromagnetism is a "mechanical" phenomenon[1] has lingering advocates. An argument that Kelvin would likely have supported is that the electrostatic field energy of a system of electrical charges at rest is "mechanical",[2] and is to be associated with the rest energy of the charges.[3,4] In particular, the interaction electrostatic field energy of an electron with other charges is considered by some people to be part of the "mechanical" rest energy of the electron.[5] This has the implication for an electron at rest that if its interaction field energy is negative and greater in magnitude than $m_0\, c^2 = 511$ keV, where m_0 is the rest mass of an electron in zero electromagnetic field and c is the speed of light in vacuum, then the rest energy, and rest mass of the electron, is negative. In this case, the acceleration of an electron in an electric field $\mathbf{E}$ would be in the direction of $\mathbf{E}$, rather than opposite to this as usually holds. If so, electrons (as well as positive ions) would be accelerated away from the positive electrode of a Van de Graaff generator [19] that is charged to potential greater than 511 keV, but such accelerated electrons are not observed.[6]

An earlier discussion of this topic by the author is at [20].

---

[1]Kelvin: *I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electro-magnetic theory* (of Maxwell). See p. 603 of [2].

[2]For a skeptical view of the concepts of potential energy and field energy, see [3].

[3]This view is an extension of arguments beginning with J.J. Thomson [4] that the "mechanical" mass of an electron is actually electromagnetic in origin.

[4]A more Maxwellian view, that the interaction energy exists separate from the "mechanical" energy of a system (while being part of the rest energy of that system), was illustrated in [5] for a system of two opposite electrical charges $\pm q$ with fixed separation d, somehow accelerated perpendicular to their line of centers. The force needed for this acceleration was computed and found to be consistent with the rest mass of the system being $2m_0 - q^2/d$, where m_0 is the rest mass of each particle. However, no claim was made that the rest mass of each charge was shifted by $-q^2/2d$.

[5]The case of an electron in an external electric scalar potential was discussed by Brillouin in [6, 7] (see also [8, 9]), and that of a permanent magnetic moment in an external magnetic field has been discussed starting with a brief remark by Frenkel in the final paragraph of [10], then eq. $(18 \cdot 9)$ of [11], eq. $(8')$, p. 1837 of [12], eq. (3.17), p. 1621 of [13], Appendix B of [14] (for comments by the author on this paper, see [15]), p. 15 of [16], p. 4 of [17] (this paper notes on p. 8 that the "mass shift" is not predicted in either the classical Foldy-Wouthuysen model or in Dirac's quantum theory of the electron), and p. 64 of [18].

[6]April 2, 2024. Presumably, such an electron would move away from the electrode until its electric-interaction energy became less in magnitude than 511 keV, and the rest mass of the electron returned to being positive. Then, the electron would be attracted to the electrode, until the (negative) interaction energy became greater in magnitude than 511 keV and the electron was again repelled from the electrode. The motion of the electron would be an "endless cycle" of repulsion and attraction in the vicinity of the electrode [8], in a kind of "electrostatic trap" (although emission of electromagnetic radiation during the oscillation would reduce the energy of the electron leading to its eventual collision with the electrode). Such behavior has not been observed.

References

[1] J.C. Maxwell, *A Dynamical Theory of the Electromagnetic Field*, Phil. Trans. Roy. Soc. London **155**, 459-512 (1865). doi/10.1098/rstl.1865.0008
kirkmcd.princeton.edu/examples/EM/maxwell_ptrsl_155_459_65.pdf

[2] G. Forbes, *Sir William Thomson on Molecular Dynamics*, Nature **31**, 601 (1885),
http://kirkmcd.princeton.edu/examples/EM/thomson_nature_31_601_85.pdf

[3] E. Hecht, *Relativity, potential energy, and mass*, Eur. J. Phys. **37**, 065804 (2016),
http://kirkmcd.princeton.edu/examples/mechanics/hecht_ejp_37_065804_16.pdf

[4] J.J. Thomson, *On the Electric and Magnetic Effects produced by the Motion of Electrified Bodies*, Phil. Mag. **11**, 229 (1881),
http://kirkmcd.princeton.edu/examples/EM/thomson_pm_11_229_81.pdf

[5] T.H. Boyer, *Electrostatic potential energy leading to an inertial mass change for a system of two point charges*, Am. J. Phys. **46**, 383 (1978),
http://kirkmcd.princeton.edu/examples/EM/boyer_ajp_46_383_78.pdf

[6] L. Brillouin, *The Actual Mass of Potential Energy, A Correction to Classical Relativity*, Proc. Nat. Acad. Sci. **53**, 475 (1965),
http://kirkmcd.princeton.edu/examples/EM/brillouin_pnas_53_475_65.pdf

[7] L. Brillouin, *The Actual Mass of Potential Energy, II*, Proc. Nat. Acad. Sci. **53**, 1280 (1965), http://kirkmcd.princeton.edu/examples/EM/brillouin_pnas_53_1280_65.pdf

[8] G.G. Karapetyan, *Radial oscillations of an electron in a Coulomb attracting field*, Open Phys. **20**, 1213 (2022),
http://kirkmcd.princeton.edu/examples/EM/karapetyan_op_20_1213_22.pdf
https://doi.org/10.1515/phys-2022-0208

[9] G.G. Karapetyan, *New Classical Relativistic Theory of a Charged Particle in an Electric Field*, Int. J. Fund. Phys. Sci. **14**, 7 (2024),
http://kirkmcd.princeton.edu/examples/EM/karapetyan_ijfps_14_7_24.pdf

[10] J. Frenkel, *Die Elektrodynamik des rotierenden Elektrons*, Z. Phys. **37**, 243 (1926),
http://kirkmcd.princeton.edu/examples/EM/frenkel_zp_37_243_26.pdf
http://kirkmcd.princeton.edu/examples/EM/frenkel_zp_37_243_26_english.pdf

[11] M. Mathisson, *Relativistic Dynamics of a Spinning Magnetic Particle*, Proc. Camb. Phil. Soc. **38**, 40 (1941), http://kirkmcd.princeton.edu/examples/EM/mathisson_pcps_38_40_41.pdf

[12] H.C. Corben, *Spin in Classical and Quantum Theory*, Phys. Rev. **121**, 1833 (1961),
http://kirkmcd.princeton.edu/examples/EM/corben_pr_121_1833_61.pdf

[13] W.G. Dixon, *On a Classical Theory of Charged Particles with Spin and the Classical Limit of the Dirac Equation*, Nuovo Cim. **36**, 1616 (1965),
http://kirkmcd.princeton.edu/examples/EM/dixon_nc_38_1616_65.pdf

[14] S.E. Gralla, A.I. Harte, and R.M. Wald, *Rigorous derivation of electromagnetic self-force*, Phys. Rev. D **83**, 024031 (2009),
http://kirkmcd.princeton.edu/examples/EM/gralla_prd_80_024031_09.pdf

[15] K.T. McDonald, *Field and Kinetic Energies of a Pair of Permanent Magnetic Dipoles* (May 21, 2017), http://kirkmcd.princeton.edu/examples/mag_energy.pdf

[16] V. Kassandrov *et al.*, *On a model of a classical relativistic particle of constant and universal mass and spin*, J. Phys. A **42**, 315204 (2009),
http://kirkmcd.princeton.edu/examples/EM/kassandrov_jpa_42_315204_09.pdf

[17] M. Wen, C.H. Keitel and H. Bauke, *Spin-one-half particles in strong electromagnetic fields: Spin effects and radiation reaction*, Phys. Rev. A **95**, 045102 (2017),
http://kirkmcd.princeton.edu/examples/QED/wen_pra_95_042102_17.pdf

[18] R.M. Wald, *Advanced Classical Electromagnetism* (Princeton U. Press, 2022),
http://kirkmcd.princeton.edu/examples/EM/wald_22_ch4.pdf
http://kirkmcd.princeton.edu/examples/EM/wald_22_ch5.pdf

[19] R.J. Van de Graaff, K.T. Compton and L.C. Van Atta, *The Electrostatic Production of High Voltage for Nuclear Investigations*, Phys. Rev. **43**, 149 (1933),
http://kirkmcd.princeton.edu/examples/accel/vandegraaff_pr_43_149_33.pdf

[20] K.T. McDonald, *Is There a Mass Shift of a Permanent Magnetic Moment in an External, Static Magnetic Field?* (Dec. 22, 2022), http://kirkmcd.princeton.edu/examples/mu.pdf