The Vector Potential is Not an Electromagnetic Momentum in General

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In science, the term "physical" should be reserved for entities that exist in Nature, and not applied to concepts such as words, numbers, mathematical symbols, and potentials, however useful they may be in discussions of "physical" objects.

Several papers [1]-[9] have prolonged a misunderstanding of Maxwell that the vector potential $\mathbf{A}$ in the Coulomb gauge (where $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$) is a physical "electromagnetic momentum". Only around 1900 was it understood that the volume density $\mathbf{p}_{\rm field}$ of electromagnetic-field momentum is the Poynting vector $\mathbf{S}$ divided by c^2, where c is the speed of light.[1] Then, the electromagnetic-field momentum is related by $\mathbf{P}_{\rm field} = \int \mathbf{p}_{\rm field}\, d{\rm Vol} = \epsilon_0 \int \mathbf{E}\times\mathbf{B}\, d{\rm Vol}$ in SI units (with permittivity ϵ_0 replaced by $1/4\pi c$ in Gaussian units, which will be used below), where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields. The field momentum is a global property of the fields and depends on the details of their distant sources.

In static examples the electromagnetic-field momentum can also be computed (following Maxwell [11]) as $\mathbf{P}_{\rm field} = \int \rho\mathbf{A}^{\rm (C)}\, d{\rm Vol}/c$ where $\mathbf{A}^{\rm (C)}(\mathbf{x}) = \int \mathbf{J}(\mathbf{x}')\, d{\rm Vol}/cr$ is the vector potential in the Coulomb gauge, ρ is the electric-charge density, $\mathbf{J}$ is the electric-current density and $r = |\mathbf{r}| = |\mathbf{x}-\mathbf{x}'|$. This reinforces that the vector potential is "useful" and "valid", but it does not imply that the vector potential is "physical", as suggested in [1]-[9].

In static examples the electromagnetic-field momentum can be computed in other ways as well. The form $\mathbf{P}_{\rm field} = \int V^{\rm (C)}\mathbf{J}\, d{\rm Vol}/c^2$, where $V^{\rm (C)} = \int \rho\, d{\rm Vol}/r$ is the electric scalar potential in the Coulomb gauge was first advocated Furry [12]. And, the form $\mathbf{P}_{\rm field} = \int (\mathbf{J}\cdot\mathbf{E})\,\mathbf{r}\, d{\rm Vol}/c^2$ where $\mathbf{E} = \int \rho\,\hat{\mathbf{r}}\, d{\rm Vol}/r^2$ was introduced by Aharonov, Pearle and Vaidman [13].

The density of momentum in the electromagnetic field is $\mathbf{p}_{\rm field} = \mathbf{S}/c^2 = \mathbf{E}\times\mathbf{B}/4\pi c$, and not $\rho\mathbf{A}^{\rm (C)}/c$ nor $V^{\rm (C)}\mathbf{J}/c^2$ nor $(\mathbf{J}\cdot\mathbf{E})\,\mathbf{r}/c^2$.

The physical electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ can be related to the electromagnetic potentials $\mathbf{A}$ and V in any gauge, so it is not reasonable to consider these potentials in the infinite variety of gauges as all physical. Maxwell did favor the Coulomb gauge, and papers such as [1]-[9] do also, implying that this gauge is the only one that is physical. Yet, even the Coulomb gauge is not unique as the transformations $\mathbf{A} \to \mathbf{A} + \boldsymbol{\nabla}\chi$ and $V \to V - \partial\chi/\partial ct$ where $\nabla^2\chi = 0$ leave $\mathbf{A}$ and V in the Coulomb gauge (*i.e.*, $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$). Those who consider the vector potential to be physical insist that its physical form is $\mathbf{A}^{\rm (C)} = \int \mathbf{J}\, d{\rm Vol}/cr$, which applies only when the current distribution is static.[2]

As a technical footnote, recall that the canonical momentum of an electric charge q with (rest) mass m and velocity $\mathbf{v}$ in an electromagnetic field can be written as $\mathbf{p}_{\rm canon} =$

[1]For a brief history of this insight, see p. 246 of [10].

[2]A static example can have nonzero electromagnetic-field momentum, but the total momentum should be zero in this case. Hence, we infer the existence of a (physical) "hidden" mechanical momentum [14] equal and opposite to the electromagnetic field momentum. This is a "relativistic" effect [15] in a static example, which is a surprising insight following from the (misguided) attempts to assign a physical significance to the vector potential.

$\mathbf{p}_{\rm mech} + q\mathbf{A}/c$ for the vector potential $\mathbf{A}$ in any gauge, with $\mathbf{p}_{\rm mech} = m\mathbf{v}/\sqrt{1-v^2/c^2}$. Then, the Hamiltonian, $H = c[m^2c^2 + (\mathbf{p}_{\rm canon} - q\mathbf{A}/c)^2]^{1/2} + qV$, of the electric charge in an electromagnetic field appears not to be gauge invariant, but the equation of motion, obtained by taking derivatives of the Hamiltonian, is gauge invariant, being $m\,\mathbf{a} = q(\mathbf{E} + \mathbf{v}/c \times \mathbf{B})$, *i.e.*, the Lorentz force law. This is perhaps the most significant descendant of Maxwell's notion that the vector potential is a kind of "electromagnetic momentum".

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